Computer algebra independent integration tests

Summer 2022 edition

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/192-7.3.2-d-x-^m-a+b-arctanh-c-x^n-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [243]. This is test number [192].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (243)	0.00 (0)
Mathematica	95.06 (231)	4.94 (12)
Maple	81.89 (199)	18.11 (44)
Maxima	63.37 (154)	36.63 (89)
Fricas	60.49 (147)	39.51 (96)
Mupad	52.67 (128)	47.33 (115)
Giac	52.26 (127)	47.74 (116)
Sympy	34.98 (85)	65.02 (158)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

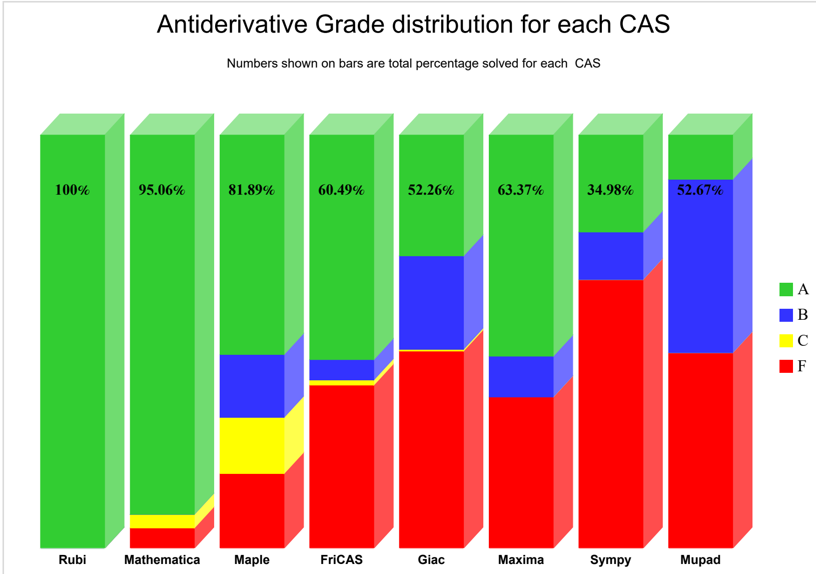
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

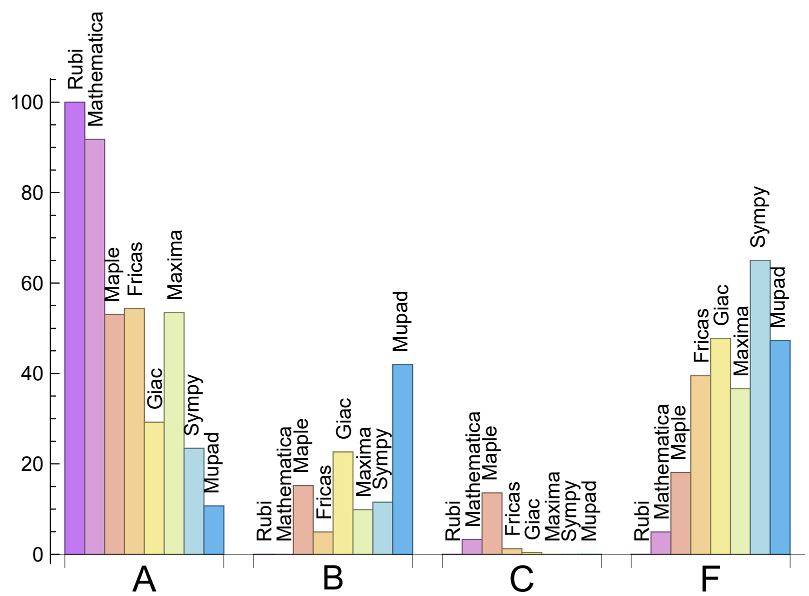
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	91.77	0.00	3.29	4.94
Fricas	54.32	4.94	1.23	39.51
Maxima	53.50	9.88	0.00	36.63
Maple	53.09	15.23	13.58	18.11
Giac	29.22	22.63	0.41	47.74
Sympy	23.46	11.52	0.00	65.02
Mupad	N/A	41.98	0.00	47.33

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	12	100.00 %	0.00 %	0.00 %
Maple	44	100.00 %	0.00 %	0.00 %
Fricas	96	100.00 %	0.00 %	0.00 %
Giac	116	100.00 %	0.00 %	0.00 %
Maxima	89	100.00 %	0.00 %	0.00 %
Sympy	158	67.09 %	32.91 %	0.00 %
Mupad	115	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

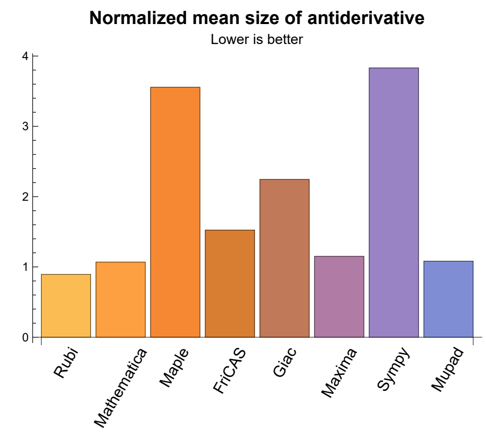
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

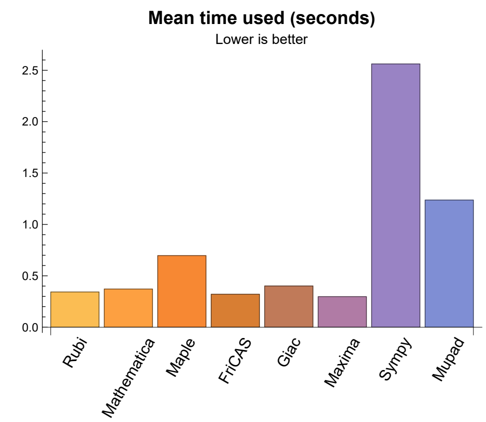
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.34	251.92	0.89	87.00	1.00
Mathematica	0.37	122.12	1.07	99.00	1.13
Maple	0.70	447.69	3.56	92.00	1.25
Maxima	0.30	127.16	1.15	62.00	1.02
Fricas	0.32	155.71	1.52	69.00	1.30
Sympy	2.56	229.52	3.83	70.00	1.26
Giac	0.40	159.84	2.24	109.00	1.41
Mupad	1.24	87.70	1.08	59.00	1.10

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
4
3
2
1
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
2.5
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{43, 44, 46, 47, 48, 49, 94, 95, 97, 98, 130, 131, 133, 134, 182, 183, 185, 186, 231, 232, 234, 235, 239, 240, 242, 243\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{90, 91\}$

Mathematica $\{\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

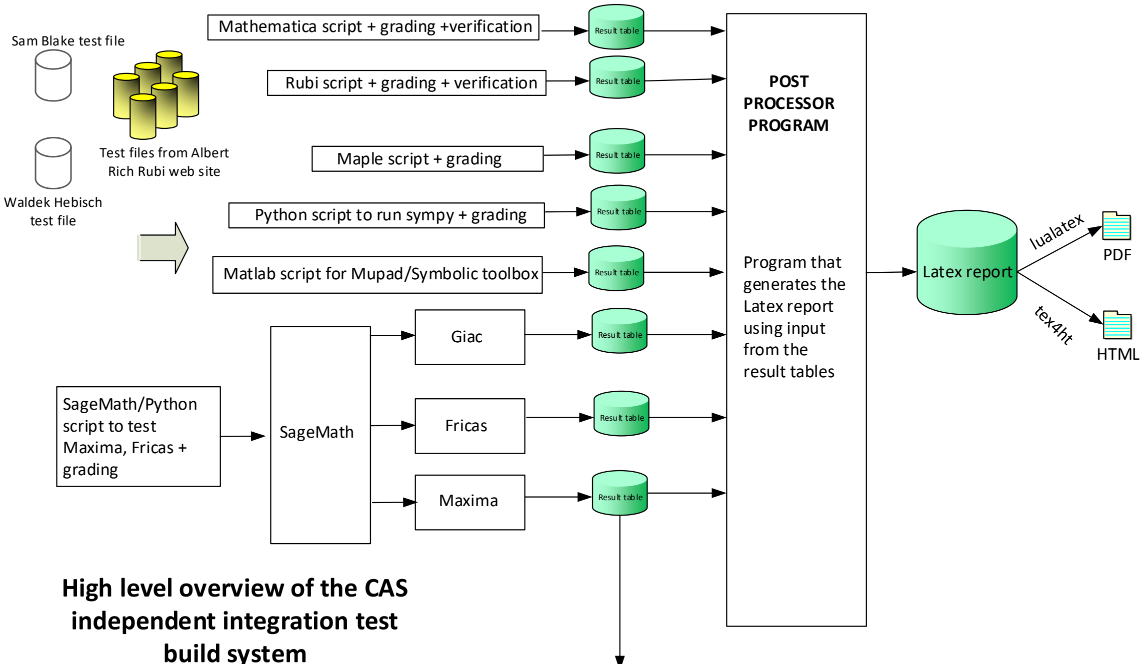
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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August 26, 2022

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 178, 179, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243 }

B grade: { }

C grade: { 31, 33, 80, 128, 151, 153, 227, 236 }

F grade: { 71, 72, 75, 76, 90, 91, 92, 93, 176, 177, 180, 181 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 18, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 67, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 119, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 148, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 182, 183, 185, 186, 187, 188, 189, 190, 192, 193, 194, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 227, 231, 232, 234, 235, 236, 238, 239, 240, 242, 243 }

B grade: { 7, 13, 15, 16, 17, 20, 21, 22, 23, 29, 54, 64, 65, 66, 70, 78, 116, 118, 122, 126, 139, 143, 144, 145, 146, 149, 155, 161, 191, 195, 196, 197, 198, 200, 201, 217, 221 }

C grade: { 19, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 77, 103, 125, 147, 150, 151, 152, 153, 154, 156, 172, 199, 202, 203, 204, 205, 206, 207, 208, 222, 233, 237 }

F grade: { 45, 68, 69, 71, 72, 73, 74, 75, 76, 79, 80, 81, 90, 91, 92, 93, 96, 117, 120, 121, 123, 124, 127, 128, 129, 132, 171, 173, 175, 176, 177, 178, 179, 180, 181, 184, 223, 224, 225, 226, 228, 229, 230, 241 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 145, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 182, 183, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 231, 232, 234, 235, 238, 239, 240, 242, 243 }

B grade: { 17, 21, 23, 66, 70, 118, 122, 149, 175, 191, 197, 198, 200, 201, 202, 203, 204, 207, 208, 217, 221, 223, 236, 237 }

C grade: { }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 45, 54, 65, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 90, 91, 92, 93, 96, 103, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 132, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 176, 177, 178, 179, 180, 181, 184, 199, 205, 206, 222, 224, 225, 226, 227, 228, 229, 230, 233, 241 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 61, 62, 63, 64, 66, 70, 83, 84, 85, 94, 95, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 122, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 145, 149, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 175, 182, 183, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 201, 209, 210, 211, 212, 214, 218, 219, 220, 231, 232, 234, 235, 238, 239, 240, 242, 243 }

B grade: { 60, 82, 86, 87, 88, 89, 198, 200, 221, 223, 227, 236 }

C grade: { 213, 215, 216 }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 45, 54, 65, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 90, 91, 92, 93, 96, 103, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 132, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 176, 177, 178, 179, 180, 181, 184, 191, 199, 202, 203, 204, 205, 206, 207, 208, 217, 222, 224, 225, 226, 228, 229, 230, 233, 237, 241 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 43, 44, 46, 47, 48, 49, 50, 52, 56, 57, 60, 64, 94, 95, 97, 135, 136, 137, 138, 140, 141, 142, 143, 145, 149, 157, 159, 163, 164, 167, 171, 182, 183, 185, 210, 231, 232, 234, 235, 238, 242 }

B grade: { 37, 51, 53, 55, 58, 59, 61, 62, 63, 66, 70, 158, 160, 162, 165, 166, 168, 169, 170, 175, 192, 193, 194, 200, 201, 209, 211, 212 }

C grade: { }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 45, 54, 65, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 176, 177, 178, 179, 180, 181, 184, 186, 187, 188, 189, 190, 191, 195, 196, 197, 198, 199, 202, 203, 204, 205, 206, 207, 208, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 233, 236, 237, 239, 240, 241, 243 }

2.1.7 Giac

A grade: { 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 59, 62, 63, 64, 94, 95, 97, 98, 99, 100, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 130, 131, 133, 134, 157, 158, 162, 163, 164, 165, 166, 167, 168, 169, 170, 182, 183, 185, 186, 216, 218, 219, 220, 231, 232, 234, 235, 239, 240, 242, 243 }

B grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 52, 53, 60, 61, 66, 85, 86, 87, 88, 89, 101, 102, 118, 135, 136, 137, 138, 140, 141, 142, 143, 145, 149, 159, 160, 171, 187, 188, 189, 190, 192, 193, 194, 209, 210, 211, 212, 214, 238 }

C grade: { 213 }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 45, 54, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 90, 91, 92, 93, 96, 103, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 184, 191, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 215, 217, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 233, 236, 237, 241 }

2.1.8 Mupad

A grade: { 43, 44, 46, 47, 48, 49, 94, 95, 97, 98, 130, 131, 133, 134, 182, 183, 185, 186, 231, 232, 234, 235, 239, 240, 242, 243 }

B grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 70, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 122, 135, 136, 137, 138, 140, 141, 142, 143, 145, 149, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 175, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 200, 201, 209, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 223, 238 }

C grade: { }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 54, 65, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 96, 103, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 132, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 176, 177, 178, 179, 180, 181, 184, 191, 199, 202, 203, 204, 205, 206, 207, 208, 210, 217, 222, 224, 225, 226, 227, 228, 229, 230, 233, 236, 237, 241 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **N.S.** in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	81	69	70	67	63	442	52
N.S.	1	1.00	1.37	1.17	1.19	1.14	1.07	7.49	0.88
time (sec)	N/A	0.022	0.010	0.060	0.291	0.335	0.386	0.458	0.839

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	62	64	55	69	68	403	53
N.S.	1	1.00	1.09	1.12	0.96	1.21	1.19	7.07	0.93
time (sec)	N/A	0.031	0.009	0.014	0.268	0.337	0.332	0.431	0.821

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	70	60	61	58	53	296	43
N.S.	1	1.00	1.46	1.25	1.27	1.21	1.10	6.17	0.90
time (sec)	N/A	0.022	0.009	0.013	0.266	0.341	0.236	0.425	0.773

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	51	55	44	58	58	258	44
N.S.	1	1.00	1.11	1.20	0.96	1.26	1.26	5.61	0.96
time (sec)	N/A	0.024	0.009	0.013	0.255	0.356	0.202	0.420	0.744

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	59	51	50	48	42	148	35
N.S.	1	1.00	1.59	1.38	1.35	1.30	1.14	4.00	0.95
time (sec)	N/A	0.012	0.008	0.013	0.263	0.340	0.176	0.413	0.727

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	29	30	42	27	156	27
N.S.	1	1.00	1.00	0.97	1.00	1.40	0.90	5.20	0.90
time (sec)	N/A	0.009	0.004	0.010	0.255	0.337	0.121	0.414	0.684

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	47	0	0	0	0	-1
N.S.	1	1.00	0.92	1.81	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.011	0.009	0.017	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	50	39	47	41	94	33
N.S.	1	1.00	1.08	1.39	1.08	1.31	1.14	2.61	0.92
time (sec)	N/A	0.018	0.008	0.019	0.262	0.401	0.312	0.408	0.700

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	59	55	45	43	36	135	46
N.S.	1	1.00	1.59	1.49	1.22	1.16	0.97	3.65	1.24
time (sec)	N/A	0.016	0.008	0.020	0.264	0.346	0.263	0.414	0.728

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	59	62	49	59	70	251	46
N.S.	1	1.00	1.09	1.15	0.91	1.09	1.30	4.65	0.85
time (sec)	N/A	0.026	0.008	0.020	0.256	0.366	0.414	0.435	0.730

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	70	64	60	52	46	292	59
N.S.	1	1.00	1.46	1.33	1.25	1.08	0.96	6.08	1.23
time (sec)	N/A	0.020	0.008	0.020	0.277	0.351	0.313	0.417	1.053

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	70	71	61	70	80	397	71
N.S.	1	1.00	1.08	1.09	0.94	1.08	1.23	6.11	1.09
time (sec)	N/A	0.028	0.009	0.022	0.263	0.380	0.574	0.410	0.911

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	164	290	215	193	211	889	171
N.S.	1	1.00	1.13	2.00	1.48	1.33	1.46	6.13	1.18
time (sec)	N/A	0.219	0.045	0.066	0.264	0.353	0.555	0.435	1.042

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	161	281	0	0	0	0	-1
N.S.	1	1.00	0.99	1.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.332	0.106	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	132	254	189	160	168	603	134
N.S.	1	1.00	1.17	2.25	1.67	1.42	1.49	5.34	1.19
time (sec)	N/A	0.166	0.035	0.031	0.261	0.347	0.387	0.435	0.916

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	122	245	0	0	0	0	-1
N.S.	1	1.00	0.94	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.183	0.056	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	90	215	158	122	114	301	89
N.S.	1	1.00	1.20	2.87	2.11	1.63	1.52	4.01	1.19
time (sec)	N/A	0.083	0.033	0.028	0.277	0.366	0.238	0.402	0.785

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	82	118	0	0	0	0	-1
N.S.	1	1.00	1.11	1.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.096	0.089	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	120	701	0	0	0	0	-1
N.S.	1	1.00	1.03	5.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.050	0.783	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	94	244	0	0	0	0	-1
N.S.	1	1.00	1.32	3.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.102	0.102	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	101	234	151	135	126	278	246
N.S.	1	1.00	1.26	2.92	1.89	1.69	1.58	3.48	3.08
time (sec)	N/A	0.096	0.040	0.043	0.268	0.356	0.400	0.434	1.490

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	145	305	0	0	0	0	-1
N.S.	1	1.00	1.12	2.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.252	0.089	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	164	271	224	173	184	612	303
N.S.	1	1.00	1.40	2.32	1.91	1.48	1.57	5.23	2.59
time (sec)	N/A	0.164	0.043	0.046	0.257	0.361	0.570	0.425	1.903

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	305	1242	0	0	0	0	-1
N.S.	1	1.00	1.23	5.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.668	0.509	2.719	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	383	1188	0	0	0	0	-1
N.S.	1	1.00	1.46	4.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.554	0.528	2.388	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	245	1157	0	0	0	0	-1
N.S.	1	1.00	1.32	6.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.420	0.347	1.262	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	250	1093	0	0	0	0	-1
N.S.	1	1.00	1.27	5.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.363	0.373	1.214	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	161	5831	0	0	0	0	-1
N.S.	1	1.00	1.31	47.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.232	0.489	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	161	245	0	0	0	0	-1
N.S.	1	1.00	1.49	2.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.306	0.242	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	178	1470	0	0	0	0	-1
N.S.	1	1.00	0.97	7.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.315	0.150	0.346	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	196	1555	0	0	0	0	-1
N.S.	1	1.00	1.92	15.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.245	0.363	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	192	4871	0	0	0	0	-1
N.S.	1	1.00	1.56	39.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.185	0.575	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	323	1716	0	0	0	0	-1
N.S.	1	1.00	1.62	8.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.345	0.638	2.553	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	295	1204	0	0	0	0	-1
N.S.	1	1.00	1.58	6.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.440	0.470	1.862	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	128	107	134	296	0	0	-1
N.S.	1	1.00	1.03	0.86	1.08	2.39	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.048	0.070	0.473	0.364	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	115	93	118	255	0	0	-1
N.S.	1	1.00	1.08	0.88	1.11	2.41	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.038	0.042	0.468	0.386	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	114	93	119	223	636	0	-1
N.S.	1	1.00	1.08	0.88	1.12	2.10	6.00	0.00	-0.01
time (sec)	N/A	0.044	0.032	0.041	0.466	0.432	5.390	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	98	68	103	211	0	88	-1
N.S.	1	1.00	1.15	0.80	1.21	2.48	0.00	1.04	-0.01
time (sec)	N/A	0.037	0.024	0.040	0.486	0.361	0.000	0.464	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	99	69	94	221	0	93	-1
N.S.	1	1.00	1.16	0.81	1.11	2.60	0.00	1.09	-0.01
time (sec)	N/A	0.037	0.029	0.041	0.464	0.360	0.000	0.445	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	107	93	101	243	0	117	-1
N.S.	1	1.00	1.00	0.87	0.94	2.27	0.00	1.09	-0.01
time (sec)	N/A	0.046	0.040	0.051	0.463	0.366	0.000	0.444	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	108	93	112	253	0	117	-1
N.S.	1	1.00	1.01	0.87	1.05	2.36	0.00	1.09	-0.01
time (sec)	N/A	0.046	0.031	0.050	0.474	0.370	0.000	0.442	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	122	107	130	272	0	135	-1
N.S.	1	1.00	0.98	0.86	1.04	2.18	0.00	1.08	-0.01
time (sec)	N/A	0.057	0.044	0.053	0.466	0.358	0.000	0.512	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	2.686	0.257	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	1.694	0.246	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	59	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.052	0.247	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.018	0.187	0.237	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.018	0.374	0.237	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.004	0.327	0.062	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.243	0.132	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	78	66	69	64	58	78	69
N.S.	1	1.00	1.44	1.22	1.28	1.19	1.07	1.44	1.28
time (sec)	N/A	0.028	0.016	0.031	0.271	0.335	7.601	0.425	1.064

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	53	45	46	62	85	57	61
N.S.	1	1.00	1.10	0.94	0.96	1.29	1.77	1.19	1.27
time (sec)	N/A	0.026	0.014	0.040	0.253	0.354	5.116	0.412	0.791

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	67	57	58	54	48	181	60
N.S.	1	1.00	1.56	1.33	1.35	1.26	1.12	4.21	1.40
time (sec)	N/A	0.021	0.014	0.042	0.258	0.362	3.614	0.406	0.939

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	42	39	37	50	71	188	52
N.S.	1	1.00	1.14	1.05	1.00	1.35	1.92	5.08	1.41
time (sec)	N/A	0.011	0.009	0.021	0.255	0.327	3.203	0.421	0.765

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	124	0	0	0	0	-1
N.S.	1	1.00	0.93	4.13	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.012	0.054	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	45	49	41	55	80	51	55
N.S.	1	1.00	1.12	1.22	1.02	1.38	2.00	1.28	1.38
time (sec)	N/A	0.020	0.011	0.031	0.261	0.385	5.460	0.434	0.855

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	65	55	51	49	41	67	52
N.S.	1	1.00	1.59	1.34	1.24	1.20	1.00	1.63	1.27
time (sec)	N/A	0.020	0.011	0.036	0.265	0.370	3.443	0.425	0.999

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	61	63	51	65	97	65	67
N.S.	1	1.00	1.09	1.12	0.91	1.16	1.73	1.16	1.20
time (sec)	N/A	0.026	0.011	0.036	0.257	0.342	8.953	0.433	0.884

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	93	53	69	197	185	73	72
N.S.	1	1.00	1.43	0.82	1.06	3.03	2.85	1.12	1.11
time (sec)	N/A	0.026	0.018	0.087	0.458	0.363	5.798	0.479	0.984

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	91	51	66	186	670	75	70
N.S.	1	1.00	1.44	0.81	1.05	2.95	10.63	1.19	1.11
time (sec)	N/A	0.024	0.015	0.055	0.482	0.362	3.381	0.461	0.851

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	57	37	55	160	702	83	55
N.S.	1	1.00	1.30	0.84	1.25	3.64	15.95	1.89	1.25
time (sec)	N/A	0.017	0.015	0.046	0.486	0.424	2.545	0.397	0.790

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	75	42	61	157	1374	79	62
N.S.	1	1.00	1.63	0.91	1.33	3.41	29.87	1.72	1.35
time (sec)	N/A	0.019	0.014	0.058	0.476	0.378	4.696	0.443	0.921

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	91	51	65	181	1904	93	71
N.S.	1	1.00	1.44	0.81	1.03	2.87	30.22	1.48	1.13
time (sec)	N/A	0.023	0.021	0.065	0.461	0.348	6.340	0.418	0.995

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	91	51	66	187	1948	91	71
N.S.	1	1.00	1.44	0.81	1.05	2.97	30.92	1.44	1.13
time (sec)	N/A	0.023	0.022	0.066	0.461	0.358	8.690	0.460	1.026

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	146	298	217	176	206	175	335
N.S.	1	1.00	1.17	2.38	1.74	1.41	1.65	1.40	2.68
time (sec)	N/A	0.187	0.048	0.075	0.264	0.368	9.249	0.519	1.733

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	132	380	0	0	0	0	-1
N.S.	1	1.00	0.90	2.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.199	0.277	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	106	287	186	138	163	361	275
N.S.	1	1.00	1.16	3.15	2.04	1.52	1.79	3.97	3.02
time (sec)	N/A	0.111	0.036	0.174	0.264	0.340	5.294	0.448	1.262

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	99	137	0	0	0	0	-1
N.S.	1	1.00	1.05	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.109	0.168	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	183	0	0	0	0	0	-1
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.229	0.050	0.021	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	119	0	0	0	0	0	-1
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.104	0.024	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	111	257	175	151	175	0	278
N.S.	1	1.00	1.26	2.92	1.99	1.72	1.99	0.00	3.16
time (sec)	N/A	0.134	0.052	0.152	0.265	0.355	7.112	0.000	1.498

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1173	1173	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.637	8.450	0.020	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1129	1129	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.464	8.319	0.023	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	958	958	566	0	0	0	0	0	-1
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.993	1.793	0.021	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	942	942	566	0	0	0	0	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.946	2.322	0.022	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1102	1102	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.268	2.296	0.022	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1176	1176	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.388	2.315	0.023	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	185	798	0	0	0	0	-1
N.S.	1	1.00	1.31	5.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.218	0.337	0.316	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	211	280	0	0	0	0	-1
N.S.	1	1.00	1.57	2.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.173	0.320	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	211	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.373	0.131	0.023	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	222	0	0	0	0	0	-1
N.S.	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.286	0.023	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	218	0	0	0	0	0	-1
N.S.	1	1.00	1.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.201	0.023	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	241	303	316	455	0	0	-1
N.S.	1	1.00	0.76	0.96	1.00	1.44	0.00	0.00	-0.00
time (sec)	N/A	0.212	0.084	0.078	0.474	0.402	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	240	303	310	399	0	0	-1
N.S.	1	1.00	0.76	0.96	0.98	1.26	0.00	0.00	-0.00
time (sec)	N/A	0.187	0.062	0.043	0.468	0.377	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	227	289	301	400	0	0	-1
N.S.	1	1.00	0.75	0.96	1.00	1.33	0.00	0.00	-0.00
time (sec)	N/A	0.164	0.049	0.040	0.467	0.432	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	227	257	296	364	0	493	-1
N.S.	1	1.00	0.80	0.90	1.04	1.28	0.00	1.73	-0.00
time (sec)	N/A	0.155	0.040	0.043	0.472	0.392	0.000	0.424	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	268	259	296	397	0	505	-1
N.S.	1	1.00	0.94	0.91	1.04	1.39	0.00	1.77	-0.00
time (sec)	N/A	0.176	0.066	0.046	0.474	0.392	0.000	0.584	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	268	279	277	428	0	516	-1
N.S.	1	1.00	0.89	0.93	0.92	1.42	0.00	1.71	-0.00
time (sec)	N/A	0.176	0.069	0.044	0.467	0.399	0.000	0.755	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	275	291	298	473	0	532	-1
N.S.	1	1.00	0.87	0.92	0.94	1.49	0.00	1.68	-0.00
time (sec)	N/A	0.189	0.055	0.052	0.471	0.378	0.000	1.871	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	281	301	297	450	0	519	-1
N.S.	1	1.00	0.89	0.95	0.94	1.42	0.00	1.64	-0.00
time (sec)	N/A	0.179	0.061	0.056	0.479	0.453	0.000	5.280	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	6327	6327	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	10.663	48.318	0.050	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	6177	6177	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	9.226	45.605	0.050	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	6334	6334	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	9.671	48.964	0.052	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	6520	6520	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	8.943	39.646	0.052	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	1.365	0.026	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.850	0.025	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	64	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.047	0.023	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.018	0.269	0.023	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.267	0.023	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	78	66	69	64	0	78	69
N.S.	1	1.00	1.44	1.22	1.28	1.19	0.00	1.44	1.28
time (sec)	N/A	0.028	0.016	0.029	0.251	0.373	0.000	0.428	1.108

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	53	45	46	62	0	57	61
N.S.	1	1.00	1.10	0.94	0.96	1.29	0.00	1.19	1.27
time (sec)	N/A	0.026	0.015	0.049	0.265	0.339	0.000	0.435	0.824

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	67	57	58	54	0	181	60
N.S.	1	1.00	1.56	1.33	1.35	1.26	0.00	4.21	1.40
time (sec)	N/A	0.022	0.013	0.049	0.253	0.326	0.000	0.424	0.957

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	42	39	37	50	0	188	52
N.S.	1	1.00	1.14	1.05	1.00	1.35	0.00	5.08	1.41
time (sec)	N/A	0.016	0.009	0.025	0.254	0.355	0.000	0.436	0.780

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	92	0	0	0	0	-1
N.S.	1	1.00	0.93	3.07	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.013	0.042	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	45	49	41	55	0	51	55
N.S.	1	1.00	1.12	1.22	1.02	1.38	0.00	1.28	1.38
time (sec)	N/A	0.019	0.011	0.034	0.269	0.336	0.000	0.418	0.853

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	65	55	51	49	0	67	52
N.S.	1	1.00	1.59	1.34	1.24	1.20	0.00	1.63	1.27
time (sec)	N/A	0.021	0.010	0.041	0.254	0.342	0.000	0.422	1.022

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	61	63	51	65	0	65	67
N.S.	1	1.00	1.09	1.12	0.91	1.16	0.00	1.16	1.20
time (sec)	N/A	0.027	0.012	0.039	0.261	0.353	0.000	0.419	0.901

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	196	184	162	981	0	207	125
N.S.	1	1.00	1.13	1.06	0.93	5.64	0.00	1.19	0.72
time (sec)	N/A	0.159	0.031	0.040	0.466	0.405	0.000	0.470	1.574

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	136	99	90	260	0	109	107
N.S.	1	1.00	1.35	0.98	0.89	2.57	0.00	1.08	1.06
time (sec)	N/A	0.068	0.030	0.029	0.457	0.389	0.000	0.415	2.757

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	187	159	155	228	0	165	118
N.S.	1	1.00	1.13	0.96	0.94	1.38	0.00	1.00	0.72
time (sec)	N/A	0.140	0.039	0.042	0.461	0.412	0.000	0.446	1.470

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	196	172	100	151	0	125	135
N.S.	1	1.00	1.70	1.50	0.87	1.31	0.00	1.09	1.17
time (sec)	N/A	0.065	0.038	0.049	0.465	0.366	0.000	0.435	3.119

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	198	186	164	248	0	208	127
N.S.	1	1.00	1.12	1.06	0.93	1.41	0.00	1.18	0.72
time (sec)	N/A	0.201	0.023	0.063	0.468	0.361	0.000	0.476	1.259

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	198	114	103	166	0	126	124
N.S.	1	1.00	1.69	0.97	0.88	1.42	0.00	1.08	1.06
time (sec)	N/A	0.065	0.021	0.029	0.463	0.367	0.000	0.423	2.438

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	187	177	155	238	0	179	118
N.S.	1	1.00	1.13	1.07	0.94	1.44	0.00	1.08	0.72
time (sec)	N/A	0.173	0.018	0.034	0.455	0.342	0.000	0.509	1.248

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	183	105	94	117	0	106	117
N.S.	1	1.00	1.76	1.01	0.90	1.12	0.00	1.02	1.12
time (sec)	N/A	0.056	0.021	0.035	0.472	0.363	0.000	0.423	2.387

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	196	172	160	196	0	187	125
N.S.	1	1.00	1.13	0.99	0.92	1.13	0.00	1.07	0.72
time (sec)	N/A	0.194	0.038	0.048	0.455	0.398	0.000	0.621	1.276

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	146	298	217	176	0	175	335
N.S.	1	1.00	1.17	2.38	1.74	1.41	0.00	1.40	2.68
time (sec)	N/A	0.193	0.047	0.171	0.279	0.357	0.000	0.464	1.596

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	132	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.209	0.029	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	106	287	186	138	0	361	275
N.S.	1	1.00	1.16	3.15	2.04	1.52	0.00	3.97	3.02
time (sec)	N/A	0.112	0.036	0.216	0.261	0.368	0.000	0.476	1.247

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	99	137	0	0	0	0	-1
N.S.	1	1.00	1.03	1.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.107	0.194	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	183	0	0	0	0	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.231	0.051	0.026	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	117	0	0	0	0	0	-1
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.109	0.029	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	111	257	175	151	0	0	278
N.S.	1	1.00	1.26	2.92	1.99	1.72	0.00	0.00	3.16
time (sec)	N/A	0.120	0.051	0.173	0.285	0.396	0.000	0.000	1.543

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	159	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.191	0.268	0.025	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	334	0	0	0	0	0	-1
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.367	0.342	0.030	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	185	799	0	0	0	0	-1
N.S.	1	1.00	1.33	5.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.201	0.371	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	191	280	0	0	0	0	-1
N.S.	1	1.00	1.47	2.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.296	0.186	0.263	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	214	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.410	0.121	0.027	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	223	0	0	0	0	0	-1
N.S.	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.223	0.269	0.026	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	218	0	0	0	0	0	-1
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.241	0.190	0.028	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	1.388	0.030	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.843	0.027	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	64	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.049	0.027	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.019	0.347	0.025	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.018	0.265	0.026	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	67	69	57	48	46	262	45
N.S.	1	1.00	1.34	1.38	1.14	0.96	0.92	5.24	0.90
time (sec)	N/A	0.022	0.010	0.201	0.258	0.350	0.160	0.426	0.779

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	50	71	42	49	49	227	42
N.S.	1	1.00	1.11	1.58	0.93	1.09	1.09	5.04	0.93
time (sec)	N/A	0.022	0.008	0.108	0.251	0.372	0.146	0.440	0.719

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	56	60	44	39	36	130	36
N.S.	1	1.00	1.44	1.54	1.13	1.00	0.92	3.33	0.92
time (sec)	N/A	0.014	0.008	0.124	0.250	0.338	0.118	0.396	0.735

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	48	29	35	24	150	27
N.S.	1	1.00	1.00	1.66	1.00	1.21	0.83	5.17	0.93
time (sec)	N/A	0.009	0.004	0.074	0.252	0.345	0.097	0.412	0.679

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	63	0	0	0	0	-1
N.S.	1	1.00	0.93	2.10	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.020	0.011	0.107	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	38	39	37	48	39	87	43
N.S.	1	1.00	1.09	1.11	1.06	1.37	1.11	2.49	1.23
time (sec)	N/A	0.015	0.009	0.095	0.258	0.364	0.300	0.415	0.745

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	60	60	52	46	44	123	49
N.S.	1	1.00	1.40	1.40	1.21	1.07	1.02	2.86	1.14
time (sec)	N/A	0.019	0.008	0.098	0.260	0.342	0.328	0.414	0.709

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	62	62	55	62	68	234	59
N.S.	1	1.00	1.09	1.09	0.96	1.09	1.19	4.11	1.04
time (sec)	N/A	0.031	0.010	0.098	0.262	0.363	0.433	0.418	0.750

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	131	310	189	149	158	552	142
N.S.	1	1.00	1.07	2.52	1.54	1.21	1.28	4.49	1.15
time (sec)	N/A	0.196	0.060	1.030	0.252	0.344	0.226	0.431	0.892

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	145	358	0	0	0	0	-1
N.S.	1	1.00	1.02	2.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.190	0.219	0.372	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	92	271	136	111	104	268	101
N.S.	1	1.00	1.11	3.27	1.64	1.34	1.25	3.23	1.22
time (sec)	N/A	0.115	0.026	0.760	0.264	0.430	0.162	0.423	0.782

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	97	287	0	0	0	0	-1
N.S.	1	1.00	1.31	3.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.083	0.225	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	114	780	0	0	0	0	-1
N.S.	1	1.00	0.86	5.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.205	0.066	3.447	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	101	137	0	0	0	0	-1
N.S.	1	1.00	1.16	1.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.133	0.585	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	119	258	165	130	124	255	235
N.S.	1	1.00	1.37	2.97	1.90	1.49	1.43	2.93	2.70
time (sec)	N/A	0.100	0.038	1.922	0.258	0.347	0.383	0.431	1.284

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	286	1337	0	0	0	0	-1
N.S.	1	1.00	1.41	6.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.453	0.451	8.020	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	316	1915	0	0	0	0	-1
N.S.	1	1.00	1.46	8.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.373	0.534	3.789	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	193	5310	0	0	0	0	-1
N.S.	1	1.00	1.43	39.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.210	3.066	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	198	1732	0	0	0	0	-1
N.S.	1	1.00	1.83	16.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.181	0.713	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	171	1631	0	0	0	0	-1
N.S.	1	1.00	0.82	7.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.340	0.126	0.708	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	215	280	0	0	0	0	-1
N.S.	1	1.00	1.71	2.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.183	1.398	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	195	6380	0	0	0	0	-1
N.S.	1	1.00	1.40	45.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.225	5.166	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	73	64	62	53	51	71	66
N.S.	1	1.00	1.35	1.19	1.15	0.98	0.94	1.31	1.22
time (sec)	N/A	0.028	0.013	0.164	0.257	0.335	3.098	0.441	1.010

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	50	65	42	52	75	52	56
N.S.	1	1.00	1.11	1.44	0.93	1.16	1.67	1.16	1.24
time (sec)	N/A	0.023	0.010	0.391	0.252	0.336	2.262	0.424	0.814

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	62	55	49	44	41	162	57
N.S.	1	1.00	1.44	1.28	1.14	1.02	0.95	3.77	1.33
time (sec)	N/A	0.021	0.010	0.210	0.258	0.339	1.677	0.433	0.865

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	39	52	34	43	61	184	47
N.S.	1	1.00	1.15	1.53	1.00	1.26	1.79	5.41	1.38
time (sec)	N/A	0.012	0.007	0.100	0.254	0.344	1.351	0.429	0.789

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	154	0	0	0	0	-1
N.S.	1	1.00	0.93	5.13	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.011	0.126	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	42	39	37	55	76	52	56
N.S.	1	1.00	1.14	1.05	1.00	1.49	2.05	1.41	1.51
time (sec)	N/A	0.016	0.009	0.100	0.248	0.357	4.295	0.412	0.842

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	64	57	56	52	49	66	59
N.S.	1	1.00	1.42	1.27	1.24	1.16	1.09	1.47	1.31
time (sec)	N/A	0.024	0.011	0.111	0.262	0.355	5.820	0.418	0.995

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	62	45	55	67	94	65	66
N.S.	1	1.00	1.09	0.79	0.96	1.18	1.65	1.14	1.16
time (sec)	N/A	0.026	0.011	0.108	0.258	0.406	8.239	0.413	0.894

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	88	55	62	170	845	67	67
N.S.	1	1.00	1.40	0.87	0.98	2.70	13.41	1.06	1.06
time (sec)	N/A	0.025	0.017	0.226	0.463	0.350	3.781	0.445	0.983

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	86	53	61	162	830	69	65
N.S.	1	1.00	1.41	0.87	1.00	2.66	13.61	1.13	1.07
time (sec)	N/A	0.024	0.014	0.155	0.472	0.354	2.842	0.426	0.895

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	54	39	51	138	632	57	52
N.S.	1	1.00	1.23	0.89	1.16	3.14	14.36	1.30	1.18
time (sec)	N/A	0.017	0.012	0.118	0.461	0.486	2.098	0.417	0.819

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	72	47	57	159	886	62	59
N.S.	1	1.00	1.57	1.02	1.24	3.46	19.26	1.35	1.28
time (sec)	N/A	0.021	0.015	0.121	0.456	0.374	3.661	0.427	0.960

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	90	57	64	189	1046	72	69
N.S.	1	1.00	1.38	0.88	0.98	2.91	16.09	1.11	1.06
time (sec)	N/A	0.026	0.022	0.131	0.464	0.425	4.995	0.452	0.992

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	90	57	65	196	994	74	69
N.S.	1	1.00	1.38	0.88	1.00	3.02	15.29	1.14	1.06
time (sec)	N/A	0.025	0.019	0.128	0.467	0.381	7.132	0.489	1.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	104	0	157	126	151	327	247
N.S.	1	1.00	1.11	0.00	1.67	1.34	1.61	3.48	2.63
time (sec)	N/A	0.121	0.041	180.000	0.259	0.420	2.301	0.422	1.209

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	107	6869	0	0	0	0	-1
N.S.	1	1.00	1.14	73.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.096	0.526	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	183	0	0	0	0	0	-1
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.047	0.064	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	114	137	0	0	0	0	-1
N.S.	1	1.00	1.15	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.123	0.642	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	131	0	183	143	172	0	262
N.S.	1	1.00	1.35	0.00	1.89	1.47	1.77	0.00	2.70
time (sec)	N/A	0.103	0.051	180.000	0.264	0.378	6.081	0.000	1.541

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1214	1214	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.926	9.129	0.066	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1172	1172	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.555	7.327	0.067	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1549	1549	565	0	0	0	0	0	-1
N.S.	1	1.00	0.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.606	2.765	0.062	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1117	1117	568	0	0	0	0	0	-1
N.S.	1	1.00	0.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.530	2.025	0.066	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1263	1263	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.054	1.930	0.065	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1337	1337	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.039	1.952	0.069	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	1.903	0.040	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	1.203	0.039	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	68	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.051	0.038	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.019	0.324	0.036	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.018	0.464	0.034	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	114	87	86	89	0	359	86
N.S.	1	1.00	1.30	0.99	0.98	1.01	0.00	4.08	0.98
time (sec)	N/A	0.030	0.039	0.065	0.257	0.395	0.000	0.442	1.434

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	101	78	78	80	0	301	58
N.S.	1	1.00	1.35	1.04	1.04	1.07	0.00	4.01	0.77
time (sec)	N/A	0.024	0.040	0.069	0.269	0.355	0.000	0.445	1.217

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	88	69	69	70	0	239	49
N.S.	1	1.00	1.42	1.11	1.11	1.13	0.00	3.85	0.79
time (sec)	N/A	0.019	0.034	0.066	0.259	0.360	0.000	0.453	1.170

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	42	50	53	56	0	174	32
N.S.	1	1.00	1.08	1.28	1.36	1.44	0.00	4.46	0.82
time (sec)	N/A	0.014	0.107	0.066	0.259	0.466	0.000	0.475	0.901

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	63	61	0	0	0	-1
N.S.	1	1.00	1.00	2.17	2.10	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.024	0.083	0.359	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	67	62	51	53	231	168	52
N.S.	1	1.00	1.68	1.55	1.28	1.32	5.78	4.20	1.30
time (sec)	N/A	0.018	0.022	0.069	0.260	0.355	3.392	0.432	1.117

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	86	71	64	64	342	356	61
N.S.	1	1.00	1.43	1.18	1.07	1.07	5.70	5.93	1.02
time (sec)	N/A	0.021	0.021	0.072	0.267	0.375	9.702	0.436	1.363

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	99	80	72	74	371	534	69
N.S.	1	1.00	1.36	1.10	0.99	1.01	5.08	7.32	0.95
time (sec)	N/A	0.023	0.022	0.070	0.262	0.346	25.829	0.444	1.386

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	224	373	265	273	0	0	453
N.S.	1	1.00	1.06	1.77	1.26	1.29	0.00	0.00	2.15
time (sec)	N/A	0.370	0.075	0.181	0.270	0.393	0.000	0.000	4.867

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	194	335	241	241	0	0	185
N.S.	1	1.00	1.12	1.94	1.39	1.39	0.00	0.00	1.07
time (sec)	N/A	0.266	0.067	0.180	0.262	0.416	0.000	0.000	1.565

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	160	297	215	207	0	0	143
N.S.	1	1.00	1.24	2.30	1.67	1.60	0.00	0.00	1.11
time (sec)	N/A	0.182	0.056	0.180	0.270	0.357	0.000	0.000	1.281

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	115	252	175	165	0	0	94
N.S.	1	1.00	1.35	2.96	2.06	1.94	0.00	0.00	1.11
time (sec)	N/A	0.102	0.039	0.174	0.263	0.361	0.000	0.000	1.058

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	164	742	0	0	0	0	-1
N.S.	1	1.00	1.13	5.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.059	2.588	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	129	275	174	157	680	0	278
N.S.	1	1.00	1.52	3.24	2.05	1.85	8.00	0.00	3.27
time (sec)	N/A	0.122	0.069	0.109	0.266	0.371	3.471	0.000	1.792

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	178	314	234	201	972	0	341
N.S.	1	1.00	1.34	2.36	1.76	1.51	7.31	0.00	2.56
time (sec)	N/A	0.196	0.079	0.117	0.261	0.378	9.711	0.000	2.707

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	418	1431	1972	0	0	0	-1
N.S.	1	1.00	1.12	3.83	5.27	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.091	0.834	19.473	0.854	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	351	1341	1579	0	0	0	-1
N.S.	1	1.00	1.15	4.41	5.19	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.731	0.578	7.973	0.770	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	285	1250	1184	0	0	0	-1
N.S.	1	1.00	1.22	5.34	5.06	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.420	0.377	7.330	0.691	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	201	5972	0	0	0	0	-1
N.S.	1	1.00	1.42	42.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.185	6.803	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	248	1542	0	0	0	0	-1
N.S.	1	1.00	1.11	6.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	0.136	5.506	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	230	4974	528	0	0	0	-1
N.S.	1	1.00	1.62	35.03	3.72	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.246	0.212	7.177	1.097	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	333	1289	703	0	0	0	-1
N.S.	1	1.00	1.42	5.51	3.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.475	0.492	8.174	1.229	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	31	35	24	36	121	170	24
N.S.	1	1.00	0.82	0.92	0.63	0.95	3.18	4.47	0.63
time (sec)	N/A	0.012	0.013	0.069	0.266	0.377	1.035	0.407	0.862

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	30	19	31	39	121	-1
N.S.	1	1.00	0.81	0.97	0.61	1.00	1.26	3.90	-0.03
time (sec)	N/A	0.009	0.009	0.039	0.252	0.390	0.548	0.414	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	25	87	72	14
N.S.	1	1.00	1.00	0.85	0.80	1.25	4.35	3.60	0.70
time (sec)	N/A	0.006	0.008	0.054	0.272	0.355	0.214	0.466	0.801

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	29	18	37	126	72	22
N.S.	1	1.00	1.00	1.21	0.75	1.54	5.25	3.00	0.92
time (sec)	N/A	0.007	0.015	0.070	0.249	0.375	0.417	0.419	0.786

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	222	194	172	1803	0	227	231
N.S.	1	1.00	1.17	1.02	0.91	9.49	0.00	1.19	1.22
time (sec)	N/A	0.249	0.041	0.054	0.471	1.179	0.000	0.490	13.380

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	75	60	58	64	0	97	110
N.S.	1	1.00	1.53	1.22	1.18	1.31	0.00	1.98	2.24
time (sec)	N/A	0.045	0.022	0.100	0.252	0.340	0.000	0.415	1.757

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	222	194	172	1848	0	0	247
N.S.	1	1.00	1.17	1.02	0.91	9.73	0.00	0.00	1.30
time (sec)	N/A	0.245	0.026	0.056	0.471	1.219	0.000	0.000	11.967

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	114	179	158	1682	0	186	107
N.S.	1	1.00	0.67	1.05	0.93	9.89	0.00	1.09	0.63
time (sec)	N/A	0.212	0.107	0.057	0.479	1.216	0.000	0.427	5.100

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	63	62	0	0	0	-1
N.S.	1	1.00	0.94	1.85	1.82	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.016	0.115	0.370	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	205	167	163	234	0	172	220
N.S.	1	1.00	1.19	0.97	0.95	1.36	0.00	1.00	1.28
time (sec)	N/A	0.163	0.030	0.053	0.485	0.369	0.000	0.488	7.456

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	220	180	168	214	0	194	228
N.S.	1	1.00	1.17	0.96	0.89	1.14	0.00	1.03	1.21
time (sec)	N/A	0.214	0.042	0.056	0.469	0.377	0.000	0.487	7.375

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	73	55	51	59	0	67	114
N.S.	1	1.00	1.55	1.17	1.09	1.26	0.00	1.43	2.43
time (sec)	N/A	0.024	0.023	0.056	0.269	0.353	0.000	0.440	1.358

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	122	258	186	179	0	0	105
N.S.	1	1.00	1.21	2.55	1.84	1.77	0.00	0.00	1.04
time (sec)	N/A	0.109	0.060	0.270	0.258	0.384	0.000	0.000	1.307

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	167	785	0	0	0	0	-1
N.S.	1	1.00	1.07	5.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.225	0.089	6.804	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	123	0	175	173	0	0	281
N.S.	1	1.00	1.28	0.00	1.82	1.80	0.00	0.00	2.93
time (sec)	N/A	0.126	0.090	0.107	0.282	0.365	0.000	0.000	2.037

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	73	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.037	0.011	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	73	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	0.034	0.013	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.094	0.011	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	65	0	141	0	0	-1
N.S.	1	1.00	1.08	1.81	0.00	3.92	0.00	0.00	-0.03
time (sec)	N/A	0.024	0.057	0.144	0.000	0.378	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	66	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.045	0.012	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	73	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.032	0.013	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	73	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.033	0.014	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.010	7.776	0.014	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.005	1.725	0.012	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	183	827	0	0	0	0	-1
N.S.	1	1.00	1.24	5.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.136	5.898	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	10.438	0.014	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.020	10.395	0.015	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	33	53	147	129	0	0	-1
N.S.	1	1.00	1.10	1.77	4.90	4.30	0.00	0.00	-0.03
time (sec)	N/A	0.015	0.031	0.128	0.321	0.357	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	95	104	0	0	0	-1
N.S.	1	1.00	0.92	3.96	4.33	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	0.010	0.096	0.259	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	30	15	22	15	101	15
N.S.	1	1.00	0.89	1.58	0.79	1.16	0.79	5.32	0.79
time (sec)	N/A	0.005	0.002	0.144	0.251	0.349	0.079	0.407	0.071

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	6.300	0.023	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	5.101	0.020	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.064	0.019	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.022	0.274	0.016	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.018	1.242	0.010	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [90] had the largest ratio of [20]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	12	0.250
2	A	4	3	1.00	12	0.250
3	A	4	3	1.00	12	0.250
4	A	4	3	1.00	12	0.250
5	A	3	3	1.00	10	0.300
6	A	3	2	1.00	8	0.250
7	A	1	1	1.00	12	0.083
8	A	5	5	1.00	12	0.417
9	A	3	3	1.00	12	0.250
10	A	4	3	1.00	12	0.250
11	A	4	3	1.00	12	0.250
12	A	4	3	1.00	12	0.250
13	A	16	7	1.00	14	0.500
14	A	14	9	1.00	14	0.643
15	A	11	7	1.00	14	0.500
16	A	9	8	1.00	14	0.571
17	A	6	5	1.00	12	0.417
18	A	5	5	1.00	10	0.500
19	A	6	5	1.00	14	0.357
20	A	4	4	1.00	14	0.286
21	A	8	7	1.00	14	0.500
22	A	8	7	1.00	14	0.500
23	A	13	8	1.00	14	0.571
24	A	33	11	1.00	14	0.786
25	A	24	11	1.00	14	0.786

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	18	10	1.00	14	0.714
27	A	12	9	1.00	14	0.643
28	A	8	8	1.00	12	0.667
29	A	5	6	1.00	10	0.600
30	A	8	6	1.00	14	0.429
31	A	5	6	1.00	14	0.429
32	A	7	6	1.00	14	0.429
33	A	14	11	1.00	14	0.786
34	A	16	8	1.00	14	0.571
35	A	7	6	1.00	16	0.375
36	A	6	6	1.00	16	0.375
37	A	6	6	1.00	16	0.375
38	A	5	5	1.00	16	0.312
39	A	5	5	1.00	16	0.312
40	A	6	6	1.00	16	0.375
41	A	6	6	1.00	16	0.375
42	A	7	6	1.00	16	0.375
43	A	0	0	0.00	0	0.000
44	A	0	0	0.00	0	0.000
45	A	2	2	1.00	14	0.143
46	A	0	0	0.00	0	0.000
47	A	0	0	0.00	0	0.000
48	A	0	0	0.00	0	0.000
49	A	0	0	0.00	0	0.000
50	A	5	4	1.00	14	0.286
51	A	4	3	1.00	14	0.214
52	A	4	4	1.00	14	0.286
53	A	2	2	1.00	12	0.167
54	A	2	2	1.00	14	0.143
55	A	5	5	1.00	14	0.357
56	A	4	4	1.00	14	0.286
57	A	4	3	1.00	14	0.214
58	A	5	5	1.00	14	0.357
59	A	5	5	1.00	14	0.357
60	A	5	4	1.00	10	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	4	4	1.00	14	0.286
62	A	5	5	1.00	14	0.357
63	A	5	5	1.00	14	0.357
64	A	12	8	1.00	16	0.500
65	A	10	9	1.00	16	0.562
66	A	7	6	1.00	16	0.375
67	A	6	6	1.00	14	0.429
68	A	7	6	1.00	16	0.375
69	A	5	5	1.00	16	0.312
70	A	9	8	1.00	16	0.500
71	A	102	26	1.00	16	1.625
72	A	86	26	1.00	16	1.625
73	A	69	21	1.00	12	1.750
74	A	47	21	1.00	16	1.313
75	A	64	24	1.00	16	1.500
76	A	77	24	1.00	16	1.500
77	A	9	9	1.00	16	0.562
78	A	6	7	1.00	14	0.500
79	A	9	7	1.00	16	0.438
80	A	6	7	1.00	16	0.438
81	A	8	7	1.00	16	0.438
82	A	16	13	1.00	18	0.722
83	A	16	13	1.00	18	0.722
84	A	15	12	1.00	18	0.667
85	A	15	12	1.00	18	0.667
86	A	15	12	1.00	18	0.667
87	A	15	12	1.00	18	0.667
88	A	16	13	1.00	18	0.722
89	A	16	13	1.00	18	0.722
90	A	238	34	1.00	20	1.700
91	A	241	33	1.00	20	1.650
92	A	197	33	1.00	20	1.650
93	A	197	33	1.00	20	1.650
94	A	0	0	0.00	0	0.000
95	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	2	2	1.00	16	0.125
97	A	0	0	0.00	0	0.000
98	A	0	0	0.00	0	0.000
99	A	5	4	1.00	14	0.286
100	A	4	3	1.00	14	0.214
101	A	4	4	1.00	14	0.286
102	A	2	2	1.00	14	0.143
103	A	2	2	1.00	14	0.143
104	A	5	5	1.00	14	0.357
105	A	4	4	1.00	14	0.286
106	A	4	3	1.00	14	0.214
107	A	12	8	1.00	14	0.571
108	A	9	8	1.00	10	0.800
109	A	11	7	1.00	14	0.500
110	A	9	9	1.00	14	0.643
111	A	12	8	1.00	14	0.571
112	A	9	9	1.00	14	0.643
113	A	11	7	1.00	12	0.583
114	A	8	8	1.00	14	0.571
115	A	12	8	1.00	14	0.571
116	A	12	8	1.00	16	0.500
117	A	10	9	1.00	16	0.562
118	A	7	6	1.00	16	0.375
119	A	6	6	1.00	16	0.375
120	A	7	6	1.00	16	0.375
121	A	5	5	1.00	16	0.312
122	A	9	8	1.00	16	0.500
123	A	9	8	1.00	16	0.500
124	A	13	10	1.00	16	0.625
125	A	9	9	1.00	16	0.562
126	A	6	7	1.00	16	0.438
127	A	9	7	1.00	16	0.438
128	A	6	7	1.00	16	0.438
129	A	8	7	1.00	16	0.438
130	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	0	0	0.00	0	0.000
132	A	2	2	1.00	16	0.125
133	A	0	0	0.00	0	0.000
134	A	0	0	0.00	0	0.000
135	A	5	4	1.00	14	0.286
136	A	5	4	1.00	14	0.286
137	A	4	4	1.00	12	0.333
138	A	4	3	1.00	10	0.300
139	A	2	2	1.00	14	0.143
140	A	2	2	1.00	14	0.143
141	A	4	4	1.00	14	0.286
142	A	5	4	1.00	14	0.286
143	A	14	9	1.00	16	0.562
144	A	9	8	1.00	16	0.500
145	A	9	8	1.00	14	0.571
146	A	6	6	1.00	12	0.500
147	A	7	6	1.00	16	0.375
148	A	6	6	1.00	16	0.375
149	A	7	6	1.00	16	0.375
150	A	17	9	1.00	16	0.562
151	A	15	12	1.00	16	0.750
152	A	8	7	1.00	14	0.500
153	A	6	7	1.00	12	0.583
154	A	9	7	1.00	16	0.438
155	A	6	7	1.00	16	0.438
156	A	9	9	1.00	16	0.562
157	A	6	5	1.00	14	0.357
158	A	5	4	1.00	14	0.286
159	A	5	5	1.00	14	0.357
160	A	3	3	1.00	12	0.250
161	A	2	2	1.00	14	0.143
162	A	2	2	1.00	14	0.143
163	A	5	5	1.00	14	0.357
164	A	5	4	1.00	14	0.286
165	A	6	6	1.00	14	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	6	6	1.00	14	0.429
167	A	6	5	1.00	10	0.500
168	A	5	5	1.00	14	0.357
169	A	6	6	1.00	14	0.429
170	A	6	6	1.00	14	0.429
171	A	9	8	1.00	16	0.500
172	A	5	5	1.00	14	0.357
173	A	7	6	1.00	16	0.375
174	A	6	6	1.00	16	0.375
175	A	7	6	1.00	16	0.375
176	A	98	34	1.00	16	2.125
177	A	80	34	1.00	16	2.125
178	A	100	30	1.00	12	2.500
179	A	72	30	1.00	16	1.875
180	A	105	31	1.00	16	1.938
181	A	130	31	1.00	16	1.938
182	A	0	0	0.00	0	0.000
183	A	0	0	0.00	0	0.000
184	A	3	3	1.00	16	0.188
185	A	0	0	0.00	0	0.000
186	A	0	0	0.00	0	0.000
187	A	7	4	1.00	16	0.250
188	A	6	4	1.00	16	0.250
189	A	5	4	1.00	14	0.286
190	A	5	4	1.00	12	0.333
191	A	2	2	1.00	16	0.125
192	A	4	4	1.00	16	0.250
193	A	5	4	1.00	16	0.250
194	A	6	4	1.00	16	0.250
195	A	22	8	1.00	18	0.444
196	A	17	8	1.00	18	0.444
197	A	12	8	1.00	16	0.500
198	A	7	6	1.00	14	0.429
199	A	7	6	1.00	18	0.333
200	A	9	8	1.00	18	0.444

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	14	9	1.00	18	0.500
202	A	54	12	1.00	18	0.667
203	A	34	12	1.00	18	0.667
204	A	19	11	1.00	16	0.688
205	A	9	9	1.00	14	0.643
206	A	9	7	1.00	18	0.389
207	A	8	7	1.00	18	0.389
208	A	17	9	1.00	18	0.500
209	A	3	2	1.00	12	0.167
210	A	3	2	1.00	12	0.167
211	A	2	2	1.00	12	0.167
212	A	4	4	1.00	12	0.333
213	A	13	9	1.00	16	0.562
214	A	5	5	1.00	16	0.312
215	A	13	9	1.00	14	0.643
216	A	13	8	1.00	12	0.667
217	A	2	2	1.00	16	0.125
218	A	12	8	1.00	16	0.500
219	A	13	9	1.00	16	0.562
220	A	5	5	1.00	16	0.312
221	A	7	6	1.00	18	0.333
222	A	7	6	1.00	18	0.333
223	A	9	8	1.00	18	0.444
224	A	2	2	1.00	14	0.143
225	A	2	2	1.00	12	0.167
226	A	3	2	1.00	10	0.200
227	A	2	2	1.00	14	0.143
228	A	2	2	1.00	14	0.143
229	A	2	2	1.00	14	0.143
230	A	2	2	1.00	14	0.143
231	A	0	0	0.00	0	0.000
232	A	0	0	0.00	0	0.000
233	A	7	6	1.00	16	0.375
234	A	0	0	0.00	0	0.000
235	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	2	2	1.00	10	0.200
237	A	2	2	1.00	10	0.200
238	A	3	3	1.00	4	0.750
239	A	0	0	0.00	0	0.000
240	A	0	0	0.00	0	0.000
241	A	3	3	1.00	16	0.188
242	A	0	0	0.00	0	0.000
243	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^5\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=59

$$\frac{bx}{6c^5}+\frac{bx^3}{18c^3}+\frac{bx^5}{30c}-\frac{b\tanh^{-1}(cx)}{6c^6}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)$$

```
[Out] 1/6*b*x/c^5+1/18*b*x^3/c^3+1/30*b*x^5/c-1/6*b*arctanh(c*x)/c^6+1/6*x^6*(a+b
*arctanh(c*x))
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6037, 308, 212}

$$\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)-\frac{b\tanh^{-1}(cx)}{6c^6}+\frac{bx}{6c^5}+\frac{bx^3}{18c^3}+\frac{bx^5}{30c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(6*c^5) + (b*x^3)/(18*c^3) + (b*x^5)/(30*c) - (b*ArcTanh[c*x])/(6*c^6
) + (x^6*(a + b*ArcTanh[c*x]))/6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int x^5(a+b\tanh^{-1}(cx))\,dx = \frac{1}{6}x^6(a+b\tanh^{-1}(cx)) - \frac{1}{6}(bc)\int \frac{x^6}{1-c^2x^2}\,dx$$
$$= \frac{1}{6}x^6(a+b\tanh^{-1}(cx)) - \frac{1}{6}(bc)\int \left(-\frac{1}{c^6}-\frac{x^2}{c^4}-\frac{x^4}{c^2}+\frac{1}{c^6(1-c^2x^2)}\right)dx$$
$$= \frac{bx}{6c^5}+\frac{bx^3}{18c^3}+\frac{bx^5}{30c}+\frac{1}{6}x^6(a+b\tanh^{-1}(cx)) - \frac{b\int \frac{1}{1-c^2x^2}\,dx}{6c^5}$$
$$= \frac{bx}{6c^5}+\frac{bx^3}{18c^3}+\frac{bx^5}{30c}-\frac{b\tanh^{-1}(cx)}{6c^6}+\frac{1}{6}x^6(a+b\tanh^{-1}(cx))$$

Mathematica [A]
time = 0.01, size = 81, normalized size = 1.37

$$\frac{bx}{6c^5}+\frac{bx^3}{18c^3}+\frac{bx^5}{30c}+\frac{ax^6}{6}+\frac{1}{6}bx^6\tanh^{-1}(cx)+\frac{b\log(1-cx)}{12c^6}-\frac{b\log(1+cx)}{12c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(6*c^5) + (b*x^3)/(18*c^3) + (b*x^5)/(30*c) + (a*x^6)/6 + (b*x^6*ArcT
anh[c*x])/6 + (b*Log[1 - c*x])/(12*c^6) - (b*Log[1 + c*x])/(12*c^6)
```

Maple [A]
time = 0.06, size = 69, normalized size = 1.17

method	result	size
derivativedivides	$\frac{\frac{c^6x^6a}{6}+\frac{b\,c^6x^6\,\mathrm{arctanh}(cx)}{6}+\frac{c^5x^5b}{30}+\frac{b\,c^3x^3}{18}+\frac{bcx}{6}+\frac{b\ln(cx-1)}{12}-\frac{b\ln(cx+1)}{12}}{c^6}$	69
default	$\frac{\frac{c^6x^6a}{6}+\frac{b\,c^6x^6\,\mathrm{arctanh}(cx)}{6}+\frac{c^5x^5b}{30}+\frac{b\,c^3x^3}{18}+\frac{bcx}{6}+\frac{b\ln(cx-1)}{12}-\frac{b\ln(cx+1)}{12}}{c^6}$	69
risch	$\frac{x^6b\ln(cx+1)}{12}-\frac{x^6b\ln(-cx+1)}{12}+\frac{x^6a}{6}+\frac{b\,x^5}{30c}+\frac{b\,x^3}{18c^3}+\frac{bx}{6c^5}+\frac{b\ln(-cx+1)}{12c^6}-\frac{b\ln(cx+1)}{12c^6}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^6*(1/6*c^6*x^6*a+1/6*b*c^6*x^6*arctanh(c*x)+1/30*c^5*x^5*b+1/18*b*c^3*x
^3+1/6*b*c*x+1/12*b*ln(c*x-1)-1/12*b*ln(c*x+1))
```

Maxima [A]
time = 0.29, size = 70, normalized size = 1.19

$$\frac{1}{6}\,ax^6+\frac{1}{180}\left(30\,x^6\,\mathrm{artanh}\,(cx)+c\left(\frac{2\,(3\,c^4x^5+5\,c^2x^3+15\,x)}{c^6}-\frac{15\,\log\,(cx+1)}{c^7}+\frac{15\,\log\,(cx-1)}{c^7}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/180*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x
)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b
```

Fricas [A]
time = 0.33, size = 67, normalized size = 1.14

$$\frac{30\,ac^6x^6+6\,bc^5x^5+10\,bc^3x^3+30\,bcx+15\,(bc^6x^6-b)\log\left(-\frac{cx+1}{cx-1}\right)}{180\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(30*a*c^6*x^6 + 6*b*c^5*x^5 + 10*b*c^3*x^3 + 30*b*c*x + 15*(b*c^6*x^6
 - b)*log(-(c*x + 1)/(c*x - 1)))/c^6
```

Sympy [A]
time = 0.39, size = 63, normalized size = 1.07

$$\begin{cases}\frac{ax^6}{6}+\frac{bx^6\operatorname{atanh}(cx)}{6}+\frac{bx^5}{30c}+\frac{bx^3}{18c^3}+\frac{bx}{6c^5}-\frac{b\operatorname{atanh}(cx)}{6c^6} & \text{for } c\neq 0\\ \frac{ax^6}{6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*x**6/6 + b*x**6*atanh(c*x)/6 + b*x**5/(30*c) + b*x**3/(18*c**3
) + b*x/(6*c**5) - b*atanh(c*x)/(6*c**6), Ne(c, 0)), (a*x**6/6, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(49) = 98.
time = 0.46, size = 442, normalized size = 7.49

$$\frac{1}{45}\,c\left(\frac{15\left(\frac{3\,(cx+1)^5b}{(cx-1)^5}+\frac{10\,(cx+1)^3b}{(cx-1)^3}+\frac{3\,(cx+1)b}{cx-1}\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^6c^7}{(cx-1)^6}-\frac{6\,(cx+1)^5c^7}{(cx-1)^5}+\frac{15\,(cx+1)^4c^7}{(cx-1)^4}-\frac{20\,(cx+1)^3c^7}{(cx-1)^3}+\frac{15\,(cx+1)^2c^7}{(cx-1)^2}-\frac{6\,(cx+1)c^7}{cx-1}+c^7}+\frac{\frac{90\,(cx+1)^5a}{(cx-1)^5}+\frac{300\,(cx+1)^3a}{(cx-1)^3}+\frac{90\,(cx+1)a}{cx-1}+\frac{45\,(cx+1)^5b}{(cx-1)^5}-\frac{135\,(cx+1)^4b}{(cx-1)^4}+\frac{230\,(cx+1)^3b}{(cx-1)^3}-\frac{210\,(cx+1)^2b}{(cx-1)^2}+\frac{93\,(cx+1)b}{cx-1}-23\,b}{\frac{(cx+1)^6c^7}{(cx-1)^6}-\frac{6\,(cx+1)^5c^7}{(cx-1)^5}+\frac{15\,(cx+1)^4c^7}{(cx-1)^4}-\frac{20\,(cx+1)^3c^7}{(cx-1)^3}+\frac{15\,(cx+1)^2c^7}{(cx-1)^2}-\frac{6\,(cx+1)c^7}{cx-1}+c^7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/45*c*(15*(3*(c*x + 1)^5*b/(c*x - 1)^5 + 10*(c*x + 1)^3*b/(c*x - 1)^3 + 3*
(c*x + 1)*b/(c*x - 1))*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^6*c^7/(c*x - 1)
^6 - 6*(c*x + 1)^5*c^7/(c*x - 1)^5 + 15*(c*x + 1)^4*c^7/(c*x - 1)^4 - 20*(c
*x + 1)^3*c^7/(c*x - 1)^3 + 15*(c*x + 1)^2*c^7/(c*x - 1)^2 - 6*(c*x + 1)*c^
7/(c*x - 1) + c^7) + (90*(c*x + 1)^5*a/(c*x - 1)^5 + 300*(c*x + 1)^3*a/(c*x
 - 1)^3 + 90*(c*x + 1)*a/(c*x - 1) + 45*(c*x + 1)^5*b/(c*x - 1)^5 - 135*(c*
```

```
x + 1)^4*b/(c*x - 1)^4 + 230*(c*x + 1)^3*b/(c*x - 1)^3 - 210*(c*x + 1)^2*b/
(c*x - 1)^2 + 93*(c*x + 1)*b/(c*x - 1) - 23*b)/((c*x + 1)^6*c^7/(c*x - 1)^6
 - 6*(c*x + 1)^5*c^7/(c*x - 1)^5 + 15*(c*x + 1)^4*c^7/(c*x - 1)^4 - 20*(c*x
 + 1)^3*c^7/(c*x - 1)^3 + 15*(c*x + 1)^2*c^7/(c*x - 1)^2 - 6*(c*x + 1)*c^7/
(c*x - 1) + c^7))
```

Mupad [B]
time = 0.84, size = 52, normalized size = 0.88

$$\frac{\frac{b\,c^3\,x^3}{18}-\frac{b\,\mathrm{atanh}(c\,x)}{6}+\frac{b\,c^5\,x^5}{30}+\frac{b\,c\,x}{6}}{c^6}+\frac{a\,x^6}{6}+\frac{b\,x^6\,\mathrm{atanh}(c\,x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atanh(c*x)),x)
```

```
[Out] ((b*c^3*x^3)/18 - (b*atanh(c*x))/6 + (b*c^5*x^5)/30 + (b*c*x)/6)/c^6 + (a*x
^6)/6 + (b*x^6*atanh(c*x))/6
```

3.2 $\int x^4\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=57

$$\frac{bx^2}{10c^3}+\frac{bx^4}{20c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{b\log\left(1-c^2x^2\right)}{10c^5}$$

```
[Out] 1/10*b*x^2/c^3+1/20*b*x^4/c+1/5*x^5*(a+b*arctanh(c*x))+1/10*b*ln(-c^2*x^2+1
)/c^5
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {6037, 272, 45}

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{bx^2}{10c^3}+\frac{b\log\left(1-c^2x^2\right)}{10c^5}+\frac{bx^4}{20c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x^2)/(10*c^3) + (b*x^4)/(20*c) + (x^5*(a + b*ArcTanh[c*x]))/5 + (b*Log[1
 - c^2*x^2])/(10*c^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int x^4(a+b\tanh^{-1}(cx))\,dx = \frac{1}{5}x^5(a+b\tanh^{-1}(cx)) - \frac{1}{5}(bc)\int \frac{x^5}{1-c^2x^2}\,dx$$
$$= \frac{1}{5}x^5(a+b\tanh^{-1}(cx)) - \frac{1}{10}(bc)\text{Subst}\left(\int \frac{x^2}{1-c^2x}\,dx, x, x^2\right)$$
$$= \frac{1}{5}x^5(a+b\tanh^{-1}(cx)) - \frac{1}{10}(bc)\text{Subst}\left(\int \left(-\frac{1}{c^4} - \frac{x}{c^2} - \frac{1}{c^4(-1+c^2x)}\right)dx, x$$
$$= \frac{bx^2}{10c^3} + \frac{bx^4}{20c} + \frac{1}{5}x^5(a+b\tanh^{-1}(cx)) + \frac{b\log(1-c^2x^2)}{10c^5}$$

Mathematica [A]
time = 0.01, size = 62, normalized size = 1.09

$$\frac{bx^2}{10c^3} + \frac{bx^4}{20c} + \frac{ax^5}{5} + \frac{1}{5}bx^5\tanh^{-1}(cx) + \frac{b\log(1-c^2x^2)}{10c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x^2)/(10*c^3) + (b*x^4)/(20*c) + (a*x^5)/5 + (b*x^5*ArcTanh[c*x])/5 + (b
*Log[1 - c^2*x^2])/(10*c^5)
```

Maple [A]
time = 0.01, size = 64, normalized size = 1.12

method	result	size
derivativedivides	$\frac{\frac{c^5x^5a}{5}+\frac{c^5x^5b\,\mathrm{arctanh}(cx)}{5}+\frac{c^4x^4b}{20}+\frac{b\,c^2x^2}{10}+\frac{b\ln(cx-1)}{10}+\frac{b\ln(cx+1)}{10}}{c^5}$	64
default	$\frac{\frac{c^5x^5a}{5}+\frac{c^5x^5b\,\mathrm{arctanh}(cx)}{5}+\frac{c^4x^4b}{20}+\frac{b\,c^2x^2}{10}+\frac{b\ln(cx-1)}{10}+\frac{b\ln(cx+1)}{10}}{c^5}$	64
risch	$\frac{x^5b\ln(cx+1)}{10}-\frac{x^5b\ln(-cx+1)}{10}+\frac{a\,x^5}{5}+\frac{b\,x^4}{20c}+\frac{b\,x^2}{10c^3}+\frac{b\ln(c^2x^2-1)}{10c^5}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(1/5*c^5*x^5*a+1/5*c^5*x^5*b*arctanh(c*x)+1/20*c^4*x^4*b+1/10*b*c^2*x
^2+1/10*b*ln(c*x-1)+1/10*b*ln(c*x+1))
```

Maxima [A]
time = 0.27, size = 55, normalized size = 0.96

$$\frac{1}{5}ax^5 + \frac{1}{20}\left(4\,x^5\,\mathrm{artanh}(cx) + c\left(\frac{c^2x^4+2\,x^2}{c^4} + \frac{2\,\log(c^2x^2-1)}{c^6}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2
*x^2 - 1)/c^6))*b
```

Fricas [A]
time = 0.34, size = 69, normalized size = 1.21

$$\frac{2\,bc^5x^5\log\left(-\frac{cx+1}{cx-1}\right)+4\,ac^5x^5+bc^4x^4+2\,bc^2x^2+2\,b\log\left(c^2x^2-1\right)}{20\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/20*(2*b*c^5*x^5*log(-(c*x + 1)/(c*x - 1)) + 4*a*c^5*x^5 + b*c^4*x^4 + 2*b
*c^2*x^2 + 2*b*log(c^2*x^2 - 1))/c^5
```

Sympy [A]
time = 0.33, size = 68, normalized size = 1.19

$$\begin{cases} \frac{ax^5}{5}+\frac{bx^5\operatorname{atanh}\left(cx\right)}{5}+\frac{bx^4}{20c}+\frac{bx^2}{10c^3}+\frac{b\log\left(x-\frac{1}{c}\right)}{5c^5}+\frac{b\operatorname{atanh}\left(cx\right)}{5c^5} & \text{for}\: c\neq 0 \\ \frac{ax^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*x**5/5 + b*x**5*atanh(c*x)/5 + b*x**4/(20*c) + b*x**2/(10*c**3
) + b*log(x - 1/c)/(5*c**5) + b*atanh(c*x)/(5*c**5), Ne(c, 0)), (a*x**5/5,
True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(49) = 98.
time = 0.43, size = 403, normalized size = 7.07

$$\frac{1}{5}\,c\left(\frac{\left(\frac{5\,(cx+1)^4b}{(cx-1)^4}+\frac{10\,(cx+1)^2b}{(cx-1)^2}+b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^5c^6}{(cx-1)^5}-\frac{5\,(cx+1)^4c^6}{(cx-1)^4}+\frac{10\,(cx+1)^3c^6}{(cx-1)^3}-\frac{10\,(cx+1)^2c^6}{(cx-1)^2}+\frac{5\,(cx+1)c^6}{cx-1}-c^6}+\frac{2\left(\frac{5\,(cx+1)^4a}{(cx-1)^4}+\frac{10\,(cx+1)^2a}{(cx-1)^2}+a+\frac{2\,(cx+1)^4b}{(cx-1)^4}-\frac{4\,(cx+1)^3b}{(cx-1)^3}+\frac{4\,(cx+1)^2b}{(cx-1)^2}-\frac{2\,(cx+1)b}{cx-1}\right)}{\frac{(cx+1)^5c^6}{(cx-1)^5}-\frac{5\,(cx+1)^4c^6}{(cx-1)^4}+\frac{10\,(cx+1)^3c^6}{(cx-1)^3}-\frac{10\,(cx+1)^2c^6}{(cx-1)^2}+\frac{5\,(cx+1)c^6}{cx-1}-c^6}-\frac{b\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^6}+\frac{b\log\left(-\frac{cx+1}{cx-1}\right)}{c^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/5*c*((5*(c*x + 1)^4*b/(c*x - 1)^4 + 10*(c*x + 1)^2*b/(c*x - 1)^2 + b)*log
(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5*c^6/(c*x - 1)^5 - 5*(c*x + 1)^4*c^6/(c*
x - 1)^4 + 10*(c*x + 1)^3*c^6/(c*x - 1)^3 - 10*(c*x + 1)^2*c^6/(c*x - 1)^2
+ 5*(c*x + 1)*c^6/(c*x - 1) - c^6) + 2*(5*(c*x + 1)^4*a/(c*x - 1)^4 + 10*(c
```

```
*x + 1)^2*a/(c*x - 1)^2 + a + 2*(c*x + 1)^4*b/(c*x - 1)^4 - 4*(c*x + 1)^3*b
/(c*x - 1)^3 + 4*(c*x + 1)^2*b/(c*x - 1)^2 - 2*(c*x + 1)*b/(c*x - 1))/((c*x
 + 1)^5*c^6/(c*x - 1)^5 - 5*(c*x + 1)^4*c^6/(c*x - 1)^4 + 10*(c*x + 1)^3*c^
6/(c*x - 1)^3 - 10*(c*x + 1)^2*c^6/(c*x - 1)^2 + 5*(c*x + 1)*c^6/(c*x - 1)
- c^6) - b*log(-(c*x + 1)/(c*x - 1) + 1)/c^6 + b*log(-(c*x + 1)/(c*x - 1))/
c^6)
```

Mupad [B]
time = 0.82, size = 53, normalized size = 0.93

$$\frac{a\,x^5}{5}+\frac{\frac{b\,\ln\left(c^2\,x^2-1\right)}{10}+\frac{b\,c^2\,x^2}{10}+\frac{b\,c^4\,x^4}{20}}{c^5}+\frac{b\,x^5\,\mathrm{atanh}\left(c\,x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atanh(c*x)),x)
```

```
[Out] (a*x^5)/5 + ((b*log(c^2*x^2 - 1))/10 + (b*c^2*x^2)/10 + (b*c^4*x^4)/20)/c^5
 + (b*x^5*atanh(c*x))/5
```

3.3 $\int x^3\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=48

$$\frac{bx}{4c^3}+\frac{bx^3}{12c}-\frac{b\tanh^{-1}(cx)}{4c^4}+\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)$$

```
[Out] 1/4*b*x/c^3+1/12*b*x^3/c-1/4*b*arctanh(c*x)/c^4+1/4*x^4*(a+b*arctanh(c*x))
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6037, 308, 212}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)-\frac{b\tanh^{-1}(cx)}{4c^4}+\frac{bx}{4c^3}+\frac{bx^3}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(4*c^3) + (b*x^3)/(12*c) - (b*ArcTanh[c*x])/(4*c^4) + (x^4*(a + b*Arc
Tanh[c*x]))/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int x^3(a+b\tanh^{-1}(cx))\,dx = \frac{1}{4}x^4(a+b\tanh^{-1}(cx)) - \frac{1}{4}(bc)\int \frac{x^4}{1-c^2x^2}\,dx$$
$$= \frac{1}{4}x^4(a+b\tanh^{-1}(cx)) - \frac{1}{4}(bc)\int \left(-\frac{1}{c^4}-\frac{x^2}{c^2}+\frac{1}{c^4(1-c^2x^2)}\right)dx$$
$$= \frac{bx}{4c^3}+\frac{bx^3}{12c}+\frac{1}{4}x^4(a+b\tanh^{-1}(cx)) - \frac{b\int \frac{1}{1-c^2x^2}\,dx}{4c^3}$$
$$= \frac{bx}{4c^3}+\frac{bx^3}{12c}-\frac{b\tanh^{-1}(cx)}{4c^4}+\frac{1}{4}x^4(a+b\tanh^{-1}(cx))$$

Mathematica [A]
time = 0.01, size = 70, normalized size = 1.46

$$\frac{bx}{4c^3}+\frac{bx^3}{12c}+\frac{ax^4}{4}+\frac{1}{4}bx^4\tanh^{-1}(cx)+\frac{b\log(1-cx)}{8c^4}-\frac{b\log(1+cx)}{8c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(4*c^3) + (b*x^3)/(12*c) + (a*x^4)/4 + (b*x^4*ArcTanh[c*x])/4 + (b*Lo
g[1 - c*x])/(8*c^4) - (b*Log[1 + c*x])/(8*c^4)
```

Maple [A]
time = 0.01, size = 60, normalized size = 1.25

method	result	size
derivativedivides	$\frac{\frac{c^4x^4a}{4}+\frac{c^4x^4b\,\mathrm{arctanh}(cx)}{4}+\frac{b\,c^3x^3}{12}+\frac{bcx}{4}+\frac{b\ln(cx-1)}{8}-\frac{b\ln(cx+1)}{8}}{c^4}$	60
default	$\frac{\frac{c^4x^4a}{4}+\frac{c^4x^4b\,\mathrm{arctanh}(cx)}{4}+\frac{b\,c^3x^3}{12}+\frac{bcx}{4}+\frac{b\ln(cx-1)}{8}-\frac{b\ln(cx+1)}{8}}{c^4}$	60
risch	$\frac{x^4b\ln(cx+1)}{8}-\frac{x^4b\ln(-cx+1)}{8}+\frac{x^4a}{4}+\frac{b\,x^3}{12c}+\frac{bx}{4c^3}+\frac{b\ln(-cx+1)}{8c^4}-\frac{b\ln(cx+1)}{8c^4}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/4*c^4*x^4*a+1/4*c^4*x^4*b*arctanh(c*x)+1/12*b*c^3*x^3+1/4*b*c*x+1/
8*b*ln(c*x-1)-1/8*b*ln(c*x+1))
```

Maxima [A]
time = 0.27, size = 61, normalized size = 1.27

$$\frac{1}{4}\,ax^4+\frac{1}{24}\left(6\,x^4\,\mathrm{artanh}\,(cx)+c\left(\frac{2\,(c^2x^3+3\,x)}{c^4}-\frac{3\,\log\,(cx+1)}{c^5}+\frac{3\,\log\,(cx-1)}{c^5}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x
 + 1)/c^5 + 3*log(c*x - 1)/c^5))*b
```

Fricas [A]
time = 0.34, size = 58, normalized size = 1.21

$$\frac{6\,ac^4x^4 + 2\,bc^3x^3 + 6\,bcx + 3\,(bc^4x^4 - b)\log\left(-\frac{cx+1}{cx-1}\right)}{24\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*c^4*x^4 + 2*b*c^3*x^3 + 6*b*c*x + 3*(b*c^4*x^4 - b)*log(-(c*x + 1
)/(c*x - 1)))/c^4
```

Sympy [A]
time = 0.24, size = 53, normalized size = 1.10

$$\begin{cases} \frac{ax^4}{4} + \frac{bx^4\operatorname{atanh}(cx)}{4} + \frac{bx^3}{12c} + \frac{bx}{4c^3} - \frac{b\operatorname{atanh}(cx)}{4c^4} & \text{for } c \neq 0 \\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*x**4/4 + b*x**4*atanh(c*x)/4 + b*x**3/(12*c) + b*x/(4*c**3) -
b*atanh(c*x)/(4*c**4), Ne(c, 0)), (a*x**4/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(40) = 80.
time = 0.42, size = 296, normalized size = 6.17

$$\frac{1}{3}c\left(\frac{3\left(\frac{(cx+1)^3b}{(cx-1)^3} + \frac{(cx+1)b}{cx-1}\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4c^5}{(cx-1)^4} - \frac{4\,(cx+1)^3c^5}{(cx-1)^3} + \frac{6\,(cx+1)^2c^5}{(cx-1)^2} - \frac{4\,(cx+1)c^5}{cx-1} + c^5} + \frac{\frac{6\,(cx+1)^3a}{(cx-1)^3} + \frac{6\,(cx+1)a}{cx-1} + \frac{3\,(cx+1)^3b}{(cx-1)^3} - \frac{6\,(cx+1)^2b}{(cx-1)^2} + \frac{5\,(cx+1)b}{cx-1} - 2\,b}{\frac{(cx+1)^4c^5}{(cx-1)^4} - \frac{4\,(cx+1)^3c^5}{(cx-1)^3} + \frac{6\,(cx+1)^2c^5}{(cx-1)^2} - \frac{4\,(cx+1)c^5}{cx-1} + c^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*c*(3*((c*x + 1)^3*b/(c*x - 1)^3 + (c*x + 1)*b/(c*x - 1))*log(-(c*x + 1)
/(c*x - 1))/((c*x + 1)^4*c^5/(c*x - 1)^4 - 4*(c*x + 1)^3*c^5/(c*x - 1)^3 +
6*(c*x + 1)^2*c^5/(c*x - 1)^2 - 4*(c*x + 1)*c^5/(c*x - 1) + c^5) + (6*(c*x
+ 1)^3*a/(c*x - 1)^3 + 6*(c*x + 1)*a/(c*x - 1) + 3*(c*x + 1)^3*b/(c*x - 1)^
3 - 6*(c*x + 1)^2*b/(c*x - 1)^2 + 5*(c*x + 1)*b/(c*x - 1) - 2*b)/((c*x + 1)
```

```
^4*c^5/(c*x - 1)^4 - 4*(c*x + 1)^3*c^5/(c*x - 1)^3 + 6*(c*x + 1)^2*c^5/(c*x
 - 1)^2 - 4*(c*x + 1)*c^5/(c*x - 1) + c^5))
```

Mupad [B]
time = 0.77, size = 43, normalized size = 0.90

$$\frac{a\,x^4}{4}+\frac{\frac{b\,c^3\,x^3}{12}-\frac{b\,\mathrm{atanh}(c\,x)}{4}+\frac{b\,c\,x}{4}}{c^4}+\frac{b\,x^4\,\mathrm{atanh}(c\,x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x)),x)
[Out] (a*x^4)/4 + ((b*c^3*x^3)/12 - (b*atanh(c*x))/4 + (b*c*x)/4)/c^4 + (b*x^4*at
anh(c*x))/4
```

3.4 $\int x^2 \left(a + b \tanh^{-1}(cx)\right) \, dx$

<u>**Optimal**</u>. Leaf size=46

$$\frac{bx^2}{6c} + \frac{1}{3}x^3\left(a + b\tanh^{-1}(cx)\right) + \frac{b\log\left(1 - c^2x^2\right)}{6c^3}$$

```
[Out] 1/6*b*x^2/c+1/3*x^3*(a+b*arctanh(c*x))+1/6*b*ln(-c^2*x^2+1)/c^3
```

<u>**Rubi**</u> [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6037, 272, 45}

$$\frac{1}{3}x^3\left(a + b\tanh^{-1}(cx)\right) + \frac{b\log\left(1 - c^2x^2\right)}{6c^3} + \frac{bx^2}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x^2)/(6*c) + (x^3*(a + b*ArcTanh[c*x]))/3 + (b*Log[1 - c^2*x^2])/(6*c^3)
```

<u>Rule 45</u>

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

<u>Rule 272</u>

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

<u>Rule 6037</u>

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

<u>Rubi steps</u>

$$\begin{aligned}\int x^2\left(a+b\tanh^{-1}(cx)\right)\,dx &= \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{3}(bc)\int\frac{x^3}{1-c^2x^2}\,dx\\ &= \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{6}(bc)\text{Subst}\left(\int\frac{x}{1-c^2x}\,dx,x,x^2\right)\\ &= \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{6}(bc)\text{Subst}\left(\int\left(-\frac{1}{c^2}-\frac{1}{c^2\left(-1+c^2x\right)}\right)\,dx,x,x^2\right)\\ &= \frac{bx^2}{6c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{b\log\left(1-c^2x^2\right)}{6c^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 1.11

$$\frac{bx^2}{6c}+\frac{ax^3}{3}+\frac{1}{3}bx^3\tanh^{-1}(cx)+\frac{b\log\left(1-c^2x^2\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x^2)/(6*c) + (a*x^3)/3 + (b*x^3*ArcTanh[c*x])/3 + (b*Log[1 - c^2*x^2])/(
6*c^3)
```

Maple [A]
time = 0.01, size = 55, normalized size = 1.20

method	result	size
derivativedivides	$\frac{\frac{c^3x^3a}{3}+\frac{b\,c^3x^3\,\mathrm{arctanh}(cx)}{3}+\frac{b\,c^2x^2}{6}+\frac{b\ln(cx-1)}{6}+\frac{b\ln(cx+1)}{6}}{c^3}$	55
default	$\frac{\frac{c^3x^3a}{3}+\frac{b\,c^3x^3\,\mathrm{arctanh}(cx)}{3}+\frac{b\,c^2x^2}{6}+\frac{b\ln(cx-1)}{6}+\frac{b\ln(cx+1)}{6}}{c^3}$	55
risch	$\frac{x^3b\ln(cx+1)}{6}-\frac{x^3b\ln(-cx+1)}{6}+\frac{x^3a}{3}+\frac{b\,x^2}{6c}+\frac{b\ln(c^2x^2-1)}{6c^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/3*c^3*x^3*a+1/3*b*c^3*x^3*arctanh(c*x)+1/6*b*c^2*x^2+1/6*b*ln(c*x-
1)+1/6*b*ln(c*x+1))
```

Maxima [A]
time = 0.26, size = 44, normalized size = 0.96

$$\frac{1}{3}\,ax^3+\frac{1}{6}\left(2\,x^3\,\mathrm{artanh}\left(cx\right)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b
```

Fricas [A]
time = 0.36, size = 58, normalized size = 1.26

$$\frac{bc^3x^3\log\left(-\frac{cx+1}{cx-1}\right)+2\,ac^3x^3+bc^2x^2+b\log\left(c^2x^2-1\right)}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(b*c^3*x^3*log(-(c*x + 1)/(c*x - 1)) + 2*a*c^3*x^3 + b*c^2*x^2 + b*log(
c^2*x^2 - 1))/c^3
```

Sympy [A]
time = 0.20, size = 58, normalized size = 1.26

$$\begin{cases} \frac{ax^3}{3}+\frac{bx^3\operatorname{atanh}(cx)}{3}+\frac{bx^2}{6c}+\frac{b\log\left(x-\frac{1}{c}\right)}{3c^3}+\frac{b\operatorname{atanh}(cx)}{3c^3} & \text{for}\ c\neq 0\\ \frac{ax^3}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*x**3/3 + b*x**3*atanh(c*x)/3 + b*x**2/(6*c) + b*log(x - 1/c)/(
3*c**3) + b*atanh(c*x)/(3*c**3), Ne(c, 0)), (a*x**3/3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(40) = 80.
time = 0.42, size = 258, normalized size = 5.61

$$\frac{1}{3}\,c\left(\frac{\left(\frac{3\,(cx+1)^2b}{(cx-1)^2}+b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^3c^4}{(cx-1)^3}-\frac{3\,(cx+1)^2c^4}{(cx-1)^2}+\frac{3\,(cx+1)c^4}{cx-1}-c^4}+\frac{2\left(\frac{3\,(cx+1)^2a}{(cx-1)^2}+a+\frac{(cx+1)^2b}{(cx-1)^2}-\frac{(cx+1)b}{cx-1}\right)}{\frac{(cx+1)^3c^4}{(cx-1)^3}-\frac{3\,(cx+1)^2c^4}{(cx-1)^2}+\frac{3\,(cx+1)c^4}{cx-1}-c^4}-\frac{b\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^4}+\frac{b\log\left(-\frac{cx+1}{cx-1}\right)}{c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*c*((3*(c*x + 1)^2*b/(c*x - 1)^2 + b)*log(-(c*x + 1)/(c*x - 1))/((c*x +
1)^3*c^4/(c*x - 1)^3 - 3*(c*x + 1)^2*c^4/(c*x - 1)^2 + 3*(c*x + 1)*c^4/(c*x
 - 1) - c^4) + 2*(3*(c*x + 1)^2*a/(c*x - 1)^2 + a + (c*x + 1)^2*b/(c*x - 1)
^2 - (c*x + 1)*b/(c*x - 1))/((c*x + 1)^3*c^4/(c*x - 1)^3 - 3*(c*x + 1)^2*c^
4/(c*x - 1)^2 + 3*(c*x + 1)*c^4/(c*x - 1) - c^4) - b*log(-(c*x + 1)/(c*x -
1) + 1)/c^4 + b*log(-(c*x + 1)/(c*x - 1))/c^4)
```

Mupad [B]
time = 0.74, size = 44, normalized size = 0.96

$$\frac{\frac{b\,\ln\left(c^2\,x^2-1\right)}{6}+\frac{b\,c^2\,x^2}{6}}{c^3}+\frac{a\,x^3}{3}+\frac{b\,x^3\,\mathrm{atanh}\left(c\,x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x)),x)
```

```
[Out] ((b*log(c^2*x^2 - 1))/6 + (b*c^2*x^2)/6)/c^3 + (a*x^3)/3 + (b*x^3*atanh(c*x
))/3
```

3.5 $\int x(a+b\tanh^{-1}(cx))\,dx$

Optimal. Leaf size=37

$$\frac{bx}{2c}-\frac{b\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}x^2(a+b\tanh^{-1}(cx))$$

```
[Out] 1/2*b*x/c-1/2*b*arctanh(c*x)/c^2+1/2*x^2*(a+b*arctanh(c*x))
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6037, 327, 212}

$$\frac{1}{2}x^2(a+b\tanh^{-1}(cx))-\frac{b\tanh^{-1}(cx)}{2c^2}+\frac{bx}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(2*c) - (b*ArcTanh[c*x])/(2*c^2) + (x^2*(a + b*ArcTanh[c*x]))/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int x(a+b\tanh^{-1}(cx))\,dx = \frac{1}{2}x^2(a+b\tanh^{-1}(cx)) - \frac{1}{2}(bc)\int \frac{x^2}{1-c^2x^2}\,dx$$
$$= \frac{bx}{2c} + \frac{1}{2}x^2(a+b\tanh^{-1}(cx)) - \frac{b\int \frac{1}{1-c^2x^2}\,dx}{2c}$$
$$= \frac{bx}{2c} - \frac{b\tanh^{-1}(cx)}{2c^2} + \frac{1}{2}x^2(a+b\tanh^{-1}(cx))$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 1.59

$$\frac{bx}{2c} + \frac{ax^2}{2} + \frac{1}{2}bx^2\tanh^{-1}(cx) + \frac{b\log(1-cx)}{4c^2} - \frac{b\log(1+cx)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(2*c) + (a*x^2)/2 + (b*x^2*ArcTanh[c*x])/2 + (b*Log[1 - c*x])/(4*c^2)
 - (b*Log[1 + c*x])/(4*c^2)
```

Maple [A]
time = 0.01, size = 51, normalized size = 1.38

method	result	size
derivativedivides	$\frac{\frac{a\,c^2x^2}{2}+\frac{\operatorname{arctanh}(cx)b\,c^2x^2}{2}+\frac{bcx}{2}+\frac{b\ln(cx-1)}{4}-\frac{b\ln(cx+1)}{4}}{c^2}$	51
default	$\frac{\frac{a\,c^2x^2}{2}+\frac{\operatorname{arctanh}(cx)b\,c^2x^2}{2}+\frac{bcx}{2}+\frac{b\ln(cx-1)}{4}-\frac{b\ln(cx+1)}{4}}{c^2}$	51
risch	$\frac{b\,x^2\ln(cx+1)}{4}-\frac{b\,x^2\ln(-cx+1)}{4}+\frac{a\,x^2}{2}+\frac{bx}{2c}+\frac{b\ln(-cx+1)}{4c^2}-\frac{b\ln(cx+1)}{4c^2}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/2*a*c^2*x^2+1/2*arctanh(c*x)*b*c^2*x^2+1/2*b*c*x+1/4*b*ln(c*x-1)-1
/4*b*ln(c*x+1))
```

Maxima [A]
time = 0.26, size = 50, normalized size = 1.35

$$\frac{1}{2}\,ax^2 + \frac{1}{4}\left(2\,x^2\,\operatorname{artanh}(cx) + c\left(\frac{2\,x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c
*x - 1)/c^3))*b
```

Fricas [A]
time = 0.34, size = 48, normalized size = 1.30

$$\frac{2\,ac^2x^2 + 2\,bcx + (bc^2x^2 - b)\log\left(-\frac{cx+1}{cx-1}\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c^2*x^2 + 2*b*c*x + (b*c^2*x^2 - b)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

Sympy [A]
time = 0.18, size = 42, normalized size = 1.14

$$\begin{cases} \frac{ax^2}{2} + \frac{bx^2\operatorname{atanh}(cx)}{2} + \frac{bx}{2c} - \frac{b\operatorname{atanh}(cx)}{2c^2} & \text{for } c \neq 0 \\ \frac{ax^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*x**2/2 + b*x**2*atanh(c*x)/2 + b*x/(2*c) - b*atanh(c*x)/(2*c**
2), Ne(c, 0)), (a*x**2/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(31) = 62.
time = 0.41, size = 148, normalized size = 4.00

$$c\left(\frac{(cx+1)b\log\left(-\frac{cx+1}{cx-1}\right)}{\left(\frac{(cx+1)^2c^3}{(cx-1)^2} - \frac{2\,(cx+1)c^3}{cx-1} + c^3\right)(cx-1)} + \frac{\frac{2\,(cx+1)a}{cx-1} + \frac{(cx+1)b}{cx-1} - b}{\frac{(cx+1)^2c^3}{(cx-1)^2} - \frac{2\,(cx+1)c^3}{cx-1} + c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] c*((c*x + 1)*b*log(-(c*x + 1)/(c*x - 1))/(((c*x + 1)^2*c^3/(c*x - 1)^2 - 2*
(c*x + 1)*c^3/(c*x - 1) + c^3)*(c*x - 1)) + (2*(c*x + 1)*a/(c*x - 1) + (c*x
 + 1)*b/(c*x - 1) - b)/((c*x + 1)^2*c^3/(c*x - 1)^2 - 2*(c*x + 1)*c^3/(c*x
- 1) + c^3))
```

Mupad [B]
time = 0.73, size = 35, normalized size = 0.95

$$\frac{a\,x^2}{2} - \frac{\frac{b\,\mathrm{atanh}(c\,x)}{2} - \frac{b\,c\,x}{2}}{c^2} + \frac{b\,x^2\,\mathrm{atanh}(c\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x)),x)
[Out] (a*x^2)/2 - ((b*atanh(c*x))/2 - (b*c*x)/2)/c^2 + (b*x^2*atanh(c*x))/2
```

3.6 $\int \left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=30

$$ax+bx\tanh^{-1}(cx)+\frac{b\log\left(1-c^2x^2\right)}{2c}$$

```
[Out] a*x+b*x*arctanh(c*x)+1/2*b*ln(-c^2*x^2+1)/c
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6021, 266}

$$ax+\frac{b\log\left(1-c^2x^2\right)}{2c}+bx\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c*x],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x] + (b*Log[1 - c^2*x^2])/(2*c)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tanh^{-1}(cx)\right)\,dx&=ax+b\int \tanh^{-1}(cx)\,dx\\&=ax+bx\tanh^{-1}(cx)-(bc)\int \frac{x}{1-c^2x^2}\,dx\\&=ax+bx\tanh^{-1}(cx)+\frac{b\log\left(1-c^2x^2\right)}{2c}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$ax + bx\tanh^{-1}(cx) + \frac{b\log(1 - c^2x^2)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c*x],x]
[Out] a*x + b*x*ArcTanh[c*x] + (b*Log[1 - c^2*x^2])/(2*c)
```

Maple [A]
time = 0.01, size = 29, normalized size = 0.97

method	result	size
default	$ax + bx\,\mathrm{arctanh}(cx) + \frac{b\ln(-c^2x^2+1)}{2c}$	29
derivativedivides	$\frac{cxa+bcx\,\mathrm{arctanh}(cx)+\frac{b\ln(-c^2x^2+1)}{2}}{c}$	32
risch	$ax + \frac{bx\ln(cx+1)}{2} - \frac{bx\ln(-cx+1)}{2} + \frac{b\ln(c^2x^2-1)}{2c}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x),x,method=_RETURNVERBOSE)
[Out] a*x+b*x*arctanh(c*x)+1/2*b*ln(-c^2*x^2+1)/c
```

Maxima [A]
time = 0.26, size = 30, normalized size = 1.00

$$ax + \frac{(2\,cx\,\mathrm{artanh}(cx) + \log(-c^2x^2 + 1))b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x),x, algorithm="maxima")
[Out] a*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b/c
```

Fricas [A]
time = 0.34, size = 42, normalized size = 1.40

$$\frac{bcx\log\left(-\frac{cx+1}{cx-1}\right) + 2\,acx + b\log(c^2x^2 - 1)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x),x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x*log(-(c*x + 1)/(c*x - 1)) + 2*a*c*x + b*log(c^2*x^2 - 1))/c
```

Sympy [A]
time = 0.12, size = 27, normalized size = 0.90

$$ax + b\left(\begin{cases} x\operatorname{atanh}(cx) + \frac{\log(cx+1)}{c} - \frac{\operatorname{atanh}(cx)}{c} & \text{for } c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x),x)
```

```
[Out] a*x + b*Piecewise((x*atanh(c*x) + log(c*x + 1)/c - atanh(c*x)/c, Ne(c, 0)),
 (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(28) = 56.
time = 0.41, size = 156, normalized size = 5.20

$$bc\left(\frac{\log\left(\frac{|-cx-1|}{|cx-1|}\right)}{c^2} - \frac{\log\left(\left|-\frac{cx+1}{cx-1}+1\right|\right)}{c^2} + \frac{\log\left(-\frac{\frac{c\left(\frac{cx+1}{cx-1}+1\right)}{\frac{(cx+1)c}{cx-1}-c}+1}{\frac{c\left(\frac{cx+1}{cx-1}+1\right)}{\frac{(cx+1)c}{cx-1}-c}-1}\right)}{c^2\left(\frac{cx+1}{cx-1}-1\right)}\right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x),x, algorithm="giac")
```

```
[Out] b*c*(log(abs(-c*x - 1)/abs(c*x - 1))/c^2 - log(abs(-(c*x + 1)/(c*x - 1) + 1
))/c^2 + log(-(c*((c*x + 1)/(c*x - 1) + 1)/((c*x + 1)*c/(c*x - 1) - c) + 1)
/(c*((c*x + 1)/(c*x - 1) + 1)/((c*x + 1)*c/(c*x - 1) - c) - 1))/(c^2*((c*x
+ 1)/(c*x - 1) - 1))) + a*x
```

Mupad [B]
time = 0.68, size = 27, normalized size = 0.90

$$a\,x + \frac{b\,\ln\left(c^2\,x^2-1\right)}{2\,c} + b\,x\,\mathrm{atanh}(c\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atanh(c*x),x)
```

```
[Out] a*x + (b*log(c^2*x^2 - 1))/(2*c) + b*x*atanh(c*x)
```

3.7 $\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx$

Optimal. Leaf size=26

$$a\log(x)-\frac{1}{2}b\text{PolyLog}(2,-cx)+\frac{1}{2}b\text{PolyLog}(2,cx)$$

```
[Out] a*ln(x)-1/2*b*polylog(2,-c*x)+1/2*b*polylog(2,c*x)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {6031}

$$a\log(x)-\frac{1}{2}b\text{Li}_2(-cx)+\frac{1}{2}b\text{Li}_2(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/x,x]
```

```
[Out] a*Log[x] - (b*PolyLog[2, -(c*x)])/2 + (b*PolyLog[2, c*x])/2
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int\frac{a+b\tanh^{-1}(cx)}{x}\,dx=a\log(x)-\frac{1}{2}b\text{Li}_2(-cx)+\frac{1}{2}b\text{Li}_2(cx)$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.92

$$a\log(x)+\frac{1}{2}b(-\text{PolyLog}(2,-cx)+\text{PolyLog}(2,cx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x,x]
```

```
[Out] a*Log[x] + (b*(-PolyLog[2, -(c*x)] + PolyLog[2, c*x]))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.
time = 0.02, size = 47, normalized size = 1.81

method	result	size
risch	$a\ln\left(-cx\right)+\frac{b\,\mathrm{dilog}\left(-cx+1\right)}{2}-\frac{b\,\mathrm{dilog}\left(cx+1\right)}{2}$	28
derivativedivides	$a\ln\left(cx\right)+b\ln\left(cx\right)\mathrm{arctanh}\left(cx\right)-\frac{b\,\mathrm{dilog}\left(cx\right)}{2}-\frac{b\,\mathrm{dilog}\left(cx+1\right)}{2}-\frac{b\ln\left(cx\right)\ln\left(cx+1\right)}{2}$	47
default	$a\ln\left(cx\right)+b\ln\left(cx\right)\mathrm{arctanh}\left(cx\right)-\frac{b\,\mathrm{dilog}\left(cx\right)}{2}-\frac{b\,\mathrm{dilog}\left(cx+1\right)}{2}-\frac{b\ln\left(cx\right)\ln\left(cx+1\right)}{2}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(c*x)+b*ln(c*x)*arctanh(c*x)-1/2*b*dilog(c*x)-1/2*b*dilog(c*x+1)-1/2*b*
ln(c*x)*ln(c*x+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{atanh}\left(cx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x,x)
```

```
[Out] Integral((a + b*atanh(c*x))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{atanh}(c\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/x,x)
[Out] int((a + b*atanh(c*x))/x, x)
```

3.8 $\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx$

Optimal. Leaf size=36

$$-\frac{a+b\tanh^{-1}(cx)}{x}+bc\log(x)-\frac{1}{2}bc\log\left(1-c^2x^2\right)$$

```
[Out] (-a-b*arctanh(c*x))/x+b*c*ln(x)-1/2*b*c*ln(-c^2*x^2+1)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6037, 272, 36, 29, 31}

$$-\frac{a+b\tanh^{-1}(cx)}{x}-\frac{1}{2}bc\log\left(1-c^2x^2\right)+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/x^2,x]
```

```
[Out] -((a + b*ArcTanh[c*x])/x) + b*c*Log[x] - (b*c*Log[1 - c^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx &= -\frac{a+b\tanh^{-1}(cx)}{x}+(bc)\int \frac{1}{x\left(1-c^2x^2\right)}\,dx\\ &= -\frac{a+b\tanh^{-1}(cx)}{x}+\frac{1}{2}(bc)\mathrm{Subst}\left(\int \frac{1}{x\left(1-c^2x\right)}\,dx,x,x^2\right)\\ &= -\frac{a+b\tanh^{-1}(cx)}{x}+\frac{1}{2}(bc)\mathrm{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)+\frac{1}{2}\left(bc^3\right)\mathrm{Subst}\left(\int \frac{1}{1-c^2x}\,dx,\right.\\ &= -\frac{a+b\tanh^{-1}(cx)}{x}+bc\log(x)-\frac{1}{2}bc\log\left(1-c^2x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.08

$$-\frac{a}{x}-\frac{b\tanh^{-1}(cx)}{x}+bc\log(x)-\frac{1}{2}bc\log\left(1-c^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x^2,x]
[Out] -(a/x) - (b*ArcTanh[c*x])/x + b*c*Log[x] - (b*c*Log[1 - c^2*x^2])/2
```

Maple [A]
time = 0.02, size = 50, normalized size = 1.39

method	result	size
derivativedivides	$c\left(-\frac{a}{cx}-\frac{b\,\mathrm{arctanh}(cx)}{cx}-\frac{b\ln(cx-1)}{2}+b\ln(cx)-\frac{b\ln(cx+1)}{2}\right)$	50
default	$c\left(-\frac{a}{cx}-\frac{b\,\mathrm{arctanh}(cx)}{cx}-\frac{b\ln(cx-1)}{2}+b\ln(cx)-\frac{b\ln(cx+1)}{2}\right)$	50
risch	$-\frac{b\ln(cx+1)}{2x}+\frac{2bc\ln(x)x-bc\ln(c^2x^2-1)x+b\ln(-cx+1)-2a}{2x}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^2,x,method=_RETURNVERBOSE)
[Out] c*(-a/c/x-b/c/x*arctanh(c*x)-1/2*b*ln(c*x-1)+b*ln(c*x)-1/2*b*ln(c*x+1))
```

Maxima [A]
time = 0.26, size = 39, normalized size = 1.08

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\text{artanh}\left(cx\right)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b - a/x
```

Fricas [A]
time = 0.40, size = 47, normalized size = 1.31

$$-\frac{bcx\log\left(c^2x^2-1\right)-2\,bcx\log\left(x\right)+b\log\left(-\frac{cx+1}{cx-1}\right)+2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*c*x*log(c^2*x^2 - 1) - 2*b*c*x*log(x) + b*log(-(c*x + 1)/(c*x - 1))
 + 2*a)/x
```

Sympy [A]
time = 0.31, size = 41, normalized size = 1.14

$$\begin{cases} -\frac{a}{x}+bc\log\left(x\right)-bc\log\left(x-\frac{1}{c}\right)-bc\,\text{atanh}\left(cx\right)-\frac{b\,\text{atanh}\left(cx\right)}{x} & \text{for}\ c\neq 0\\ -\frac{a}{x} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**2,x)
```

```
[Out] Piecewise((-a/x + b*c*log(x) - b*c*log(x - 1/c) - b*c*atanh(c*x) - b*atanh(
c*x)/x, Ne(c, 0)), (-a/x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(34) = 68.
time = 0.41, size = 94, normalized size = 2.61

$$\left(b\log\left(-\frac{cx+1}{cx-1}-1\right)-b\log\left(-\frac{cx+1}{cx-1}\right)+\frac{b\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{cx+1}{cx-1}+1}+\frac{2\,a}{\frac{cx+1}{cx-1}+1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] (b*log(-(c*x + 1)/(c*x - 1) - 1) - b*log(-(c*x + 1)/(c*x - 1)) + b*log(-(c*
x + 1)/(c*x - 1))/((c*x + 1)/(c*x - 1) + 1) + 2*a/((c*x + 1)/(c*x - 1) + 1)
)*c
```

Mupad [B]
time = 0.70, size = 33, normalized size = 0.92

$$b\,c\,\ln\left(x\right)-\frac{a+b\,\mathrm{atanh}(c\,x)}{x}-\frac{b\,c\,\ln\left(c^2\,x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/x^2,x)
```

```
[Out] b*c*log(x) - (a + b*atanh(c*x))/x - (b*c*log(c^2*x^2 - 1))/2
```

3.9 $\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx$

Optimal. Leaf size=37

$$-\frac{bc}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)-\frac{a+b\tanh^{-1}(cx)}{2x^2}$$

```
[Out] -1/2*b*c/x+1/2*b*c^2*arctanh(c*x)+1/2*(-a-b*arctanh(c*x))/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6037, 331, 212}

$$-\frac{a+b\tanh^{-1}(cx)}{2x^2}+\frac{1}{2}bc^2\tanh^{-1}(cx)-\frac{bc}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/x^3,x]
```

```
[Out] -1/2*(b*c)/x + (b*c^2*ArcTanh[c*x])/2 - (a + b*ArcTanh[c*x])/(2*x^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx = -\frac{a+b\tanh^{-1}(cx)}{2x^2} + \frac{1}{2}(bc)\int \frac{1}{x^2(1-c^2x^2)}\,dx$$
$$= -\frac{bc}{2x} - \frac{a+b\tanh^{-1}(cx)}{2x^2} + \frac{1}{2}(bc^3)\int \frac{1}{1-c^2x^2}\,dx$$
$$= -\frac{bc}{2x} + \frac{1}{2}bc^2\tanh^{-1}(cx) - \frac{a+b\tanh^{-1}(cx)}{2x^2}$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 1.59

$$-\frac{a}{2x^2} - \frac{bc}{2x} - \frac{b\tanh^{-1}(cx)}{2x^2} - \frac{1}{4}bc^2\log(1-cx) + \frac{1}{4}bc^2\log(1+cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*c)/(2*x) - (b*ArcTanh[c*x])/(2*x^2) - (b*c^2*Log[1 - c*x])/
4 + (b*c^2*Log[1 + c*x])/4
```

Maple [A]
time = 0.02, size = 55, normalized size = 1.49

method	result	size
derivativedivides	$c^2\left(-\frac{a}{2c^2x^2} - \frac{b\,\mathrm{arctanh}(cx)}{2c^2x^2} - \frac{b\ln(cx-1)}{4} + \frac{b\ln(cx+1)}{4} - \frac{b}{2cx}\right)$	55
default	$c^2\left(-\frac{a}{2c^2x^2} - \frac{b\,\mathrm{arctanh}(cx)}{2c^2x^2} - \frac{b\ln(cx-1)}{4} + \frac{b\ln(cx+1)}{4} - \frac{b}{2cx}\right)$	55
risch	$-\frac{b\ln(cx+1)}{4x^2} - \frac{b\,x^2\ln(-cx+1)c^2 - b\,c^2\ln(-cx-1)x^2 + 2bcx - b\ln(-cx+1) + 2a}{4x^2}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a/c^2/x^2-1/2*b/c^2/x^2*arctanh(c*x)-1/4*b*ln(c*x-1)+1/4*b*ln(c*x
+1)-1/2*b/c/x)
```

Maxima [A]
time = 0.26, size = 45, normalized size = 1.22

$$\frac{1}{4}\left(\left(c\log(cx+1) - c\log(cx-1) - \frac{2}{x}\right)c - \frac{2\,\mathrm{artanh}(cx)}{x^2}\right)b - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b - 1/
2*a/x^2
```

Fricas [A]
time = 0.35, size = 43, normalized size = 1.16

$$-\frac{2\,bcx-\left(bc^2x^2-b\right)\log\left(-\frac{cx+1}{cx-1}\right)+2\,a}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*c*x - (b*c^2*x^2 - b)*log(-(c*x + 1)/(c*x - 1)) + 2*a)/x^2
```

Sympy [A]
time = 0.26, size = 36, normalized size = 0.97

$$-\frac{a}{2x^2}+\frac{bc^2\,\mathrm{atanh}\,(cx)}{2}-\frac{bc}{2x}-\frac{b\,\mathrm{atanh}\,(cx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**3,x)
```

```
[Out] -a/(2*x**2) + b*c**2*atanh(c*x)/2 - b*c/(2*x) - b*atanh(c*x)/(2*x**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(31) = 62.
time = 0.41, size = 135, normalized size = 3.65

$$\left(\frac{(cx+1)bc\log\left(-\frac{cx+1}{cx-1}\right)}{(cx-1)\left(\frac{(cx+1)^2}{(cx-1)^2}+\frac{2\,(cx+1)}{cx-1}+1\right)}+\frac{\frac{2\,(cx+1)ac}{cx-1}+\frac{(cx+1)bc}{cx-1}+bc}{\frac{(cx+1)^2}{(cx-1)^2}+\frac{2\,(cx+1)}{cx-1}+1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] ((c*x + 1)*b*c*log(-(c*x + 1)/(c*x - 1))/((c*x - 1)*((c*x + 1)^2/(c*x - 1)^
2 + 2*(c*x + 1)/(c*x - 1) + 1)) + (2*(c*x + 1)*a*c/(c*x - 1) + (c*x + 1)*b*
c/(c*x - 1) + b*c)/((c*x + 1)^2/(c*x - 1)^2 + 2*(c*x + 1)/(c*x - 1) + 1))*c
```

Mupad [B]
time = 0.73, size = 46, normalized size = 1.24

$$\frac{b\,c\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{-c^2}}\right)\,\sqrt{-c^2}}{2}-\frac{\frac{a}{2}+\frac{b\,\mathrm{atanh}(c\,x)}{2}+\frac{b\,c\,x}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/x^3,x)
```

```
[Out] (b*c*atan((c^2*x)/(-c^2)^(1/2))*(-c^2)^(1/2))/2 - (a/2 + (b*atanh(c*x))/2 +
 (b*c*x)/2)/x^2
```

3.10 $\int \frac{a+b\tanh^{-1}(cx)}{x^4}\,dx$

Optimal. Leaf size=54

$$-\frac{bc}{6x^2}-\frac{a+b\tanh^{-1}(cx)}{3x^3}+\frac{1}{3}bc^3\log(x)-\frac{1}{6}bc^3\log\left(1-c^2x^2\right)$$

```
[Out] -1/6*b*c/x^2+1/3*(-a-b*arctanh(c*x))/x^3+1/3*b*c^3*ln(x)-1/6*b*c^3*ln(-c^2*
x^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6037, 272, 46}

$$-\frac{a+b\tanh^{-1}(cx)}{3x^3}+\frac{1}{3}bc^3\log(x)-\frac{1}{6}bc^3\log\left(1-c^2x^2\right)-\frac{bc}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/x^4,x]
```

```
[Out] -1/6*(b*c)/x^2 - (a + b*ArcTanh[c*x])/(3*x^3) + (b*c^3*Log[x])/3 - (b*c^3*L
og[1 - c^2*x^2])/6
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^4}\,dx = -\frac{a+b\tanh^{-1}(cx)}{3x^3}+\frac{1}{3}(bc)\int \frac{1}{x^3(1-c^2x^2)}\,dx$$
$$= -\frac{a+b\tanh^{-1}(cx)}{3x^3}+\frac{1}{6}(bc)\text{Subst}\left(\int \frac{1}{x^2(1-c^2x)}\,dx,x,x^2\right)$$
$$= -\frac{a+b\tanh^{-1}(cx)}{3x^3}+\frac{1}{6}(bc)\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{c^2}{x}-\frac{c^4}{-1+c^2x}\right)\,dx,x,x^2\right)$$
$$= -\frac{bc}{6x^2}-\frac{a+b\tanh^{-1}(cx)}{3x^3}+\frac{1}{3}bc^3\log(x)-\frac{1}{6}bc^3\log\left(1-c^2x^2\right)$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 1.09

$$-\frac{a}{3x^3}-\frac{bc}{6x^2}-\frac{b\tanh^{-1}(cx)}{3x^3}+\frac{1}{3}bc^3\log(x)-\frac{1}{6}bc^3\log\left(1-c^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*c)/(6*x^2) - (b*ArcTanh[c*x])/(3*x^3) + (b*c^3*Log[x])/3 - (b*c^3*Log[1 - c^2*x^2])/6
```

Maple [A]
time = 0.02, size = 62, normalized size = 1.15

method	result	size
derivativedivides	$c^3\left(-\frac{a}{3c^3x^3}-\frac{b\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{b\ln(cx-1)}{6}-\frac{b}{6c^2x^2}+\frac{b\ln(cx)}{3}-\frac{b\ln(cx+1)}{6}\right)$	62
default	$c^3\left(-\frac{a}{3c^3x^3}-\frac{b\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{b\ln(cx-1)}{6}-\frac{b}{6c^2x^2}+\frac{b\ln(cx)}{3}-\frac{b\ln(cx+1)}{6}\right)$	62
risch	$-\frac{b\ln(cx+1)}{6x^3}+\frac{2b\,c^3\ln(x)x^3-b\,c^3\ln(c^2x^2-1)x^3-bcx+b\ln(-cx+1)-2a}{6x^3}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/3*a/c^3/x^3-1/3*b/c^3/x^3*arctanh(c*x)-1/6*b*ln(c*x-1)-1/6*b/c^2/x^2+1/3*b*ln(c*x)-1/6*b*ln(c*x+1))
```

Maxima [A]
time = 0.26, size = 49, normalized size = 0.91

$$-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\mathrm{artanh}\left(cx\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b - 1/3*a/x^3
```

Fricas [A]
time = 0.37, size = 59, normalized size = 1.09

$$-\frac{bc^3x^3\log\left(c^2x^2-1\right)-2\,bc^3x^3\log\left(x\right)+bcx+b\log\left(-\frac{cx+1}{cx-1}\right)+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*c^3*x^3*log(c^2*x^2 - 1) - 2*b*c^3*x^3*log(x) + b*c*x + b*log(-(c*x
 + 1)/(c*x - 1)) + 2*a)/x^3
```

Sympy [A]
time = 0.41, size = 70, normalized size = 1.30

$$\begin{cases} -\frac{a}{3x^3}+\frac{bc^3\log\left(x\right)}{3}-\frac{bc^3\log\left(x-\frac{1}{c}\right)}{3}-\frac{bc^3\operatorname{atanh}\left(cx\right)}{3}-\frac{bc}{6x^2}-\frac{b\operatorname{atanh}\left(cx\right)}{3x^3} & \text{for}\ c\neq 0\\ -\frac{a}{3x^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3) + b*c**3*log(x)/3 - b*c**3*log(x - 1/c)/3 - b*c**3*a
tanh(c*x)/3 - b*c/(6*x**2) - b*atanh(c*x)/(3*x**3), Ne(c, 0)), (-a/(3*x**3)
, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(46) = 92.
time = 0.44, size = 251, normalized size = 4.65

$$\frac{1}{3}\left(bc^2\log\left(-\frac{cx+1}{cx-1}-1\right)-bc^2\log\left(-\frac{cx+1}{cx-1}\right)+\frac{\left(\frac{3\,(cx+1)^2bc^2}{(cx-1)^2}+bc^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^3}{(cx-1)^3}+\frac{3\,(cx+1)^2}{(cx-1)^2}+\frac{3\,(cx+1)}{cx-1}+1}+\frac{2\left(\frac{3\,(cx+1)^2ac^2}{(cx-1)^2}+ac^2+\frac{(cx+1)^2bc^2}{(cx-1)^2}+\frac{(cx+1)bc^2}{cx-1}\right)}{\frac{(cx+1)^3}{(cx-1)^3}+\frac{3\,(cx+1)^2}{(cx-1)^2}+\frac{3\,(cx+1)}{cx-1}+1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(b*c^2*log(-(c*x + 1)/(c*x - 1) - 1) - b*c^2*log(-(c*x + 1)/(c*x - 1))
+ (3*(c*x + 1)^2*b*c^2/(c*x - 1)^2 + b*c^2)*log(-(c*x + 1)/(c*x - 1))/((c*x
 + 1)^3/(c*x - 1)^3 + 3*(c*x + 1)^2/(c*x - 1)^2 + 3*(c*x + 1)/(c*x - 1) + 1
) + 2*(3*(c*x + 1)^2*a*c^2/(c*x - 1)^2 + a*c^2 + (c*x + 1)^2*b*c^2/(c*x - 1
```

```
)^2 + (c*x + 1)*b*c^2/(c*x - 1))/((c*x + 1)^3/(c*x - 1)^3 + 3*(c*x + 1)^2/(
c*x - 1)^2 + 3*(c*x + 1)/(c*x - 1) + 1))*c
```

Mupad [B]
time = 0.73, size = 46, normalized size = 0.85

$$\frac{b\,c^3\,\ln\left(x\right)}{3}-\frac{b\,c^3\,\ln\left(c^2\,x^2-1\right)}{6}-\frac{\frac{a}{3}+\frac{b\,\mathrm{atanh}\left(c\,x\right)}{3}+\frac{b\,c\,x}{6}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/x^4,x)
```

```
[Out] (b*c^3*log(x))/3 - (b*c^3*log(c^2*x^2 - 1))/6 - (a/3 + (b*atanh(c*x))/3 + (
b*c*x)/6)/x^3
```

3.11 $\int \frac{a+b\tanh^{-1}(cx)}{x^5}\,dx$

Optimal. Leaf size=48

$$-\frac{bc}{12x^3}-\frac{bc^3}{4x}+\frac{1}{4}bc^4\tanh^{-1}(cx)-\frac{a+b\tanh^{-1}(cx)}{4x^4}$$

```
[Out] -1/12*b*c/x^3-1/4*b*c^3/x+1/4*b*c^4*arctanh(c*x)+1/4*(-a-b*arctanh(c*x))/x^
4
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6037, 331, 212}

$$-\frac{a+b\tanh^{-1}(cx)}{4x^4}+\frac{1}{4}bc^4\tanh^{-1}(cx)-\frac{bc^3}{4x}-\frac{bc}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/x^5,x]
```

```
[Out] -1/12*(b*c)/x^3 - (b*c^3)/(4*x) + (b*c^4*ArcTanh[c*x])/4 - (a + b*ArcTanh[c
*x])/(4*x^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx)}{x^5}\,dx &= -\frac{a+b\tanh^{-1}(cx)}{4x^4}+\frac{1}{4}(bc)\int \frac{1}{x^4\left(1-c^2x^2\right)}\,dx\\ &= -\frac{bc}{12x^3}-\frac{a+b\tanh^{-1}(cx)}{4x^4}+\frac{1}{4}\left(bc^3\right)\int \frac{1}{x^2\left(1-c^2x^2\right)}\,dx\\ &= -\frac{bc}{12x^3}-\frac{bc^3}{4x}-\frac{a+b\tanh^{-1}(cx)}{4x^4}+\frac{1}{4}\left(bc^5\right)\int \frac{1}{1-c^2x^2}\,dx\\ &= -\frac{bc}{12x^3}-\frac{bc^3}{4x}+\frac{1}{4}bc^4\tanh^{-1}(cx)-\frac{a+b\tanh^{-1}(cx)}{4x^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 70, normalized size = 1.46

$$-\frac{a}{4x^4}-\frac{bc}{12x^3}-\frac{bc^3}{4x}-\frac{b\tanh^{-1}(cx)}{4x^4}-\frac{1}{8}bc^4\log(1-cx)+\frac{1}{8}bc^4\log(1+cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x^5,x]
[Out] -1/4*a/x^4 - (b*c)/(12*x^3) - (b*c^3)/(4*x) - (b*ArcTanh[c*x])/(4*x^4) - (b
*c^4*Log[1 - c*x])/8 + (b*c^4*Log[1 + c*x])/8
```

Maple [A]
time = 0.02, size = 64, normalized size = 1.33

method	result	size
derivativedivides	$c^4\left(-\frac{a}{4c^4x^4}-\frac{b\,\mathrm{arctanh}(cx)}{4c^4x^4}-\frac{b\ln(cx-1)}{8}-\frac{b}{12c^3x^3}-\frac{b}{4cx}+\frac{b\ln(cx+1)}{8}\right)$	64
default	$c^4\left(-\frac{a}{4c^4x^4}-\frac{b\,\mathrm{arctanh}(cx)}{4c^4x^4}-\frac{b\ln(cx-1)}{8}-\frac{b}{12c^3x^3}-\frac{b}{4cx}+\frac{b\ln(cx+1)}{8}\right)$	64
risch	$-\frac{b\ln(cx+1)}{8x^4}-\frac{3x^4b\ln(-cx+1)c^4-3b\,c^4\ln(-cx-1)x^4+6b\,c^3x^3+2bcx-3b\ln(-cx+1)+6a}{24x^4}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^5,x,method=_RETURNVERBOSE)
[Out] c^4*(-1/4*a/c^4/x^4-1/4*b/c^4/x^4*arctanh(c*x)-1/8*b*ln(c*x-1)-1/12*b/c^3/x
^3-1/4*b/c/x+1/8*b*ln(c*x+1))
```

Maxima [A]
time = 0.28, size = 60, normalized size = 1.25

$$\frac{1}{24}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c-\frac{6\ \mathrm{artanh}\left(cx\right)}{x^4}\right)b-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c -
 6*arctanh(c*x)/x^4)*b - 1/4*a/x^4
```

Fricas [A]
time = 0.35, size = 52, normalized size = 1.08

$$-\frac{6\,bc^3x^3+2\,bcx-3\,(bc^4x^4-b)\log\left(-\frac{cx+1}{cx-1}\right)+6\,a}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(6*b*c^3*x^3 + 2*b*c*x - 3*(b*c^4*x^4 - b)*log(-(c*x + 1)/(c*x - 1))
+ 6*a)/x^4
```

Sympy [A]
time = 0.31, size = 46, normalized size = 0.96

$$-\frac{a}{4x^4}+\frac{bc^4\operatorname{atanh}(cx)}{4}-\frac{bc^3}{4x}-\frac{bc}{12x^3}-\frac{b\operatorname{atanh}(cx)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**5,x)
```

```
[Out] -a/(4*x**4) + b*c**4*atanh(c*x)/4 - b*c**3/(4*x) - b*c/(12*x**3) - b*atanh(
c*x)/(4*x**4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(40) = 80.
time = 0.42, size = 292, normalized size = 6.08

$$\frac{1}{3}c\left(\frac{3\left(\frac{(cx+1)^3bc^3}{(cx-1)^3}+\frac{(cx+1)bc^3}{cx-1}\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4}{(cx-1)^4}+\frac{4\,(cx+1)^3}{(cx-1)^3}+\frac{6\,(cx+1)^2}{(cx-1)^2}+\frac{4\,(cx+1)}{cx-1}+1}+\frac{\frac{6\,(cx+1)^3ac^3}{(cx-1)^3}+\frac{6\,(cx+1)ac^3}{cx-1}+\frac{3\,(cx+1)^3bc^3}{(cx-1)^3}+\frac{6\,(cx+1)^2bc^3}{(cx-1)^2}+\frac{5\,(cx+1)bc^3}{cx-1}+2\,bc^3}{\frac{(cx+1)^4}{(cx-1)^4}+\frac{4\,(cx+1)^3}{(cx-1)^3}+\frac{6\,(cx+1)^2}{(cx-1)^2}+\frac{4\,(cx+1)}{cx-1}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] 1/3*c*(3*((c*x + 1)^3*b*c^3/(c*x - 1)^3 + (c*x + 1)*b*c^3/(c*x - 1))*log(-(
c*x + 1)/(c*x - 1))/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 +
6*(c*x + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1) + (6*(c*x + 1)^3*a*c
^3/(c*x - 1)^3 + 6*(c*x + 1)*a*c^3/(c*x - 1) + 3*(c*x + 1)^3*b*c^3/(c*x - 1
)^3 + 6*(c*x + 1)^2*b*c^3/(c*x - 1)^2 + 5*(c*x + 1)*b*c^3/(c*x - 1) + 2*b*c
```

```
^3)/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x + 1)^2/(c
*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1))
```

Mupad [B]
time = 1.05, size = 59, normalized size = 1.23

$$\frac{b\,\ln\left(1-c\,x\right)}{8\,x^4}-\frac{b\,\ln\left(c\,x+1\right)}{8\,x^4}-\frac{b\,c^3\,x^3+\frac{b\,c\,x}{3}+a}{4\,x^4}-\frac{b\,c^4\,\mathrm{atan}(c\,x\,1\mathrm{i})\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/x^5,x)
```

```
[Out] (b*log(1 - c*x))/(8*x^4) - (b*c^4*atan(c*x*1i)*1i)/4 - (b*log(c*x + 1))/(8*
x^4) - (a + b*c^3*x^3 + (b*c*x)/3)/(4*x^4)
```

3.12 $\int \frac{a+b\tanh^{-1}(cx)}{x^6}\,dx$

Optimal. Leaf size=65

$$-\frac{bc}{20x^4}-\frac{bc^3}{10x^2}-\frac{a+b\tanh^{-1}(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{1}{10}bc^5\log\left(1-c^2x^2\right)$$

```
[Out] -1/20*b*c/x^4-1/10*b*c^3/x^2+1/5*(-a-b*arctanh(c*x))/x^5+1/5*b*c^5*ln(x)-1/
10*b*c^5*ln(-c^2*x^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6037, 272, 46}

$$-\frac{a+b\tanh^{-1}(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{bc^3}{10x^2}-\frac{1}{10}bc^5\log\left(1-c^2x^2\right)-\frac{bc}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/x^6,x]
```

```
[Out] -1/20*(b*c)/x^4 - (b*c^3)/(10*x^2) - (a + b*ArcTanh[c*x])/(5*x^5) + (b*c^5*
Log[x])/5 - (b*c^5*Log[1 - c^2*x^2])/10
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^6}\,dx = -\frac{a+b\tanh^{-1}(cx)}{5x^5}+\frac{1}{5}(bc)\int \frac{1}{x^5(1-c^2x^2)}\,dx$$
$$= -\frac{a+b\tanh^{-1}(cx)}{5x^5}+\frac{1}{10}(bc)\text{Subst}\left(\int \frac{1}{x^3(1-c^2x)}\,dx,x,x^2\right)$$
$$= -\frac{a+b\tanh^{-1}(cx)}{5x^5}+\frac{1}{10}(bc)\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{c^2}{x^2}+\frac{c^4}{x}-\frac{c^6}{-1+c^2x}\right)\,dx,x,x^2\right)$$
$$= -\frac{bc}{20x^4}-\frac{bc^3}{10x^2}-\frac{a+b\tanh^{-1}(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{1}{10}bc^5\log\left(1-c^2x^2\right)$$

Mathematica [A]
time = 0.01, size = 70, normalized size = 1.08

$$-\frac{a}{5x^5}-\frac{bc}{20x^4}-\frac{bc^3}{10x^2}-\frac{b\tanh^{-1}(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{1}{10}bc^5\log\left(1-c^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x^6,x]
```

```
[Out] -1/5*a/x^5 - (b*c)/(20*x^4) - (b*c^3)/(10*x^2) - (b*ArcTanh[c*x])/(5*x^5) +
 (b*c^5*Log[x])/5 - (b*c^5*Log[1 - c^2*x^2])/10
```

Maple [A]
time = 0.02, size = 71, normalized size = 1.09

method	result	size
derivativedivides	$c^5\left(-\frac{a}{5c^5x^5}-\frac{b\,\mathrm{arctanh}(cx)}{5c^5x^5}-\frac{b\ln(cx-1)}{10}-\frac{b}{20c^4x^4}-\frac{b}{10c^2x^2}+\frac{b\ln(cx)}{5}-\frac{b\ln(cx+1)}{10}\right)$	71
default	$c^5\left(-\frac{a}{5c^5x^5}-\frac{b\,\mathrm{arctanh}(cx)}{5c^5x^5}-\frac{b\ln(cx-1)}{10}-\frac{b}{20c^4x^4}-\frac{b}{10c^2x^2}+\frac{b\ln(cx)}{5}-\frac{b\ln(cx+1)}{10}\right)$	71
risch	$-\frac{b\ln(cx+1)}{10x^5}+\frac{4b\,c^5\ln(x)x^5-2b\,c^5\ln(c^2x^2-1)x^5-2b\,c^3x^3-bcx+2b\ln(-cx+1)-4a}{20x^5}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(-1/5*a/c^5/x^5-1/5*b/c^5/x^5*arctanh(c*x)-1/10*b*ln(c*x-1)-1/20*b/c^4/
x^4-1/10*b/c^2/x^2+1/5*b*ln(c*x)-1/10*b*ln(c*x+1))
```

Maxima [A]
time = 0.26, size = 61, normalized size = 0.94

$$-\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2-1\right)-2\,c^4\log\left(x^2\right)+\frac{2\,c^2x^2+1}{x^4}\right)c+\frac{4\,\mathrm{artanh}\left(cx\right)}{x^5}\right)b-\frac{a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 - 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c +
4*arctanh(c*x)/x^5)*b - 1/5*a/x^5
```

Fricas [A]
time = 0.38, size = 70, normalized size = 1.08

$$-\frac{2\,bc^5x^5\log\left(c^2x^2-1\right)-4\,bc^5x^5\log\left(x\right)+2\,bc^3x^3+bcx+2\,b\log\left(-\frac{cx+1}{cx-1}\right)+4\,a}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(2*b*c^5*x^5*log(c^2*x^2 - 1) - 4*b*c^5*x^5*log(x) + 2*b*c^3*x^3 + b*
c*x + 2*b*log(-(c*x + 1)/(c*x - 1)) + 4*a)/x^5
```

Sympy [A]
time = 0.57, size = 80, normalized size = 1.23

$$\begin{cases} -\frac{a}{5x^5}+\frac{bc^5\log\left(x\right)}{5}-\frac{bc^5\log\left(x-\frac{1}{c}\right)}{5}-\frac{bc^5\operatorname{atanh}\left(cx\right)}{5}-\frac{bc^3}{10x^2}-\frac{bc}{20x^4}-\frac{b\operatorname{atanh}\left(cx\right)}{5x^5} & \text{for}\: c\neq 0\\ -\frac{a}{5x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**6,x)
```

```
[Out] Piecewise((-a/(5*x**5) + b*c**5*log(x)/5 - b*c**5*log(x - 1/c)/5 - b*c**5*a
tanh(c*x)/5 - b*c**3/(10*x**2) - b*c/(20*x**4) - b*atanh(c*x)/(5*x**5), Ne(
c, 0)), (-a/(5*x**5), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(55) = 110.
time = 0.41, size = 397, normalized size = 6.11

$$\frac{1}{5}\left(bc^4\log\left(-\frac{cx+1}{cx-1}-1\right)-bc^4\log\left(-\frac{cx+1}{cx-1}\right)+\frac{\left(\frac{5\,(cx+1)^4bc^4}{(cx-1)^4}+\frac{10\,(cx+1)^2bc^4}{(cx-1)^2}+bc^4\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^5}{(cx-1)^5}+\frac{5\,(cx+1)^4}{(cx-1)^4}+\frac{10\,(cx+1)^3}{(cx-1)^3}+\frac{10\,(cx+1)^2}{(cx-1)^2}+\frac{5\,(cx+1)}{cx-1}+1}+\frac{2\left(\frac{5\,(cx+1)^4ac^4}{(cx-1)^4}+\frac{10\,(cx+1)^2ac^4}{(cx-1)^2}+ac^4+\frac{2\,(cx+1)^4bc^4}{(cx-1)^4}+\frac{4\,(cx+1)^3bc^4}{(cx-1)^3}+\frac{4\,(cx+1)^2bc^4}{(cx-1)^2}+\frac{2\,(cx+1)bc^4}{cx-1}\right)}{\frac{(cx+1)^5}{(cx-1)^5}+\frac{5\,(cx+1)^4}{(cx-1)^4}+\frac{10\,(cx+1)^3}{(cx-1)^3}+\frac{10\,(cx+1)^2}{(cx-1)^2}+\frac{5\,(cx+1)}{cx-1}+1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^6,x, algorithm="giac")
```

```
[Out] 1/5*(b*c^4*log(-(c*x + 1)/(c*x - 1) - 1) - b*c^4*log(-(c*x + 1)/(c*x - 1))
+ (5*(c*x + 1)^4*b*c^4/(c*x - 1)^4 + 10*(c*x + 1)^2*b*c^4/(c*x - 1)^2 + b*c
^4)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5/(c*x - 1)^5 + 5*(c*x + 1)^4/(c*x
 - 1)^4 + 10*(c*x + 1)^3/(c*x - 1)^3 + 10*(c*x + 1)^2/(c*x - 1)^2 + 5*(c*x
+ 1)/(c*x - 1) + 1) + 2*(5*(c*x + 1)^4*a*c^4/(c*x - 1)^4 + 10*(c*x + 1)^2*a
```

```
*c^4/(c*x - 1)^2 + a*c^4 + 2*(c*x + 1)^4*b*c^4/(c*x - 1)^4 + 4*(c*x + 1)^3*
b*c^4/(c*x - 1)^3 + 4*(c*x + 1)^2*b*c^4/(c*x - 1)^2 + 2*(c*x + 1)*b*c^4/(c*
x - 1))/((c*x + 1)^5/(c*x - 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*(c*x + 1)
^3/(c*x - 1)^3 + 10*(c*x + 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x - 1) + 1))*c
```

Mupad [B]
time = 0.91, size = 71, normalized size = 1.09

$$\frac{b\,c^5\,\ln\left(x\right)}{5}-\frac{b\,c^5\,\ln\left(c^2\,x^2-1\right)}{10}-\frac{\frac{b\,c^3\,x^3}{2}+\frac{b\,c\,x}{4}+a}{5\,x^5}-\frac{b\,\ln\left(c\,x+1\right)}{10\,x^5}+\frac{b\,\ln\left(1-c\,x\right)}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/x^6,x)
```

```
[Out] (b*c^5*log(x))/5 - (b*c^5*log(c^2*x^2 - 1))/10 - (a + (b*c^3*x^3)/2 + (b*c*
x)/4)/(5*x^5) - (b*log(c*x + 1))/(10*x^5) + (b*log(1 - c*x))/(10*x^5)
```

3.13 $\int x^5 \left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=145

$$\frac{abx}{3c^5}+\frac{4b^2x^2}{45c^4}+\frac{b^2x^4}{60c^2}+\frac{b^2x\tanh^{-1}(cx)}{3c^5}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{9c^3}+\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)}{15c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{6c^6}$$

```
[Out] 1/3*a*b*x/c^5+4/45*b^2*x^2/c^4+1/60*b^2*x^4/c^2+1/3*b^2*x*arctanh(c*x)/c^5+
1/9*b*x^3*(a+b*arctanh(c*x))/c^3+1/15*b*x^5*(a+b*arctanh(c*x))/c-1/6*(a+b*a
rctanh(c*x))^2/c^6+1/6*x^6*(a+b*arctanh(c*x))^2+23/90*b^2*ln(-c^2*x^2+1)/c^
6
```

Rubi [A]
time = 0.22, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$,
Rules used = {6037, 6127, 272, 45, 6021, 266, 6095}

$$-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{6c^6}+\frac{abx}{3c^5}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{9c^3}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)}{15c}+\frac{b^2x\tanh^{-1}(cx)}{3c^5}+\frac{4b^2x^2}{45c^4}+\frac{b^2x^4}{60c^2}+\frac{23b^2\log\left(1-c^2x^2\right)}{90c^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a*b*x)/(3*c^5) + (4*b^2*x^2)/(45*c^4) + (b^2*x^4)/(60*c^2) + (b^2*x*ArcTan
h[c*x])/(3*c^5) + (b*x^3*(a + b*ArcTanh[c*x]))/(9*c^3) + (b*x^5*(a + b*ArcT
anh[c*x]))/(15*c) - (a + b*ArcTanh[c*x])^2/(6*c^6) + (x^6*(a + b*ArcTanh[c*
x])^2)/6 + (23*b^2*Log[1 - c^2*x^2])/(90*c^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$
\begin{aligned}
\int x^5 \left(a+b\tanh^{-1}(cx)\right)^2 \, dx &= \frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{3}(bc)\int \frac{x^6\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\,dx \\
&= \frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{b\int x^4\left(a+b\tanh^{-1}(cx)\right)\,dx}{3c}-\frac{b\int \frac{x^4\left(a+b\tanh^{-1}(cx\right.}{1-c^2x^2}}{3c} \\
&= \frac{bx^5\left(a+b\tanh^{-1}(cx)\right)}{15c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{15}b^2\int \frac{x^5}{1-c^2x^2}\,dx+ \\
&= \frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{9c^3}+\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)}{15c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^2 \\
&= \frac{abx}{3c^5}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{9c^3}+\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)}{15c}-\frac{\left(a+b\tanh^{-1}(cx\right.}{6c^6} \\
&= \frac{abx}{3c^5}+\frac{b^2x^2}{30c^4}+\frac{b^2x^4}{60c^2}+\frac{b^2x\tanh^{-1}(cx)}{3c^5}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{9c^3}+\frac{bx^5\left(a+b\right.}{} \\
&= \frac{abx}{3c^5}+\frac{4b^2x^2}{45c^4}+\frac{b^2x^4}{60c^2}+\frac{b^2x\tanh^{-1}(cx)}{3c^5}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{9c^3}+\frac{bx^5\left(a+\right.}{}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 164, normalized size = 1.13

$$
\frac{60abcx+16b^2c^2x^2+20abc^3x^3+3b^2c^4x^4+12abc^5x^5+30a^2c^6x^6+4bcx(15ac^5x^5+b(15+5c^2x^2+3c^4x^4))\tanh^{-1}(cx)+30b^2(-1+c^6x^6)\tanh^{-1}(cx)^2+2b(15a+23b)\log(1-cx)-30ab\log(1+cx)+46b^2\log(1+cx)}{180c^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (60*a*b*c*x + 16*b^2*c^2*x^2 + 20*a*b*c^3*x^3 + 3*b^2*c^4*x^4 + 12*a*b*c^5*
x^5 + 30*a^2*c^6*x^6 + 4*b*c*x*(15*a*c^5*x^5 + b*(15 + 5*c^2*x^2 + 3*c^4*x^
4))*ArcTanh[c*x] + 30*b^2*(-1 + c^6*x^6)*ArcTanh[c*x]^2 + 2*b*(15*a + 23*b)
*Log[1 - c*x] - 30*a*b*Log[1 + c*x] + 46*b^2*Log[1 + c*x])/(180*c^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(127) = 254$.
time = 0.07, size = 290, normalized size = 2.00

method	result
derivativedivides	$\frac{\frac{c^6x^6a^2}{6}+\frac{b^2c^6x^6\operatorname{arctanh}(cx)^2}{6}+\frac{b^2\operatorname{arctanh}(cx)c^5x^5}{15}+\frac{b^2\operatorname{arctanh}(cx)c^3x^3}{9}+\frac{b^2\operatorname{arctanh}(cx)cx}{3}+\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{6}-\frac{b^2\operatorname{arctan}}{}}{}$
default	$\frac{\frac{c^6x^6a^2}{6}+\frac{b^2c^6x^6\operatorname{arctanh}(cx)^2}{6}+\frac{b^2\operatorname{arctanh}(cx)c^5x^5}{15}+\frac{b^2\operatorname{arctanh}(cx)c^3x^3}{9}+\frac{b^2\operatorname{arctanh}(cx)cx}{3}+\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{6}-\frac{b^2\operatorname{arctan}}{}}{}$

risch	$\frac{b^2(x^6c^6-1)\ln(cx+1)^2}{24c^6} + \frac{b(-15x^6b\ln(-cx+1)c^6+30c^6x^6a+6c^5x^5b+10b\,c^3x^3+30bcx+15b\ln(-cx+1))\ln(cx+1)}{180c^6} + b^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^6*(1/6*c^6*x^6*a^2+1/6*b^2*c^6*x^6*arctanh(c*x)^2+1/15*b^2*arctanh(c*x)
*c^5*x^5+1/9*b^2*arctanh(c*x)*c^3*x^3+1/3*b^2*arctanh(c*x)*c*x+1/6*b^2*arct
anh(c*x)*ln(c*x-1)-1/6*b^2*arctanh(c*x)*ln(c*x+1)-1/12*b^2*ln(c*x-1)*ln(1/2
*c*x+1/2)+1/24*b^2*ln(c*x-1)^2-1/12*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/12*b^2
*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+1/24*b^2*ln(c*x+1)^2+1/60*b^2*c^4*x^4+4/4
5*b^2*c^2*x^2+23/90*b^2*ln(c*x-1)+23/90*b^2*ln(c*x+1)+1/3*a*b*c^6*x^6*arcta
nh(c*x)+1/15*c^5*x^5*a*b+1/9*a*b*c^3*x^3+1/3*a*b*c*x+1/6*a*b*ln(c*x-1)-1/6*
a*b*ln(c*x+1))
```

Maxima [A]

time = 0.26, size = 215, normalized size = 1.48

$$\frac{1}{6}b^2x^6\operatorname{artanh}(cx)^2 + \frac{1}{6}a^2x^6 + \frac{1}{90}\left(30\,x^6\operatorname{artanh}(cx) + c\left(\frac{2\,(3\,c^4x^5+5\,c^2x^3+15\,x)}{c^6} - \frac{15\,\log(cx+1)}{c^7} + \frac{15\,\log(cx-1)}{c^7}\right)\right)ab + \frac{1}{360}\left(4\,c\left(\frac{2\,(3\,c^4x^5+5\,c^2x^3+15\,x)}{c^6} - \frac{15\,\log(cx+1)}{c^7} + \frac{15\,\log(cx-1)}{c^7}\right)\operatorname{artanh}(cx) + \frac{6\,c^4x^4+32\,c^2x^2-2\,(15\,\log(cx-1)-46)\log(cx+1)+15\,\log(cx+1)^2+15\,\log(cx-1)^2+92\,\log(cx-1)}{c^6}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6*arctanh(c*x)^2 + 1/6*a^2*x^6 + 1/90*(30*x^6*arctanh(c*x) + c*(2
*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)
/c^7))*a*b + 1/360*(4*c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x
+ 1)/c^7 + 15*log(c*x - 1)/c^7)*arctanh(c*x) + (6*c^4*x^4 + 32*c^2*x^2 - 2*
(15*log(c*x - 1) - 46)*log(c*x + 1) + 15*log(c*x + 1)^2 + 15*log(c*x - 1)^2
 + 92*log(c*x - 1))/c^6)*b^2
```

Fricas [A]

time = 0.35, size = 193, normalized size = 1.33

$$\frac{60\,a^2c^6x^6 + 24\,abc^5x^5 + 6\,b^2c^4x^4 + 40\,abc^3x^3 + 32\,b^2c^2x^2 + 120\,abcx + 15\,(b^2c^6x^6-b^2)\log\left(-\frac{cx+1}{cx-1}\right)^2 - 4\,(15\,ab-23\,b^2)\log(cx+1) + 4\,(15\,ab+23\,b^2)\log(cx-1) + 4\,(15\,abc^6x^6+3\,b^2c^5x^5+5\,b^2c^3x^3+15\,b^2cx)\log\left(-\frac{cx+1}{cx-1}\right)}{360\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/360*(60*a^2*c^6*x^6 + 24*a*b*c^5*x^5 + 6*b^2*c^4*x^4 + 40*a*b*c^3*x^3 + 3
2*b^2*c^2*x^2 + 120*a*b*c*x + 15*(b^2*c^6*x^6 - b^2)*log(-(c*x + 1)/(c*x -
1))^2 - 4*(15*a*b - 23*b^2)*log(c*x + 1) + 4*(15*a*b + 23*b^2)*log(c*x - 1)
 + 4*(15*a*b*c^6*x^6 + 3*b^2*c^5*x^5 + 5*b^2*c^3*x^3 + 15*b^2*c*x)*log(-(c*
x + 1)/(c*x - 1)))/c^6
```

Sympy [A]

time = 0.56, size = 211, normalized size = 1.46

$$\begin{cases} \frac{a^2x^6}{6} + \frac{abx^6\operatorname{atanh}(cx)}{3} + \frac{abx^5}{15c} + \frac{abx^3}{9c^3} + \frac{abx}{3c^5} - \frac{ab\operatorname{atanh}(cx)}{3c^6} + \frac{b^2x^6\operatorname{atanh}^2(cx)}{6} + \frac{b^2x^5\operatorname{atanh}(cx)}{15c} + \frac{b^2x^4}{60c^2} + \frac{b^2x^3\operatorname{atanh}(cx)}{9c^3} + \frac{4b^2x^2}{45c^4} + \frac{b^2x\operatorname{atanh}(cx)}{3c^5} + \frac{23b^2\log\left(x-\frac{1}{c}\right)}{45c^6} - \frac{b^2\operatorname{atanh}^2(cx)}{6c^6} + \frac{23b^2\operatorname{atanh}(cx)}{45c^6} & \text{for}\ c \neq 0 \\ \frac{a^2x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**6/6 + a*b*x**6*atanh(c*x)/3 + a*b*x**5/(15*c) + a*b*x**3
/(9*c**3) + a*b*x/(3*c**5) - a*b*atanh(c*x)/(3*c**6) + b**2*x**6*atanh(c*x)
**2/6 + b**2*x**5*atanh(c*x)/(15*c) + b**2*x**4/(60*c**2) + b**2*x**3*atanh
(c*x)/(9*c**3) + 4*b**2*x**2/(45*c**4) + b**2*x*atanh(c*x)/(3*c**5) + 23*b*
*2*log(x - 1/c)/(45*c**6) - b**2*atanh(c*x)**2/(6*c**6) + 23*b**2*atanh(c*x
)/(45*c**6), Ne(c, 0)), (a**2*x**6/6, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 889 vs. 2(127) = 254.
time = 0.43, size = 889, normalized size = 6.13

$$\frac{1}{90}\left(\frac{15\left(\frac{3(cx+1)^5b^2}{(cx-1)^5}+\frac{10(cx+1)^3b^2}{(cx-1)^3}+\frac{3(cx+1)b^2}{cx-1}\right)\log\left(-\frac{cx+1}{cx-1}\right)^2}{\frac{(cx+1)^6c^7}{(cx-1)^6}-\frac{6(cx+1)^5c^7}{(cx-1)^5}+\frac{15(cx+1)^4c^7}{(cx-1)^4}-\frac{20(cx+1)^3c^7}{(cx-1)^3}+\frac{15(cx+1)^2c^7}{(cx-1)^2}-\frac{6(cx+1)c^7}{cx-1}+c^7}+\frac{2\left(\frac{90(cx+1)^5ab}{(cx-1)^5}+\frac{300(cx+1)^3ab}{(cx-1)^3}+\frac{90(cx+1)ab}{cx-1}+\frac{45(cx+1)^5b^2}{(cx-1)^5}-\frac{135(cx+1)^4b^2}{(cx-1)^4}+\frac{230(cx+1)^3b^2}{(cx-1)^3}-\frac{210(cx+1)^2b^2}{(cx-1)^2}+\frac{93(cx+1)b^2}{cx-1}-23b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^6c^7}{(cx-1)^6}-\frac{6(cx+1)^5c^7}{(cx-1)^5}+\frac{15(cx+1)^4c^7}{(cx-1)^4}-\frac{20(cx+1)^3c^7}{(cx-1)^3}+\frac{15(cx+1)^2c^7}{(cx-1)^2}-\frac{6(cx+1)c^7}{cx-1}+c^7}+\frac{4\left(\frac{45(cx+1)^5a^2}{(cx-1)^5}+\frac{150(cx+1)^3a^2}{(cx-1)^3}+\frac{45(cx+1)a^2}{cx-1}+\frac{45(cx+1)^5ab}{(cx-1)^5}-\frac{135(cx+1)^4ab}{(cx-1)^4}+\frac{230(cx+1)^3ab}{(cx-1)^3}-\frac{210(cx+1)^2ab}{(cx-1)^2}+\frac{93(cx+1)ab}{cx-1}-23ab+\frac{11(cx+1)^5b^2}{(cx-1)^5}-\frac{38(cx+1)^4b^2}{(cx-1)^4}+\frac{54(cx+1)^3b^2}{(cx-1)^3}-\frac{38(cx+1)^2b^2}{(cx-1)^2}+\frac{11(cx+1)b^2}{cx-1}\right)}{\frac{(cx+1)^6c^7}{(cx-1)^6}-\frac{6(cx+1)^5c^7}{(cx-1)^5}+\frac{15(cx+1)^4c^7}{(cx-1)^4}-\frac{20(cx+1)^3c^7}{(cx-1)^3}+\frac{15(cx+1)^2c^7}{(cx-1)^2}-\frac{6(cx+1)c^7}{cx-1}+c^7}-\frac{46b^2\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^7}+\frac{46b^2\log\left(-\frac{cx+1}{cx-1}\right)}{c^7}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/90*(15*(3*(c*x + 1)^5*b^2/(c*x - 1)^5 + 10*(c*x + 1)^3*b^2/(c*x - 1)^3 +
3*(c*x + 1)*b^2/(c*x - 1))*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)^6*c^7/(c*
x - 1)^6 - 6*(c*x + 1)^5*c^7/(c*x - 1)^5 + 15*(c*x + 1)^4*c^7/(c*x - 1)^4 -
 20*(c*x + 1)^3*c^7/(c*x - 1)^3 + 15*(c*x + 1)^2*c^7/(c*x - 1)^2 - 6*(c*x +
 1)*c^7/(c*x - 1) + c^7) + 2*(90*(c*x + 1)^5*a*b/(c*x - 1)^5 + 300*(c*x + 1
)^3*a*b/(c*x - 1)^3 + 90*(c*x + 1)*a*b/(c*x - 1) + 45*(c*x + 1)^5*b^2/(c*x
- 1)^5 - 135*(c*x + 1)^4*b^2/(c*x - 1)^4 + 230*(c*x + 1)^3*b^2/(c*x - 1)^3
- 210*(c*x + 1)^2*b^2/(c*x - 1)^2 + 93*(c*x + 1)*b^2/(c*x - 1) - 23*b^2)*lo
g(-(c*x + 1)/(c*x - 1))/((c*x + 1)^6*c^7/(c*x - 1)^6 - 6*(c*x + 1)^5*c^7/(c
*x - 1)^5 + 15*(c*x + 1)^4*c^7/(c*x - 1)^4 - 20*(c*x + 1)^3*c^7/(c*x - 1)^3
 + 15*(c*x + 1)^2*c^7/(c*x - 1)^2 - 6*(c*x + 1)*c^7/(c*x - 1) + c^7) + 4*(4
5*(c*x + 1)^5*a^2/(c*x - 1)^5 + 150*(c*x + 1)^3*a^2/(c*x - 1)^3 + 45*(c*x +
 1)*a^2/(c*x - 1) + 45*(c*x + 1)^5*a*b/(c*x - 1)^5 - 135*(c*x + 1)^4*a*b/(c
*x - 1)^4 + 230*(c*x + 1)^3*a*b/(c*x - 1)^3 - 210*(c*x + 1)^2*a*b/(c*x - 1)
^2 + 93*(c*x + 1)*a*b/(c*x - 1) - 23*a*b + 11*(c*x + 1)^5*b^2/(c*x - 1)^5 -
 38*(c*x + 1)^4*b^2/(c*x - 1)^4 + 54*(c*x + 1)^3*b^2/(c*x - 1)^3 - 38*(c*x
+ 1)^2*b^2/(c*x - 1)^2 + 11*(c*x + 1)*b^2/(c*x - 1))/((c*x + 1)^6*c^7/(c*x
- 1)^6 - 6*(c*x + 1)^5*c^7/(c*x - 1)^5 + 15*(c*x + 1)^4*c^7/(c*x - 1)^4 - 2
0*(c*x + 1)^3*c^7/(c*x - 1)^3 + 15*(c*x + 1)^2*c^7/(c*x - 1)^2 - 6*(c*x + 1
)*c^7/(c*x - 1) + c^7) - 46*b^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^7 + 46*b^2*
log(-(c*x + 1)/(c*x - 1))/c^7)*c
```

Mupad [B]
time = 1.04, size = 171, normalized size = 1.18

$$\frac{46\,b^2\,\ln\left(c^2\,x^2-1\right)-30\,b^2\,\mathrm{atanh}(c\,x)^2+30\,a^2\,c^6\,x^6+16\,b^2\,c^2\,x^2+3\,b^2\,c^4\,x^4-60\,a\,b\,\mathrm{atanh}(c\,x)+20\,b^2\,c^3\,x^3\,\mathrm{atanh}(c\,x)+12\,b^2\,c^5\,x^5\,\mathrm{atanh}(c\,x)+60\,b^2\,c\,x\,\mathrm{atanh}(c\,x)+30\,b^2\,c^6\,x^6\,\mathrm{atanh}(c\,x)^2+20\,a\,b\,c^3\,x^3+12\,a\,b\,c^5\,x^5+60\,a\,b\,c\,x+60\,a\,b\,c^6\,x^6\,\mathrm{atanh}(c\,x)}{180\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atanh(c*x))^2,x)
```

```
[Out] (46*b^2*log(c^2*x^2 - 1) - 30*b^2*atanh(c*x)^2 + 30*a^2*c^6*x^6 + 16*b^2*c^
2*x^2 + 3*b^2*c^4*x^4 - 60*a*b*atanh(c*x) + 20*b^2*c^3*x^3*atanh(c*x) + 12*
b^2*c^5*x^5*atanh(c*x) + 60*b^2*c*x*atanh(c*x) + 30*b^2*c^6*x^6*atanh(c*x)^
2 + 20*a*b*c^3*x^3 + 12*a*b*c^5*x^5 + 60*a*b*c*x + 60*a*b*c^6*x^6*atanh(c*x
))/(180*c^6)
```

3.14 $\int x^4 \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=162

$$\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}-\frac{3b^2\tanh^{-1}(cx)}{10c^5}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{bx^4\left(a+b\tanh^{-1}(cx)\right)}{10c}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{5c^5}+\frac{1}{5}x^5\left(\right.$$

```
[Out] 3/10*b^2*x/c^4+1/30*b^2*x^3/c^2-3/10*b^2*arctanh(c*x)/c^5+1/5*b*x^2*(a+b*ar
ctanh(c*x))/c^3+1/10*b*x^4*(a+b*arctanh(c*x))/c+1/5*(a+b*arctanh(c*x))^2/c^
5+1/5*x^5*(a+b*arctanh(c*x))^2-2/5*b*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^5-
1/5*b^2*polylog(2,1-2/(-c*x+1))/c^5
```

Rubi [A]
time = 0.21, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6037, 6127, 308, 212, 327, 6131, 6055, 2449, 2352}

$$\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{5c^5}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{5c^5}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{bx^4\left(a+b\tanh^{-1}(cx)\right)}{10c}-\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{5c^5}-\frac{3b^2\tanh^{-1}(cx)}{10c^5}+\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (3*b^2*x)/(10*c^4) + (b^2*x^3)/(30*c^2) - (3*b^2*ArcTanh[c*x])/(10*c^5) + (
b*x^2*(a + b*ArcTanh[c*x]))/(5*c^3) + (b*x^4*(a + b*ArcTanh[c*x]))/(10*c) +
 (a + b*ArcTanh[c*x])^2/(5*c^5) + (x^5*(a + b*ArcTanh[c*x])^2)/5 - (2*b*(a
+ b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^5) - (b^2*PolyLog[2, 1 - 2/(1 - c*
x)])/(5*c^5)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^4\left(a+b\tanh^{-1}(cx)\right)^2\,dx &= \frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{5}(2bc)\int \frac{x^5\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\,dx\\
&= \frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{(2b)\int x^3\left(a+b\tanh^{-1}(cx)\right)\,dx}{5c}-\frac{(2b)\int \frac{x^3(a+b\tan}{1-c}}{5c}\\
&= \frac{bx^4\left(a+b\tanh^{-1}(cx)\right)}{10c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{10}b^2\int \frac{x^4}{1-c^2x^2}\,dx+\\
&= \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{bx^4\left(a+b\tanh^{-1}(cx)\right)}{10c}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{5c^5}+\frac{1}{5}\\
&= \frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{bx^4\left(a+b\tanh^{-1}(cx)\right)}{10c}+\frac{(a+b\tan}{5}\\
&= \frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}-\frac{3b^2\tanh^{-1}(cx)}{10c^5}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{bx^4\left(a+b\tanh^{-1}\right.}{10c}\\
&= \frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}-\frac{3b^2\tanh^{-1}(cx)}{10c^5}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{bx^4\left(a+b\tanh^{-1}\right.}{10c}
\end{aligned}
$$

Mathematica [A]
time = 0.33, size = 161, normalized size = 0.99

$$
\frac{-9ab+9b^2cx+6abc^2x^2+b^2c^3x^3+3abc^4x^4+6a^2c^5x^5+6b^2(-1+c^5x^5)\tanh^{-1}(cx)^2+3b\tanh^{-1}(cx)\left(4ac^5x^5+b(-3+2c^2x^2+c^4x^4)-4b\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+6ab\log\left(-1+c^2x^2\right)+6b^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)}{30c^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (-9*a*b + 9*b^2*c*x + 6*a*b*c^2*x^2 + b^2*c^3*x^3 + 3*a*b*c^4*x^4 + 6*a^2*c
^5*x^5 + 6*b^2*(-1 + c^5*x^5)*ArcTanh[c*x]^2 + 3*b*ArcTanh[c*x]*(4*a*c^5*x^
5 + b*(-3 + 2*c^2*x^2 + c^4*x^4) - 4*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 6*a*
b*Log[-1 + c^2*x^2] + 6*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])])/(30*c^5)
```

Maple [A]
time = 0.11, size = 281, normalized size = 1.73

method	result
derivativedivides	$\frac{c^5x^5a^2}{5}+\frac{c^5x^5b^2\,\mathrm{arctanh}(cx)^2}{5}+\frac{b^2\,\mathrm{arctanh}(cx)c^4x^4}{10}+\frac{b^2\,\mathrm{arctanh}(cx)c^2x^2}{5}+\frac{b^2\,\mathrm{arctanh}(cx)\ln(cx-1)}{5}+\frac{b^2\,\mathrm{arctanh}(cx)\ln(cx+1)}{5}+b^2$
default	$\frac{c^5x^5a^2}{5}+\frac{c^5x^5b^2\,\mathrm{arctanh}(cx)^2}{5}+\frac{b^2\,\mathrm{arctanh}(cx)c^4x^4}{10}+\frac{b^2\,\mathrm{arctanh}(cx)c^2x^2}{5}+\frac{b^2\,\mathrm{arctanh}(cx)\ln(cx-1)}{5}+\frac{b^2\,\mathrm{arctanh}(cx)\ln(cx+1)}{5}+b^2$
risch	$\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}+\frac{b^2\ln(-cx+1)^2x^5}{20}-\frac{b^2\ln(-cx+1)^2}{20c^5}+\frac{137b^2\ln(-cx+1)}{300c^5}-\frac{137ab}{150c^5}-\frac{23b^2\ln(cx-1)}{75c^5}+\frac{b^2\ln(cx}{10c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(1/5*c^5*x^5*a^2+1/5*c^5*x^5*b^2*arctanh(c*x)^2+1/10*b^2*arctanh(c*x)
*c^4*x^4+1/5*b^2*arctanh(c*x)*c^2*x^2+1/5*b^2*arctanh(c*x)*ln(c*x-1)+1/5*b^
2*arctanh(c*x)*ln(c*x+1)+1/30*b^2*c^3*x^3+3/10*b^2*c*x+3/20*b^2*ln(c*x-1)-3
/20*b^2*ln(c*x+1)-1/5*b^2*dilog(1/2*c*x+1/2)-1/10*b^2*ln(c*x-1)*ln(1/2*c*x+
1/2)+1/20*b^2*ln(c*x-1)^2-1/10*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+1/10*b^
2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/20*b^2*ln(c*x+1)^2+2/5*c^5*x^5*a*b*arctanh(c
*x)+1/10*c^4*x^4*a*b+1/5*a*b*c^2*x^2+1/5*a*b*ln(c*x-1)+1/5*a*b*ln(c*x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5 + 1/10*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c
^2*x^2 - 1)/c^6))*a*b - 1/36000*(24*c^6*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c
^10 - 15*log(c*x + 1)/c^11 + 15*log(c*x - 1)/c^11) - 45*c^5*((c^2*x^4 + 2*x
^2)/c^8 + 2*log(c^2*x^2 - 1)/c^10) - 1080000*c^5*integrate(1/150*x^5*log(c*
x + 1)/(c^6*x^2 - c^4), x) + 50*c^4*(2*(c^2*x^3 + 3*x)/c^8 - 3*log(c*x + 1)
/c^9 + 3*log(c*x - 1)/c^9) - 300*c^3*(x^2/c^6 + log(c^2*x^2 - 1)/c^8) + 900
*c^2*(2*x/c^6 - log(c*x + 1)/c^7 + log(c*x - 1)/c^7) - 540000*c*integrate(1
/150*x*log(c*x + 1)/(c^6*x^2 - c^4), x) - 60*(30*c^5*x^5*log(c*x + 1)^2 + (
12*c^5*x^5 - 15*c^4*x^4 + 20*c^3*x^3 - 30*c^2*x^2 + 60*c*x - 60*(c^5*x^5 +
1)*log(c*x + 1))*log(-c*x + 1))/c^5 - (72*(c*x - 1)^5*(25*log(-c*x + 1)^2 -
 10*log(-c*x + 1) + 2) + 1125*(c*x - 1)^4*(8*log(-c*x + 1)^2 - 4*log(-c*x +
 1) + 1) + 2000*(c*x - 1)^3*(9*log(-c*x + 1)^2 - 6*log(-c*x + 1) + 2) + 900
0*(c*x - 1)^2*(2*log(-c*x + 1)^2 - 2*log(-c*x + 1) + 1) + 9000*(c*x - 1)*(l
og(-c*x + 1)^2 - 2*log(-c*x + 1) + 2))/c^5 + 1800*log(150*c^6*x^2 - 150*c^4
)/c^5 - 540000*integrate(1/150*log(c*x + 1)/(c^6*x^2 - c^4), x))*b^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^4*arctanh(c*x)^2 + 2*a*b*x^4*arctanh(c*x) + a^2*x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4(a + b\,\mathrm{atanh}\,(cx))^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x))**2,x)
```

```
[Out] Integral(x**4*(a + b*atanh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,(a + b\,\mathrm{atanh}(c\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atanh(c*x))^2,x)
```

```
[Out] int(x^4*(a + b*atanh(c*x))^2, x)
```

3.15 $\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=113

$$\frac{abx}{2c^3}+\frac{b^2x^2}{12c^2}+\frac{b^2x\tanh^{-1}(cx)}{2c^3}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{b^2\log(\ldots$$

```
[Out] 1/2*a*b*x/c^3+1/12*b^2*x^2/c^2+1/2*b^2*x*arctanh(c*x)/c^3+1/6*b*x^3*(a+b*ar
ctanh(c*x))/c-1/4*(a+b*arctanh(c*x))^2/c^4+1/4*x^4*(a+b*arctanh(c*x))^2+1/3
*b^2*ln(-c^2*x^2+1)/c^4
```

Rubi [A]
time = 0.17, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {6037, 6127, 272, 45, 6021, 266, 6095}

$$-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4c^4}+\frac{abx}{2c^3}+\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{b^2x\tanh^{-1}(cx)}{2c^3}+\frac{b^2x^2}{12c^2}+\frac{b^2\log\left(1-c^2x^2\right)}{3c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a*b*x)/(2*c^3) + (b^2*x^2)/(12*c^2) + (b^2*x*ArcTanh[c*x])/(2*c^3) + (b*x^
3*(a + b*ArcTanh[c*x]))/(6*c) - (a + b*ArcTanh[c*x])^2/(4*c^4) + (x^4*(a +
b*ArcTanh[c*x])^2)/4 + (b^2*Log[1 - c^2*x^2])/(3*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
```

```
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx &= \frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{2}(bc)\int\frac{x^4\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\,dx\\
&= \frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{b\int x^2\left(a+b\tanh^{-1}(cx)\right)\,dx}{2c}-\frac{b\int\frac{x^2\left(a+b\tanh^{-1}(cx\right.}{1-c^2x^2}}{2c}\\
&= \frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{6}b^2\int\frac{x^3}{1-c^2x^2}\,dx+\frac{b}{}\\
&= \frac{abx}{2c^3}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}x^4\left(a+b\tanh^{-1}(c\right.\\
&= \frac{abx}{2c^3}+\frac{b^2x\tanh^{-1}(cx)}{2c^3}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}\\
&= \frac{abx}{2c^3}+\frac{b^2x^2}{12c^2}+\frac{b^2x\tanh^{-1}(cx)}{2c^3}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}-\frac{\left(a+b\tanh^{-1}(cx\right.}{4c^4}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 132, normalized size = 1.17

$$\frac{6abcx+b^2c^2x^2+2abc^3x^3+3a^2c^4x^4+2bcx(3ac^3x^3+b(3+c^2x^2))\tanh^{-1}(cx)+3b^2(-1+c^4x^4)\tanh^{-1}(cx)^2+b(3a+4b)\log(1-cx)-3ab\log(1+cx)+4b^2\log(1+cx)}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (6*a*b*c*x + b^2*c^2*x^2 + 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + 2*b*c*x*(3*a*c^3
*x^3 + b*(3 + c^2*x^2))*ArcTanh[c*x] + 3*b^2*(-1 + c^4*x^4)*ArcTanh[c*x]^2
+ b*(3*a + 4*b)*Log[1 - c*x] - 3*a*b*Log[1 + c*x] + 4*b^2*Log[1 + c*x])/(12
*c^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(99) = 198$.
time = 0.03, size = 254, normalized size = 2.25

method	result
derivativedivides	$\frac{c^4x^4a^2}{4}+\frac{b^2c^4x^4\operatorname{arctanh}(cx)^2}{4}+\frac{b^2\operatorname{arctanh}(cx)c^3x^3}{6}+\frac{b^2\operatorname{arctanh}(cx)cx}{2}+\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{4}-\frac{b^2\operatorname{arctanh}(cx)\ln(cx+1)}{4}-\frac{b^2\ln(c}{}$
default	$\frac{c^4x^4a^2}{4}+\frac{b^2c^4x^4\operatorname{arctanh}(cx)^2}{4}+\frac{b^2\operatorname{arctanh}(cx)c^3x^3}{6}+\frac{b^2\operatorname{arctanh}(cx)cx}{2}+\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{4}-\frac{b^2\operatorname{arctanh}(cx)\ln(cx+1)}{4}-\frac{b^2\ln(c}{}$
risch	$\frac{b^2(c^4x^4-1)\ln(cx+1)^2}{16c^4}+\frac{b(-3x^4b\ln(-cx+1)c^4+6c^4x^4a+2bc^3x^3+6bcx+3b\ln(-cx+1))\ln(cx+1)}{24c^4}+\frac{\ln(-cx+1)^2b^2x^4}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/4*c^4*x^4*a^2+1/4*b^2*c^4*x^4*arctanh(c*x)^2+1/6*b^2*arctanh(c*x)*
c^3*x^3+1/2*b^2*arctanh(c*x)*c*x+1/4*b^2*arctanh(c*x)*ln(c*x-1)-1/4*b^2*arc
tanh(c*x)*ln(c*x+1)-1/8*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)+1/16*b^2*ln(c*x-1)^2+
1/8*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-1/8*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)
+1/16*b^2*ln(c*x+1)^2+1/12*b^2*c^2*x^2+1/3*b^2*ln(c*x-1)+1/3*b^2*ln(c*x+1)+
1/2*c^4*x^4*a*b*arctanh(c*x)+1/6*a*b*c^3*x^3+1/2*a*b*c*x+1/4*a*b*ln(c*x-1)-
1/4*a*b*ln(c*x+1))
```

Maxima [A]
time = 0.26, size = 189, normalized size = 1.67

$$\frac{1}{4}b^2x^4\operatorname{artanh}(cx)^2+\frac{1}{4}a^2x^4+\frac{1}{12}\left(6x^4\operatorname{artanh}(cx)+c\left(\frac{2(c^2x^3+3x)}{c^4}-\frac{3\log(cx+1)}{c^5}+\frac{3\log(cx-1)}{c^5}\right)\right)ab+\frac{1}{48}\left(4c\left(\frac{2(c^2x^3+3x)}{c^4}-\frac{3\log(cx+1)}{c^5}+\frac{3\log(cx-1)}{c^5}\right)\operatorname{artanh}(cx)+\frac{4c^2x^2-2(3\log(cx-1)-8)\log(cx+1)+3\log(cx+1)^2+3\log(cx-1)^2+16\log(cx-1)}{c^4}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctanh(c*x)^2 + 1/4*a^2*x^4 + 1/12*(6*x^4*arctanh(c*x) + c*(2*
(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b + 1/48*
```

```
(4*c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*arct
anh(c*x) + (4*c^2*x^2 - 2*(3*log(c*x - 1) - 8)*log(c*x + 1) + 3*log(c*x + 1
)^2 + 3*log(c*x - 1)^2 + 16*log(c*x - 1))/c^4)*b^2
```

Fricas [A]
time = 0.35, size = 160, normalized size = 1.42

$$\frac{12\,a^2c^4x^4 + 8\,abc^3x^3 + 4\,b^2c^2x^2 + 24\,abcx + 3\,(b^2c^4x^4 - b^2)\log\left(-\frac{cx+1}{cx-1}\right)^2 - 4\,(3\,ab - 4\,b^2)\log\left(cx+1\right) + 4\,(3\,ab + 4\,b^2)\log\left(cx-1\right) + 4\,(3\,abc^4x^4 + b^2c^3x^3 + 3\,b^2cx)\log\left(-\frac{cx+1}{cx-1}\right)}{48\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(12*a^2*c^4*x^4 + 8*a*b*c^3*x^3 + 4*b^2*c^2*x^2 + 24*a*b*c*x + 3*(b^2*
c^4*x^4 - b^2)*log(-(c*x + 1)/(c*x - 1))^2 - 4*(3*a*b - 4*b^2)*log(c*x + 1)
 + 4*(3*a*b + 4*b^2)*log(c*x - 1) + 4*(3*a*b*c^4*x^4 + b^2*c^3*x^3 + 3*b^2*
c*x)*log(-(c*x + 1)/(c*x - 1)))/c^4
```

Sympy [A]
time = 0.39, size = 168, normalized size = 1.49

$$\begin{cases} \frac{a^2x^4}{4} + \frac{abx^4\operatorname{atanh}(cx)}{2} + \frac{abx^3}{6c} + \frac{abx}{2c^3} - \frac{ab\operatorname{atanh}(cx)}{2c^4} + \frac{b^2x^4\operatorname{atanh}^2(cx)}{4} + \frac{b^2x^3\operatorname{atanh}(cx)}{6c} + \frac{b^2x^2}{12c^2} + \frac{b^2x\operatorname{atanh}(cx)}{2c^3} + \frac{2b^2\log\left(x-\frac{1}{c}\right)}{3c^4} - \frac{b^2\operatorname{atanh}^2(cx)}{4c^4} + \frac{2b^2\operatorname{atanh}(cx)}{3c^4} & \text{for}\; c \neq 0 \\ \frac{a^2x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**4/4 + a*b*x**4*atanh(c*x)/2 + a*b*x**3/(6*c) + a*b*x/(2*
c**3) - a*b*atanh(c*x)/(2*c**4) + b**2*x**4*atanh(c*x)**2/4 + b**2*x**3*ata
nh(c*x)/(6*c) + b**2*x**2/(12*c**2) + b**2*x*atanh(c*x)/(2*c**3) + 2*b**2*l
og(x - 1/c)/(3*c**4) - b**2*atanh(c*x)**2/(4*c**4) + 2*b**2*atanh(c*x)/(3*c
**4), Ne(c, 0)), (a**2*x**4/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(99) = 198.
time = 0.43, size = 603, normalized size = 5.34

$$\frac{1}{6}\left(\frac{3\left(\frac{(cx+1)^3b^2}{(cx-1)^3} + \frac{(cx+1)b^2}{cx-1}\right)\log\left(-\frac{cx+1}{cx-1}\right)^2}{\frac{(cx+1)^4c^5}{(cx-1)^4} - \frac{4\,(cx+1)^3c^5}{(cx-1)^3} + \frac{6\,(cx+1)^2c^5}{(cx-1)^2} - \frac{4\,(cx+1)c^5}{cx-1} + c^5} + \frac{2\left(\frac{6\,(cx+1)^3ab}{(cx-1)^3} + \frac{6\,(cx+1)ab}{cx-1} + \frac{3\,(cx+1)^3b^2}{(cx-1)^3} - \frac{6\,(cx+1)^2b^2}{(cx-1)^2} + \frac{5\,(cx+1)b^2}{cx-1} - 2\,b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4c^5}{(cx-1)^4} - \frac{4\,(cx+1)^3c^5}{(cx-1)^3} + \frac{6\,(cx+1)^2c^5}{(cx-1)^2} - \frac{4\,(cx+1)c^5}{cx-1} + c^5} + \frac{2\left(\frac{6\,(cx+1)^3a^2}{(cx-1)^3} + \frac{6\,(cx+1)a^2}{cx-1} + \frac{6\,(cx+1)^3ab}{(cx-1)^3} - \frac{12\,(cx+1)^2ab}{(cx-1)^2} + \frac{10\,(cx+1)ab}{cx-1} - 4\,ab + \frac{(cx+1)^3b^2}{(cx-1)^3} - \frac{2\,(cx+1)^2b^2}{(cx-1)^2} + \frac{(cx+1)b^2}{cx-1}\right)}{\frac{(cx+1)^4c^5}{(cx-1)^4} - \frac{4\,(cx+1)^3c^5}{(cx-1)^3} + \frac{6\,(cx+1)^2c^5}{(cx-1)^2} - \frac{4\,(cx+1)c^5}{cx-1} + c^5} - \frac{4\,b^2\log\left(-\frac{cx+1}{cx-1} + 1\right)}{c^5} + \frac{4\,b^2\log\left(-\frac{cx+1}{cx-1}\right)}{c^5}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*((c*x + 1)^3*b^2/(c*x - 1)^3 + (c*x + 1)*b^2/(c*x - 1))*log(-(c*x +
1)/(c*x - 1))^2/((c*x + 1)^4*c^5/(c*x - 1)^4 - 4*(c*x + 1)^3*c^5/(c*x - 1)^
3 + 6*(c*x + 1)^2*c^5/(c*x - 1)^2 - 4*(c*x + 1)*c^5/(c*x - 1) + c^5) + 2*(6
*(c*x + 1)^3*a*b/(c*x - 1)^3 + 6*(c*x + 1)*a*b/(c*x - 1) + 3*(c*x + 1)^3*b^
2/(c*x - 1)^3 - 6*(c*x + 1)^2*b^2/(c*x - 1)^2 + 5*(c*x + 1)*b^2/(c*x - 1) -
```

```
 2*b^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^4*c^5/(c*x - 1)^4 - 4*(c*x + 1
)^3*c^5/(c*x - 1)^3 + 6*(c*x + 1)^2*c^5/(c*x - 1)^2 - 4*(c*x + 1)*c^5/(c*x
- 1) + c^5) + 2*(6*(c*x + 1)^3*a^2/(c*x - 1)^3 + 6*(c*x + 1)*a^2/(c*x - 1)
+ 6*(c*x + 1)^3*a*b/(c*x - 1)^3 - 12*(c*x + 1)^2*a*b/(c*x - 1)^2 + 10*(c*x
+ 1)*a*b/(c*x - 1) - 4*a*b + (c*x + 1)^3*b^2/(c*x - 1)^3 - 2*(c*x + 1)^2*b^
2/(c*x - 1)^2 + (c*x + 1)*b^2/(c*x - 1))/((c*x + 1)^4*c^5/(c*x - 1)^4 - 4*(
c*x + 1)^3*c^5/(c*x - 1)^3 + 6*(c*x + 1)^2*c^5/(c*x - 1)^2 - 4*(c*x + 1)*c^
5/(c*x - 1) + c^5) - 4*b^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^5 + 4*b^2*log(-(
c*x + 1)/(c*x - 1))/c^5)*c
```

Mupad [B]

time = 0.92, size = 134, normalized size = 1.19

$$\frac{4\,b^2\,\ln\left(c^2\,x^2-1\right)-3\,b^2\,\mathrm{atanh}(c\,x)^2+3\,a^2\,c^4\,x^4+b^2\,c^2\,x^2-6\,a\,b\,\mathrm{atanh}(c\,x)+2\,b^2\,c^3\,x^3\,\mathrm{atanh}(c\,x)+6\,b^2\,c\,x\,\mathrm{atanh}(c\,x)+3\,b^2\,c^4\,x^4\,\mathrm{atanh}(c\,x)^2+2\,a\,b\,c^3\,x^3+6\,a\,b\,c\,x+6\,a\,b\,c^4\,x^4\,\mathrm{atanh}(c\,x)}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))^2,x)
```

```
[Out] (4*b^2*log(c^2*x^2 - 1) - 3*b^2*atanh(c*x)^2 + 3*a^2*c^4*x^4 + b^2*c^2*x^2
- 6*a*b*atanh(c*x) + 2*b^2*c^3*x^3*atanh(c*x) + 6*b^2*c*x*atanh(c*x) + 3*b^
2*c^4*x^4*atanh(c*x)^2 + 2*a*b*c^3*x^3 + 6*a*b*c*x + 6*a*b*c^4*x^4*atanh(c*
x))/(12*c^4)
```

3.16 $\int x^2 \left(a+b \tanh ^{-1}(c x)\right)^2 \, dx$

Optimal. Leaf size=130

$$\frac{b^2 x}{3 c^2}-\frac{b^2 \tanh ^{-1}(c x)}{3 c^3}+\frac{b x^2 \left(a+b \tanh ^{-1}(c x)\right)}{3 c}+\frac{\left(a+b \tanh ^{-1}(c x)\right)^2}{3 c^3}+\frac{1}{3} x^3 \left(a+b \tanh ^{-1}(c x)\right)^2-\frac{2 b \left(a+b \tanh\right.}{}$$

```
[Out] 1/3*b^2*x/c^2-1/3*b^2*arctanh(c*x)/c^3+1/3*b*x^2*(a+b*arctanh(c*x))/c+1/3*(
a+b*arctanh(c*x))^2/c^3+1/3*x^3*(a+b*arctanh(c*x))^2-2/3*b*(a+b*arctanh(c*x
))*ln(2/(-c*x+1))/c^3-1/3*b^2*polylog(2,1-2/(-c*x+1))/c^3
```

Rubi [A]
time = 0.15, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6037, 6127, 327, 212, 6131, 6055, 2449, 2352}

$$\frac{\left(a+b \tanh ^{-1}(c x)\right)^2}{3 c^3}-\frac{2 b \log \left(\frac{2}{1-c x}\right) \left(a+b \tanh ^{-1}(c x)\right)}{3 c^3}+\frac{1}{3} x^3 \left(a+b \tanh ^{-1}(c x)\right)^2+\frac{b x^2 \left(a+b \tanh ^{-1}(c x)\right)}{3 c}-\frac{b^2 \text{Li}_2\left(1-\frac{2}{1-c x}\right)}{3 c^3}-\frac{b^2 \tanh ^{-1}(c x)}{3 c^3}+\frac{b^2 x}{3 c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (b^2*x)/(3*c^2) - (b^2*ArcTanh[c*x])/(3*c^3) + (b*x^2*(a + b*ArcTanh[c*x]))
/(3*c) + (a + b*ArcTanh[c*x])^2/(3*c^3) + (x^3*(a + b*ArcTanh[c*x])^2)/3 -
(2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^3) - (b^2*PolyLog[2, 1 - 2
/(1 - c*x)])/(3*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2(a+b\tanh^{-1}(cx))^2\,dx = \frac{1}{3}x^3(a+b\tanh^{-1}(cx))^2 - \frac{1}{3}(2bc)\int \frac{x^3(a+b\tanh^{-1}(cx))}{1-c^2x^2}\,dx$$

$$= \frac{1}{3}x^3(a+b\tanh^{-1}(cx))^2 + \frac{(2b)\int x(a+b\tanh^{-1}(cx))\,dx}{3c} - \frac{(2b)\int \frac{x(a+b\tanh}{1-c^2x}}{3c}$$

$$= \frac{bx^2(a+b\tanh^{-1}(cx))}{3c} + \frac{(a+b\tanh^{-1}(cx))^2}{3c^3} + \frac{1}{3}x^3(a+b\tanh^{-1}(cx))^2 -$$

$$= \frac{b^2x}{3c^2} + \frac{bx^2(a+b\tanh^{-1}(cx))}{3c} + \frac{(a+b\tanh^{-1}(cx))^2}{3c^3} + \frac{1}{3}x^3(a+b\tanh^{-1}(cx$$

$$= \frac{b^2x}{3c^2} - \frac{b^2\tanh^{-1}(cx)}{3c^3} + \frac{bx^2(a+b\tanh^{-1}(cx))}{3c} + \frac{(a+b\tanh^{-1}(cx))^2}{3c^3} + \frac{1}{3}x$$

$$= \frac{b^2x}{3c^2} - \frac{b^2\tanh^{-1}(cx)}{3c^3} + \frac{bx^2(a+b\tanh^{-1}(cx))}{3c} + \frac{(a+b\tanh^{-1}(cx))^2}{3c^3} + \frac{1}{3}x$$

Mathematica [A]
time = 0.18, size = 122, normalized size = 0.94

$$\frac{b^2cx + abc^2x^2 + a^2c^3x^3 + b^2(-1+c^3x^3)\tanh^{-1}(cx)^2 + b\tanh^{-1}(cx)\left(-b+bc^2x^2+2ac^3x^3-2b\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right) + ab\log(-1+c^2x^2) + b^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (b^2*c*x + a*b*c^2*x^2 + a^2*c^3*x^3 + b^2*(-1 + c^3*x^3)*ArcTanh[c*x]^2 +
b*ArcTanh[c*x]*(-b + b*c^2*x^2 + 2*a*c^3*x^3 - 2*b*Log[1 + E^(-2*ArcTanh[c*
x])]) + a*b*Log[-1 + c^2*x^2] + b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])])/(3*c^
3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(116) = 232.
time = 0.06, size = 245, normalized size = 1.88

method	result
derivativedivides	$\frac{c^3x^3a^2}{3} + \frac{b^2c^3x^3\operatorname{arctanh}(cx)^2}{3} + \frac{b^2\operatorname{arctanh}(cx)c^2x^2}{3} + \frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{3} + \frac{b^2\operatorname{arctanh}(cx)\ln(cx+1)}{3} + \frac{b^2cx}{3} + \frac{b^2\ln(cx-1)}{6} - \frac{b^2}{}$
default	$\frac{c^3x^3a^2}{3} + \frac{b^2c^3x^3\operatorname{arctanh}(cx)^2}{3} + \frac{b^2\operatorname{arctanh}(cx)c^2x^2}{3} + \frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{3} + \frac{b^2\operatorname{arctanh}(cx)\ln(cx+1)}{3} + \frac{b^2cx}{3} + \frac{b^2\ln(cx-1)}{6} - \frac{b^2}{}$
risch	$\frac{b^2x}{3c^2} + \frac{a^2x^3}{3} + \frac{b^2\ln(cx+1)^2x^3}{12} + \frac{b^2\ln(cx+1)^2}{12c^3} - \frac{b^2\ln(cx+1)}{6c^3} - \frac{11ab}{9c^3} + \frac{b^2\ln(-cx+1)^2x^3}{12} - \frac{b^2\ln(-cx+1)^2}{12c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/3*c^3*x^3*a^2+1/3*b^2*c^3*x^3*arctanh(c*x)^2+1/3*b^2*arctanh(c*x)*
c^2*x^2+1/3*b^2*arctanh(c*x)*ln(c*x-1)+1/3*b^2*arctanh(c*x)*ln(c*x+1)+1/3*b
^2*c*x+1/6*b^2*ln(c*x-1)-1/6*b^2*ln(c*x+1)-1/3*b^2*dilog(1/2*c*x+1/2)-1/6*b
^2*ln(c*x-1)*ln(1/2*c*x+1/2)+1/12*b^2*ln(c*x-1)^2-1/6*b^2*ln(-1/2*c*x+1/2)*
ln(1/2*c*x+1/2)+1/6*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/12*b^2*ln(c*x+1)^2+2/3
*a*b*c^3*x^3*arctanh(c*x)+1/3*a*b*c^2*x^2+1/3*a*b*ln(c*x-1)+1/3*a*b*ln(c*x+
1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))
*a*b - 1/216*(2*c^4*(2*(c^2*x^3 + 3*x)/c^6 - 3*log(c*x + 1)/c^7 + 3*log(c*x
 - 1)/c^7) - 3*c^3*(x^2/c^4 + log(c^2*x^2 - 1)/c^6) - 648*c^3*integrate(1/9
*x^3*log(c*x + 1)/(c^4*x^2 - c^2), x) + 9*c^2*(2*x/c^4 - log(c*x + 1)/c^5 +
 log(c*x - 1)/c^5) - 324*c*integrate(1/9*x*log(c*x + 1)/(c^4*x^2 - c^2), x)
 - 6*(3*c^3*x^3*log(c*x + 1)^2 + (2*c^3*x^3 - 3*c^2*x^2 + 6*c*x - 6*(c^3*x^
3 + 1)*log(c*x + 1))*log(-c*x + 1))/c^3 - (2*(c*x - 1)^3*(9*log(-c*x + 1)^2
 - 6*log(-c*x + 1) + 2) + 27*(c*x - 1)^2*(2*log(-c*x + 1)^2 - 2*log(-c*x +
1) + 1) + 54*(c*x - 1)*(log(-c*x + 1)^2 - 2*log(-c*x + 1) + 2))/c^3 + 18*lo
g(9*c^4*x^2 - 9*c^2)/c^3 - 324*integrate(1/9*log(c*x + 1)/(c^4*x^2 - c^2),
x))*b^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a + b\operatorname{atanh}(cx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))**2,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{atanh}(c\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))^2,x)
```

```
[Out] int(x^2*(a + b*atanh(c*x))^2, x)
```

3.17 $\int x\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=75

$$\frac{abx}{c}+\frac{b^2x\tanh^{-1}(cx)}{c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{b^2\log\left(1-c^2x^2\right)}{2c^2}$$

```
[Out] a*b*x/c+b^2*x*arctanh(c*x)/c-1/2*(a+b*arctanh(c*x))^2/c^2+1/2*x^2*(a+b*arct
anh(c*x))^2+1/2*b^2*ln(-c^2*x^2+1)/c^2
```

Rubi [A]
time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.417$, Rules used = {6037, 6127, 6021, 266, 6095}

$$-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{abx}{c}+\frac{b^2\log\left(1-c^2x^2\right)}{2c^2}+\frac{b^2x\tanh^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a*b*x)/c + (b^2*x*ArcTanh[c*x])/c - (a + b*ArcTanh[c*x])^2/(2*c^2) + (x^2*
(a + b*ArcTanh[c*x])^2)/2 + (b^2*Log[1 - c^2*x^2])/(2*c^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}
\int x\left(a+b\tanh^{-1}(cx)\right)^2\,dx &= \frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2-(bc)\int\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\,dx\\
&= \frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{b\int\left(a+b\tanh^{-1}(cx)\right)\,dx}{c}-\frac{b\int\frac{a+b\tanh^{-1}(cx)}{1-c^2x^2}\,dx}{c}\\
&= \frac{abx}{c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{b^2\int\tanh^{-1}(cx)\,dx}{c}\\
&= \frac{abx}{c}+\frac{b^2x\tanh^{-1}(cx)}{c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2-b^2\\
&= \frac{abx}{c}+\frac{b^2x\tanh^{-1}(cx)}{c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{b^2}{}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 90, normalized size = 1.20

$$\frac{2abcx+a^2c^2x^2+2bcx(b+acx)\tanh^{-1}(cx)+b^2(-1+c^2x^2)\tanh^{-1}(cx)^2+b(a+b)\log(1-cx)-ab\log(1+cx)+b^2\log(1+cx)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (2*a*b*c*x + a^2*c^2*x^2 + 2*b*c*x*(b + a*c*x)*ArcTanh[c*x] + b^2*(-1 + c^2
*x^2)*ArcTanh[c*x]^2 + b*(a + b)*Log[1 - c*x] - a*b*Log[1 + c*x] + b^2*Log[
1 + c*x])/(2*c^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(69) = 138$.
time = 0.03, size = 215, normalized size = 2.87

method	result
risch	$\frac{b^2(c^2x^2-1)\ln(cx+1)^2}{8c^2}+\frac{b(-b\,x^2\ln(-cx+1)c^2+2a\,c^2x^2+2bcx+b\ln(-cx+1))\ln(cx+1)}{4c^2}+\frac{\ln(-cx+1)^2b^2x^2}{8}-\ln(-c$
derivativedivides	$\frac{c^2x^2a^2}{2}+\frac{b^2c^2x^2\,\mathrm{arctanh}(cx)^2}{2}+b^2\,\mathrm{arctanh}(cx)cx+\frac{b^2\,\mathrm{arctanh}(cx)\ln(cx-1)}{2}-\frac{b^2\,\mathrm{arctanh}(cx)\ln(cx+1)}{2}-\frac{b^2\ln(cx-1)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{4}+b$
default	$\frac{c^2x^2a^2}{2}+\frac{b^2c^2x^2\,\mathrm{arctanh}(cx)^2}{2}+b^2\,\mathrm{arctanh}(cx)cx+\frac{b^2\,\mathrm{arctanh}(cx)\ln(cx-1)}{2}-\frac{b^2\,\mathrm{arctanh}(cx)\ln(cx+1)}{2}-\frac{b^2\ln(cx-1)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{4}+b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/2*c^2*x^2*a^2+1/2*b^2*c^2*x^2*arctanh(c*x)^2+b^2*arctanh(c*x)*c*x+
1/2*b^2*arctanh(c*x)*ln(c*x-1)-1/2*b^2*arctanh(c*x)*ln(c*x+1)-1/4*b^2*ln(c*
x-1)*ln(1/2*c*x+1/2)+1/8*b^2*ln(c*x-1)^2+1/2*b^2*ln(c*x-1)+1/2*b^2*ln(c*x+1
)+1/4*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-1/4*b^2*ln(-1/2*c*x+1/2)*ln(c*x+
1)+1/8*b^2*ln(c*x+1)^2+a*b*c^2*x^2*arctanh(c*x)+a*b*c*x+1/2*a*b*ln(c*x-1)-1
/2*a*b*ln(c*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(69) = 138$.
time = 0.28, size = 158, normalized size = 2.11

$$\frac{1}{2}b^2x^2\,\mathrm{artanh}\,(cx)^2+\frac{1}{2}a^2x^2+\frac{1}{2}\left(2\,x^2\,\mathrm{artanh}\,(cx)+c\left(\frac{2\,x}{c^2}-\frac{\log\,(cx+1)}{c^3}+\frac{\log\,(cx-1)}{c^3}\right)\right)ab+\frac{1}{8}\left(4\,c\left(\frac{2\,x}{c^2}-\frac{\log\,(cx+1)}{c^3}+\frac{\log\,(cx-1)}{c^3}\right)\mathrm{artanh}\,(cx)-\frac{2\,(\log\,(cx-1)-2)\log\,(cx+1)-\log\,(cx+1)^2-\log\,(cx-1)^2-4\,\log\,(cx-1)}{c^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arctanh(c*x)^2 + 1/2*a^2*x^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*x
/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a*b + 1/8*(4*c*(2*x/c^2 - log(
c*x + 1)/c^3 + log(c*x - 1)/c^3)*arctanh(c*x) - (2*(log(c*x - 1) - 2)*log(c
*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*log(c*x - 1))/c^2)*b^2
```

Fricas [A]
time = 0.37, size = 122, normalized size = 1.63

$$\frac{4\,a^2c^2x^2+8\,abcx+(b^2c^2x^2-b^2)\log\left(-\frac{cx+1}{cx-1}\right)^2-4\,(ab-b^2)\log\,(cx+1)+4\,(ab+b^2)\log\,(cx-1)+4\,(abc^2x^2+b^2cx)\log\left(-\frac{cx+1}{cx-1}\right)}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a^2*c^2*x^2 + 8*a*b*c*x + (b^2*c^2*x^2 - b^2)*log(-(c*x + 1)/(c*x -
1))^2 - 4*(a*b - b^2)*log(c*x + 1) + 4*(a*b + b^2)*log(c*x - 1) + 4*(a*b*c^
2*x^2 + b^2*c*x)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

Sympy [A]
time = 0.24, size = 114, normalized size = 1.52

$$\begin{cases} \frac{a^2x^2}{2} + abx^2 \operatorname{atanh}(cx) + \frac{abx}{c} - \frac{ab \operatorname{atanh}(cx)}{c^2} + \frac{b^2x^2 \operatorname{atanh}^2(cx)}{2} + \frac{b^2x \operatorname{atanh}(cx)}{c} + \frac{b^2 \log\left(x - \frac{1}{c}\right)}{c^2} - \frac{b^2 \operatorname{atanh}^2(cx)}{2c^2} + \frac{b^2 \operatorname{atanh}(cx)}{c^2} & \text{for } c \neq 0 \\ \frac{a^2x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**2/2 + a*b*x**2*atanh(c*x) + a*b*x/c - a*b*atanh(c*x)/c**
2 + b**2*x**2*atanh(c*x)**2/2 + b**2*x*atanh(c*x)/c + b**2*log(x - 1/c)/c**
2 - b**2*atanh(c*x)**2/(2*c**2) + b**2*atanh(c*x)/c**2, Ne(c, 0)), (a**2*x*
*2/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(69) = 138.
time = 0.40, size = 301, normalized size = 4.01

$$\frac{1}{2}\left(\frac{(cx+1)b^2\log\left(-\frac{cx+1}{cx-1}\right)^2}{\left(\frac{(cx+1)^2c^3}{(cx-1)^2} - \frac{2(cx+1)c^3}{cx-1} + c^3\right)(cx-1)} + \frac{2\left(\frac{2(cx+1)ab}{cx-1} + \frac{(cx+1)b^2}{cx-1} - b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^2c^3}{(cx-1)^2} - \frac{2(cx+1)c^3}{cx-1} + c^3} + \frac{4\left(\frac{(cx+1)a^2}{cx-1} + \frac{(cx+1)ab}{cx-1} - ab\right)}{\frac{(cx+1)^2c^3}{(cx-1)^2} - \frac{2(cx+1)c^3}{cx-1} + c^3} - \frac{2\,b^2\log\left(-\frac{cx+1}{cx-1} + 1\right)}{c^3} + \frac{2\,b^2\log\left(-\frac{cx+1}{cx-1}\right)}{c^3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/2*((c*x + 1)*b^2*log(-(c*x + 1)/(c*x - 1))^2/(((c*x + 1)^2*c^3/(c*x - 1)^
2 - 2*(c*x + 1)*c^3/(c*x - 1) + c^3)*(c*x - 1)) + 2*(2*(c*x + 1)*a*b/(c*x -
 1) + (c*x + 1)*b^2/(c*x - 1) - b^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^2
*c^3/(c*x - 1)^2 - 2*(c*x + 1)*c^3/(c*x - 1) + c^3) + 4*((c*x + 1)*a^2/(c*x
 - 1) + (c*x + 1)*a*b/(c*x - 1) - a*b)/((c*x + 1)^2*c^3/(c*x - 1)^2 - 2*(c*
x + 1)*c^3/(c*x - 1) + c^3) - 2*b^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^3 + 2*b
^2*log(-(c*x + 1)/(c*x - 1))/c^3)*c
```

Mupad [B]
time = 0.78, size = 89, normalized size = 1.19

$$\frac{a^2\,x^2}{2} - \frac{\frac{b^2\,\mathrm{atanh}(c\,x)^2}{2} - \frac{b^2\,\ln(c^2\,x^2-1)}{2} - c\,(x\,\mathrm{atanh}(c\,x)\,b^2 + a\,x\,b) + a\,b\,\mathrm{atanh}(c\,x)}{c^2} + \frac{b^2\,x^2\,\mathrm{atanh}(c\,x)^2}{2} + a\,b\,x^2\,\mathrm{atanh}(c\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))^2,x)
```

```
[Out] (a^2*x^2)/2 - ((b^2*atanh(c*x)^2)/2 - (b^2*log(c^2*x^2 - 1))/2 - c*(b^2*x*a
tanh(c*x) + a*b*x) + a*b*atanh(c*x))/c^2 + (b^2*x^2*atanh(c*x)^2)/2 + a*b*x
^2*atanh(c*x)
```

3.18 $\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=74

$$\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c}+x\left(a+b\tanh^{-1}(cx)\right)^2-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{c}-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}$$

```
[Out] (a+b*arctanh(c*x))^2/c+x*(a+b*arctanh(c*x))^2-2*b*(a+b*arctanh(c*x))*ln(2/(
-c*x+1))/c-b^2*polylog(2,1-2/(-c*x+1))/c
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6021, 6131, 6055, 2449, 2352}

$$x\left(a+b\tanh^{-1}(cx)\right)^2+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}-\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a + b*ArcTanh[c*x])^2/c + x*(a + b*ArcTanh[c*x])^2 - (2*b*(a + b*ArcTanh[c
*x])*Log[2/(1 - c*x)])/c - (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/c
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
```

```
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}(cx)\right)^2 \, dx = x\left(a+b\tanh^{-1}(cx)\right)^2-(2bc)\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\, dx$$
$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c}+x\left(a+b\tanh^{-1}(cx)\right)^2-(2b)\int \frac{a+b\tanh^{-1}(cx)}{1-cx}\, dx$$
$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c}+x\left(a+b\tanh^{-1}(cx)\right)^2-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right.}{c}$$
$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c}+x\left(a+b\tanh^{-1}(cx)\right)^2-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right.}{c}$$
$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c}+x\left(a+b\tanh^{-1}(cx)\right)^2-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right.}{c}$$

Mathematica [A]
time = 0.10, size = 82, normalized size = 1.11

$$\frac{b^2(-1+cx)\tanh^{-1}(cx)^2+2b\tanh^{-1}(cx)\left(acx-b\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+a(acx+b\log\left(1-c^2x^2\right))+b^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (b^2*(-1 + c*x)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*c*x - b*Log[1 + E^(-2*
ArcTanh[c*x])]) + a*(a*c*x + b*Log[1 - c^2*x^2]) + b^2*PolyLog[2, -E^(-2*Ar
cTanh[c*x])])/c
```

Maple [A]
time = 0.09, size = 118, normalized size = 1.59

method	result

derivativedivides	$\frac{cx\,a^2+b^2cx\,\mathrm{arctanh}(cx)^2+b^2\,\mathrm{arctanh}(cx)^2-2\,\mathrm{arctanh}(cx)\ln\left(1+\frac{(cx+1)^2}{-c^2x^2+1}\right)b^2-\mathrm{polylog}\left(2,-\frac{(cx+1)^2}{-c^2x^2+1}\right)b^2+2abcx\,\mathrm{arctanl}}{c}$
default	$\frac{cx\,a^2+b^2cx\,\mathrm{arctanh}(cx)^2+b^2\,\mathrm{arctanh}(cx)^2-2\,\mathrm{arctanh}(cx)\ln\left(1+\frac{(cx+1)^2}{-c^2x^2+1}\right)b^2-\mathrm{polylog}\left(2,-\frac{(cx+1)^2}{-c^2x^2+1}\right)b^2+2abcx\,\mathrm{arctanl}}{c}$
risch	$-\frac{a^2}{c}-\frac{b^2}{c}+a^2x-\frac{b^2\,\mathrm{dilog}\left(\frac{cx}{2}+\frac{1}{2}\right)}{c}-\frac{b^2\ln(cx-1)}{c}+\frac{b^2\ln(cx+1)^2x}{4}+\frac{b^2\ln(cx+1)^2}{4c}-\frac{2ab}{c}+\frac{\ln(-cx+1)^2x}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(c*x*a^2+b^2*c*x*arctanh(c*x)^2+b^2*arctanh(c*x)^2-2*arctanh(c*x)*ln(1+
(c*x+1)^2/(-c^2*x^2+1))*b^2-polylog(2,-(c*x+1)^2/(-c^2*x^2+1))*b^2+2*a*b*c*
x*arctanh(c*x)+a*b*ln(-c^2*x^2+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) - 6*c*integrate(x
*log(c*x + 1)/(c^2*x^2 - 1), x) - (c*x - 1)*(log(-c*x + 1)^2 - 2*log(-c*x +
 1) + 2)/c - (c*x*log(c*x + 1)^2 + 2*(c*x - (c*x + 1)*log(c*x + 1))*log(-c*
x + 1))/c + log(c^2*x^2 - 1)/c - 2*integrate(log(c*x + 1)/(c^2*x^2 - 1), x)
)*b^2 + a^2*x + (2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a*b/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{atanh}\,(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{atanh}(c\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2,x)
```

```
[Out] int((a + b*atanh(c*x))^2, x)
```

3.19 $\int \frac{(a+b\tanh^{-1}(cx))^2}{x}\,dx$

Optimal. Leaf size=117

$$2(a+b\tanh^{-1}(cx))^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-b(a+b\tanh^{-1}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)+b(a+b\tanh^{-1}($$

```
[Out] -2*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))-b*(a+b*arctanh(c*x))*polylog
(2,1-2/(-c*x+1))+b*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))+1/2*b^2*poly
log(3,1-2/(-c*x+1))-1/2*b^2*polylog(3,-1+2/(-c*x+1))
```

Rubi [A]
time = 0.19, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6033, 6199, 6095, 6205, 6745}

$$-b\text{Li}_2\left(1-\frac{2}{1-cx}\right)(a+b\tanh^{-1}(cx))+b\text{Li}_2\left(\frac{2}{1-cx}-1\right)(a+b\tanh^{-1}(cx))+2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)(a+b\tanh^{-1}(cx))^2+\frac{1}{2}b^2\text{Li}_3\left(1-\frac{2}{1-cx}\right)-\frac{1}{2}b^2\text{Li}_3\left(\frac{2}{1-cx}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/x,x]
```

```
[Out] 2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - b*(a + b*ArcTanh[c*x])*
PolyLog[2, 1 - 2/(1 - c*x)] + b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 -
 c*x)] + (b^2*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*PolyLog[3, -1 + 2/(1 -
c*x)])/2
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
```

```
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx = 2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-(4bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)\tanh}{1-c^2x^2}$$

$$= 2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)+(2bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log}{1-c^2x^2}$$

$$= 2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-b\left(a+b\tanh^{-1}(cx)\right)\text{Li}_2\left(1-\frac{}{1}\right.$$

$$= 2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-b\left(a+b\tanh^{-1}(cx)\right)\text{Li}_2\left(1-\frac{}{1}\right.$$

Mathematica [A]
time = 0.05, size = 120, normalized size = 1.03

$$2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(\frac{1+cx}{-1+cx}\right)+\frac{1}{2}b\left(2\left(a+b\tanh^{-1}(cx)\right)\text{PolyLog}\left(2,\frac{1+cx}{1-cx}\right)-2\left(a+b\tanh^{-1}(cx)\right)\text{PolyLog}\left(2,\frac{1+cx}{-1+cx}\right)+b\left(-\text{PolyLog}\left(3,\frac{1+cx}{1-cx}\right)+\text{PolyLog}\left(3,\frac{1+cx}{-1+cx}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/x,x]
```

```
[Out] 2*(a + b*ArcTanh[c*x])^2*ArcTanh[(1 + c*x)/(-1 + c*x)] + (b*(2*(a + b*ArcTa
nh[c*x])*PolyLog[2, (1 + c*x)/(1 - c*x)] - 2*(a + b*ArcTanh[c*x])*PolyLog[2
, (1 + c*x)/(-1 + c*x)] + b*(-PolyLog[3, (1 + c*x)/(1 - c*x)] + PolyLog[3,
(1 + c*x)/(-1 + c*x)])))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.78, size = 701, normalized size = 5.99

method	result
derivativedivides	$a^2 \ln(cx) + b^2 \ln(cx) \operatorname{arctanh}(cx)^2 - b^2 \operatorname{arctanh}(cx) \operatorname{polylog}\left(2, -\frac{(cx+1)^2}{-c^2x^2+1}\right) + \frac{b^2 \operatorname{polylog}\left(3\right.}{2}$
default	$a^2 \ln(cx) + b^2 \ln(cx) \operatorname{arctanh}(cx)^2 - b^2 \operatorname{arctanh}(cx) \operatorname{polylog}\left(2, -\frac{(cx+1)^2}{-c^2x^2+1}\right) + \frac{b^2 \operatorname{polylog}\left(3\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*ln(c*x)+b^2*ln(c*x)*arctanh(c*x)^2-b^2*arctanh(c*x)*polylog(2,-(c*x+1)^
2/(-c^2*x^2+1))+1/2*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-b^2*arctanh(c*x)
^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1
)^(1/2))+2*b^2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2*pol
ylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+b^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^
2+1)^(1/2))+2*b^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2
*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*I*b^2*Pi*csgn(I*((c*x+1)^2/(-c^
2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^
2*arctanh(c*x)^2+1/2*I*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^
2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+1/2*I*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+
1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)
/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2-1/2*I*b^2*Pi*csgn(I/(1+(c*x+1)^
2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1
)))^2*arctanh(c*x)^2-a*b*ln(c*x)*ln(c*x+1)+2*a*b*ln(c*x)*arctanh(c*x)-a*b*d
ilog(c*x)-a*b*dilog(c*x+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + integrate(1/4*b^2*(log(c*x + 1) - log(-c*x + 1))^2/x + a*b*(lo
g(c*x + 1) - log(-c*x + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x,x, algorithm="fricas")
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(cx\right)\right)^{2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x,x)
[Out] Integral((a + b*atanh(c*x))**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/x,x)
[Out] int((a + b*atanh(c*x))^2/x, x)
```

3.20 $\int \frac{(a+b\tanh^{-1}(cx))^2}{x^2}\,dx$

Optimal. Leaf size=71

$$c(a+b\tanh^{-1}(cx))^2-\frac{(a+b\tanh^{-1}(cx))^2}{x}+2bc(a+b\tanh^{-1}(cx))\log\left(2-\frac{2}{1+cx}\right)-b^2c\text{PolyLog}\Big(2,-1+$$

```
[Out] c*(a+b*arctanh(c*x))^2-(a+b*arctanh(c*x))^2/x+2*b*c*(a+b*arctanh(c*x))*ln(2
-2/(c*x+1))-b^2*c*polylog(2,-1+2/(c*x+1))
```

Rubi [A]
time = 0.10, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6037, 6135, 6079, 2497}

$$c(a+b\tanh^{-1}(cx))^2-\frac{(a+b\tanh^{-1}(cx))^2}{x}+2bc\log\left(2-\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))+b^2(-c)\text{Li}_2\left(\frac{2}{cx+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/x^2,x]
```

```
[Out] c*(a + b*ArcTanh[c*x])^2 - (a + b*ArcTanh[c*x])^2/x + 2*b*c*(a + b*ArcTanh[
c*x])*Log[2 - 2/(1 + c*x)] - b^2*c*PolyLog[2, -1 + 2/(1 + c*x)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
```

```
2*d^2 - e^2, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x} + (2bc)\int \frac{a+b\tanh^{-1}(cx)}{x\left(1-c^2x^2\right)}\,dx$$
$$= c\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x} + (2bc)\int \frac{a+b\tanh^{-1}(cx)}{x(1+cx)}\,dx$$
$$= c\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 2bc\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\right.$$
$$= c\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 2bc\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\right.$$

Mathematica [**A**]
time = 0.10, size = 94, normalized size = 1.32

$$\frac{b^2(-1+cx)\tanh^{-1}(cx)^2 + 2b\tanh^{-1}(cx)\left(-a+bcx\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right) - a\left(a-2bcx\log(cx)+bcx\log\left(1-c^2x^2\right)\right) - b^2cx\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/x^2,x]
```

```
[Out] (b^2*(-1 + c*x)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(-a + b*c*x*Log[1 - E^(-2
*ArcTanh[c*x])]) - a*(a - 2*b*c*x*Log[c*x] + b*c*x*Log[1 - c^2*x^2]) - b^2*
c*x*PolyLog[2, E^(-2*ArcTanh[c*x])])/x
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(71) = 142$.
time = 0.10, size = 244, normalized size = 3.44

method	result
derivativedivides	$c\left(-\frac{a^2}{cx} - \frac{b^2\,\text{arctanh}(cx)^2}{cx} - b^2\,\text{arctanh}\,(cx)\ln(cx-1) + 2b^2\ln(cx)\,\text{arctanh}\,(cx) - b^2\,\text{arcta}\right.$

default	$c\left(-\frac{a^2}{cx}-\frac{b^2\,\mathrm{arctanh}(cx)^2}{cx}-b^2\,\mathrm{arctanh}\,(cx)\ln\,(cx-1)+2b^2\ln\,(cx)\,\mathrm{arctanh}\,(cx)-b^2\,\mathrm{arctan}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(c*x))^2/x^2,x,method=_RETURNVERBOSE)

[Out] c*(-a^2/c/x-b^2/c/x*arctanh(c*x)^2-b^2*arctanh(c*x)*ln(c*x-1)+2*b^2*ln(c*x)*arctanh(c*x)-b^2*arctanh(c*x)*ln(c*x+1)+b^2*dilog(1/2*c*x+1/2)+1/2*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)-1/4*b^2*ln(c*x-1)^2-1/2*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/2*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+1/4*b^2*ln(c*x+1)^2-b^2*dilog(c*x)-b^2*dilog(c*x+1)-b^2*ln(c*x)*ln(c*x+1)-2*a*b/c/x*arctanh(c*x)-a*b*ln(c*x-1)+2*a*b*ln(c*x)-a*b*ln(c*x+1))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^2/x^2,x, algorithm="maxima")

[Out] -(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b - 1/4*b^2*(log(-c*x + 1)^2/x + integrate(-((c*x - 1)*log(c*x + 1)^2 + 2*(c*x - (c*x - 1)*log(c*x + 1))*log(-c*x + 1))/(c*x^3 - x^2), x)) - a^2/x

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^2/x^2,x, algorithm="fricas")

[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/x^2, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\,\mathrm{atanh}\,(cx)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x))**2/x**2,x)

[Out] Integral((a + b*atanh(c*x))**2/x**2, x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)^2/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/x^2,x)
[Out] int((a + b*atanh(c*x))^2/x^2, x)
```

3.21 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=80

$$-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+b^2c^2\log(x)-\frac{1}{2}b^2c^2\log\left(1-c^2x^2\right)$$

```
[Out] -b*c*(a+b*arctanh(c*x))/x+1/2*c^2*(a+b*arctanh(c*x))^2-1/2*(a+b*arctanh(c*x
))^2/x^2+b^2*c^2*ln(x)-1/2*b^2*c^2*ln(-c^2*x^2+1)
```

Rubi [A]
time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6037, 6129, 272, 36, 29, 31, 6095}

$$\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{1}{2}b^2c^2\log\left(1-c^2x^2\right)+b^2c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/x^3,x]
```

```
[Out] -((b*c*(a + b*ArcTanh[c*x]))/x) + (c^2*(a + b*ArcTanh[c*x])^2)/2 - (a + b*A
rcTanh[c*x])^2/(2*x^2) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 - c^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx &= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+(bc)\int \frac{a+b\tanh^{-1}(cx)}{x^2\left(1-c^2x^2\right)}\,dx\\
&= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+(bc)\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx+\left(bc^3\right)\int \frac{a+b\tanh^{-1}(c}{1-c^2x^2}\\
&= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+(b\\
&= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+\frac{1}{2}\\
&= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+\frac{1}{2}\\
&= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+b^2\end{aligned}$$

Mathematica [A]
time = 0.04, size = 101, normalized size = 1.26

$$-\frac{a^2+2abcx+2b(a+bcx)\tanh^{-1}(cx)-b^2(-1+c^2x^2)\tanh^{-1}(cx)^2-2b^2c^2x^2\log(x)+b(a+b)c^2x^2\log(1-cx)-(a-b)bc^2x^2\log(1+cx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/x^3,x]
```

```
[Out] -1/2*(a^2 + 2*a*b*c*x + 2*b*(a + b*c*x)*ArcTanh[c*x] - b^2*(-1 + c^2*x^2)*A
rcTanh[c*x]^2 - 2*b^2*c^2*x^2*Log[x] + b*(a + b)*c^2*x^2*Log[1 - c*x] - (a
- b)*b*c^2*x^2*Log[1 + c*x])/x^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(74) = 148$.
time = 0.04, size = 234, normalized size = 2.92

method	result
risch	$\frac{b^2(c^2x^2-1)\ln(cx+1)^2}{8x^2} - \frac{b(b\,x^2\ln(-cx+1)c^2+2bcx-b\ln(-cx+1)+2a)\ln(cx+1)}{4x^2} + \frac{b^2c^2x^2\ln(-cx+1)^2+4b\,c^2\ln(-cx$
derivativedivides	$c^2\Big(-\frac{a^2}{2c^2x^2} - \frac{b^2\operatorname{arctanh}(cx)^2}{2c^2x^2} - \frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{2} + \frac{b^2\operatorname{arctanh}(cx)\ln(cx+1)}{2} - \frac{b^2\operatorname{arctanh}(cx)}{cx} + \frac{b^2\ln($
default	$c^2\Big(-\frac{a^2}{2c^2x^2} - \frac{b^2\operatorname{arctanh}(cx)^2}{2c^2x^2} - \frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{2} + \frac{b^2\operatorname{arctanh}(cx)\ln(cx+1)}{2} - \frac{b^2\operatorname{arctanh}(cx)}{cx} + \frac{b^2\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a^2/c^2/x^2-1/2*b^2/c^2/x^2*arctanh(c*x)^2-1/2*b^2*arctanh(c*x)*l
n(c*x-1)+1/2*b^2*arctanh(c*x)*ln(c*x+1)-b^2*arctanh(c*x)/c/x+1/4*b^2*ln(c*x
-1)*ln(1/2*c*x+1/2)-1/8*b^2*ln(c*x-1)^2-1/2*b^2*ln(c*x-1)+b^2*ln(c*x)-1/2*b
^2*ln(c*x+1)-1/4*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+1/4*b^2*ln(-1/2*c*x+1
/2)*ln(c*x+1)-1/8*b^2*ln(c*x+1)^2-a*b/c^2/x^2*arctanh(c*x)-1/2*a*b*ln(c*x-1
)+1/2*a*b*ln(c*x+1)-a*b/c/x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(74) = 148$.
time = 0.27, size = 151, normalized size = 1.89

$$\frac{1}{2}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\operatorname{artanh}\left(cx\right)}{x^2}\right)ab+\frac{1}{8}\left(\left(2\left(\log\left(cx-1\right)-2\right)\log\left(cx+1\right)-\log\left(cx+1\right)^2-\log\left(cx-1\right)^2-4\,\log\left(cx-1\right)+8\,\log\left(x\right)\right)c^2+4\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c\,\operatorname{artanh}\left(cx\right)\right)b^2-\frac{b^2\operatorname{artanh}\left(cx\right)^2}{2\,x^2}-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b +
1/8*((2*(log(c*x - 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 -
 4*log(c*x - 1) + 8*log(x))*c^2 + 4*(c*log(c*x + 1) - c*log(c*x - 1) - 2/x)
*c*arctanh(c*x))*b^2 - 1/2*b^2*arctanh(c*x)^2/x^2 - 1/2*a^2/x^2
```

Fricas [A]
time = 0.36, size = 135, normalized size = 1.69

$$\frac{8\,b^2c^2x^2\log\left(x\right)+4\left(ab-b^2\right)c^2x^2\log\left(cx+1\right)-4\left(ab+b^2\right)c^2x^2\log\left(cx-1\right)-8\,abcx+\left(b^2c^2x^2-b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)^2-4\,a^2-4\left(b^2cx+ab\right)\log\left(-\frac{cx+1}{cx-1}\right)}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b^2*c^2*x^2*log(x) + 4*(a*b - b^2)*c^2*x^2*log(c*x + 1) - 4*(a*b + b
^2)*c^2*x^2*log(c*x - 1) - 8*a*b*c*x + (b^2*c^2*x^2 - b^2)*log(-(c*x + 1)/(
c*x - 1))^2 - 4*a^2 - 4*(b^2*c*x + a*b)*log(-(c*x + 1)/(c*x - 1)))/x^2
```

Sympy [A]
time = 0.40, size = 126, normalized size = 1.58

$$\begin{cases} -\frac{a^2}{2x^2} + abc^2 \operatorname{atanh}(cx) - \frac{abc}{x} - \frac{ab\operatorname{atanh}(cx)}{x^2} + b^2c^2\log(x) - b^2c^2\log\left(x - \frac{1}{c}\right) + \frac{b^2c^2\operatorname{atanh}^2(cx)}{2} - b^2c^2\operatorname{atanh}(cx) - \frac{b^2c\operatorname{atanh}(cx)}{x} - \frac{b^2\operatorname{atanh}^2(cx)}{2x^2} & \text{for}\ c \neq 0 \\ -\frac{a^2}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**3,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) + a*b*c**2*atanh(c*x) - a*b*c/x - a*b*atanh(c*x)/
x**2 + b**2*c**2*log(x) - b**2*c**2*log(x - 1/c) + b**2*c**2*atanh(c*x)**2/
2 - b**2*c**2*atanh(c*x) - b**2*c*atanh(c*x)/x - b**2*atanh(c*x)**2/(2*x**2
), Ne(c, 0)), (-a**2/(2*x**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(74) = 148.
time = 0.43, size = 278, normalized size = 3.48

$$\frac{1}{2}\left(2b^2c\log\left(-\frac{cx+1}{cx-1}-1\right) - 2b^2c\log\left(-\frac{cx+1}{cx-1}\right) + \frac{(cx+1)b^2c\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx-1)\left(\frac{(cx+1)^2}{(cx-1)^2} + \frac{2(cx+1)}{cx-1} + 1\right)} + \frac{2\left(\frac{2(cx+1)abc}{cx-1} + \frac{(cx+1)b^2c}{cx-1} + b^2c\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^2}{(cx-1)^2} + \frac{2(cx+1)}{cx-1} + 1} + \frac{4\left(\frac{(cx+1)a^2c}{cx-1} + \frac{(cx+1)abc}{cx-1} + abc\right)}{\frac{(cx+1)^2}{(cx-1)^2} + \frac{2(cx+1)}{cx-1} + 1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*c*log(-(c*x + 1)/(c*x - 1) - 1) - 2*b^2*c*log(-(c*x + 1)/(c*x -
1)) + (c*x + 1)*b^2*c*log(-(c*x + 1)/(c*x - 1))^2/((c*x - 1)*((c*x + 1)^2/(
c*x - 1)^2 + 2*(c*x + 1)/(c*x - 1) + 1)) + 2*(2*(c*x + 1)*a*b*c/(c*x - 1) +
 (c*x + 1)*b^2*c/(c*x - 1) + b^2*c)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^2/
(c*x - 1)^2 + 2*(c*x + 1)/(c*x - 1) + 1) + 4*((c*x + 1)*a^2*c/(c*x - 1) + (
c*x + 1)*a*b*c/(c*x - 1) + a*b*c)/((c*x + 1)^2/(c*x - 1)^2 + 2*(c*x + 1)/(c
*x - 1) + 1))*c
```

Mupad [B]
time = 1.49, size = 246, normalized size = 3.08

$$\frac{b^2c^2\ln(cx+1)^2}{8} - \frac{a^2}{2x^2} + \frac{b^2c^2\ln(1-cx)^2}{8} - \frac{b^2\ln(cx+1)^2}{8x^2} - \frac{b^2\ln(1-cx)^2}{8x^2} + b^2c^2\ln(x) - \frac{b^2c^2\ln(cx-1)}{2} - \frac{b^2c^2\ln(cx+1)}{2} - \frac{ab\ln(cx+1)}{2x^2} + \frac{ab\ln(1-cx)}{2x^2} + \frac{b^2\ln(cx+1)\ln(1-cx)}{4x^2} - \frac{abc}{x} - \frac{b^2c\ln(cx+1)}{2x} + \frac{b^2c\ln(1-cx)}{2x} - \frac{abc^2\ln(cx-1)}{2} + \frac{abc^2\ln(cx+1)}{2} - \frac{b^2c^2\ln(cx+1)\ln(1-cx)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/x^3,x)
```

```
[Out] (b^2*c^2*log(c*x + 1)^2)/8 - a^2/(2*x^2) + (b^2*c^2*log(1 - c*x)^2)/8 - (b^
2*log(c*x + 1)^2)/(8*x^2) - (b^2*log(1 - c*x)^2)/(8*x^2) + b^2*c^2*log(x) -
 (b^2*c^2*log(c*x - 1))/2 - (b^2*c^2*log(c*x + 1))/2 - (a*b*log(c*x + 1))/(
2*x^2) + (a*b*log(1 - c*x))/(2*x^2) + (b^2*log(c*x + 1)*log(1 - c*x))/(4*x^
2) - (a*b*c)/x - (b^2*c*log(c*x + 1))/(2*x) + (b^2*c*log(1 - c*x))/(2*x) -
(a*b*c^2*log(c*x - 1))/2 + (a*b*c^2*log(c*x + 1))/2 - (b^2*c^2*log(c*x + 1)
*log(1 - c*x))/4
```

3.22 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=130

$$-\frac{b^2c^2}{3x}+\frac{1}{3}b^2c^3\tanh^{-1}(cx)-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3x^2}+\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}+\frac{2}{3}bc^3(a+$$

```
[Out] -1/3*b^2*c^2/x+1/3*b^2*c^3*arctanh(c*x)-1/3*b*c*(a+b*arctanh(c*x))/x^2+1/3*
c^3*(a+b*arctanh(c*x))^2-1/3*(a+b*arctanh(c*x))^2/x^3+2/3*b*c^3*(a+b*arctan
h(c*x))*ln(2-2/(c*x+1))-1/3*b^2*c^3*polylog(2,-1+2/(c*x+1))
```

Rubi [A]
time = 0.17, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6037, 6129, 331, 212, 6135, 6079, 2497}

$$\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{3}bc^3\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{1}{3}b^2c^3\text{Li}_2\left(\frac{2}{cx+1}-1\right)+\frac{1}{3}b^2c^3\tanh^{-1}(cx)-\frac{b^2c^2}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/x^4,x]
```

```
[Out] -1/3*(b^2*c^2)/x + (b^2*c^3*ArcTanh[c*x])/3 - (b*c*(a + b*ArcTanh[c*x]))/(3
*x^2) + (c^3*(a + b*ArcTanh[c*x])^2)/3 - (a + b*ArcTanh[c*x])^2/(3*x^3) + (
2*b*c^3*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/3 - (b^2*c^3*PolyLog[2,
-1 + 2/(1 + c*x)])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx &= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}(2bc)\int \frac{a+b\tanh^{-1}(cx)}{x^3\left(1-c^2x^2\right)}\,dx \\
&= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}(2bc)\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx+\frac{1}{3}\left(2bc^3\right)\int \frac{a+b\tan}{x(1-} \\
&= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3x^2}+\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3} \\
&= -\frac{b^2c^2}{3x}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3x^2}+\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right.}{3x^3} \\
&= -\frac{b^2c^2}{3x}+\frac{1}{3}b^2c^3\tanh^{-1}(cx)-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3x^2}+\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^2
\end{aligned}
$$

Mathematica [A]
time = 0.25, size = 145, normalized size = 1.12

$$-\frac{a^2+abcx+b^2c^2x^2+b^2(1-c^3x^3)\tanh^{-1}(cx)^2+b\tanh^{-1}(cx)\left(2a+bcx-bc^3x^3-2bc^3x^3\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)-2abc^3x^3\log(cx)+abc^3x^3\log\left(1-c^2x^2\right)+b^2c^3x^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/x^4,x]
```

```
[Out] -1/3*(a^2 + a*b*c*x + b^2*c^2*x^2 + b^2*(1 - c^3*x^3)*ArcTanh[c*x]^2 + b*Ar
cTanh[c*x]*(2*a + b*c*x - b*c^3*x^3 - 2*b*c^3*x^3*Log[1 - E^(-2*ArcTanh[c*x
])]) - 2*a*b*c^3*x^3*Log[c*x] + a*b*c^3*x^3*Log[1 - c^2*x^2] + b^2*c^3*x^3*
PolyLog[2, E^(-2*ArcTanh[c*x])])/x^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(116) = 232$.
time = 0.09, size = 305, normalized size = 2.35

method	result
derivativedivides	$c^3\left(-\frac{a^2}{3c^3x^3}-\frac{b^2\operatorname{arctanh}(cx)^2}{3c^3x^3}-\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{3}-\frac{b^2\operatorname{arctanh}(cx)}{3c^2x^2}+\frac{2b^2\ln(cx)\operatorname{arctanh}(cx)}{3}-\frac{b^2\operatorname{arc}}{}\right.$
default	$c^3\left(-\frac{a^2}{3c^3x^3}-\frac{b^2\operatorname{arctanh}(cx)^2}{3c^3x^3}-\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{3}-\frac{b^2\operatorname{arctanh}(cx)}{3c^2x^2}+\frac{2b^2\ln(cx)\operatorname{arctanh}(cx)}{3}-\frac{b^2\operatorname{arc}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/3*a^2/c^3/x^3-1/3*b^2/c^3/x^3*arctanh(c*x)^2-1/3*b^2*arctanh(c*x)*l
n(c*x-1)-1/3*b^2*arctanh(c*x)/c^2/x^2+2/3*b^2*ln(c*x)*arctanh(c*x)-1/3*b^2*
```

```
arctanh(c*x)*ln(c*x+1)-1/6*b^2*ln(c*x-1)+1/6*b^2*ln(c*x+1)-1/3*b^2/c/x+1/3*
b^2*dilog(1/2*c*x+1/2)+1/6*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)-1/12*b^2*ln(c*x-1)
^2-1/6*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/6*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1
/2)+1/12*b^2*ln(c*x+1)^2-1/3*b^2*dilog(c*x)-1/3*b^2*dilog(c*x+1)-1/3*b^2*ln
(c*x)*ln(c*x+1)-2/3*a*b/c^3/x^3*arctanh(c*x)-1/3*a*b*ln(c*x-1)-1/3*a*b/c^2/
x^2+2/3*a*b*ln(c*x)-1/3*a*b*ln(c*x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*a*b - 1/12*b^2*(log(-c*x + 1)^2/x^3 + 3*integrate(-1/3*(3*(c*x - 1)*log(c*
x + 1)^2 + 2*(c*x - 3*(c*x - 1)*log(c*x + 1))*log(-c*x + 1))/(c*x^5 - x^4),
 x)) - 1/3*a^2/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}{\left(c x \right)}\right)^{2}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**4,x)
```

```
[Out] Integral((a + b*atanh(c*x))**2/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/x^4, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/x^4,x)
```

```
[Out] int((a + b*atanh(c*x))^2/x^4, x)
```

3.23 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^5}\,dx$

Optimal. Leaf size=117

$$-\frac{b^2c^2}{12x^2}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{bc^3\left(a+b\tanh^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}+\frac{2}{3}b^2c$$

```
[Out] -1/12*b^2*c^2/x^2-1/6*b*c*(a+b*arctanh(c*x))/x^3-1/2*b*c^3*(a+b*arctanh(c*x
))/x+1/4*c^4*(a+b*arctanh(c*x))^2-1/4*(a+b*arctanh(c*x))^2/x^4+2/3*b^2*c^4*
ln(x)-1/3*b^2*c^4*ln(-c^2*x^2+1)
```

Rubi [A]
time = 0.16, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6037, 6129, 272, 46, 36, 29, 31, 6095}

$$\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc^3\left(a+b\tanh^{-1}(cx)\right)}{2x}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}+\frac{2}{3}b^2c^4\log(x)-\frac{b^2c^2}{12x^2}-\frac{1}{3}b^2c^4\log\left(1-c^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/x^5,x]
```

```
[Out] -1/12*(b^2*c^2)/x^2 - (b*c*(a + b*ArcTanh[c*x]))/(6*x^3) - (b*c^3*(a + b*Ar
cTanh[c*x]))/(2*x) + (c^4*(a + b*ArcTanh[c*x])^2)/4 - (a + b*ArcTanh[c*x])^
2/(4*x^4) + (2*b^2*c^4*Log[x])/3 - (b^2*c^4*Log[1 - c^2*x^2])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^5}\,dx &= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}+\frac{1}{2}(bc)\int \frac{a+b\tanh^{-1}(cx)}{x^4\left(1-c^2x^2\right)}\,dx \\
&= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}+\frac{1}{2}(bc)\int \frac{a+b\tanh^{-1}(cx)}{x^4}\,dx+\frac{1}{2}\left(bc^3\right)\int \frac{a+b\tanh^{-1}\cdots}{x^2\left(1-c^2\cdots\right.} \\
&= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}+\frac{1}{6}\left(b^2c^2\right)\int \frac{1}{x^3\left(1-c^2x^2\right)}\,dx+\cdots \\
&= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{bc^3\left(a+b\tanh^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^2-\cdots \\
&= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{bc^3\left(a+b\tanh^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^2-\cdots \\
&= -\frac{b^2c^2}{12x^2}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{bc^3\left(a+b\tanh^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tanh^{-1}(c\cdots\right. \\
&= -\frac{b^2c^2}{12x^2}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{bc^3\left(a+b\tanh^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tanh^{-1}(c\cdots\right.
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 164, normalized size = 1.40

$$
-\frac{3a^2+2abcx+b^2c^2x^2+6abc^3x^3+2b\left(3a+bcx+3bc^3x^3\right)\tanh^{-1}(cx)-3b^2\left(-1+c^4x^4\right)\tanh^{-1}(cx)^2-8b^2c^4x^4\log(x)+3abc^4x^4\log(1-cx)+4b^2c^4x^4\log(1-cx)-3abc^4x^4\log(1+cx)+4b^2c^4x^4\log(1+cx)}{12x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/x^5,x]
```

```
[Out] -1/12*(3*a^2 + 2*a*b*c*x + b^2*c^2*x^2 + 6*a*b*c^3*x^3 + 2*b*(3*a + b*c*x +
 3*b*c^3*x^3)*ArcTanh[c*x] - 3*b^2*(-1 + c^4*x^4)*ArcTanh[c*x]^2 - 8*b^2*c^
4*x^4*Log[x] + 3*a*b*c^4*x^4*Log[1 - c*x] + 4*b^2*c^4*x^4*Log[1 - c*x] - 3*
a*b*c^4*x^4*Log[1 + c*x] + 4*b^2*c^4*x^4*Log[1 + c*x])/x^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(103) = 206$.
time = 0.05, size = 271, normalized size = 2.32

method	result
derivativedivides	$c^4\left(-\frac{a^2}{4c^4x^4}-\frac{b^2\operatorname{arctanh}(cx)^2}{4c^4x^4}-\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{4}-\frac{b^2\operatorname{arctanh}(cx)}{6c^3x^3}-\frac{b^2\operatorname{arctanh}(cx)}{2cx}+\frac{b^2\operatorname{arctanh}(cx)\cdots}{4}\right.$ ⋯
default	$c^4\left(-\frac{a^2}{4c^4x^4}-\frac{b^2\operatorname{arctanh}(cx)^2}{4c^4x^4}-\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{4}-\frac{b^2\operatorname{arctanh}(cx)}{6c^3x^3}-\frac{b^2\operatorname{arctanh}(cx)}{2cx}+\frac{b^2\operatorname{arctanh}(cx)\cdots}{4}\right.$ ⋯
risch	$\frac{b^2\left(c^4x^4-1\right)\ln(cx+1)^2}{16x^4}-\frac{b\left(3x^4b\ln(-cx+1)c^4+6b\,c^3x^3+2bcx-3b\ln(-cx+1)+6a\right)\ln(cx+1)}{24x^4}+\frac{3b^2c^4x^4\ln(-cx+1)^2+\cdots}{}$ ⋯

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(-1/4*a^2/c^4/x^4-1/4*b^2/c^4/x^4*arctanh(c*x)^2-1/4*b^2*arctanh(c*x)*l
n(c*x-1)-1/6*b^2*arctanh(c*x)/c^3/x^3-1/2*b^2*arctanh(c*x)/c/x+1/4*b^2*arct
anh(c*x)*ln(c*x+1)+1/8*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)-1/16*b^2*ln(c*x-1)^2+1
/8*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/8*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-
1/16*b^2*ln(c*x+1)^2-1/3*b^2*ln(c*x-1)-1/12*b^2/c^2/x^2+2/3*b^2*ln(c*x)-1/3
*b^2*ln(c*x+1)-1/2*a*b/c^4/x^4*arctanh(c*x)-1/4*a*b*ln(c*x-1)-1/6*a*b/c^3/x
^3-1/2*a*b/c/x+1/4*a*b*ln(c*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(103) = 206$.
time $= 0.26$, size $= 224$, normalized size $= 1.91$

$$\frac{1}{12}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c-\frac{6\,\operatorname{artanh}\left(cx\right)}{x^4}\right)ab+\frac{1}{48}\left(\left(32\,c^2\log\left(x\right)-\frac{3\,c^2x^2\log\left(cx+1\right)^2+3\,c^2x^2\log\left(cx-1\right)^2+16\,c^2x^2\log\left(cx-1\right)-2\left(3\,c^2x^2\log\left(cx-1\right)-8\,c^2x^2\right)\log\left(cx+1\right)+4}{x^2}\right)c^2+4\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c\,\operatorname{artanh}\left(cx\right)\right)b^2-\frac{b^2\operatorname{artanh}\left(cx\right)^2}{4\,x^4}-\frac{a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c -
 6*arctanh(c*x)/x^4)*a*b + 1/48*((32*c^2*log(x) - (3*c^2*x^2*log(c*x + 1)^2
 + 3*c^2*x^2*log(c*x - 1)^2 + 16*c^2*x^2*log(c*x - 1) - 2*(3*c^2*x^2*log(c*
x - 1) - 8*c^2*x^2)*log(c*x + 1) + 4)/x^2)*c^2 + 4*(3*c^3*log(c*x + 1) - 3*
c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c*arctanh(c*x))*b^2 - 1/4*b^2*arc
tanh(c*x)^2/x^4 - 1/4*a^2/x^4
```

Fricas [A]
time $= 0.36$, size $= 173$, normalized size $= 1.48$

$$\frac{32\,b^2c^4x^4\log\left(x\right)+4\left(3\,ab-4\,b^2\right)c^4x^4\log\left(cx+1\right)-4\left(3\,ab+4\,b^2\right)c^4x^4\log\left(cx-1\right)-24\,abc^3x^3-4\,b^2c^2x^2-8\,abcx+3\left(b^2c^4x^4-b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)^2-12\,a^2-4\left(3\,b^2c^3x^3+b^2cx+3\,ab\right)\log\left(-\frac{cx+1}{cx-1}\right)}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/48*(32*b^2*c^4*x^4*log(x) + 4*(3*a*b - 4*b^2)*c^4*x^4*log(c*x + 1) - 4*(3
*a*b + 4*b^2)*c^4*x^4*log(c*x - 1) - 24*a*b*c^3*x^3 - 4*b^2*c^2*x^2 - 8*a*b
*c*x + 3*(b^2*c^4*x^4 - b^2)*log(-(c*x + 1)/(c*x - 1))^2 - 12*a^2 - 4*(3*b^
2*c^3*x^3 + b^2*c*x + 3*a*b)*log(-(c*x + 1)/(c*x - 1)))/x^4
```

Sympy [A]
time $= 0.57$, size $= 184$, normalized size $= 1.57$

$$\begin{cases} -\frac{a^2}{4x^4}+\frac{abc^4\operatorname{atanh}\left(cx\right)}{2}-\frac{abc^3}{2x}-\frac{abc}{6x^3}-\frac{ab\operatorname{atanh}\left(cx\right)}{2x^4}+\frac{2b^2c^4\log\left(x\right)}{3}-\frac{2b^2c^4\log\left(x-\frac{1}{c}\right)}{3}+\frac{b^2c^4\operatorname{atanh}^2\left(cx\right)}{4}-\frac{2b^2c^4\operatorname{atanh}\left(cx\right)}{3}-\frac{b^2c^3\operatorname{atanh}\left(cx\right)}{2x}-\frac{b^2c^2}{12x^2}-\frac{b^2c\operatorname{atanh}\left(cx\right)}{6x^3}-\frac{b^2\operatorname{atanh}^2\left(cx\right)}{4x^4} & \text{for}\: c \neq 0 \\ -\frac{a^2}{4x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**5,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) + a*b*c**4*atanh(c*x)/2 - a*b*c**3/(2*x) - a*b*c/
(6*x**3) - a*b*atanh(c*x)/(2*x**4) + 2*b**2*c**4*log(x)/3 - 2*b**2*c**4*log
(x - 1/c)/3 + b**2*c**4*atanh(c*x)**2/4 - 2*b**2*c**4*atanh(c*x)/3 - b**2*c
**3*atanh(c*x)/(2*x) - b**2*c**2/(12*x**2) - b**2*c*atanh(c*x)/(6*x**3) - b
**2*atanh(c*x)**2/(4*x**4), Ne(c, 0)), (-a**2/(4*x**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(103) = 206$.
time = 0.42, size = 612, normalized size = 5.23

$$\frac{1}{6}\left(4\,b^2c^3\log\left(-\frac{cx+1}{cx-1}-1\right)-4\,b^2c^3\log\left(-\frac{cx+1}{cx-1}\right)+\frac{3\left(\frac{(cx+1)^3b^2c^3}{(cx-1)^3}+\frac{(cx+1)b^2c^3}{cx-1}\right)\log\left(-\frac{cx+1}{cx-1}\right)^2}{\frac{(cx+1)^4}{(cx-1)^4}+\frac{4\,(cx+1)^3}{(cx-1)^3}+\frac{6\,(cx+1)^2}{(cx-1)^2}+\frac{4\,(cx+1)}{cx-1}+1}+\frac{2\left(\frac{6\,(cx+1)^3abc^3}{(cx-1)^3}+\frac{6\,(cx+1)abc^3}{cx-1}+\frac{3\,(cx+1)^3b^2c^3}{(cx-1)^3}+\frac{6\,(cx+1)^2b^2c^3}{(cx-1)^2}+\frac{5\,(cx+1)b^2c^3}{cx-1}+2\,b^2c^3\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4}{(cx-1)^4}+\frac{4\,(cx+1)^3}{(cx-1)^3}+\frac{6\,(cx+1)^2}{(cx-1)^2}+\frac{4\,(cx+1)}{cx-1}+1}+\frac{2\left(\frac{6\,(cx+1)^3a^2c^3}{(cx-1)^3}+\frac{6\,(cx+1)a^2c^3}{cx-1}+\frac{6\,(cx+1)^3abc^3}{(cx-1)^3}+\frac{12\,(cx+1)^2abc^3}{(cx-1)^2}+\frac{10\,(cx+1)abc^3}{cx-1}+4\,abc^3+\frac{(cx+1)^3b^2c^3}{(cx-1)^3}+\frac{2\,(cx+1)^2b^2c^3}{(cx-1)^2}+\frac{(cx+1)b^2c^3}{cx-1}\right)}{\frac{(cx+1)^4}{(cx-1)^4}+\frac{4\,(cx+1)^3}{(cx-1)^3}+\frac{6\,(cx+1)^2}{(cx-1)^2}+\frac{4\,(cx+1)}{cx-1}+1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] 1/6*(4*b^2*c^3*log(-(c*x + 1)/(c*x - 1) - 1) - 4*b^2*c^3*log(-(c*x + 1)/(c*
x - 1)) + 3*((c*x + 1)^3*b^2*c^3/(c*x - 1)^3 + (c*x + 1)*b^2*c^3/(c*x - 1))
*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x
- 1)^3 + 6*(c*x + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1) + 2*(6*(c*x
 + 1)^3*a*b*c^3/(c*x - 1)^3 + 6*(c*x + 1)*a*b*c^3/(c*x - 1) + 3*(c*x + 1)^3
*b^2*c^3/(c*x - 1)^3 + 6*(c*x + 1)^2*b^2*c^3/(c*x - 1)^2 + 5*(c*x + 1)*b^2*
c^3/(c*x - 1) + 2*b^2*c^3)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^4/(c*x - 1)
^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c
*x - 1) + 1) + 2*(6*(c*x + 1)^3*a^2*c^3/(c*x - 1)^3 + 6*(c*x + 1)*a^2*c^3/(
c*x - 1) + 6*(c*x + 1)^3*a*b*c^3/(c*x - 1)^3 + 12*(c*x + 1)^2*a*b*c^3/(c*x
- 1)^2 + 10*(c*x + 1)*a*b*c^3/(c*x - 1) + 4*a*b*c^3 + (c*x + 1)^3*b^2*c^3/(
c*x - 1)^3 + 2*(c*x + 1)^2*b^2*c^3/(c*x - 1)^2 + (c*x + 1)*b^2*c^3/(c*x - 1
))/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x + 1)^2/(c*
x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1))*c
```

Mupad [B]
time = 1.90, size = 303, normalized size = 2.59

$$\frac{b^2\,c^4\ln\left(c\,x+1\right)^2}{16}-\frac{a^2}{4\,x^4}+\frac{b^2\,c^4\ln\left(1-c\,x\right)^2}{16}-\frac{b^2\ln\left(c\,x+1\right)^2}{16\,x^4}-\frac{b^2\ln\left(1-c\,x\right)^2}{16\,x^4}-\frac{b^2\,c^2}{12\,x^2}+\frac{2\,b^2\,c^4\,\ln\left(x\right)}{3}-\frac{b^2\,c^4\,\ln\left(c\,x-1\right)}{3}-\frac{b^2\,c^4\,\ln\left(c\,x+1\right)}{3}-\frac{a\,b\,\ln\left(c\,x+1\right)}{4\,x^4}+\frac{a\,b\,\ln\left(1-c\,x\right)}{4\,x^4}+\frac{b^2\,\ln\left(c\,x+1\right)\,\ln\left(1-c\,x\right)}{8\,x^4}-\frac{a\,b\,c}{6\,x^3}-\frac{b^2\,c\,\ln\left(c\,x+1\right)}{12\,x^3}+\frac{b^2\,c\,\ln\left(1-c\,x\right)}{12\,x^3}-\frac{a\,b\,c^3}{2\,x}-\frac{b^2\,c^3\,\ln\left(c\,x+1\right)}{4\,x}+\frac{b^2\,c^3\,\ln\left(1-c\,x\right)}{4\,x}-\frac{a\,b\,c^4\,\ln\left(c\,x-1\right)}{4}+\frac{a\,b\,c^4\,\ln\left(c\,x+1\right)}{4}-\frac{b^2\,c^4\,\ln\left(c\,x+1\right)\,\ln\left(1-c\,x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/x^5,x)
```

```
[Out] (b^2*c^4*log(c*x + 1)^2)/16 - a^2/(4*x^4) + (b^2*c^4*log(1 - c*x)^2)/16 - (
b^2*log(c*x + 1)^2)/(16*x^4) - (b^2*log(1 - c*x)^2)/(16*x^4) - (b^2*c^2)/(1
2*x^2) + (2*b^2*c^4*log(x))/3 - (b^2*c^4*log(c*x - 1))/3 - (b^2*c^4*log(c*x
 + 1))/3 - (a*b*log(c*x + 1))/(4*x^4) + (a*b*log(1 - c*x))/(4*x^4) + (b^2*l
```

```
og(c*x + 1)*log(1 - c*x))/(8*x^4) - (a*b*c)/(6*x^3) - (b^2*c*log(c*x + 1))/
(12*x^3) + (b^2*c*log(1 - c*x))/(12*x^3) - (a*b*c^3)/(2*x) - (b^2*c^3*log(c
*x + 1))/(4*x) + (b^2*c^3*log(1 - c*x))/(4*x) - (a*b*c^4*log(c*x - 1))/4 +
(a*b*c^4*log(c*x + 1))/4 - (b^2*c^4*log(c*x + 1)*log(1 - c*x))/8
```

3.24 $\int x^5 \left(a+b\tanh^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=247

$$\frac{19b^3x}{60c^5}+\frac{b^3x^3}{60c^3}-\frac{19b^3\tanh^{-1}(cx)}{60c^6}+\frac{4b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4\left(a+b\tanh^{-1}(cx)\right)}{20c^2}+\frac{23b\left(a+b\tanh^{-1}(cx)\right)^2}{30c^6}$$

```
[Out] 19/60*b^3*x/c^5+1/60*b^3*x^3/c^3-19/60*b^3*arctanh(c*x)/c^6+4/15*b^2*x^2*(a
+b*arctanh(c*x))/c^4+1/20*b^2*x^4*(a+b*arctanh(c*x))/c^2+23/30*b*(a+b*arcta
nh(c*x))^2/c^6+1/2*b*x*(a+b*arctanh(c*x))^2/c^5+1/6*b*x^3*(a+b*arctanh(c*x)
)^2/c^3+1/10*b*x^5*(a+b*arctanh(c*x))^2/c-1/6*(a+b*arctanh(c*x))^3/c^6+1/6*
x^6*(a+b*arctanh(c*x))^3-23/15*b^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^6-23
/30*b^3*polylog(2,1-2/(-c*x+1))/c^6
```

Rubi [A]
time = 0.67, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6037, 6127, 308, 212, 327, 6131, 6055, 2449, 2352, 6021, 6095}

$$-\frac{23b^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{15c^6}+\frac{4b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4\left(a+b\tanh^{-1}(cx)\right)}{20c^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{6c^6}+\frac{23b\left(a+b\tanh^{-1}(cx)\right)^2}{30c^6}+\frac{bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c^5}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)^2}{6c^3}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^3+\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)^2}{10c}-\frac{23b^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{30c^6}-\frac{19b^3\tanh^{-1}(cx)}{60c^6}+\frac{19b^3x}{60c^5}+\frac{b^3x^3}{60c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (19*b^3*x)/(60*c^5) + (b^3*x^3)/(60*c^3) - (19*b^3*ArcTanh[c*x])/(60*c^6) +
 (4*b^2*x^2*(a + b*ArcTanh[c*x]))/(15*c^4) + (b^2*x^4*(a + b*ArcTanh[c*x]))
/(20*c^2) + (23*b*(a + b*ArcTanh[c*x])^2)/(30*c^6) + (b*x*(a + b*ArcTanh[c*
x])^2)/(2*c^5) + (b*x^3*(a + b*ArcTanh[c*x])^2)/(6*c^3) + (b*x^5*(a + b*Arc
Tanh[c*x])^2)/(10*c) - (a + b*ArcTanh[c*x])^3/(6*c^6) + (x^6*(a + b*ArcTanh
[c*x])^3)/6 - (23*b^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(15*c^6) - (23
*b^3*PolyLog[2, 1 - 2/(1 - c*x)])/(30*c^6)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^5\left(a+b\tanh^{-1}(cx)\right)^3\,dx=\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^3-\frac{1}{2}(bc)\int\frac{x^6\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx$$

$$=\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^3+\frac{b\int x^4\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{2c}-\frac{b\int\frac{x^4\left(a+b\tanh^{-1}(cx)\right.}{1-c^2x^2}}{2c}$$

$$=\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)^2}{10c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^3-\frac{1}{5}b^2\int\frac{x^5\left(a+b\tanh^{-1}(\right.}{1-c^2x^2}$$

$$=\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)^2}{6c^3}+\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)^2}{10c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^3$$

$$=\frac{b^2x^4\left(a+b\tanh^{-1}(cx)\right)}{20c^2}+\frac{bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c^5}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)^2}{6c^3}$$

$$=\frac{4b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4\left(a+b\tanh^{-1}(cx)\right)}{20c^2}+\frac{23b\left(a+b\tanh^{-1}(cx)\right)}{30c^6}$$

$$=\frac{19b^3x}{60c^5}+\frac{b^3x^3}{60c^3}+\frac{4b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4\left(a+b\tanh^{-1}(cx)\right)}{20c^2}+\frac{23b(a}{}$$

$$=\frac{19b^3x}{60c^5}+\frac{b^3x^3}{60c^3}-\frac{19b^3\tanh^{-1}(cx)}{60c^6}+\frac{4b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4\left(a+b\tan\right.}{20c^2}$$

$$=\frac{19b^3x}{60c^5}+\frac{b^3x^3}{60c^3}-\frac{19b^3\tanh^{-1}(cx)}{60c^6}+\frac{4b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4\left(a+b\tan\right.}{20c^2}$$

Mathematica [A]
time = 0.51, size = 305, normalized size = 1.23

$$\frac{-19ab^2+30a^2bcx+19b^3cx+16ab^2c^2x^2+10a^2bc^3x^3+b^3c^3x^3+3ab^2c^4x^4+6a^2bc^5x^5+10a^3c^6x^6+2b^2(b(-23+15cx+5c^3x^3+3c^5x^5)+15a(-1+c^6x^6))\tanh^{-1}(cx)^2+10b^3(-1+c^6x^6)\tanh^{-1}(cx)^3+b\tanh^{-1}(cx)\left(30a^2c^6x^6+4abcx(15+5c^2x^2+3c^4x^4)+b^2(-19+16c^2x^2+3c^4x^4)-92b^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+15a^2b\log(1-cx)-15a^2b\log(1+cx)+46ab^2\log(1-c^2x^2)+46b^3\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)}{60c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (-19*a*b^2 + 30*a^2*b*c*x + 19*b^3*c*x + 16*a*b^2*c^2*x^2 + 10*a^2*b*c^3*x^
3 + b^3*c^3*x^3 + 3*a*b^2*c^4*x^4 + 6*a^2*b*c^5*x^5 + 10*a^3*c^6*x^6 + 2*b^
2*(b*(-23 + 15*c*x + 5*c^3*x^3 + 3*c^5*x^5) + 15*a*(-1 + c^6*x^6))*ArcTanh[
c*x]^2 + 10*b^3*(-1 + c^6*x^6)*ArcTanh[c*x]^3 + b*ArcTanh[c*x]*(30*a^2*c^6*
x^6 + 4*a*b*c*x*(15 + 5*c^2*x^2 + 3*c^4*x^4) + b^2*(-19 + 16*c^2*x^2 + 3*c^
4*x^4) - 92*b^2*Log[1 + E^(-2*ArcTanh[c*x])]) + 15*a^2*b*Log[1 - c*x] - 15*
a^2*b*Log[1 + c*x] + 46*a*b^2*Log[1 - c^2*x^2] + 46*b^3*PolyLog[2, -E^(-2*A
rcTanh[c*x])])/(60*c^6)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.72, size = 1242, normalized size = 5.03

method	result	size
derivativedivides	Expression too large to display	1242
default	Expression too large to display	1242
risch	Expression too large to display	1362

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^6*(1/6*c^6*x^6*a^3+1/4*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2
+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-1/3*b^3+1/8*I*b^3*arctanh(c*x)^2
*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+1/8*
I*b^3*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2
/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/8*I*b^3*arctanh(c*x)^2*Pi*csgn
(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x
^2+1)))^2+1/4*a^2*b*ln(c*x-1)-1/4*a^2*b*ln(c*x+1)+1/2*b^3*arctanh(c*x)^2*ln
((c*x+1)/(-c^2*x^2+1)^(1/2))+1/4*b^3*arctanh(c*x)^2*ln(c*x-1)-1/4*b^3*arcta
nh(c*x)^2*ln(c*x+1)-23/15*b^3*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2
))-23/15*b^3*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/8*a*b^2*ln(c
*x-1)^2+1/8*a*b^2*ln(c*x+1)^2+23/30*a*b^2*ln(c*x-1)+23/30*a*b^2*ln(c*x+1)+1
9/60*b^3*c*x+1/60*b^3*c^3*x^3-1/2*a*b^2*arctanh(c*x)*ln(c*x+1)-1/4*a*b^2*ln
(c*x-1)*ln(1/2*c*x+1/2)-1/4*I*b^3*arctanh(c*x)^2*Pi+1/10*a^2*b*c^5*x^5+1/6*
a^2*b*c^3*x^3+1/2*a^2*b*c*x+1/10*b^3*c^5*x^5*arctanh(c*x)^2+1/6*b^3*c^6*x^6
*arctanh(c*x)^3+1/6*b^3*c^3*x^3*arctanh(c*x)^2+1/2*b^3*c*x*arctanh(c*x)^2+4
/15*b^3*arctanh(c*x)*c^2*x^2+1/20*b^3*arctanh(c*x)*c^4*x^4+1/20*a*b^2*c^4*x
^4+4/15*a*b^2*c^2*x^2-1/4*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/4*a*b^2*ln(-1/
2*c*x+1/2)*ln(1/2*c*x+1/2)+1/2*a*b^2*arctanh(c*x)*ln(c*x-1)-19/60*b^3*arcta
nh(c*x)-23/15*b^3*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-23/15*b^3*dilog(1+I
*(c*x+1)/(-c^2*x^2+1)^(1/2))+23/30*b^3*arctanh(c*x)^2-1/6*b^3*arctanh(c*x)^
3-1/8*I*b^3*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*
```

```
x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))
)+1/8*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3+1/2*a^2*b*c^6
*x^6*arctanh(c*x)+1/2*a*b^2*c^6*x^6*arctanh(c*x)^2+1/5*a*b^2*c^5*x^5*arctan
h(c*x)+1/3*a*b^2*c^3*x^3*arctanh(c*x)+a*b^2*c*x*arctanh(c*x)+1/4*I*b^3*arct
anh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/4*I*b^3*arctanh(c*x)^2
*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+1/8*I*b^3*arctanh(c*x)^2*Pi*csgn(I
*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*b^2*x^6*arctanh(c*x)^2 + 1/6*a^3*x^6 + 1/60*(30*x^6*arctanh(c*x) + c*
(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x -
1)/c^7))*a^2*b + 1/120*(4*c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(
c*x + 1)/c^7 + 15*log(c*x - 1)/c^7)*arctanh(c*x) + (6*c^4*x^4 + 32*c^2*x^2
- 2*(15*log(c*x - 1) - 46)*log(c*x + 1) + 15*log(c*x + 1)^2 + 15*log(c*x -
1)^2 + 92*log(c*x - 1))/c^6)*a*b^2 - 1/1728000*(500*c^7*((2*c^4*x^6 + 3*c^2
*x^4 + 6*x^2)/c^11 + 6*log(c^2*x^2 - 1)/c^13) + 728*c^6*(2*(3*c^4*x^5 + 5*c
^2*x^3 + 15*x)/c^11 - 15*log(c*x + 1)/c^12 + 15*log(c*x - 1)/c^12) + 1485*c
^5*((c^2*x^4 + 2*x^2)/c^9 + 2*log(c^2*x^2 - 1)/c^11) - 622080000*c^5*integr
ate(1/3600*x^5*log(c*x + 1)/(c^7*x^2 - c^5), x) + 9750*c^4*(2*(c^2*x^3 + 3*
x)/c^9 - 3*log(c*x + 1)/c^10 + 3*log(c*x - 1)/c^10) - 2700*c^3*(x^2/c^7 + l
og(c^2*x^2 - 1)/c^9) - 1036800000*c^3*integrate(1/3600*x^3*log(c*x + 1)/(c^
7*x^2 - c^5), x) + 227700*c^2*(2*x/c^7 - log(c*x + 1)/c^8 + log(c*x - 1)/c^
8) - 5495040000*c*integrate(1/3600*x*log(c*x + 1)/(c^7*x^2 - c^5), x) + (10
00*(36*log(-c*x + 1)^3 - 18*log(-c*x + 1)^2 + 6*log(-c*x + 1) - 1)*(c*x - 1
)^6 + 1728*(125*log(-c*x + 1)^3 - 75*log(-c*x + 1)^2 + 30*log(-c*x + 1) - 6
)*(c*x - 1)^5 + 16875*(32*log(-c*x + 1)^3 - 24*log(-c*x + 1)^2 + 12*log(-c*
x + 1) - 3)*(c*x - 1)^4 + 80000*(9*log(-c*x + 1)^3 - 9*log(-c*x + 1)^2 + 6*
log(-c*x + 1) - 2)*(c*x - 1)^3 + 135000*(4*log(-c*x + 1)^3 - 6*log(-c*x + 1
)^2 + 6*log(-c*x + 1) - 3)*(c*x - 1)^2 + 216000*(log(-c*x + 1)^3 - 3*log(-c
*x + 1)^2 + 6*log(-c*x + 1) - 6)*(c*x - 1))/c^6 - 60*(600*(c^6*x^6 - 1)*log
(c*x + 1)^3 + 240*(3*c^5*x^5 + 5*c^3*x^3 + 15*c*x)*log(c*x + 1)^2 - 30*(10*
c^6*x^6 - 12*c^5*x^5 + 15*c^4*x^4 - 20*c^3*x^3 + 30*c^2*x^2 - 60*c*x - 60*(
c^6*x^6 - 1)*log(c*x + 1) + 37)*log(-c*x + 1)^2 + (100*c^6*x^6 + 264*c^5*x^
5 - 165*c^4*x^4 + 1140*c^3*x^3 - 1230*c^2*x^2 - 1800*(c^6*x^6 - 1)*log(c*x
+ 1)^2 + 8820*c*x - 480*(3*c^5*x^5 + 5*c^3*x^3 + 15*c*x + 23)*log(c*x + 1))
*log(-c*x + 1))/c^6 + 264600*log(3600*c^7*x^2 - 3600*c^5)/c^6 - 2384640000*
integrate(1/3600*log(c*x + 1)/(c^7*x^2 - c^5), x))*b^3
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^5*arctanh(c*x)^3 + 3*a*b^2*x^5*arctanh(c*x)^2 + 3*a^2*b*x^5*
arctanh(c*x) + a^3*x^5, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5(a+b\operatorname{atanh}(cx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral(x**5*(a + b*atanh(c*x))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*x^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,(a+b\operatorname{atanh}(c\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atanh(c*x))^3,x)
```

```
[Out] int(x^5*(a + b*atanh(c*x))^3, x)
```

3.25 $\int x^4 \left(a+b\tanh^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=262

$$\frac{9ab^2x}{10c^4}+\frac{b^3x^2}{20c^3}+\frac{9b^3x\tanh^{-1}(cx)}{10c^4}+\frac{b^2x^3\left(a+b\tanh^{-1}(cx)\right)}{10c^2}-\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{20c^5}+\frac{3bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{10c^3}+$$

```
[Out] 9/10*a*b^2*x/c^4+1/20*b^3*x^2/c^3+9/10*b^3*x*arctanh(c*x)/c^4+1/10*b^2*x^3*
(a+b*arctanh(c*x))/c^2-9/20*b*(a+b*arctanh(c*x))^2/c^5+3/10*b*x^2*(a+b*arct
anh(c*x))^2/c^3+3/20*b*x^4*(a+b*arctanh(c*x))^2/c+1/5*(a+b*arctanh(c*x))^3/
c^5+1/5*x^5*(a+b*arctanh(c*x))^3-3/5*b*(a+b*arctanh(c*x))^2*ln(2/(-c*x+1))/
c^5+1/2*b^3*ln(-c^2*x^2+1)/c^5-3/5*b^2*(a+b*arctanh(c*x))*polylog(2,1-2/(-c
*x+1))/c^5+3/10*b^3*polylog(3,1-2/(-c*x+1))/c^5
```

Rubi [A]
time = 0.55, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6037, 6127, 272, 45, 6021, 266, 6095, 6131, 6055, 6205, 6745}

$$-\frac{3b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{5c^5}+\frac{9ab^2x}{10c^4}+\frac{b^2x^3\left(a+b\tanh^{-1}(cx)\right)}{10c^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{5c^5}-\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{20c^5}-\frac{3b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{5c^5}+\frac{3bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{10c^3}+\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^3+\frac{3bx^4\left(a+b\tanh^{-1}(cx)\right)^2}{20c}+\frac{3b^3\text{Li}_3\left(1-\frac{2}{1-cx}\right)}{10c^5}+\frac{9b^3x\tanh^{-1}(cx)}{10c^4}+\frac{b^3x^2}{20c^3}+\frac{b^3\log\left(1-c^2x^2\right)}{2c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (9*a*b^2*x)/(10*c^4) + (b^3*x^2)/(20*c^3) + (9*b^3*x*ArcTanh[c*x])/(10*c^4)
 + (b^2*x^3*(a + b*ArcTanh[c*x]))/(10*c^2) - (9*b*(a + b*ArcTanh[c*x])^2)/(
20*c^5) + (3*b*x^2*(a + b*ArcTanh[c*x])^2)/(10*c^3) + (3*b*x^4*(a + b*ArcTa
nh[c*x])^2)/(20*c) + (a + b*ArcTanh[c*x])^3/(5*c^5) + (x^5*(a + b*ArcTanh[c
*x])^3)/5 - (3*b*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/(5*c^5) + (b^3*Lo
g[1 - c^2*x^2])/(2*c^5) - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 -
 c*x)])/(5*c^5) + (3*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/(10*c^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
```

```
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int x^4\left(a+b\tanh^{-1}(cx)\right)^3\,dx &= \frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^3-\frac{1}{5}(3bc)\int\frac{x^5\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx\\
&= \frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^3+\frac{(3b)\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{5c}-\frac{(3b)\int\frac{x^3(a+b\tan}{1-c}}{5c}\\
&= \frac{3bx^4\left(a+b\tanh^{-1}(cx)\right)^2}{20c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^3-\frac{1}{10}\left(3b^2\right)\int\frac{x^4\left(a+b\tanh\right.}{1-c}\\
&= \frac{3bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{10c^3}+\frac{3bx^4\left(a+b\tanh^{-1}(cx)\right)^2}{20c}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{5c^5}-\\
&= \frac{b^2x^3\left(a+b\tanh^{-1}(cx)\right)}{10c^2}+\frac{3bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{10c^3}+\frac{3bx^4\left(a+b\tanh^{-1}(cx)\right.}{20c}\\
&= \frac{9ab^2x}{10c^4}+\frac{b^2x^3\left(a+b\tanh^{-1}(cx)\right)}{10c^2}-\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{20c^5}+\frac{3bx^2\left(a+b\tanh\right.}{10c^3}\\
&= \frac{9ab^2x}{10c^4}+\frac{9b^3x\tanh^{-1}(cx)}{10c^4}+\frac{b^2x^3\left(a+b\tanh^{-1}(cx)\right)}{10c^2}-\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{20c^5}\\
&= \frac{9ab^2x}{10c^4}+\frac{b^3x^2}{20c^3}+\frac{9b^3x\tanh^{-1}(cx)}{10c^4}+\frac{b^2x^3\left(a+b\tanh^{-1}(cx)\right)}{10c^2}-\frac{9b\left(a+b\tanh\right.}{20c^5}
\end{aligned}$$

Mathematica [A]
time = 0.53, size = 383, normalized size = 1.46

$$\frac{-b^3+18ab^2cx+6a^2bc^2x^2+b^3c^2x^2+2ab^2c^3x^3+3a^2bc^4x^4+4a^3c^5x^5-18ab^2\tanh^{-1}(cx)+18b^3cx\tanh^{-1}(cx)+12ab^2c^2x^2\tanh^{-1}(cx)+2b^3c^3x^3\tanh^{-1}(cx)+6ab^2c^4x^4\tanh^{-1}(cx)+12a^2bc^5x^5\tanh^{-1}(cx)-12ab^2\tanh^{-1}(cx)^2-9b^3\tanh^{-1}(cx)^2+6b^3c^2x^2\tanh^{-1}(cx)^2+3b^3c^4x^4\tanh^{-1}(cx)^2+12ab^2c^5x^5\tanh^{-1}(cx)^2-4b^3\tanh^{-1}(cx)^3+4b^3c^5x^5\tanh^{-1}(cx)^3-24ab^2\tanh^{-1}(cx)\log\left(1+e^{-2\tanh^{-1}(cx)}\right)-12b^3\tanh^{-1}(cx)^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)+6a^2b\log\left(1-c^2x^2\right)+10b^3\log\left(1-c^2x^2\right)+12b^2\left(a+b\tanh^{-1}(cx)\right)\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+6b^3\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)}{20c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (-b^3 + 18*a*b^2*c*x + 6*a^2*b*c^2*x^2 + b^3*c^2*x^2 + 2*a*b^2*c^3*x^3 + 3*
a^2*b*c^4*x^4 + 4*a^3*c^5*x^5 - 18*a*b^2*ArcTanh[c*x] + 18*b^3*c*x*ArcTanh[
c*x] + 12*a*b^2*c^2*x^2*ArcTanh[c*x] + 2*b^3*c^3*x^3*ArcTanh[c*x] + 6*a*b^2
*c^4*x^4*ArcTanh[c*x] + 12*a^2*b*c^5*x^5*ArcTanh[c*x] - 12*a*b^2*ArcTanh[c*
x]^2 - 9*b^3*ArcTanh[c*x]^2 + 6*b^3*c^2*x^2*ArcTanh[c*x]^2 + 3*b^3*c^4*x^4*
ArcTanh[c*x]^2 + 12*a*b^2*c^5*x^5*ArcTanh[c*x]^2 - 4*b^3*ArcTanh[c*x]^3 + 4
*b^3*c^5*x^5*ArcTanh[c*x]^3 - 24*a*b^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c
*x])] - 12*b^3*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 6*a^2*b*Log[1
- c^2*x^2] + 10*b^3*Log[1 - c^2*x^2] + 12*b^2*(a + b*ArcTanh[c*x])*PolyLog[
2, -E^(-2*ArcTanh[c*x])] + 6*b^3*PolyLog[3, -E^(-2*ArcTanh[c*x])])/(20*c^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.39, size = 1188, normalized size = 4.53

method	result	size
derivativedivides	Expression too large to display	1188
default	Expression too large to display	1188

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(-3/20*I*b^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*c
sgn(I*(c*x+1)^2/(c^2*x^2-1))-1/20*b^3+1/5*c^5*x^5*b^3*arctanh(c*x)^3+3/20*a
^2*b*c^4*x^4+3/10*a^2*b*c^2*x^2+3/20*b^3*c^4*x^4*arctanh(c*x)^2+3/10*b^3*ar
ctanh(c*x)^2*c^2*x^2+1/10*b^3*arctanh(c*x)*c^3*x^3+9/10*b^3*arctanh(c*x)*c*
x-3/10*I*b^3*Pi*arctanh(c*x)^2+3/20*I*b^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^
2/(c^2*x^2-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2
-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))-3/5*a*b^2*dilog(1/2*c*x+1/2)-3/5*b^3*arctan
h(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-3/5*b^3*ln(2)*arctanh(c*x)^2+3/5*
c^5*x^5*a*b^2*arctanh(c*x)^2+3/5*c^5*x^5*a^2*b*arctanh(c*x)+3/10*a*b^2*c^4*
x^4*arctanh(c*x)+3/5*a*b^2*c^2*x^2*arctanh(c*x)-3/20*I*b^3*Pi*arctanh(c*x)^
2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-3/10*I*b^3*Pi*arctanh(c*x)^2*csgn(I/(1+(c
*x+1)^2/(-c^2*x^2+1)))^3-3/20*I*b^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2
*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+3/10*I*b^3*Pi*arctanh(c*x)^2*csgn(I/(
1+(c*x+1)^2/(-c^2*x^2+1)))^2-b^3*ln(1+(c*x+1)^2/(-c^2*x^2+1))+3/10*b^3*poly
log(3,-(c*x+1)^2/(-c^2*x^2+1))+1/20*c^2*b^3*x^2+3/10*a^2*b*ln(c*x-1)+3/10*a
^2*b*ln(c*x+1)-3/5*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+3/10*b
^3*arctanh(c*x)^2*ln(c*x-1)+3/10*b^3*arctanh(c*x)^2*ln(c*x+1)+3/20*a*b^2*ln
(c*x-1)^2-3/20*a*b^2*ln(c*x+1)^2+9/20*a*b^2*ln(c*x-1)-9/20*a*b^2*ln(c*x+1)+
3/5*a*b^2*arctanh(c*x)*ln(c*x+1)-3/10*a*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)+3/10*
a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/10*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2
```

```
)+3/5*a*b^2*arctanh(c*x)*ln(c*x-1)+1/5*a^3*c^5*x^5+b^3*arctanh(c*x)-9/20*b^
3*arctanh(c*x)^2+1/5*b^3*arctanh(c*x)^3-3/10*I*b^3*Pi*arctanh(c*x)^2*csgn(I
*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+3/20*I*b^3*Pi*
arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(
1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/20*I*b^3*Pi*arctanh(c*x)^2*csgn(I/(1+(c*x+1)
^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^
2+1/10*a*b^2*c^3*x^3+9/10*b^2*c*x*a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/5*a^3*x^5 + 3/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c
^2*x^2 - 1)/c^6))*a^2*b - 1/80*(2*(b^3*c^5*x^5 - b^3)*log(-c*x + 1)^3 - 3*(
4*a*b^2*c^5*x^5 + b^3*c^4*x^4 + 2*b^3*c^2*x^2 + 2*(b^3*c^5*x^5 + b^3)*log(c
*x + 1))*log(-c*x + 1)^2)/c^5 - integrate(-1/40*(5*(b^3*c^5*x^5 - b^3*c^4*x
^4)*log(c*x + 1)^3 + 30*(a*b^2*c^5*x^5 - a*b^2*c^4*x^4)*log(c*x + 1)^2 - 3*
(4*a*b^2*c^5*x^5 + b^3*c^4*x^4 + 2*b^3*c^2*x^2 + 5*(b^3*c^5*x^5 - b^3*c^4*x
^4)*log(c*x + 1)^2 - 2*(10*a*b^2*c^4*x^4 - (10*a*b^2*c^5 + b^3*c^5)*x^5 - b
^3)*log(c*x + 1))*log(-c*x + 1))/(c^5*x - c^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^4*arctanh(c*x)^3 + 3*a*b^2*x^4*arctanh(c*x)^2 + 3*a^2*b*x^4*
arctanh(c*x) + a^3*x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{4}(a+b\operatorname{atanh}(cx))^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral(x**4*(a + b*atanh(c*x))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,(a+b\,\mathrm{atanh}(c\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atanh(c*x))^3,x)
```

```
[Out] int(x^4*(a + b*atanh(c*x))^3, x)
```

3.26 $\int x^3 \left(a+b \tanh^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=185

$$\frac{b^3 x}{4c^3}-\frac{b^3 \tanh^{-1}(cx)}{4c^4}+\frac{b^2 x^2 \left(a+b \tanh^{-1}(cx)\right)}{4c^2}+\frac{b \left(a+b \tanh^{-1}(cx)\right)^2}{c^4}+\frac{3bx \left(a+b \tanh^{-1}(cx)\right)^2}{4c^3}+\frac{bx^3 \left(a+b \tan\right.}{4c}$$

```
[Out] 1/4*b^3*x/c^3-1/4*b^3*arctanh(c*x)/c^4+1/4*b^2*x^2*(a+b*arctanh(c*x))/c^2+b
*(a+b*arctanh(c*x))^2/c^4+3/4*b*x*(a+b*arctanh(c*x))^2/c^3+1/4*b*x^3*(a+b*a
rctanh(c*x))^2/c-1/4*(a+b*arctanh(c*x))^3/c^4+1/4*x^4*(a+b*arctanh(c*x))^3-
2*b^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^4-b^3*polylog(2,1-2/(-c*x+1))/c^4
```

Rubi [A]
time = 0.42, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6037, 6127, 327, 212, 6131, 6055, 2449, 2352, 6021, 6095}

$$-\frac{2b^2 \log \left(\frac{2}{1-cx}\right) \left(a+b \tanh^{-1}(cx)\right)}{c^4}+\frac{b^2 x^2 \left(a+b \tanh^{-1}(cx)\right)}{4c^2}-\frac{\left(a+b \tanh^{-1}(cx)\right)^3}{4c^4}+\frac{b \left(a+b \tanh^{-1}(cx)\right)^2}{c^4}+\frac{3bx \left(a+b \tanh^{-1}(cx)\right)^2}{4c^3}+\frac{1}{4}x^4 \left(a+b \tanh^{-1}(cx)\right)^3+\frac{bx^3 \left(a+b \tanh^{-1}(cx)\right)^2}{4c}-\frac{b^3 \text{Li}_2\left(1-\frac{2}{1-cx}\right)}{c^4}-\frac{b^3 \tanh^{-1}(cx)}{4c^4}+\frac{b^3 x}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (b^3*x)/(4*c^3) - (b^3*ArcTanh[c*x])/(4*c^4) + (b^2*x^2*(a + b*ArcTanh[c*x]
))/(4*c^2) + (b*(a + b*ArcTanh[c*x])^2)/c^4 + (3*b*x*(a + b*ArcTanh[c*x])^2
)/(4*c^3) + (b*x^3*(a + b*ArcTanh[c*x])^2)/(4*c) - (a + b*ArcTanh[c*x])^3/(
4*c^4) + (x^4*(a + b*ArcTanh[c*x])^3)/4 - (2*b^2*(a + b*ArcTanh[c*x])*Log[2
/(1 - c*x)])/c^4 - (b^3*PolyLog[2, 1 - 2/(1 - c*x)])/c^4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
```

```
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(a+b\tanh^{-1}(cx)\right)^3\,dx &= \frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^3-\frac{1}{4}(3bc)\int\frac{x^4\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx\\
&= \frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^3+\frac{(3b)\int x^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{4c}-\frac{(3b)\int\frac{x^2(a+b\tan}{1-c}}{4c}\\
&= \frac{bx^3\left(a+b\tanh^{-1}(cx)\right)^2}{4c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^3-\frac{1}{2}b^2\int\frac{x^3\left(a+b\tanh^{-1}(\right.}{1-c^2x^2}\\
&= \frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{4c^3}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)^2}{4c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{4c^4}+\\
&= \frac{b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{c^4}+\frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{4c^3}+\\
&= \frac{b^3x}{4c^3}+\frac{b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{c^4}+\frac{3bx\left(a+b\tanh^{-1}(c\right.}{4c^3}\\
&= \frac{b^3x}{4c^3}-\frac{b^3\tanh^{-1}(cx)}{4c^4}+\frac{b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{c^4}+\frac{3b}{}\\
&= \frac{b^3x}{4c^3}-\frac{b^3\tanh^{-1}(cx)}{4c^4}+\frac{b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{c^4}+\frac{3b}{}
\end{aligned}
$$

Mathematica [A]
time = 0.35, size = 245, normalized size = 1.32

$$
\frac{-2ab^2+6a^2bcx+2b^3cx+2ab^2c^2x^2+2a^2bc^3x^3+2a^3c^4x^4+2b^2(b(-4+3cx+c^3x^3)+3a(-1+c^4x^4))\tanh^{-1}(cx)^2+2b^3(-1+c^4x^4)\tanh^{-1}(cx)^3+2b\tanh^{-1}(cx)\left(3a^2c^4x^4+b^2(-1+c^2x^2)+2abcx(3+c^2x^2)-8b^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+3a^2b\log(1-cx)-3a^2b\log(1+cx)+8ab^2\log\left(1-c^2x^2\right)+8b^3\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)}{8c^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (-2*a*b^2 + 6*a^2*b*c*x + 2*b^3*c*x + 2*a*b^2*c^2*x^2 + 2*a^2*b*c^3*x^3 + 2
*a^3*c^4*x^4 + 2*b^2*(b*(-4 + 3*c*x + c^3*x^3) + 3*a*(-1 + c^4*x^4))*ArcTan
h[c*x]^2 + 2*b^3*(-1 + c^4*x^4)*ArcTanh[c*x]^3 + 2*b*ArcTanh[c*x]*(3*a^2*c^
4*x^4 + b^2*(-1 + c^2*x^2) + 2*a*b*c*x*(3 + c^2*x^2) - 8*b^2*Log[1 + E^(-2*
ArcTanh[c*x])]) + 3*a^2*b*Log[1 - c*x] - 3*a^2*b*Log[1 + c*x] + 8*a*b^2*Log
[1 - c^2*x^2] + 8*b^3*PolyLog[2, -E^(-2*ArcTanh[c*x])])/(8*c^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.26, size = 1157, normalized size = 6.25

method	result
risch	$\frac{b^3x}{4c^3}+\frac{3\ln(-cx+1)^2ab^2x^4}{16}-\frac{3\ln(-cx+1)a^2bx^4}{8}-\frac{b^3\ln(\frac{cx}{2}+\frac{1}{2})\ln(-cx+1)}{c^4}+\frac{b^2\ln(-cx-1)a}{c^4}-\frac{3b^3(-cx+1)^4}{128}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/4*c^4*x^4*a^3+3/16*I*b^3*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c
^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/4*
b^3+3/4*a*b^2*c^4*x^4*arctanh(c*x)^2+3/4*c^4*x^4*a^2*b*arctanh(c*x)+3/16*I*
b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1
)))^3+3/16*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3+3/8*I*b^
3*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/8*I*b^3*arctanh(
c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-3/16*I*b^3*arctanh(c*x)^2*Pi
*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c
*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))+3/8*a^2*b*ln(c*x-1)-3/8*a^2
*b*ln(c*x+1)+3/4*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+3/8*b^3*
arctanh(c*x)^2*ln(c*x-1)-3/8*b^3*arctanh(c*x)^2*ln(c*x+1)-2*b^3*arctanh(c*x
)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^3*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^
2*x^2+1)^(1/2))+3/16*a*b^2*ln(c*x-1)^2+3/16*a*b^2*ln(c*x+1)^2+a*b^2*ln(c*x-
1)+a*b^2*ln(c*x+1)+1/4*b^3*c*x-3/4*a*b^2*arctanh(c*x)*ln(c*x+1)-3/8*a*b^2*l
n(c*x-1)*ln(1/2*c*x+1/2)+1/4*a^2*b*c^3*x^3+3/4*a^2*b*c*x+1/4*b^3*c^3*x^3*ar
ctanh(c*x)^2+3/4*b^3*c*x*arctanh(c*x)^2+1/4*b^3*arctanh(c*x)*c^2*x^2+1/4*a*
b^2*c^2*x^2-3/8*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+3/8*a*b^2*ln(-1/2*c*x+1/2)
*ln(1/2*c*x+1/2)+3/4*a*b^2*arctanh(c*x)*ln(c*x-1)-1/4*b^3*arctanh(c*x)-2*b^
3*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^3*dilog(1+I*(c*x+1)/(-c^2*x^2+1
)^(1/2))+b^3*arctanh(c*x)^2-1/4*b^3*arctanh(c*x)^3-3/8*I*b^3*arctanh(c*x)^2
*Pi+1/4*c^4*x^4*b^3*arctanh(c*x)^3-3/16*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x
+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))
^2+3/8*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c
*x+1)^2/(c^2*x^2-1))^2+3/16*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^
2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+1/2*a*b^2*c^3*x^3*arctanh(c*x)+
3/2*a*b^2*c*x*arctanh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arctanh(c*x)^2 + 1/4*a^3*x^4 + 1/8*(6*x^4*arctanh(c*x) + c*(2
*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a^2*b + 1/
16*(4*c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*a
rctanh(c*x) + (4*c^2*x^2 - 2*(3*log(c*x - 1) - 8)*log(c*x + 1) + 3*log(c*x
+ 1)^2 + 3*log(c*x - 1)^2 + 16*log(c*x - 1))/c^4)*a*b^2 - 1/9216*(27*c^5*((
c^2*x^4 + 2*x^2)/c^7 + 2*log(c^2*x^2 - 1)/c^9) + 74*c^4*(2*(c^2*x^3 + 3*x)/
c^7 - 3*log(c*x + 1)/c^8 + 3*log(c*x - 1)/c^8) + 60*c^3*(x^2/c^5 + log(c^2*
x^2 - 1)/c^7) - 221184*c^3*integrate(1/96*x^3*log(c*x + 1)/(c^5*x^2 - c^3),
 x) + 1692*c^2*(2*x/c^5 - log(c*x + 1)/c^6 + log(c*x - 1)/c^6) - 1105920*c*
integrate(1/96*x*log(c*x + 1)/(c^5*x^2 - c^3), x) + (9*(32*log(-c*x + 1)^3
- 24*log(-c*x + 1)^2 + 12*log(-c*x + 1) - 3)*(c*x - 1)^4 + 128*(9*log(-c*x
+ 1)^3 - 9*log(-c*x + 1)^2 + 6*log(-c*x + 1) - 2)*(c*x - 1)^3 + 432*(4*log(
-c*x + 1)^3 - 6*log(-c*x + 1)^2 + 6*log(-c*x + 1) - 3)*(c*x - 1)^2 + 1152*(
log(-c*x + 1)^3 - 3*log(-c*x + 1)^2 + 6*log(-c*x + 1) - 6)*(c*x - 1))/c^4 -
 12*(24*(c^4*x^4 - 1)*log(c*x + 1)^3 + 48*(c^3*x^3 + 3*c*x)*log(c*x + 1)^2
- 6*(3*c^4*x^4 - 4*c^3*x^3 + 6*c^2*x^2 - 12*c*x - 12*(c^4*x^4 - 1)*log(c*x
+ 1) + 7)*log(-c*x + 1)^2 + (9*c^4*x^4 + 28*c^3*x^3 - 18*c^2*x^2 - 72*(c^4*
x^4 - 1)*log(c*x + 1)^2 + 300*c*x - 96*(c^3*x^3 + 3*c*x + 4)*log(c*x + 1))*
log(-c*x + 1))/c^4 + 1800*log(96*c^5*x^2 - 96*c^3)/c^4 - 442368*integrate(1
/96*log(c*x + 1)/(c^5*x^2 - c^3), x))*b^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctanh(c*x)^3 + 3*a*b^2*x^3*arctanh(c*x)^2 + 3*a^2*b*x^3*
arctanh(c*x) + a^3*x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a + b\operatorname{atanh}(cx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral(x**3*(a + b*atanh(c*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,(a+b\,\mathrm{atanh}(c\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))^3,x)
```

```
[Out] int(x^3*(a + b*atanh(c*x))^3, x)
```

3.27 $\int x^2\left(a+b\tanh^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=197

$$\frac{ab^2x}{c^2}+\frac{b^3x\tanh^{-1}(cx)}{c^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3c^3}+\frac{1}{3}x^3\left(a+b\tan\right.$$

```
[Out] a*b^2*x/c^2+b^3*x*arctanh(c*x)/c^2-1/2*b*(a+b*arctanh(c*x))^2/c^3+1/2*b*x^2
*(a+b*arctanh(c*x))^2/c+1/3*(a+b*arctanh(c*x))^3/c^3+1/3*x^3*(a+b*arctanh(c
*x))^3-b*(a+b*arctanh(c*x))^2*ln(2/(-c*x+1))/c^3+1/2*b^3*ln(-c^2*x^2+1)/c^3
-b^2*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/c^3+1/2*b^3*polylog(3,1-2/(
-c*x+1))/c^3
```

Rubi [A]
time = 0.36, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.643$,
Rules used = {6037, 6127, 6021, 266, 6095, 6131, 6055, 6205, 6745}

$$-\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3}+\frac{ab^2x}{c^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3c^3}-\frac{b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{c^3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^3+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+\frac{b^3\text{Li}_3\left(1-\frac{2}{1-cx}\right)}{2c^3}+\frac{b^3x\tanh^{-1}(cx)}{c^2}+\frac{b^3\log\left(1-c^2x^2\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (a*b^2*x)/c^2 + (b^3*x*ArcTanh[c*x])/c^2 - (b*(a + b*ArcTanh[c*x])^2)/(2*c^
3) + (b*x^2*(a + b*ArcTanh[c*x])^2)/(2*c) + (a + b*ArcTanh[c*x])^3/(3*c^3)
+ (x^3*(a + b*ArcTanh[c*x])^3)/3 - (b*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x
)])/c^3 + (b^3*Log[1 - c^2*x^2])/(2*c^3) - (b^2*(a + b*ArcTanh[c*x])*PolyLo
g[2, 1 - 2/(1 - c*x)])/c^3 + (b^3*PolyLog[3, 1 - 2/(1 - c*x)])/(2*c^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
```

```
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}(cx)\right)^3\,dx = \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^3-(bc)\int\frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx$$

$$= \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^3+\frac{b\int x\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{c}-\frac{b\int\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}}{c}$$

$$= \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3c^3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^3-b$$

$$= \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3c^3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^3-$$

$$= \frac{ab^2x}{c^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tanh^{-1}(cx)\right)}{3c^3}$$

$$= \frac{ab^2x}{c^2}+\frac{b^3x\tanh^{-1}(cx)}{c^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+$$

$$= \frac{ab^2x}{c^2}+\frac{b^3x\tanh^{-1}(cx)}{c^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+$$

Mathematica [A]
time = 0.37, size = 250, normalized size = 1.27

$$\frac{3a^2bc^2x^2+2a^3c^3x^3+6a^2bc^3x^3\tanh^{-1}(cx)+3a^2b\log\left(1-c^2x^2\right)+6ab^2\left(cx+\left(-1+c^3x^3\right)\tanh^{-1}(cx)^2+\tanh^{-1}(cx)\left(-1+c^2x^2-2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\right)+b^3\left(6cx\tanh^{-1}(cx)-3\tanh^{-1}(cx)^2+3c^2x^2\tanh^{-1}(cx)^2-2\tanh^{-1}(cx)^3+2c^3x^3\tanh^{-1}(cx)^3-6\tanh^{-1}(cx)^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)+3\log\left(1-c^2x^2\right)+6\tanh^{-1}(cx)\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+3\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (3*a^2*b*c^2*x^2 + 2*a^3*c^3*x^3 + 6*a^2*b*c^3*x^3*ArcTanh[c*x] + 3*a^2*b*L
og[1 - c^2*x^2] + 6*a*b^2*(c*x + (-1 + c^3*x^3)*ArcTanh[c*x]^2 + ArcTanh[c*
x]*(-1 + c^2*x^2 - 2*Log[1 + E^(-2*ArcTanh[c*x])]) + PolyLog[2, -E^(-2*ArcT
anh[c*x])]) + b^3*(6*c*x*ArcTanh[c*x] - 3*ArcTanh[c*x]^2 + 3*c^2*x^2*ArcTan
h[c*x]^2 - 2*ArcTanh[c*x]^3 + 2*c^3*x^3*ArcTanh[c*x]^3 - 6*ArcTanh[c*x]^2*L
og[1 + E^(-2*ArcTanh[c*x])] + 3*Log[1 - c^2*x^2] + 6*ArcTanh[c*x]*PolyLog[2
, -E^(-2*ArcTanh[c*x])] + 3*PolyLog[3, -E^(-2*ArcTanh[c*x])]))/(6*c^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.21, size = 1093, normalized size = 5.55

method	result	size
derivativedivides	Expression too large to display	1093
default	Expression too large to display	1093

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/2*a^2*b*c^2*x^2+1/2*b^3*arctanh(c*x)^2*c^2*x^2+b^3*arctanh(c*x)*c*
x-a*b^2*dilog(1/2*c*x+1/2)-b^3*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+
1))-b^3*ln(2)*arctanh(c*x)^2+a*b^2*c^2*x^2*arctanh(c*x)-b^3*ln(1+(c*x+1)^2/
(-c^2*x^2+1))+1/2*b^3*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+1/2*a^2*b*ln(c*x-1
)+1/2*a^2*b*ln(c*x+1)-b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+1/2
*b^3*arctanh(c*x)^2*ln(c*x-1)+1/2*b^3*arctanh(c*x)^2*ln(c*x+1)+1/4*a*b^2*ln
(c*x-1)^2-1/4*a*b^2*ln(c*x+1)^2+1/2*a*b^2*ln(c*x-1)-1/2*a*b^2*ln(c*x+1)+a*b
^2*arctanh(c*x)*ln(c*x+1)-1/2*a*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)+1/2*a*b^2*ln(
-1/2*c*x+1/2)*ln(c*x+1)-1/2*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+a*b^2*ar
ctanh(c*x)*ln(c*x-1)+1/4*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-
1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))-1/2*I*b^3*ar
ctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)
^(1/2))-1/4*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c
*x+1)/(-c^2*x^2+1)^(1/2))^2+b^3*arctanh(c*x)-1/2*b^3*arctanh(c*x)^2+1/3*b^3
*arctanh(c*x)^3+a*b^2*c^3*x^3*arctanh(c*x)^2+a^2*b*c^3*x^3*arctanh(c*x)-1/4
*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^
2+1)))^3-1/2*I*b^3*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-1
/4*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3+1/2*I*b^3*arctan
h(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/4*I*b^3*arctanh(c*x)^2*P
i*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I/(1+(c*x
+1)^2/(-c^2*x^2+1)))+1/4*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-
1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c
*x+1)^2/(c^2*x^2-1))-1/2*I*b^3*arctanh(c*x)^2*Pi+1/3*b^3*c^3*x^3*arctanh(c*
x)^3+1/3*a^3*c^3*x^3+b^2*c*x*a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*x^3 + 1/2*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))
*a^2*b - 1/24*((b^3*c^3*x^3 - b^3)*log(-c*x + 1)^3 - 3*(2*a*b^2*c^3*x^3 + b
^3*c^2*x^2 + (b^3*c^3*x^3 + b^3)*log(c*x + 1))*log(-c*x + 1)^2)/c^3 - integ
rate(-1/8*((b^3*c^3*x^3 - b^3*c^2*x^2)*log(c*x + 1)^3 + 6*(a*b^2*c^3*x^3 -
a*b^2*c^2*x^2)*log(c*x + 1)^2 - (4*a*b^2*c^3*x^3 + 2*b^3*c^2*x^2 + 3*(b^3*c
^3*x^3 - b^3*c^2*x^2)*log(c*x + 1)^2 - 2*(6*a*b^2*c^2*x^2 - (6*a*b^2*c^3 +
b^3*c^3)*x^3 - b^3)*log(c*x + 1))*log(-c*x + 1))/(c^3*x - c^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctanh(c*x)^3 + 3*a*b^2*x^2*arctanh(c*x)^2 + 3*a^2*b*x^2*
arctanh(c*x) + a^3*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a + b\,\mathrm{atanh}\,(cx))^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(a+b\,\mathrm{atanh}(c\,x)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))^3,x)
```

```
[Out] int(x^2*(a + b*atanh(c*x))^3, x)
```

3.28 $\int x\left(a+b\tanh^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=123

$$\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^3-\frac{3b^2\left(a+b\right.}{}$$

```
[Out] 3/2*b*(a+b*arctanh(c*x))^2/c^2+3/2*b*x*(a+b*arctanh(c*x))^2/c-1/2*(a+b*arct
anh(c*x))^3/c^2+1/2*x^2*(a+b*arctanh(c*x))^3-3*b^2*(a+b*arctanh(c*x))*ln(2/
(-c*x+1))/c^2-3/2*b^3*polylog(2,1-2/(-c*x+1))/c^2
```

Rubi [A]
time = 0.18, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.667$, Rules used = {6037, 6127, 6021, 6131, 6055, 2449, 2352, 6095}

$$-\frac{3b^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2}+\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^3+\frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{3b^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (3*b*(a + b*ArcTanh[c*x])^2)/(2*c^2) + (3*b*x*(a + b*ArcTanh[c*x])^2)/(2*c)
 - (a + b*ArcTanh[c*x])^3/(2*c^2) + (x^2*(a + b*ArcTanh[c*x])^3)/2 - (3*b^2
*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c^2 - (3*b^3*PolyLog[2, 1 - 2/(1 -
c*x)])/(2*c^2)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}(cx)\right)^3\,dx=\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^3-\frac{1}{2}(3bc)\int\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx$$
$$=\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^3+\frac{(3b)\int\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{2c}-\frac{(3b)\int\frac{(a+b\tanh^{-1}(}{1-c^2x^2}}{2c}$$
$$=\frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^3-$$
$$=\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}$$
$$=\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}$$
$$=\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}$$
$$=\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}$$

Mathematica [A]
time = 0.23, size = 161, normalized size = 1.31

$$\frac{6b^2(-1+cx)(a+b+acx)\tanh^{-1}(cx)^2+2b^3(-1+c^2x^2)\tanh^{-1}(cx)^3+6b\tanh^{-1}(cx)\left(acx(2b+acx)-2b^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+a(6abcx+2a^2c^2x^2+3ab\log(1-cx)-3ab\log(1+cx)+6b^2\log(1-c^2x^2))+6b^3\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (6*b^2*(-1 + c*x)*(a + b + a*c*x)*ArcTanh[c*x]^2 + 2*b^3*(-1 + c^2*x^2)*Arc
Tanh[c*x]^3 + 6*b*ArcTanh[c*x]*(a*c*x*(2*b + a*c*x) - 2*b^2*Log[1 + E^(-2*A
rcTanh[c*x])]) + a*(6*a*b*c*x + 2*a^2*c^2*x^2 + 3*a*b*Log[1 - c*x] - 3*a*b*
Log[1 + c*x] + 6*b^2*Log[1 - c^2*x^2]) + 6*b^3*PolyLog[2, -E^(-2*ArcTanh[c*
x])])/(4*c^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 5831, normalized size = 47.41

method	result
risch	$\frac{3b^2(-cx+1)^2\ln(-cx+1)a}{8c^2}+\frac{3a^2bx}{2c}+\frac{3b^3\ln(-cx+1)^2x}{16c}-\frac{9b^3\ln(-cx+1)x}{16c}-\frac{3a\,b^2\ln(-cx+1)^2}{8c^2}+\frac{9a\,b^2\ln(-cx}{8c^2}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a*b^2*x^2*arctanh(c*x)^2 + 1/2*a^3*x^2 + 3/4*(2*x^2*arctanh(c*x) + c*(2
*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a^2*b + 3/8*(4*c*(2*x/c^2 -
log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*arctanh(c*x) - (2*(log(c*x - 1) - 2)*l
og(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*log(c*x - 1))/c^2)*a*b^2
- 1/64*(3*c^3*(x^2/c^3 + log(c^2*x^2 - 1)/c^5) + 21*c^2*(2*x/c^3 - log(c*x
+ 1)/c^4 + log(c*x - 1)/c^4) - 576*c*integrate(1/4*x*log(c*x + 1)/(c^3*x^2
- c), x) - 2*(12*c*x*log(c*x + 1)^2 + 2*(c^2*x^2 - 1)*log(c*x + 1)^3 - 3*(c
^2*x^2 - 2*c*x - 2*(c^2*x^2 - 1)*log(c*x + 1) + 1)*log(-c*x + 1)^2 + 3*(c^2
*x^2 - 2*(c^2*x^2 - 1)*log(c*x + 1)^2 + 6*c*x - 8*(c*x + 1)*log(c*x + 1))*l
og(-c*x + 1))/c^2 + ((4*log(-c*x + 1)^3 - 6*log(-c*x + 1)^2 + 6*log(-c*x +
1) - 3)*(c*x - 1)^2 + 8*(log(-c*x + 1)^3 - 3*log(-c*x + 1)^2 + 6*log(-c*x +
 1) - 6)*(c*x - 1))/c^2 + 18*log(4*c^3*x^2 - 4*c)/c^2 - 192*integrate(1/4*l
og(c*x + 1)/(c^3*x^2 - c), x))*b^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctanh(c*x)^3 + 3*a*b^2*x*arctanh(c*x)^2 + 3*a^2*b*x*arctan
h(c*x) + a^3*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b\,\mathrm{atanh}\,(cx))^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral(x*(a + b*atanh(c*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))^3,x)
```

```
[Out] int(x*(a + b*atanh(c*x))^3, x)
```

3.29 $\int \left(a+b\tanh^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=108

$$\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c}+x\left(a+b\tanh^{-1}(cx)\right)^3-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-cx}\right)}{c}-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)\text{PolyL}}{c}$$

```
[Out] (a+b*arctanh(c*x))^3/c+x*(a+b*arctanh(c*x))^3-3*b*(a+b*arctanh(c*x))^2*ln(2
/(-c*x+1))/c-3*b^2*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/c+3/2*b^3*pol
ylog(3,1-2/(-c*x+1))/c
```

Rubi [A]
time = 0.16, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6021, 6131, 6055, 6095, 6205, 6745}

$$-\frac{3b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+x\left(a+b\tanh^{-1}(cx)\right)^3+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c}-\frac{3b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{c}+\frac{3b^3\text{Li}_3\left(1-\frac{2}{1-cx}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (a + b*ArcTanh[c*x])^3/c + x*(a + b*ArcTanh[c*x])^3 - (3*b*(a + b*ArcTanh[c
*x])^2*Log[2/(1 - c*x)])/c - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(
1 - c*x)])/c + (3*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/(2*c)
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\tanh^{-1}(cx)\right)^3\,dx &= x\left(a+b\tanh^{-1}(cx)\right)^3-(3bc)\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx\\
&= \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c}+x\left(a+b\tanh^{-1}(cx)\right)^3-(3b)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1-cx}\,dx\\
&= \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c}+x\left(a+b\tanh^{-1}(cx)\right)^3-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-c\right.}}{c}\\
&= \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c}+x\left(a+b\tanh^{-1}(cx)\right)^3-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-c\right.}}{c}\\
&= \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c}+x\left(a+b\tanh^{-1}(cx)\right)^3-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-c\right.}}{c}
\end{aligned}$$

Mathematica [A]
time = 0.31, size = 161, normalized size = 1.49

$$\frac{2a^3cx+6a^2bcx\tanh^{-1}(cx)+3a^2b\log\left(1-c^2x^2\right)+6ab^2\left(\tanh^{-1}(cx)\left((-1+cx)\tanh^{-1}(cx)-2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\right)+b^3\left(2\tanh^{-1}(cx)^2\left((-1+cx)\tanh^{-1}(cx)-3\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+6\tanh^{-1}(cx)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+3\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3,x]
```

[Out] (2*a^3*c*x + 6*a^2*b*c*x*ArcTanh[c*x] + 3*a^2*b*Log[1 - c^2*x^2] + 6*a*b^2*(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log[1 + E^(-2*ArcTanh[c*x])]) + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + b^3*(2*ArcTanh[c*x]^2*((-1 + c*x)*ArcTanh[c*x] - 3*Log[1 + E^(-2*ArcTanh[c*x])]) + 6*ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 3*PolyLog[3, -E^(-2*ArcTanh[c*x])]))/(2*c)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(106) = 212$.
time = 0.24, size = 245, normalized size = 2.27

method	result
derivativedivides	$\frac{a^3cx+b^3cx\operatorname{arctanh}(cx)^3+b^3\operatorname{arctanh}(cx)^3-3b^3\operatorname{arctanh}(cx)^2\ln\left(1+\frac{(cx+1)^2}{-c^2x^2+1}\right)-3b^3\operatorname{arctanh}(cx)\operatorname{polylog}\left(2,-\frac{(cx+1)^2}{-c^2x^2+1}\right)}{}$
default	$\frac{a^3cx+b^3cx\operatorname{arctanh}(cx)^3+b^3\operatorname{arctanh}(cx)^3-3b^3\operatorname{arctanh}(cx)^2\ln\left(1+\frac{(cx+1)^2}{-c^2x^2+1}\right)-3b^3\operatorname{arctanh}(cx)\operatorname{polylog}\left(2,-\frac{(cx+1)^2}{-c^2x^2+1}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)

[Out] 1/c*(a^3*c*x+b^3*c*x*arctanh(c*x)^3+b^3*arctanh(c*x)^3-3*b^3*arctanh(c*x)^2*ln(1+(c*x+1)^2/(-c^2*x^2+1))-3*b^3*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+3/2*b^3*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+3*a*b^2*c*x*arctanh(c*x)^2+3*a*b^2*arctanh(c*x)^2-6*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))*a*b^2-3*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))*a*b^2+3*a^2*b*c*x*arctanh(c*x)+3/2*a^2*b*ln(-c^2*x^2+1))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^3,x, algorithm="maxima")

[Out] a^3*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a^2*b/c - 1/8*((b^3*c*x - b^3)*log(-c*x + 1)^3 - 3*(2*a*b^2*c*x + (b^3*c*x + b^3)*log(c*x + 1))*log(-c*x + 1)^2)/c - integrate(-1/8*((b^3*c*x - b^3)*log(c*x + 1)^3 + 6*(a*b^2*c*x - a*b^2)*log(c*x + 1)^2 - 3*(4*a*b^2*c*x + (b^3*c*x - b^3)*log(c*x + 1)^2 - 2*(2*a*b^2 - b^3 - (2*a*b^2*c + b^3*c)*x)*log(c*x + 1))*log(-c*x + 1))/(c*x - 1), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x)
 + a^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{atanh}\,(cx))^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{atanh}(c\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3,x)
```

```
[Out] int((a + b*atanh(c*x))^3, x)
```

3.30 $\int \frac{(a+b\tanh^{-1}(cx))^3}{x}\,dx$

Optimal. Leaf size=184

$$2(a+b\tanh^{-1}(cx))^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-\frac{3}{2}b(a+b\tanh^{-1}(cx))^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)+\frac{3}{2}b(a+b\tanh$$

```
[Out] -2*(a+b*arctanh(c*x))^3*arctanh(-1+2/(-c*x+1))-3/2*b*(a+b*arctanh(c*x))^2*p
olylog(2,1-2/(-c*x+1))+3/2*b*(a+b*arctanh(c*x))^2*polylog(2,-1+2/(-c*x+1))+
3/2*b^2*(a+b*arctanh(c*x))*polylog(3,1-2/(-c*x+1))-3/2*b^2*(a+b*arctanh(c*x
))*polylog(3,-1+2/(-c*x+1))-3/4*b^3*polylog(4,1-2/(-c*x+1))+3/4*b^3*polylog
(4,-1+2/(-c*x+1))
```

Rubi [A]
time = 0.32, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6033, 6199, 6095, 6205, 6209, 6745}

$$\frac{3}{2}b^2\text{Li}_3\left(1-\frac{2}{1-cx}\right)(a+b\tanh^{-1}(cx))-\frac{3}{2}b^2\text{Li}_3\left(\frac{2}{1-cx}-1\right)(a+b\tanh^{-1}(cx))-\frac{3}{2}b\text{Li}_2\left(1-\frac{2}{1-cx}\right)(a+b\tanh^{-1}(cx))^2+\frac{3}{2}b\text{Li}_2\left(\frac{2}{1-cx}-1\right)(a+b\tanh^{-1}(cx))^2+2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)(a+b\tanh^{-1}(cx))^3-\frac{3}{4}b^3\text{Li}_4\left(1-\frac{2}{1-cx}\right)+\frac{3}{4}b^3\text{Li}_4\left(\frac{2}{1-cx}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/x,x]
```

```
[Out] 2*(a + b*ArcTanh[c*x])^3*ArcTanh[1 - 2/(1 - c*x)] - (3*b*(a + b*ArcTanh[c*x
])^2*PolyLog[2, 1 - 2/(1 - c*x)])/2 + (3*b*(a + b*ArcTanh[c*x])^2*PolyLog[2
, -1 + 2/(1 - c*x)])/2 + (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, 1 - 2/(1 -
c*x)])/2 - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, -1 + 2/(1 - c*x)])/2 - (3
*b^3*PolyLog[4, 1 - 2/(1 - c*x)])/4 + (3*b^3*PolyLog[4, -1 + 2/(1 - c*x)])/
4
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
```

```
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6209

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}\,dx = 2\left(a+b\tanh^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-(6bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2 \tan}{1-c^2x^2}$$

$$= 2\left(a+b\tanh^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)+(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2 \log}{1-c^2x^2}$$

$$= 2\left(a+b\tanh^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-\frac{3}{2}b\left(a+b\tanh^{-1}(cx)\right)^2\mathrm{Li}_2\Big(1-$$

$$= 2\left(a+b\tanh^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-\frac{3}{2}b\left(a+b\tanh^{-1}(cx)\right)^2\mathrm{Li}_2\Big(1-$$

$$= 2\left(a+b\tanh^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-\frac{3}{2}b\left(a+b\tanh^{-1}(cx)\right)^2\mathrm{Li}_2\Big(1-$$

Mathematica [A]
time = 0.15, size = 178, normalized size = 0.97

$$2\left(a+b\tanh^{-1}(cx)\right)^3\tanh^{-1}\left(\frac{1+cx}{-1+cx}\right)+\frac{3}{4}b\left(2\left(a+b\tanh^{-1}(cx)\right)^2\mathrm{PolyLog}\left(2,\frac{1+cx}{1-cx}\right)-2\left(a+b\tanh^{-1}(cx)\right)^2\mathrm{PolyLog}\left(2,\frac{1+cx}{-1+cx}\right)+b\left(-2\left(a+b\tanh^{-1}(cx)\right)\mathrm{PolyLog}\left(3,\frac{1+cx}{1-cx}\right)+2\left(a+b\tanh^{-1}(cx)\right)\mathrm{PolyLog}\left(3,\frac{1+cx}{-1+cx}\right)+b\left(\mathrm{PolyLog}\left(4,\frac{1+cx}{1-cx}\right)-\mathrm{PolyLog}\left(4,\frac{1+cx}{-1+cx}\right)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/x,x]
```

```
[Out] 2*(a + b*ArcTanh[c*x])^3*ArcTanh[(1 + c*x)/(-1 + c*x)] + (3*b*(2*(a + b*Arc
Tanh[c*x])^2*PolyLog[2, (1 + c*x)/(1 - c*x)] - 2*(a + b*ArcTanh[c*x])^2*Pol
yLog[2, (1 + c*x)/(-1 + c*x)] + b*(-2*(a + b*ArcTanh[c*x])*PolyLog[3, (1 +
c*x)/(1 - c*x)] + 2*(a + b*ArcTanh[c*x])*PolyLog[3, (1 + c*x)/(-1 + c*x)] +
 b*(PolyLog[4, (1 + c*x)/(1 - c*x)] - PolyLog[4, (1 + c*x)/(-1 + c*x)]))))/
4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.35, size = 1470, normalized size = 7.99

method	result	size
derivativedivides	Expression too large to display	1470
default	Expression too large to display	1470

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*b^3*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^
2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^3-3/4*b^3*polylog(4,-(c*
x+1)^2/(-c^2*x^2+1))+6*b^3*polylog(4,(c*x+1)/(-c^2*x^2+1)^(1/2))+6*b^3*poly
log(4,-(c*x+1)/(-c^2*x^2+1)^(1/2))+a^3*ln(c*x)+3/2*I*a*b^2*Pi*csgn(I*((c*x+
1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/
(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2+1/2*I*b^3*Pi*csg
n(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^3
-3/2*I*a*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*
x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-3/2*I*a*b^2*Pi*csgn(
I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^
2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+1/2*I*b^3*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+
1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)
/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^3-3/2*a^2*b*dilog(c*x)-3/2*a^2*b*
dilog(c*x+1)+3/2*a*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-6*a*b^2*polylog(3
,(c*x+1)/(-c^2*x^2+1)^(1/2))-6*a*b^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))
+b^3*ln(c*x)*arctanh(c*x)^3-b^3*arctanh(c*x)^3*ln((c*x+1)^2/(-c^2*x^2+1)-1)
-3/2*b^3*arctanh(c*x)^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+3/2*b^3*arctanh(
c*x)*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+b^3*arctanh(c*x)^3*ln(1-(c*x+1)/(-c
^2*x^2+1)^(1/2))+3*b^3*arctanh(c*x)^2*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))
-6*b^3*arctanh(c*x)*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+b^3*arctanh(c*x)^
3*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+3*b^3*arctanh(c*x)^2*polylog(2,-(c*x+1)/
(-c^2*x^2+1)^(1/2))-6*b^3*arctanh(c*x)*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2
))-1/2*I*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*
x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^3+3/2*I*a*b^2*Pi*csgn(
```

```
I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+3
*a*b^2*ln(c*x)*arctanh(c*x)^2-3*a*b^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c
^2*x^2+1))-3*a*b^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+3*a*b^2*arct
anh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+6*a*b^2*arctanh(c*x)*polylog(2,
(c*x+1)/(-c^2*x^2+1)^(1/2))+3*a*b^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1
)^(1/2))+6*a*b^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-3/2*a^
2*b*ln(c*x)*ln(c*x+1)+3*a^2*b*ln(c*x)*arctanh(c*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + integrate(1/8*b^3*(log(c*x + 1) - log(-c*x + 1))^3/x + 3/4*a*b
^2*(log(c*x + 1) - log(-c*x + 1))^2/x + 3/2*a^2*b*(log(c*x + 1) - log(-c*x
+ 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/x,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atanh}(c\,x)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/x,x)
```

```
[Out] int((a + b*atanh(c*x))^3/x, x)
```

3.31 $\int \frac{(a+b\tanh^{-1}(cx))^3}{x^2}\,dx$

Optimal. Leaf size=102

$$c(a+b\tanh^{-1}(cx))^3-\frac{(a+b\tanh^{-1}(cx))^3}{x}+3bc(a+b\tanh^{-1}(cx))^2\log\left(2-\frac{2}{1+cx}\right)-3b^2c(a+b\tanh^{-1}($$

```
[Out] c*(a+b*arctanh(c*x))^3-(a+b*arctanh(c*x))^3/x+3*b*c*(a+b*arctanh(c*x))^2*ln
(2-2/(c*x+1))-3*b^2*c*(a+b*arctanh(c*x))*polylog(2,-1+2/(c*x+1))-3/2*b^3*c*
polylog(3,-1+2/(c*x+1))
```

Rubi [A]
time = 0.19, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6037, 6135, 6079, 6095, 6203, 6745}

$$-3b^2c\text{Li}_2\left(\frac{2}{cx+1}-1\right)(a+b\tanh^{-1}(cx))+c(a+b\tanh^{-1}(cx))^3-\frac{(a+b\tanh^{-1}(cx))^3}{x}+3bc\log\left(2-\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))^2-\frac{3}{2}b^3c\text{Li}_3\left(\frac{2}{cx+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/x^2,x]
```

```
[Out] c*(a + b*ArcTanh[c*x])^3 - (a + b*ArcTanh[c*x])^3/x + 3*b*c*(a + b*ArcTanh[
c*x])^2*Log[2 - 2/(1 + c*x)] - 3*b^2*c*(a + b*ArcTanh[c*x])*PolyLog[2, -1 +
 2/(1 + c*x)] - (3*b^3*c*PolyLog[3, -1 + 2/(1 + c*x)])/2
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^2}\,dx &= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}+(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x\left(1-c^2x^2\right)}\,dx\\
&= c\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}+(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(1+cx)}\,dx\\
&= c\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}+3bc\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\right.\\
&= c\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}+3bc\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\right.\\
&= c\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}+3bc\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\right.
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.24, size = 196, normalized size = 1.92

$$-\frac{a^3}{x}-\frac{3a^2b\tanh^{-1}(cx)}{x}+3a^2bc\log(x)-\frac{3}{2}a^2bc\log\left(1-c^2x^2\right)+3ab^2c\left(\tanh^{-1}(cx)\left(\tanh^{-1}(cx)-\frac{\tanh^{-1}(cx)}{cx}+2\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)-\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)\right)+b^3c\left(\frac{i\pi^3}{8}-\tanh^{-1}(cx)^3-\frac{\tanh^{-1}(cx)^3}{cx}+3\tanh^{-1}(cx)^2\log\left(1-e^{2\tanh^{-1}(cx)}\right)+3\tanh^{-1}(cx)\text{PolyLog}\left(2,e^{2\tanh^{-1}(cx)}\right)-\frac{3}{2}\text{PolyLog}\left(3,e^{2\tanh^{-1}(cx)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/x^2,x]
```

```
[Out] -(a^3/x) - (3*a^2*b*ArcTanh[c*x])/x + 3*a^2*b*c*Log[x] - (3*a^2*b*c*Log[1 -
 c^2*x^2])/2 + 3*a*b^2*c*(ArcTanh[c*x]*(ArcTanh[c*x] - ArcTanh[c*x]/(c*x) +
 2*Log[1 - E^(-2*ArcTanh[c*x])]) - PolyLog[2, E^(-2*ArcTanh[c*x])]) + b^3*c
*((I/8)*Pi^3 - ArcTanh[c*x]^3 - ArcTanh[c*x]^3/(c*x) + 3*ArcTanh[c*x]^2*Log
[1 - E^(2*ArcTanh[c*x])] + 3*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] -
(3*PolyLog[3, E^(2*ArcTanh[c*x])])/2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 1555, normalized size = 15.25

method	result	size
derivativedivides	Expression too large to display	1555
default	Expression too large to display	1555

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-a^3/c/x+3/2*I*b^3*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c
^2*x^2+1)))^3*arctanh(c*x)^2+3/2*I*b^3*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^
2/(-c^2*x^2+1)))^3-3/2*I*b^3*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^
2+1)))^2+3/4*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*c
sgn(I*(c*x+1)^2/(c^2*x^2-1))+3*a*b^2*dilog(1/2*c*x+1/2)+3*b^3*ln(2)*arctanh
(c*x)^2-3/4*I*b^3*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn
(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x
^2+1)))+3/2*I*b^3*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2
/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)
))*arctanh(c*x)^2-3*a*b^2/c/x*arctanh(c*x)^2-3*a^2*b/c/x*arctanh(c*x)+3/4*I
*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3+3/4*I*b^3*arctanh(c*
x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-b^3/c/x*
arctanh(c*x)^3+6*a*b^2*arctanh(c*x)*ln(c*x)-3*a*b^2*ln(c*x)*ln(c*x+1)+3/2*I
*b^3*arctanh(c*x)^2*Pi-3/2*a^2*b*ln(c*x-1)-3/2*a^2*b*ln(c*x+1)+3*b^3*arctan
h(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-3/2*b^3*arctanh(c*x)^2*ln(c*x-1)-3/
2*b^3*arctanh(c*x)^2*ln(c*x+1)-3/4*a*b^2*ln(c*x-1)^2+3/4*a*b^2*ln(c*x+1)^2-
3*a*b^2*arctanh(c*x)*ln(c*x+1)+3/2*a*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)-3/2*a*b^
2*ln(-1/2*c*x+1/2)*ln(c*x+1)+3/2*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-3*a
*b^2*arctanh(c*x)*ln(c*x-1)-3*a*b^2*dilog(c*x)-3*a*b^2*dilog(c*x+1)+3*a^2*b
*ln(c*x)-3*b^3*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+6*b^3*arctanh(c*
x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+3*b^3*arctanh(c*x)^2*ln(1-(c*x+1)/
(-c^2*x^2+1)^(1/2))+6*b^3*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2
))+3*b^3*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+3*b^3*ln(c*x)*arct
anh(c*x)^2-6*b^3*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-6*b^3*polylog(3,(c*
x+1)/(-c^2*x^2+1)^(1/2))-b^3*arctanh(c*x)^3-3/2*I*b^3*Pi*csgn(I/(1+(c*x+1)^
2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1
```

```
)))^2*arctanh(c*x)^2-3/4*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-
1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/2*I*b^3*Pi
*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c
*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+3/2*I*b^3*arctanh(c*x)^2*Pi*csgn(I*
(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+3/4*I*b^3*arcta
nh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1
)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^2,x, algorithm="maxima")
```

```
[Out] -3/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a^2*b - a^3/x - 1
/8*((b^3*c*x - b^3)*log(-c*x + 1)^3 + 3*(2*a*b^2 + (b^3*c*x + b^3)*log(c*x
+ 1))*log(-c*x + 1)^2)/x - integrate(-1/8*((b^3*c*x - b^3)*log(c*x + 1)^3 +
 6*(a*b^2*c*x - a*b^2)*log(c*x + 1)^2 + 3*(4*a*b^2*c*x - (b^3*c*x - b^3)*lo
g(c*x + 1)^2 + 2*(b^3*c^2*x^2 + 2*a*b^2 - (2*a*b^2*c - b^3*c)*x)*log(c*x +
1))*log(-c*x + 1))/(c*x^3 - x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{atanh}(cx))^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/x^2,x)
```

```
[Out] int((a + b*atanh(c*x))^3/x^2, x)
```

3.32 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^3}\,dx$

Optimal. Leaf size=123

$$\frac{3}{2}bc^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2x^2}+3b^2c^2(a+$$

```
[Out] 3/2*b*c^2*(a+b*arctanh(c*x))^2-3/2*b*c*(a+b*arctanh(c*x))^2/x+1/2*c^2*(a+b*
arctanh(c*x))^3-1/2*(a+b*arctanh(c*x))^3/x^2+3*b^2*c^2*(a+b*arctanh(c*x))*l
n(2-2/(c*x+1))-3/2*b^3*c^2*polylog(2,-1+2/(c*x+1))
```

Rubi [A]
time = 0.21, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6037, 6129, 6135, 6079, 2497, 6095}

$$3b^2c^2\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{2}bc^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2x^2}-\frac{3bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x}-\frac{3}{2}b^3c^2\text{Li}_2\left(\frac{2}{cx+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/x^3,x]
```

```
[Out] (3*b*c^2*(a + b*ArcTanh[c*x])^2)/2 - (3*b*c*(a + b*ArcTanh[c*x])^2)/(2*x) +
 (c^2*(a + b*ArcTanh[c*x])^3)/2 - (a + b*ArcTanh[c*x])^3/(2*x^2) + 3*b^2*c^
2*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - (3*b^3*c^2*PolyLog[2, -1 + 2/
(1 + c*x)])/2
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
```

```
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^3}\,dx &= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2x^2}+\frac{1}{2}(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2\left(1-c^2x^2\right)}\,dx \\
&= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2x^2}+\frac{1}{2}(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx+\frac{1}{2}\left(3bc^3\right)\int \frac{(a+ \\
&= -\frac{3bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2x^2}+ \\
&= \frac{3}{2}bc^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^3 \\
&= \frac{3}{2}bc^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^3 \\
&= \frac{3}{2}bc^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^3
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 192, normalized size = 1.56

$$
\frac{6b^2(-1+cx)(a+acx+bcx)\tanh^{-1}(cx)^2+2b^3\left(-1+c^2x^2\right)\tanh^{-1}(cx)^3-6b\tanh^{-1}(cx)\left(a^2+2abcx-2b^2c^2x^2\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)+a\left(-2a^2-6abcx-3abc^2x^2\log(1-cx)+3abc^2x^2\log(1+cx)+12b^2c^2x^2\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)\right)-6b^3c^2x^2\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)}{4x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/x^3,x]
```

```
[Out] (6*b^2*(-1 + c*x)*(a + a*c*x + b*c*x)*ArcTanh[c*x]^2 + 2*b^3*(-1 + c^2*x^2)
*ArcTanh[c*x]^3 - 6*b*ArcTanh[c*x]*(a^2 + 2*a*b*c*x - 2*b^2*c^2*x^2*Log[1 -
 E^(-2*ArcTanh[c*x])]) + a*(-2*a^2 - 6*a*b*c*x - 3*a*b*c^2*x^2*Log[1 - c*x]
 + 3*a*b*c^2*x^2*Log[1 + c*x] + 12*b^2*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]]
) - 6*b^3*c^2*x^2*PolyLog[2, E^(-2*ArcTanh[c*x])])/(4*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.58, size = 4871, normalized size = 39.60

method	result	size
derivativedivides	Expression too large to display	4871
default	Expression too large to display	4871

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a^3/c^2/x^2-3/4*I*b^3*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(
I*(c*x+1)^2/(c^2*x^2-1))^2*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-3/8*I*b^3*ar
ctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+
3/4*I*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*dilog(1+(c*x+1)/(-c^2*x^2
+1)^(1/2))-3/8*I*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2
/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arct
anh(c*x)*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))-3/4*I*b^3*Pi*csgn(I/(1+(c*x+1)^2/
(-c^2*x^2+1)))^3*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))-3/4*I*b^3*Pi*csgn(I/(1+(
c*x+1)^2/(-c^2*x^2+1)))^3*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-3/8*I*b^3*P
i*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*dilog(1+(c*x+1
)/(-c^2*x^2+1)^(1/2))+3/8*I*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*dilog((c
*x+1)/(-c^2*x^2+1)^(1/2))-3/8*I*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*dilo
g(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+3/8*I*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^
3*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+3/8*I*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*
x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))
+3/8*I*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*po
lylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+3/8*I*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-
1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))+3/8*I*b^
3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))
-3/4*I*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*polylog(2,-(c*x+1)/(-c^2
*x^2+1)^(1/2))+3/4*I*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*polylog(2,
(c*x+1)/(-c^2*x^2+1)^(1/2))+3/4*I*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))
^2*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+3/4*I*b^3*Pi*csgn(I/(1+(c*x+1)^2/
(-c^2*x^2+1)))^2*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))-3/4*I*b^3*Pi*csgn(I/(1+(
c*x+1)^2/(-c^2*x^2+1)))^2*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-3/4*I*b^3*Pi*
```

```
arctanh(c*x)*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))-3/4*I*b^3*arctanh(c*x)^2*Pi*c
sgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+3/4*I*b^3*arctanh(c*x)^2*Pi*csgn(I/(1+(
c*x+1)^2/(-c^2*x^2+1)))^3-3/8*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2
*x^2-1))^3-3/2*a^2*b/c^2/x^2*arctanh(c*x)-3/2*a*b^2/c^2/x^2*arctanh(c*x)^2-
3*a*b^2*arctanh(c*x)/c/x-1/2*b^3/c^2/x^2*arctanh(c*x)^3-3/4*I*b^3*Pi*dilog(
(c*x+1)/(-c^2*x^2+1)^(1/2))+3/4*I*b^3*arctanh(c*x)^2*Pi+3/4*I*b^3*Pi*dilog(
1+(c*x+1)/(-c^2*x^2+1)^(1/2))-3/4*I*b^3*Pi*polylog(2,-(c*x+1)/(-c^2*x^2+1)^
(1/2))-3/4*I*b^3*Pi*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-3/4*a^2*b*ln(c*x-
1)+3/4*a^2*b*ln(c*x+1)-3/2*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2)
)-3/4*b^3*arctanh(c*x)^2*ln(c*x-1)+3/4*b^3*arctanh(c*x)^2*ln(c*x+1)-3/8*a*b
^2*ln(c*x-1)^2-3/8*a*b^2*ln(c*x+1)^2-3/2*a*b^2*ln(c*x-1)-3/2*a*b^2*ln(c*x+1
)+3/2*a*b^2*arctanh(c*x)*ln(c*x+1)+3/4*a*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)+3/4*
a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/4*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)
-3/2*a*b^2*arctanh(c*x)*ln(c*x-1)-3/2*b^3*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))
+3/2*b^3*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+3/2*b^3*polylog(2,(c*x+1)/(-c^
2*x^2+1)^(1/2))+3/2*b^3*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-3/2*a^2*b/c/
x-3/2*b^3*arctanh(c*x)^2/c/x-3/2*b^3*arctanh(c*x)^2+1/2*b^3*arctanh(c*x)^3+
3*a*b^2*ln(c*x)+3*b^3*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+3/2*b^3
*arctanh(c*x)*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+3/8*I*b^3*Pi*csgn(I*(c*x+1)^
2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)*ln(1-(c*x+1)/(-c^2
*x^2+1)^(1/2))+3/4*I*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*
x)*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))-3/4*I*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*
x^2+1)))^3*arctanh(c*x)*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))-3/8*I*b^3*Pi*csgn(
I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c
^2*x^2+1)))^2*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-3/8*I*b^3*Pi*csgn(I*(c*x+
1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^
2*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-3/8*I*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*
x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*polylog(
2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+3/8*I*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3
*arctanh(c*x)*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+3/4*I*b^3*Pi*csgn(I*(c*x+1)/
(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*dilog((c*x+1)/(-c^2*x^2
+1)^(1/2))+3/8*I*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2
/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*polylog(2,-(c*x+1)/(-c^2*x^2+1)^
(1/2))+3/4*I*b^3*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^
2*x^2-1))^2*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+3/8*I*b^3*Pi*csgn(I*(c*x
+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))
^2*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-3/8*I*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x
^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*dilog((c*
x+1)/(-c^2*x^2+1)^(1/2))-3/8*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x
^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))-3/4*I*b^3*arctanh(c*x)^2*Pi*cs
gn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-3/8*I*b^3*
Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a^2*b
+ 3/8*((2*(log(c*x - 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2
 - 4*log(c*x - 1) + 8*log(x))*c^2 + 4*(c*log(c*x + 1) - c*log(c*x - 1) - 2/
x)*c*arctanh(c*x))*a*b^2 - 1/16*b^3*(((c^2*x^2 - 1)*log(-c*x + 1)^3 + 3*(2*
c*x - (c^2*x^2 - 1)*log(c*x + 1))*log(-c*x + 1)^2)/x^2 + 2*integrate(-((c*x
 - 1)*log(c*x + 1)^3 + 3*(2*c^2*x^2 - (c*x - 1)*log(c*x + 1)^2 - (c^3*x^3 -
 c*x)*log(c*x + 1))*log(-c*x + 1))/(c*x^4 - x^3), x)) - 3/2*a*b^2*arctanh(c
*x)^2/x^2 - 1/2*a^3/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atanh}\,(cx))^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/x**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atanh}(c\,x))^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/x^3,x)
```

```
[Out] int((a + b*atanh(c*x))^3/x^3, x)
```

3.33 $\int \frac{(a+b\tanh^{-1}(cx))^3}{x^4}\,dx$

Optimal. Leaf size=200

$$-\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}bc^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^3-\frac{(a+$$

```
[Out] -b^2*c^2*(a+b*arctanh(c*x))/x+1/2*b*c^3*(a+b*arctanh(c*x))^2-1/2*b*c*(a+b*a
rctanh(c*x))^2/x^2+1/3*c^3*(a+b*arctanh(c*x))^3-1/3*(a+b*arctanh(c*x))^3/x^
3+b^3*c^3*ln(x)-1/2*b^3*c^3*ln(-c^2*x^2+1)+b*c^3*(a+b*arctanh(c*x))^2*ln(2-
2/(c*x+1))-b^2*c^3*(a+b*arctanh(c*x))*polylog(2,-1+2/(c*x+1))-1/2*b^3*c^3*p
olylog(3,-1+2/(c*x+1))
```

Rubi [A]

time = 0.34, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6037, 6129, 272, 36, 29, 31, 6095, 6135, 6079, 6203, 6745}

$$-b^2c^3\text{Li}_2\left(\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^3+\frac{1}{2}bc^3\left(a+b\tanh^{-1}(cx)\right)^2+bc^3\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3x^3}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{1}{2}b^3c^3\text{Li}_3\left(\frac{2}{cx+1}-1\right)+b^3c^3\log(x)-\frac{1}{2}b^3c^3\log\left(1-c^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/x^4,x]
```

```
[Out] -((b^2*c^2*(a + b*ArcTanh[c*x]))/x) + (b*c^3*(a + b*ArcTanh[c*x])^2)/2 - (b
*c*(a + b*ArcTanh[c*x])^2)/(2*x^2) + (c^3*(a + b*ArcTanh[c*x])^3)/3 - (a +
b*ArcTanh[c*x])^3/(3*x^3) + b^3*c^3*Log[x] - (b^3*c^3*Log[1 - c^2*x^2])/2 +
 b*c^3*(a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)] - b^2*c^3*(a + b*ArcTanh
[c*x])*PolyLog[2, -1 + 2/(1 + c*x)] - (b^3*c^3*PolyLog[3, -1 + 2/(1 + c*x)]
)/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
```

```
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^4}\,dx &= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3x^3}+(bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3\left(1-c^2x^2\right)}\,dx\\ &= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3x^3}+(bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx+\left(bc^3\right)\int \frac{\left(a+b\tanh^{-1}\right.}{x\left(1-c^2\right.}\\ &= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3x^3}+\left(b^2\right.\\ &= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3x^3}+bc^3\\ &= -\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}bc^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+\\ &= -\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}bc^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+\\ &= -\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}bc^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+\\ &= -\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}bc^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.64, size = 323, normalized size = 1.62

$$-\frac{8a^3+12a^2bcx+24a^2b\tanh^{-1}(cx)-24a^2bc^3x^3\log(x)+12a^2bc^3x^3\log\left(1-c^2x^2\right)+24ab^2\left(c^2x^2+\left(1-c^3x^3\right)\tanh^{-1}(cx)^2-cx\tanh^{-1}(cx)\left(-1+c^2x^2+2c^2x^2\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)+c^3x^3\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)\right)+b^3\left(-ic^3\pi^3x^3+24c^2x^2\tanh^{-1}(cx)+12cx\tanh^{-1}(cx)^2-12c^3x^3\tanh^{-1}(cx)^2+8\tanh^{-1}(cx)^3+8c^3x^3\tanh^{-1}(cx)^3-24c^3x^3\tanh^{-1}(cx)^2\log\left(1-e^{2\tanh^{-1}(cx)}\right)-24c^3x^3\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)-24c^3x^3\tanh^{-1}(cx)\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}(cx)}\right)+12c^3x^3\mathrm{PolyLog}\left(3,e^{2\tanh^{-1}(cx)}\right)\right)}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/x^4,x]
```

```
[Out] -1/24*(8*a^3 + 12*a^2*b*c*x + 24*a^2*b*ArcTanh[c*x] - 24*a^2*b*c^3*x^3*Log[
x] + 12*a^2*b*c^3*x^3*Log[1 - c^2*x^2] + 24*a*b^2*(c^2*x^2 + (1 - c^3*x^3)*
ArcTanh[c*x]^2 - c*x*ArcTanh[c*x]*(-1 + c^2*x^2 + 2*c^2*x^2*Log[1 - E^(-2*A
```

```
rcTanh[c*x])]) + c^3*x^3*PolyLog[2, E^(-2*ArcTanh[c*x])]) + b^3*((-I)*c^3*P
i^3*x^3 + 24*c^2*x^2*ArcTanh[c*x] + 12*c*x*ArcTanh[c*x]^2 - 12*c^3*x^3*ArcT
anh[c*x]^2 + 8*ArcTanh[c*x]^3 + 8*c^3*x^3*ArcTanh[c*x]^3 - 24*c^3*x^3*ArcTa
nh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 24*c^3*x^3*Log[(c*x)/Sqrt[1 - c^2*x
^2]] - 24*c^3*x^3*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + 12*c^3*x^3*
PolyLog[3, E^(2*ArcTanh[c*x])]))/x^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.55, size = 1716, normalized size = 8.58

method	result	size
derivativedivides	Expression too large to display	1716
default	Expression too large to display	1716

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/3*a^3/c^3/x^3-1/2*I*b^3*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2
*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+1
/4*I*b^3*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1
)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/2*I*b^3*arctanh(c*x)^2*Pi*c
sgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*((c*x
+1)^2/(-c^2*x^2+1)-1))-1/4*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^
2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+1/2*I*b^3*
arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+
1)^(1/2))+1/4*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*
(c*x+1)/(-c^2*x^2+1)^(1/2))^2+a*b^2*dilog(1/2*c*x+1/2)+b^3*ln(2)*arctanh(c*
x)^2+2*a*b^2*arctanh(c*x)*ln(c*x)-a*b^2*ln(c*x)*ln(c*x+1)-1/2*a^2*b/c^2/x^2
+1/2*I*b^3*arctanh(c*x)^2*Pi-1/2*b^3/c^2/x^2*arctanh(c*x)^2-b^3*arctanh(c*x
)/c/x-a*b^2/c/x-1/2*a^2*b*ln(c*x-1)-1/2*a^2*b*ln(c*x+1)+b^3*arctanh(c*x)^2*
ln((c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*b^3*arctanh(c*x)^2*ln(c*x-1)-1/2*b^3*arc
tanh(c*x)^2*ln(c*x+1)-1/4*a*b^2*ln(c*x-1)^2+1/4*a*b^2*ln(c*x+1)^2-1/2*a*b^2
*ln(c*x-1)+1/2*a*b^2*ln(c*x+1)-a*b^2*arctanh(c*x)*ln(c*x+1)+1/2*a*b^2*ln(c*
x-1)*ln(1/2*c*x+1/2)+b^3*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+b^3*ln((c*x+1)/(-
c^2*x^2+1)^(1/2)-1)-1/2*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/2*a*b^2*ln(-1/2*
c*x+1/2)*ln(1/2*c*x+1/2)-a*b^2*arctanh(c*x)*ln(c*x-1)-a*b^2*dilog(c*x)-a*b^
2*dilog(c*x+1)+a^2*b*ln(c*x)-b^3*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1
)+2*b^3*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+b^3*arctanh(c*x)
^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*b^3*arctanh(c*x)*polylog(2,-(c*x+1)/(
-c^2*x^2+1)^(1/2))+b^3*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+b^3*
ln(c*x)*arctanh(c*x)^2-2*b^3*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^3*p
olylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-b^3*arctanh(c*x)+1/2*b^3*arctanh(c*x)^
2-1/3*b^3*arctanh(c*x)^3+1/2*I*b^3*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-
c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*
csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))-1/4*I*b^3*arctanh(c*x)^2*Pi*csgn(I/(1+(c
```

```
*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+
1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))-a*b^2/c^3/x^3*arctanh(c*x)^2-a^2*b/c^3/x
^3*arctanh(c*x)+1/4*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1
+(c*x+1)^2/(-c^2*x^2+1)))^3+1/4*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c
^2*x^2-1))^3-1/2*I*b^3*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))
^2+1/2*I*b^3*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+1/2*I*b
^3*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x
^2+1)))^3-a*b^2/c^2/x^2*arctanh(c*x)-1/3*b^3/c^3/x^3*arctanh(c*x)^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*a^2*b - 1/3*a^3/x^3 - 1/24*((b^3*c^3*x^3 - b^3)*log(-c*x + 1)^3 + 3*(b^3*c
*x + 2*a*b^2 + (b^3*c^3*x^3 + b^3)*log(c*x + 1))*log(-c*x + 1)^2)/x^3 - int
egrate(-1/8*((b^3*c*x - b^3)*log(c*x + 1)^3 + 6*(a*b^2*c*x - a*b^2)*log(c*x
 + 1)^2 + (2*b^3*c^2*x^2 + 4*a*b^2*c*x - 3*(b^3*c*x - b^3)*log(c*x + 1)^2 +
 2*(b^3*c^4*x^4 + 6*a*b^2 - (6*a*b^2*c - b^3*c)*x)*log(c*x + 1))*log(-c*x +
 1))/(c*x^5 - x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}{\left(c x \right)}\right)^{3}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/x**4,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atanh}(c\,x))^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/x^4,x)
```

```
[Out] int((a + b*atanh(c*x))^3/x^4, x)
```

3.34 $\int \frac{(a+b\tanh^{-1}(cx))^3}{x^5}\,dx$

Optimal. Leaf size=187

$$-\frac{b^3c^3}{4x}+\frac{1}{4}b^3c^4\tanh^{-1}(cx)-\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{4x^2}+bc^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{4x^3}-\frac{3bc^3\left(a\right.}{}$$

```
[Out] -1/4*b^3*c^3/x+1/4*b^3*c^4*arctanh(c*x)-1/4*b^2*c^2*(a+b*arctanh(c*x))/x^2+
b*c^4*(a+b*arctanh(c*x))^2-1/4*b*c*(a+b*arctanh(c*x))^2/x^3-3/4*b*c^3*(a+b*
arctanh(c*x))^2/x+1/4*c^4*(a+b*arctanh(c*x))^3-1/4*(a+b*arctanh(c*x))^3/x^4
+2*b^2*c^4*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-b^3*c^4*polylog(2,-1+2/(c*x+1
))
```

Rubi [A]
time = 0.44, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6037, 6129, 331, 212, 6135, 6079, 2497, 6095}

$$2b^2c^4\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{4x^2}+\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^3+bc^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{3bc^3\left(a+b\tanh^{-1}(cx)\right)^2}{4x}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{4x^4}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{4x^3}-b^3c^4\text{Li}_2\left(\frac{2}{cx+1}-1\right)+\frac{1}{4}b^3c^4\tanh^{-1}(cx)-\frac{b^3c^3}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/x^5,x]
```

```
[Out] -1/4*(b^3*c^3)/x + (b^3*c^4*ArcTanh[c*x])/4 - (b^2*c^2*(a + b*ArcTanh[c*x])
)/(4*x^2) + b*c^4*(a + b*ArcTanh[c*x])^2 - (b*c*(a + b*ArcTanh[c*x])^2)/(4*
x^3) - (3*b*c^3*(a + b*ArcTanh[c*x])^2)/(4*x) + (c^4*(a + b*ArcTanh[c*x])^3
)/4 - (a + b*ArcTanh[c*x])^3/(4*x^4) + 2*b^2*c^4*(a + b*ArcTanh[c*x])*Log[2
 - 2/(1 + c*x)] - b^3*c^4*PolyLog[2, -1 + 2/(1 + c*x)]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^5}\,dx &= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{4x^4}+\frac{1}{4}(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4\left(1-c^2x^2\right)}\,dx \\
&= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{4x^4}+\frac{1}{4}(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx+\frac{1}{4}\left(3bc^3\right)\int \frac{(a+b}{x^2\,(} \\
&= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{4x^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{4x^4}+\frac{1}{2}\left(b^2c^2\right)\int \frac{a+b\tanh^{-1}(cx)}{x^3\left(1-c^2x^2\right)} \\
&= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{4x^3}-\frac{3bc^3\left(a+b\tanh^{-1}(cx)\right)^2}{4x}+\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^3 \\
&= -\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{4x^2}+bc^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{4x^3}- \\
&= -\frac{b^3c^3}{4x}-\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{4x^2}+bc^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc\left(a+b\tanh^{-1}(c\right.}{4x^3} \\
&= -\frac{b^3c^3}{4x}+\frac{1}{4}b^3c^4\tanh^{-1}(cx)-\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{4x^2}+bc^4\left(a+b\tanh^{-1}(cx)\right)^2
\end{aligned}$$

Mathematica [A]
time = 0.47, size = 295, normalized size = 1.58

$$-\frac{2a^3+2a^2bcx+2ab^2c^2x^2+6a^2bc^3x^3+2b^3c^3x^3-2ab^2c^4x^4+2b^2\left(bcx\left(1+3c^2x^2-4c^3x^3\right)+a\left(3-3c^4x^4\right)\right)\tanh^{-1}(cx)^2-2b^3\left(-1+c^4x^4\right)\tanh^{-1}(cx)^3+2b\tanh^{-1}(cx)\left(3a^2+b^2c^2x^2\left(1-c^2x^2\right)+2abcx\left(1+3c^2x^2\right)-8b^2c^4x^4\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)+3a^2bc^4x^4\log(1-cx)-3a^2bc^4x^4\log(1+cx)-16ab^2c^4x^4\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+8b^3c^4x^4\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/x^5,x]
```

```
[Out] -1/8*(2*a^3 + 2*a^2*b*c*x + 2*a*b^2*c^2*x^2 + 6*a^2*b*c^3*x^3 + 2*b^3*c^3*x
^3 - 2*a*b^2*c^4*x^4 + 2*b^2*(b*c*x*(1 + 3*c^2*x^2 - 4*c^3*x^3) + a*(3 - 3*
c^4*x^4))*ArcTanh[c*x]^2 - 2*b^3*(-1 + c^4*x^4)*ArcTanh[c*x]^3 + 2*b*ArcTan
h[c*x]*(3*a^2 + b^2*c^2*x^2*(1 - c^2*x^2) + 2*a*b*c*x*(1 + 3*c^2*x^2) - 8*b
^2*c^4*x^4*Log[1 - E^(-2*ArcTanh[c*x])]) + 3*a^2*b*c^4*x^4*Log[1 - c*x] - 3
*a^2*b*c^4*x^4*Log[1 + c*x] - 16*a*b^2*c^4*x^4*Log[(c*x)/Sqrt[1 - c^2*x^2]]
 + 8*b^3*c^4*x^4*PolyLog[2, E^(-2*ArcTanh[c*x])])/x^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.86, size = 1204, normalized size = 6.44

method	result	size
derivativedivides	Expression too large to display	1204
default	Expression too large to display	1204

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(3/16*I*b^3*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I
*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2
+1)))-3/16*I*b^3*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(
I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/16*I*b^3*arctanh(c*
x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))-
3/8*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+
1)^2/(c^2*x^2-1))^2+3/16*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-
1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/2*a*b^2*ar
ctanh(c*x)/c/x-1/4*a^3/c^4/x^4-1/4*b^3/(c*x+1-(-c^2*x^2+1)^(1/2))*(-c^2*x^2
+1)^(1/2)+1/4*b^3/((-c^2*x^2+1)^(1/2)+c*x+1)*(-c^2*x^2+1)^(1/2)-3/8*a^2*b*l
n(c*x-1)+3/8*a^2*b*ln(c*x+1)-3/4*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)
^(1/2))-3/8*b^3*arctanh(c*x)^2*ln(c*x-1)+3/8*b^3*arctanh(c*x)^2*ln(c*x+1)-3
/16*a*b^2*ln(c*x-1)^2-3/16*a*b^2*ln(c*x+1)^2-a*b^2*ln(c*x-1)-a*b^2*ln(c*x+1
)+3/4*a*b^2*arctanh(c*x)*ln(c*x+1)+3/8*a*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)+3/8*
a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/8*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)
-3/4*a*b^2*arctanh(c*x)*ln(c*x-1)-2*b^3*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))+2
*b^3*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-3/4*a^2*b/c/x-3/4*b^3*arctanh(c*x)
^2/c/x+1/4*b^3*arctanh(c*x)-b^3*arctanh(c*x)^2+1/4*b^3*arctanh(c*x)^3+2*a*b
^2*ln(c*x)+2*b^3*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-3/16*I*b^3*a
rctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-3/8*I*b^3*arctanh(c*x)^2*P
i*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+3/8*I*b^3*arctanh(c*x)^2*Pi*csgn(I/(
1+(c*x+1)^2/(-c^2*x^2+1)))^3-3/16*I*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/
(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-3/4*a^2*b/c^4/x^4*arctanh(c*x)-3/
4*a*b^2/c^4/x^4*arctanh(c*x)^2-1/2*a*b^2*arctanh(c*x)/c^3/x^3-1/4*b^3/c^3/x
^3*arctanh(c*x)^2-1/4*b^3*arctanh(c*x)/c^2/x^2-1/4*a*b^2/c^2/x^2-1/4*a^2*b/
c^3/x^3+3/8*I*b^3*arctanh(c*x)^2*Pi-1/4*b^3/c^4/x^4*arctanh(c*x)^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c -
6*arctanh(c*x)/x^4)*a^2*b + 1/16*((32*c^2*log(x) - (3*c^2*x^2*log(c*x + 1)^
2 + 3*c^2*x^2*log(c*x - 1)^2 + 16*c^2*x^2*log(c*x - 1) - 2*(3*c^2*x^2*log(c
*x - 1) - 8*c^2*x^2)*log(c*x + 1) + 4)/x^2)*c^2 + 4*(3*c^3*log(c*x + 1) - 3
*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c*arctanh(c*x))*a*b^2 - 1/32*b^3
```

```
*(((c^4*x^4 - 1)*log(-c*x + 1)^3 + (6*c^3*x^3 + 2*c*x - 3*(c^4*x^4 - 1)*log
(c*x + 1))*log(-c*x + 1)^2)/x^4 + 4*integrate(-1/2*(2*(c*x - 1)*log(c*x + 1
)^3 + (6*c^4*x^4 + 2*c^2*x^2 - 6*(c*x - 1)*log(c*x + 1)^2 - 3*(c^5*x^5 - c*
x)*log(c*x + 1))*log(-c*x + 1))/(c*x^6 - x^5), x)) - 3/4*a*b^2*arctanh(c*x)
^2/x^4 - 1/4*a^3/x^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))^3}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/x**5,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atanh}(c\,x))^3}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/x^5,x)
```

```
[Out] int((a + b*atanh(c*x))^3/x^5, x)
```

3.35 $\int (dx)^{5/2} \left(a+b\tanh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=124

$$\frac{4bd^2\sqrt{dx}}{7c^3}+\frac{4b(dx)^{5/2}}{35c}-\frac{2bd^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{2bd^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}}$$

```
[Out] 4/35*b*(d*x)^(5/2)/c-2/7*b*d^(5/2)*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))/c^(7
/2)+2/7*(d*x)^(7/2)*(a+b*arctanh(c*x))/d-2/7*b*d^(5/2)*arctanh(c^(1/2)*(d*x
)^(1/2)/d^(1/2))/c^(7/2)+4/7*b*d^2*(d*x)^(1/2)/c^3
```

Rubi [A]
time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6049, 327, 335, 218, 214, 211}

$$\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{2bd^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}}-\frac{2bd^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}}+\frac{4bd^2\sqrt{dx}}{7c^3}+\frac{4b(dx)^{5/2}}{35c}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (4*b*d^2*Sqrt[d*x])/(7*c^3) + (4*b*(d*x)^(5/2))/(35*c) - (2*b*d^(5/2)*ArcTa
n[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/2)) + (2*(d*x)^(7/2)*(a + b*ArcTanh
[c*x]))/(7*d) - (2*b*d^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/
2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\int (dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)\,dx = \frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{(2bc)\int\frac{(dx)^{7/2}}{1-c^2x^2}\,dx}{7d}$$

$$= \frac{4b(dx)^{5/2}}{35c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{(2bd)\int\frac{(dx)^{3/2}}{1-c^2x^2}\,dx}{7c}$$

$$= \frac{4bd^2\sqrt{dx}}{7c^3}+\frac{4b(dx)^{5/2}}{35c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{(2bd^3)\int\frac{1}{\sqrt{dx}\,(1-}}{7c^3}$$

$$= \frac{4bd^2\sqrt{dx}}{7c^3}+\frac{4b(dx)^{5/2}}{35c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{(4bd^2)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{4bd^2\sqrt{dx}}{7c^3}+\frac{4b(dx)^{5/2}}{35c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{(2bd^3)\,\mathrm{Subst}\left(\int\right.}{7}$$

$$= \frac{4bd^2\sqrt{dx}}{7c^3}+\frac{4b(dx)^{5/2}}{35c}-\frac{2bd^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}}+\frac{2(dx)^{7/2}\left(a+b\tan\right.}{7d}$$

Mathematica [A]
time = 0.05, size = 128, normalized size = 1.03

$$\frac{(dx)^{5/2}\left(20b\sqrt{c}\sqrt{x}+4bc^{5/2}x^{5/2}+10ac^{7/2}x^{7/2}-10b\mathrm{ArcTan}\left(\sqrt{c}\sqrt{x}\right)+10bc^{7/2}x^{7/2}\tanh^{-1}(cx)+5b\log\left(1-\sqrt{c}\sqrt{x}\right)-5b\log\left(1+\sqrt{c}\sqrt{x}\right)\right)}{35c^{7/2}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] ((d*x)^(5/2)*(20*b*Sqrt[c]*Sqrt[x] + 4*b*c^(5/2)*x^(5/2) + 10*a*c^(7/2)*x^(
7/2) - 10*b*ArcTan[Sqrt[c]*Sqrt[x]] + 10*b*c^(7/2)*x^(7/2)*ArcTanh[c*x] + 5
*b*Log[1 - Sqrt[c]*Sqrt[x]] - 5*b*Log[1 + Sqrt[c]*Sqrt[x]]))/(35*c^(7/2)*x^
(5/2))
```

Maple [A]
time = 0.07, size = 107, normalized size = 0.86

method	result	size
derivativedivides	$\frac{\frac{2(dx)^{\frac{7}{2}} a}{7}+\frac{2b(dx)^{\frac{7}{2}} \operatorname{arctanh}(cx)}{7}+\frac{4bd(dx)^{\frac{5}{2}}}{35c}+\frac{4b\,d^{3}\sqrt{dx}}{7c^{3}}-\frac{2b\,d^{4}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{7c^{3}\sqrt{dc}}-\frac{2b\,d^{4}\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{7c^{3}\sqrt{dc}}}{d}$	107
default	$\frac{\frac{2(dx)^{\frac{7}{2}} a}{7}+\frac{2b(dx)^{\frac{7}{2}} \operatorname{arctanh}(cx)}{7}+\frac{4bd(dx)^{\frac{5}{2}}}{35c}+\frac{4b\,d^{3}\sqrt{dx}}{7c^{3}}-\frac{2b\,d^{4}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{7c^{3}\sqrt{dc}}-\frac{2b\,d^{4}\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{7c^{3}\sqrt{dc}}}{d}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/7*(d*x)^(7/2)*a+1/7*b*(d*x)^(7/2)*arctanh(c*x)+2/35*b*d/c*(d*x)^(5/2
)+2/7*b*d^3/c^3*(d*x)^(1/2)-1/7*b*d^4/c^3/(d*c)^(1/2)*arctan(c*(d*x)^(1/2)/
(d*c)^(1/2))-1/7*b*d^4/c^3/(d*c)^(1/2)*arctanh(c*(d*x)^(1/2)/(d*c)^(1/2)))
```

Maxima [A]
time = 0.47, size = 134, normalized size = 1.08

$$\frac{10\,(dx)^{\frac{7}{2}}\,a+\left(10\,(dx)^{\frac{7}{2}}\operatorname{artanh}(cx)-\frac{\left(\frac{10\,d^{5}\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,c^{4}}-\frac{5\,d^{5}\log\left(\frac{\sqrt{dx}\,c-\sqrt{cd}}{\sqrt{dx}\,c+\sqrt{cd}}\right)}{\sqrt{cd}\,c^{4}}-\frac{4\left((dx)^{\frac{5}{2}}c^{2}d^{2}+5\sqrt{dx}\,d^{4}\right)}{c^{4}}\right)c}{d}\right)b}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/35*(10*(d*x)^(7/2)*a + (10*(d*x)^(7/2)*arctanh(c*x) - (10*d^5*arctan(sqrt
(d*x)*c/sqrt(c*d))/(sqrt(c*d)*c^4) - 5*d^5*log((sqrt(d*x)*c - sqrt(c*d))/(s
qrt(d*x)*c + sqrt(c*d)))/(sqrt(c*d)*c^4) - 4*((d*x)^(5/2)*c^2*d^2 + 5*sqrt(
d*x)*d^4)/c^4)*c/d)*b)/d
```

Fricas [A]
time = 0.36, size = 296, normalized size = 2.39

$$\left[-\frac{10\,bd^{2}\sqrt{\frac{d}{c}}\arctan\left(\frac{\sqrt{dx}\,c\sqrt{\frac{d}{c}}}{d}\right)-5\,bd^{2}\sqrt{\frac{d}{c}}\log\left(\frac{cdx-2\,\sqrt{dx}\,c\sqrt{\frac{d}{c}}+d}{cx-1}\right)-\left(5\,bc^{3}d^{2}x^{3}\log\left(-\frac{cx+1}{cx-1}\right)+10\,ac^{3}d^{2}x^{3}+4\,bc^{2}d^{2}x^{2}+20\,bd^{2}\right)\sqrt{dx}}{35\,c^{3}},\ \frac{10\,bd^{2}\sqrt{-\frac{d}{c}}\arctan\left(\frac{\sqrt{dx}\,c\sqrt{-\frac{d}{c}}}{d}\right)+5\,bd^{2}\sqrt{-\frac{d}{c}}\log\left(\frac{cdx-2\,\sqrt{dx}\,c\sqrt{-\frac{d}{c}}-d}{cx+1}\right)+\left(5\,bc^{3}d^{2}x^{3}\log\left(-\frac{cx+1}{cx-1}\right)+10\,ac^{3}d^{2}x^{3}+4\,bc^{2}d^{2}x^{2}+20\,bd^{2}\right)\sqrt{dx}}{35\,c^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] [-1/35*(10*b*d^2*sqrt(d/c)*arctan(sqrt(d*x)*c*sqrt(d/c)/d) - 5*b*d^2*sqrt(d
/c)*log((c*d*x - 2*sqrt(d*x)*c*sqrt(d/c) + d)/(c*x - 1)) - (5*b*c^3*d^2*x^3
*log(-(c*x + 1)/(c*x - 1)) + 10*a*c^3*d^2*x^3 + 4*b*c^2*d^2*x^2 + 20*b*d^2)
*sqrt(d*x))/c^3, 1/35*(10*b*d^2*sqrt(-d/c)*arctan(sqrt(d*x)*c*sqrt(-d/c)/d)
 + 5*b*d^2*sqrt(-d/c)*log((c*d*x - 2*sqrt(d*x)*c*sqrt(-d/c) - d)/(c*x + 1))
 + (5*b*c^3*d^2*x^3*log(-(c*x + 1)/(c*x - 1)) + 10*a*c^3*d^2*x^3 + 4*b*c^2*
d^2*x^2 + 20*b*d^2)*sqrt(d*x))/c^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{5}{2}}\left(a+b\,\mathrm{atanh}\,(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(a+b*atanh(c*x)),x)
```

```
[Out] Integral((d*x)**(5/2)*(a + b*atanh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(5/2)*(b*arctanh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{atanh}(c\,x)\right)\,{\left(d\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d*x)^(5/2),x)
```

```
[Out] int((a + b*atanh(c*x))*(d*x)^(5/2), x)
```

3.36 $\int (dx)^{3/2} \left(a+b\tanh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=106

$$\frac{4b(dx)^{3/2}}{15c}+\frac{2bd^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}-\frac{2bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}}$$

```
[Out] 4/15*b*(d*x)^(3/2)/c+2/5*b*d^(3/2)*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))/c^(5
/2)+2/5*(d*x)^(5/2)*(a+b*arctanh(c*x))/d-2/5*b*d^(3/2)*arctanh(c^(1/2)*(d*x
)^(1/2)/d^(1/2))/c^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6049, 327, 335, 304, 211, 214}

$$\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}+\frac{2bd^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}}-\frac{2bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}}+\frac{4b(dx)^{3/2}}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (4*b*(d*x)^(3/2))/(15*c) + (2*b*d^(3/2)*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]]
)/(5*c^(5/2)) + (2*(d*x)^(5/2)*(a + b*ArcTanh[c*x]))/(5*d) - (2*b*d^(3/2)*A
rcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(5*c^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\int (dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)\,dx=\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}-\frac{(2bc)\int\frac{(dx)^{5/2}}{1-c^2x^2}\,dx}{5d}$$

$$=\frac{4b(dx)^{3/2}}{15c}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}-\frac{(2bd)\int\frac{\sqrt{dx}}{1-c^2x^2}\,dx}{5c}$$

$$=\frac{4b(dx)^{3/2}}{15c}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}-\frac{(4b)\text{Subst}\left(\int\frac{x^2}{1-\frac{c^2x^4}{d^2}}\,dx,x,\sqrt{d}\right.}{5c}$$

$$=\frac{4b(dx)^{3/2}}{15c}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}-\frac{\left(2bd^2\right)\text{Subst}\left(\int\frac{1}{d-cx^2}\,dx,x,\sqrt{}\right.}{5c^2}$$

$$=\frac{4b(dx)^{3/2}}{15c}+\frac{2bd^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}-$$

Mathematica [A]
time = 0.04, size = 115, normalized size = 1.08

$$\frac{(dx)^{3/2}\left(4bc^{3/2}x^{3/2}+6ac^{5/2}x^{5/2}+6b\text{ArcTan}\left(\sqrt{c}\sqrt{x}\right)+6bc^{5/2}x^{5/2}\tanh^{-1}(cx)+3b\log\left(1-\sqrt{c}\sqrt{x}\right)-3b\log\left(1+\sqrt{c}\sqrt{x}\right)\right)}{15c^{5/2}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] ((d*x)^(3/2)*(4*b*c^(3/2)*x^(3/2) + 6*a*c^(5/2)*x^(5/2) + 6*b*ArcTan[Sqrt[c
]*Sqrt[x]] + 6*b*c^(5/2)*x^(5/2)*ArcTanh[c*x] + 3*b*Log[1 - Sqrt[c]*Sqrt[x]
] - 3*b*Log[1 + Sqrt[c]*Sqrt[x]]))/(15*c^(5/2)*x^(3/2))
```

Maple [A]
time = 0.04, size = 93, normalized size = 0.88

method	result	size
derivativedivides	$\frac{\frac{2(dx)^{\frac{5}{2}}a}{5}+\frac{2b(dx)^{\frac{5}{2}}\operatorname{arctanh}(cx)}{5}+\frac{4bd(dx)^{\frac{3}{2}}}{15c}+\frac{2b\,d^{3}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{5c^{2}\sqrt{dc}}-\frac{2b\,d^{3}\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{5c^{2}\sqrt{dc}}}{d}$	93
default	$\frac{\frac{2(dx)^{\frac{5}{2}}a}{5}+\frac{2b(dx)^{\frac{5}{2}}\operatorname{arctanh}(cx)}{5}+\frac{4bd(dx)^{\frac{3}{2}}}{15c}+\frac{2b\,d^{3}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{5c^{2}\sqrt{dc}}-\frac{2b\,d^{3}\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{5c^{2}\sqrt{dc}}}{d}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/5*(d*x)^(5/2)*a+1/5*b*(d*x)^(5/2)*arctanh(c*x)+2/15*b*d*(d*x)^(3/2)/
c+1/5*b*d^3/c^2/(d*c)^(1/2)*arctan(c*(d*x)^(1/2)/(d*c)^(1/2))-1/5*b*d^3/c^2
/(d*c)^(1/2)*arctanh(c*(d*x)^(1/2)/(d*c)^(1/2)))
```

Maxima [A]
time = 0.47, size = 118, normalized size = 1.11

$$\frac{6\,(dx)^{\frac{5}{2}}a+\left(6\,(dx)^{\frac{5}{2}}\operatorname{artanh}(cx)+\frac{\left(\frac{4\,(dx)^{\frac{3}{2}}d^{2}}{c^{2}}+\frac{6\,d^{4}\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,c^{3}}+\frac{3\,d^{4}\log\left(\frac{\sqrt{dx}\,c-\sqrt{cd}}{\sqrt{dx}\,c+\sqrt{cd}}\right)}{\sqrt{cd}\,c^{3}}\right)c}{d}\right)b}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/15*(6*(d*x)^(5/2)*a + (6*(d*x)^(5/2)*arctanh(c*x) + (4*(d*x)^(3/2)*d^2/c^
2 + 6*d^4*arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*c^3) + 3*d^4*log((sqrt(d
*x)*c - sqrt(c*d))/(sqrt(d*x)*c + sqrt(c*d)))/(sqrt(c*d)*c^3))*c/d)*b)/d
```

Fricas [A]
time = 0.39, size = 255, normalized size = 2.41

$$\left[\frac{6\,bd\sqrt{\frac{d}{c}}\arctan\left(\frac{\sqrt{dx}\,c\sqrt{\frac{d}{c}}}{d}\right) + 3\,bd\sqrt{\frac{d}{c}}\log\left(\frac{cdx - 2\,\sqrt{dx}\,c\sqrt{\frac{d}{c}} + d}{cx - 1}\right) + \left(3\,bc^{2}dx^{2}\log\left(-\frac{cx+1}{cx-1}\right) + 6\,ac^{2}dx^{2} + 4\,bcdx\right)\sqrt{dx}}{15\,c^{2}}, \frac{6\,bd\sqrt{-\frac{d}{c}}\arctan\left(\frac{\sqrt{dx}\,c\sqrt{-\frac{d}{c}}}{d}\right) + 3\,bd\sqrt{-\frac{d}{c}}\log\left(\frac{cdx + 2\,\sqrt{dx}\,c\sqrt{-\frac{d}{c}} - d}{cx + 1}\right) + \left(3\,bc^{2}dx^{2}\log\left(-\frac{cx+1}{cx-1}\right) + 6\,ac^{2}dx^{2} + 4\,bcdx\right)\sqrt{dx}}{15\,c^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] [1/15*(6*b*d*sqrt(d/c)*arctan(sqrt(d*x)*c*sqrt(d/c)/d) + 3*b*d*sqrt(d/c)*lo
g((c*d*x - 2*sqrt(d*x)*c*sqrt(d/c) + d)/(c*x - 1)) + (3*b*c^2*d*x^2*log(-(c
*x + 1)/(c*x - 1)) + 6*a*c^2*d*x^2 + 4*b*c*d*x)*sqrt(d*x))/c^2, 1/15*(6*b*d
*sqrt(-d/c)*arctan(sqrt(d*x)*c*sqrt(-d/c)/d) + 3*b*d*sqrt(-d/c)*log((c*d*x
+ 2*sqrt(d*x)*c*sqrt(-d/c) - d)/(c*x + 1)) + (3*b*c^2*d*x^2*log(-(c*x + 1)/
(c*x - 1)) + 6*a*c^2*d*x^2 + 4*b*c*d*x)*sqrt(d*x))/c^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}} \left(a + b \operatorname{atanh}\left(cx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(a+b*atanh(c*x)),x)
```

```
[Out] Integral((d*x)**(3/2)*(a + b*atanh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*(b*arctanh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)\,{\left(d\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d*x)^(3/2),x)
```

```
[Out] int((a + b*atanh(c*x))*(d*x)^(3/2), x)
```

3.37 $\int \sqrt{dx} \left(a+b\tanh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=106

$$\frac{4b\sqrt{dx}}{3c}-\frac{2b\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}}+\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d}-\frac{2b\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}}$$

```
[Out] 2/3*(d*x)^(3/2)*(a+b*arctanh(c*x))/d-2/3*b*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/
2))*d^(1/2)/c^(3/2)-2/3*b*arctanh(c^(1/2)*(d*x)^(1/2)/d^(1/2))*d^(1/2)/c^(3
/2)+4/3*b*(d*x)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6049, 327, 335, 218, 214, 211}

$$\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d}-\frac{2b\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}}-\frac{2b\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}}+\frac{4b\sqrt{dx}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (4*b*Sqrt[d*x])/(3*c) - (2*b*Sqrt[d]*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(
3*c^(3/2)) + (2*(d*x)^(3/2)*(a + b*ArcTanh[c*x]))/(3*d) - (2*b*Sqrt[d]*ArcT
anh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{dx}\,\left(a+b\tanh^{-1}(cx)\right)\,dx &= \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d}-\frac{(2bc)\int \frac{(dx)^{3/2}}{1-c^2x^2}\,dx}{3d}\\ &= \frac{4b\sqrt{dx}}{3c}+\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d}-\frac{(2bd)\int \frac{1}{\sqrt{dx}\,(1-c^2x^2)}\,dx}{3c}\\ &= \frac{4b\sqrt{dx}}{3c}+\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d}-\frac{(4b)\text{Subst}\left(\int \frac{1}{1-\frac{c^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right.}{3c}\\ &= \frac{4b\sqrt{dx}}{3c}+\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d}-\frac{(2bd)\text{Subst}\left(\int \frac{1}{d-cx^2}\,dx,x,\sqrt{dx}\right)}{3c}\\ &= \frac{4b\sqrt{dx}}{3c}-\frac{2b\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}}+\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d}-\frac{2b}{}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 114, normalized size = 1.08

$$\frac{\sqrt{dx}\left(4b\sqrt{c}\,\sqrt{x}+2ac^{3/2}x^{3/2}-2b\text{ArcTan}\left(\sqrt{c}\,\sqrt{x}\right)+2bc^{3/2}x^{3/2}\tanh^{-1}(cx)+b\log\left(1-\sqrt{c}\,\sqrt{x}\right)-b\log\left(1+\sqrt{c}\,\sqrt{x}\right)\right)}{3c^{3/2}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (Sqrt[d*x]*(4*b*Sqrt[c]*Sqrt[x] + 2*a*c^(3/2)*x^(3/2) - 2*b*ArcTan[Sqrt[c]*
Sqrt[x]] + 2*b*c^(3/2)*x^(3/2)*ArcTanh[c*x] + b*Log[1 - Sqrt[c]*Sqrt[x]] -
b*Log[1 + Sqrt[c]*Sqrt[x]]))/(3*c^(3/2)*Sqrt[x])
```

Maple [A]
time = 0.04, size = 93, normalized size = 0.88

method	result	size
derivativedivides	$\frac{\frac{2(dx)^{\frac{3}{2}}a}{3}+\frac{2b(dx)^{\frac{3}{2}}\operatorname{arctanh}(cx)}{3}+\frac{4bd\sqrt{dx}}{3c}-\frac{2bd^2\arctan\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{3c\sqrt{dc}}-\frac{2bd^2\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{3c\sqrt{dc}}}{d}$	93
default	$\frac{\frac{2(dx)^{\frac{3}{2}}a}{3}+\frac{2b(dx)^{\frac{3}{2}}\operatorname{arctanh}(cx)}{3}+\frac{4bd\sqrt{dx}}{3c}-\frac{2bd^2\arctan\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{3c\sqrt{dc}}-\frac{2bd^2\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{3c\sqrt{dc}}}{d}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/3*(d*x)^(3/2)*a+1/3*b*(d*x)^(3/2)*arctanh(c*x)+2/3*b*d*(d*x)^(1/2)/c
-1/3*b*d^2/c/(d*c)^(1/2)*arctan(c*(d*x)^(1/2)/(d*c)^(1/2))-1/3*b*d^2/c/(d*c
)^(1/2)*arctanh(c*(d*x)^(1/2)/(d*c)^(1/2)))
```

Maxima [A]
time = 0.47, size = 119, normalized size = 1.12

$$\frac{2\,(dx)^{\frac{3}{2}}\,a+\left(2\,(dx)^{\frac{3}{2}}\operatorname{artanh}\left(cx\right)-\frac{\left(\frac{2\,d^3\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,c^2}-\frac{d^3\log\left(\frac{\sqrt{dx}\,c-\sqrt{cd}}{\sqrt{dx}\,c+\sqrt{cd}}\right)}{\sqrt{cd}\,c^2}-\frac{4\,\sqrt{dx}\,d^2}{c^2}\right)c}{d}\right)b}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*(2*(d*x)^(3/2)*a + (2*(d*x)^(3/2)*arctanh(c*x) - (2*d^3*arctan(sqrt(d*x
)*c/sqrt(c*d))/(sqrt(c*d)*c^2) - d^3*log((sqrt(d*x)*c - sqrt(c*d))/(sqrt(d*
x)*c + sqrt(c*d)))/(sqrt(c*d)*c^2) - 4*sqrt(d*x)*d^2/c^2)*c/d)*b)/d
```

Fricas [A]
time = 0.43, size = 223, normalized size = 2.10

$$\left[-\frac{2\,b\sqrt{\frac{d}{c}}\arctan\left(\frac{\sqrt{dx}\,c\sqrt{\frac{d}{c}}}{d}\right) - b\sqrt{\frac{d}{c}}\log\left(\frac{cdx-2\sqrt{dx}\,c\sqrt{\frac{d}{c}}+d}{cx-1}\right) - \left(bcx\log\left(-\frac{cx+1}{cx-1}\right)+2\,acx+4\,b\right)\sqrt{dx}}{3\,c},\ \frac{2\,b\sqrt{-\frac{d}{c}}\arctan\left(\frac{\sqrt{dx}\,c\sqrt{-\frac{d}{c}}}{d}\right)+b\sqrt{-\frac{d}{c}}\log\left(\frac{cdx-2\sqrt{dx}\,c\sqrt{-\frac{d}{c}}-d}{cx+1}\right)+\left(bcx\log\left(-\frac{cx+1}{cx-1}\right)+2\,acx+4\,b\right)\sqrt{dx}}{3\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] [-1/3*(2*b*sqrt(d/c)*arctan(sqrt(d*x)*c*sqrt(d/c)/d) - b*sqrt(d/c)*log((c*d
*x - 2*sqrt(d*x)*c*sqrt(d/c) + d)/(c*x - 1)) - (b*c*x*log(-(c*x + 1)/(c*x -
 1)) + 2*a*c*x + 4*b)*sqrt(d*x))/c, 1/3*(2*b*sqrt(-d/c)*arctan(sqrt(d*x)*c*
sqrt(-d/c)/d) + b*sqrt(-d/c)*log((c*d*x - 2*sqrt(d*x)*c*sqrt(-d/c) - d)/(c*
x + 1)) + (b*c*x*log(-(c*x + 1)/(c*x - 1)) + 2*a*c*x + 4*b)*sqrt(d*x))/c]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(100) = 200$.
time = 5.39, size = 636, normalized size = 6.00

$$\frac{2a(dx)^{\frac{3}{2}}}{3d}+\frac{2b\begin{cases}\frac{c^2\left(\frac{d}{c}\right)^{\frac{3}{2}}\sqrt{-\frac{d}{c}}\log\left(\sqrt{dx}+\sqrt{-\frac{d}{c}}\right)}{12c^2\sqrt{\frac{d}{c}}+12c^2\sqrt{-\frac{d}{c}}}+\frac{4c^2\sqrt{\frac{d}{c}}(dx)^{\frac{3}{2}}\operatorname{atanh}(cx)}{12c^2\sqrt{\frac{d}{c}}+12c^2\sqrt{-\frac{d}{c}}}-\frac{c^2\sqrt{\frac{d}{c}}\left(-\frac{d}{c}\right)^{\frac{3}{2}}\log\left(\sqrt{dx}+\sqrt{-\frac{d}{c}}\right)}{12c^2\sqrt{\frac{d}{c}}+12c^2\sqrt{-\frac{d}{c}}}+\frac{4c^2(dx)^{\frac{3}{2}}\sqrt{-\frac{d}{c}}\operatorname{atanh}(cx)}{12c^2\sqrt{\frac{d}{c}}+12c^2\sqrt{-\frac{d}{c}}}+\frac{8cd\sqrt{\frac{d}{c}}\sqrt{dx}}{12c^2\sqrt{\frac{d}{c}}+12c^2\sqrt{-\frac{d}{c}}}+\frac{4cd\sqrt{\frac{d}{c}}\sqrt{-\frac{d}{c}}\log\left(-\sqrt{\frac{d}{c}}+\sqrt{dx}\right)}{12c^2\sqrt{\frac{d}{c}}+12c^2\sqrt{-\frac{d}{c}}}-\frac{6cd\sqrt{\frac{d}{c}}\sqrt{-\frac{d}{c}}\log\left(\sqrt{dx}+\sqrt{-\frac{d}{c}}\right)}{12c^2\sqrt{\frac{d}{c}}+12c^2\sqrt{-\frac{d}{c}}}+\frac{4cd\sqrt{\frac{d}{c}}\sqrt{-\frac{d}{c}}\operatorname{atanh}(cx)}{12c^2\sqrt{\frac{d}{c}}+12c^2\sqrt{-\frac{d}{c}}}+\frac{8cd\sqrt{dx}\sqrt{-\frac{d}{c}}}{12c^2\sqrt{\frac{d}{c}}+12c^2\sqrt{-\frac{d}{c}}}+\frac{4d^2\log\left(-\sqrt{\frac{d}{c}}+\sqrt{dx}\right)}{12c^2\sqrt{\frac{d}{c}}+12c^2\sqrt{-\frac{d}{c}}}-\frac{4d^2\log\left(\sqrt{dx}-\sqrt{-\frac{d}{c}}\right)}{12c^2\sqrt{\frac{d}{c}}+12c^2\sqrt{-\frac{d}{c}}}+\frac{4d^2\operatorname{atanh}(cx)}{12c^2\sqrt{\frac{d}{c}}+12c^2\sqrt{-\frac{d}{c}}} & \text{for}\ c\neq 0\\ 0 & \text{otherwise}\end{cases}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(a+b*atanh(c*x)),x)
```

```
[Out] 2*a*(d*x)**(3/2)/(3*d) + 2*b*Piecewise((c**2*(d/c)**(3/2)*sqrt(-d/c)*log(sq
rt(d*x) + sqrt(-d/c))/(12*c**2*sqrt(d/c) + 12*c**2*sqrt(-d/c)) + 4*c**2*sqr
t(d/c)*(d*x)**(3/2)*atanh(c*x)/(12*c**2*sqrt(d/c) + 12*c**2*sqrt(-d/c)) - c
**2*sqrt(d/c)*(-d/c)**(3/2)*log(sqrt(d*x) + sqrt(-d/c))/(12*c**2*sqrt(d/c)
+ 12*c**2*sqrt(-d/c)) + 4*c**2*(d*x)**(3/2)*sqrt(-d/c)*atanh(c*x)/(12*c**2*
sqrt(d/c) + 12*c**2*sqrt(-d/c)) + 8*c*d*sqrt(d/c)*sqrt(d*x)/(12*c**2*sqrt(d
/c) + 12*c**2*sqrt(-d/c)) + 4*c*d*sqrt(d/c)*sqrt(-d/c)*log(-sqrt(d/c) + sqr
t(d*x))/(12*c**2*sqrt(d/c) + 12*c**2*sqrt(-d/c)) - 6*c*d*sqrt(d/c)*sqrt(-d/
c)*log(sqrt(d*x) + sqrt(-d/c))/(12*c**2*sqrt(d/c) + 12*c**2*sqrt(-d/c)) + 4
*c*d*sqrt(d/c)*sqrt(-d/c)*atanh(c*x)/(12*c**2*sqrt(d/c) + 12*c**2*sqrt(-d/c
)) + 8*c*d*sqrt(d*x)*sqrt(-d/c)/(12*c**2*sqrt(d/c) + 12*c**2*sqrt(-d/c)) +
4*d**2*log(-sqrt(d/c) + sqrt(d*x))/(12*c**2*sqrt(d/c) + 12*c**2*sqrt(-d/c))
 - 4*d**2*log(sqrt(d*x) - sqrt(-d/c))/(12*c**2*sqrt(d/c) + 12*c**2*sqrt(-d/
c)) + 4*d**2*atanh(c*x)/(12*c**2*sqrt(d/c) + 12*c**2*sqrt(-d/c)), Ne(c, 0))
, (0, True))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(d*x)*(b*arctanh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{atanh}(c\,x))\,\sqrt{d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d*x)^(1/2),x)
[Out] int((a + b*atanh(c*x))*(d*x)^(1/2), x)
```

3.38 $\int \frac{a+b\tanh^{-1}(cx)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=85

$$\frac{2b\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}+\frac{2\sqrt{dx}\,(a+b\tanh^{-1}(cx))}{d}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}$$

```
[Out] 2*b*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))/c^(1/2)/d^(1/2)-2*b*arctanh(c^(1/2)
*(d*x)^(1/2)/d^(1/2))/c^(1/2)/d^(1/2)+2*(a+b*arctanh(c*x))*(d*x)^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6049, 335, 304, 211, 214}

$$\frac{2\sqrt{dx}\,(a+b\tanh^{-1}(cx))}{d}+\frac{2b\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/Sqrt[d*x],x]
```

```
[Out] (2*b*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[c]*Sqrt[d]) + (2*Sqrt[d*x]*
(a + b*ArcTanh[c*x]))/d - (2*b*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[
c]*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx)}{\sqrt{dx}}\,dx &= \frac{2\sqrt{dx}\,\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{(2bc)\int \frac{\sqrt{dx}}{1-c^2x^2}\,dx}{d}\\ &= \frac{2\sqrt{dx}\,\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{(4bc)\text{Subst}\left(\int \frac{x^2}{1-\frac{c^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{d^2}\\ &= \frac{2\sqrt{dx}\,\left(a+b\tanh^{-1}(cx)\right)}{d}-(2b)\text{Subst}\left(\int \frac{1}{d-cx^2}\,dx,x,\sqrt{dx}\right)+(2b)\text{Subst}\left(\int\right.\\ &= \frac{2b\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\,\sqrt{d}}+\frac{2\sqrt{dx}\,\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\,\sqrt{d}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 98, normalized size = 1.15

$$\frac{\sqrt{x}\,\left(2a\sqrt{c}\,\sqrt{x}\,+2b\text{ArcTan}\left(\sqrt{c}\,\sqrt{x}\,\right)+2b\sqrt{c}\,\sqrt{x}\,\tanh^{-1}(cx)+b\log\left(1-\sqrt{c}\,\sqrt{x}\,\right)-b\log\left(1+\sqrt{c}\,\sqrt{x}\,\right)\right)}{\sqrt{c}\,\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/Sqrt[d*x],x]
[Out] (Sqrt[x]*(2*a*Sqrt[c]*Sqrt[x] + 2*b*ArcTan[Sqrt[c]*Sqrt[x]] + 2*b*Sqrt[c]*S
qrt[x]*ArcTanh[c*x] + b*Log[1 - Sqrt[c]*Sqrt[x]] - b*Log[1 + Sqrt[c]*Sqrt[x
]]))/(Sqrt[c]*Sqrt[d*x])
```

Maple [A]
time = 0.04, size = 68, normalized size = 0.80

method	result	size
derivativedivides	$\frac{2\sqrt{dx}\,a+2b\sqrt{dx}\,\operatorname{arctanh}(cx)+\frac{2bd\arctan\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{\sqrt{dc}}-\frac{2bd\,\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{\sqrt{dc}}}{d}$	68
default	$\frac{2\sqrt{dx}\,a+2b\sqrt{dx}\,\operatorname{arctanh}(cx)+\frac{2bd\arctan\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{\sqrt{dc}}-\frac{2bd\,\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{\sqrt{dc}}}{d}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*((d*x)^(1/2)*a+b*(d*x)^(1/2)*arctanh(c*x)+b*d/(d*c)^(1/2)*arctan(c*(d*x
)^(1/2)/(d*c)^(1/2))-b*d/(d*c)^(1/2)*arctanh(c*(d*x)^(1/2)/(d*c)^(1/2)))
```

Maxima [A]
time = 0.49, size = 103, normalized size = 1.21

$$\frac{\left(2\sqrt{dx}\,\operatorname{artanh}(cx)+\frac{\left(\frac{2d^2\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,c}+\frac{d^2\log\left(\frac{\sqrt{dx}\,c-\sqrt{cd}}{\sqrt{dx}\,c+\sqrt{cd}}\right)}{\sqrt{cd}\,c}\right)c}{d}\right)b+2\sqrt{dx}\,a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] ((2*sqrt(d*x)*arctanh(c*x) + (2*d^2*arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d
)*c) + d^2*log((sqrt(d*x)*c - sqrt(c*d))/(sqrt(d*x)*c + sqrt(c*d)))/(sqrt(c
*d)*c))*c/d)*b + 2*sqrt(d*x)*a)/d
```

Fricas [A]
time = 0.36, size = 211, normalized size = 2.48

$$\left[-\frac{2\sqrt{cd}\,b\arctan\left(\frac{\sqrt{cd}\,\sqrt{dx}}{cdx}\right)-\sqrt{cd}\,b\log\left(\frac{cdx-2\sqrt{cd}\,\sqrt{dx}+d}{cx-1}\right)-\left(bc\log\left(-\frac{cx+1}{cx-1}\right)+2\,ac\right)\sqrt{dx}}{cd},\ \frac{2\sqrt{-cd}\,b\arctan\left(\frac{\sqrt{-cd}\,\sqrt{dx}}{cdx}\right)-\sqrt{-cd}\,b\log\left(\frac{cdx-2\sqrt{-cd}\,\sqrt{dx}-d}{cx+1}\right)+\left(bc\log\left(-\frac{cx+1}{cx-1}\right)+2\,ac\right)\sqrt{dx}}{cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-(2*sqrt(c*d)*b*arctan(sqrt(c*d)*sqrt(d*x)/(c*d*x)) - sqrt(c*d)*b*log((c*d
*x - 2*sqrt(c*d)*sqrt(d*x) + d)/(c*x - 1)) - (b*c*log(-(c*x + 1)/(c*x - 1))
```

```
 + 2*a*c)*sqrt(d*x))/(c*d), (2*sqrt(-c*d)*b*arctan(sqrt(-c*d)*sqrt(d*x)/(c*
d*x)) - sqrt(-c*d)*b*log((c*d*x - 2*sqrt(-c*d)*sqrt(d*x) - d)/(c*x + 1)) +
(b*c*log(-(c*x + 1)/(c*x - 1)) + 2*a*c)*sqrt(d*x))/(c*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atanh}(cx)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(d*x)**(1/2),x)
```

```
[Out] Integral((a + b*atanh(c*x))/sqrt(d*x), x)
```

Giac [**A**]
time = 0.46, size = 88, normalized size = 1.04

$$\frac{\left(2\,cd\left(\frac{\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,c} + \frac{\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{-cd}}\right)}{\sqrt{-cd}\,c}\right) + \sqrt{dx}\,\log\left(-\frac{cx+1}{cx-1}\right)\right)b + 2\,\sqrt{dx}\,a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] ((2*c*d*(arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*c) + arctan(sqrt(d*x)*c/s
qrt(-c*d))/(sqrt(-c*d)*c)) + sqrt(d*x)*log(-(c*x + 1)/(c*x - 1)))*b + 2*sqr
t(d*x)*a)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{atanh}(c\,x)}{\sqrt{d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d*x)^(1/2),x)
```

```
[Out] int((a + b*atanh(c*x))/(d*x)^(1/2), x)
```

3.39 $\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2b\sqrt{c}\,\mathrm{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{d\sqrt{dx}}+\frac{2b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

```
[Out] 2*b*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))*c^(1/2)/d^(3/2)+2*b*arctanh(c^(1/2)
*(d*x)^(1/2)/d^(1/2))*c^(1/2)/d^(3/2)-2*(a+b*arctanh(c*x))/d/(d*x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6049, 335, 218, 214, 211}

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{d\sqrt{dx}}+\frac{2b\sqrt{c}\,\mathrm{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{2b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d*x)^(3/2),x]
```

```
[Out] (2*b*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/d^(3/2) - (2*(a + b*ArcTa
nh[c*x]))/(d*Sqrt[d*x]) + (2*b*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]]
)/d^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{3/2}}\,dx &= -\frac{2\left(a+b\tanh^{-1}(cx)\right)}{d\sqrt{dx}}+\frac{(2bc)\int \frac{1}{\sqrt{dx}\,(1-c^2x^2)}\,dx}{d}\\
&= -\frac{2\left(a+b\tanh^{-1}(cx)\right)}{d\sqrt{dx}}+\frac{(4bc)\text{Subst}\left(\int \frac{1}{1-\frac{c^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{d^2}\\
&= -\frac{2\left(a+b\tanh^{-1}(cx)\right)}{d\sqrt{dx}}+\frac{(2bc)\text{Subst}\left(\int \frac{1}{d-cx^2}\,dx,x,\sqrt{dx}\right)}{d}+\frac{(2bc)\text{Subst}\left(\int \frac{1}{d+cx^2}\right.}{d}\\
&= \frac{2b\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{d\sqrt{dx}}+\frac{2b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 99, normalized size = 1.16

$$\frac{x\left(-2a+2b\sqrt{c}\,\sqrt{x}\,\text{ArcTan}\left(\sqrt{c}\,\sqrt{x}\right)-2b\tanh^{-1}(cx)-b\sqrt{c}\,\sqrt{x}\,\log\left(1-\sqrt{c}\,\sqrt{x}\right)+b\sqrt{c}\,\sqrt{x}\,\log\left(1+\sqrt{c}\,\sqrt{x}\right)\right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d*x)^(3/2),x]
[Out] (x*(-2*a + 2*b*Sqrt[c]*Sqrt[x]*ArcTan[Sqrt[c]*Sqrt[x]] - 2*b*ArcTanh[c*x] -
 b*Sqrt[c]*Sqrt[x]*Log[1 - Sqrt[c]*Sqrt[x]] + b*Sqrt[c]*Sqrt[x]*Log[1 + Sqr
t[c]*Sqrt[x]]))/(d*x)^(3/2)
```

Maple [A]
time = 0.04, size = 69, normalized size = 0.81

method	result	size
derivativedivides	$\frac{-\frac{2a}{\sqrt{dx}}-\frac{2b\,\mathrm{arctanh}(cx)}{\sqrt{dx}}+\frac{2bc\arctan\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{\sqrt{dc}}+\frac{2bc\,\mathrm{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{\sqrt{dc}}}{d}$	69
default	$\frac{-\frac{2a}{\sqrt{dx}}-\frac{2b\,\mathrm{arctanh}(cx)}{\sqrt{dx}}+\frac{2bc\arctan\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{\sqrt{dc}}+\frac{2bc\,\mathrm{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{\sqrt{dc}}}{d}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-a/(d*x)^(1/2)-b/(d*x)^(1/2)*arctanh(c*x)+b*c/(d*c)^(1/2)*arctan(c*(d*
x)^(1/2)/(d*c)^(1/2))+b*c/(d*c)^(1/2)*arctanh(c*(d*x)^(1/2)/(d*c)^(1/2)))
```

Maxima [A]
time = 0.46, size = 94, normalized size = 1.11

$$\frac{b\left(\frac{\left(\frac{2\,d\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}}-\frac{d\log\left(\frac{\sqrt{dx}\,c-\sqrt{cd}}{\sqrt{dx}\,c+\sqrt{cd}}\right)}{\sqrt{cd}}\right)c}{d}-\frac{2\,\mathrm{artanh}(cx)}{\sqrt{dx}}\right)-\frac{2\,a}{\sqrt{dx}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] (b*((2*d*arctan(sqrt(d*x)*c/sqrt(c*d))/sqrt(c*d) - d*log((sqrt(d*x)*c - sqr
t(c*d))/(sqrt(d*x)*c + sqrt(c*d)))/sqrt(c*d))*c/d - 2*arctanh(c*x)/sqrt(d*x
)) - 2*a/sqrt(d*x))/d
```

Fricas [A]
time = 0.36, size = 221, normalized size = 2.60

$$\left[-\frac{2\,bdx\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{\frac{c}{d}}}{cx}\right)-bdx\sqrt{\frac{c}{d}}\log\left(\frac{cx+2\,\sqrt{dx}\sqrt{\frac{c}{d}}+1}{cx-1}\right)+\sqrt{dx}\left(b\log\left(-\frac{cx+1}{cx-1}\right)+2\,a\right)}{d^2x},\ -\frac{2\,bdx\sqrt{-\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{c}{d}}}{cx}\right)-bdx\sqrt{-\frac{c}{d}}\log\left(\frac{cx+2\,\sqrt{dx}\sqrt{-\frac{c}{d}}-1}{cx+1}\right)+\sqrt{dx}\left(b\log\left(-\frac{cx+1}{cx-1}\right)+2\,a\right)}{d^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-(2*b*d*x*sqrt(c/d)*arctan(sqrt(d*x)*sqrt(c/d)/(c*x)) - b*d*x*sqrt(c/d)*lo
g((c*x + 2*sqrt(d*x)*sqrt(c/d) + 1)/(c*x - 1)) + sqrt(d*x)*(b*log(-(c*x + 1
)/(c*x - 1)) + 2*a))/(d^2*x), -(2*b*d*x*sqrt(-c/d)*arctan(sqrt(d*x)*sqrt(-c
/d)/(c*x)) - b*d*x*sqrt(-c/d)*log((c*x + 2*sqrt(d*x)*sqrt(-c/d) - 1)/(c*x +
 1)) + sqrt(d*x)*(b*log(-(c*x + 1)/(c*x - 1)) + 2*a))/(d^2*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{atanh}\,(cx)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(d*x)**(3/2),x)
```

```
[Out] Integral((a + b*atanh(c*x))/(d*x)**(3/2), x)
```

Giac [A]
time = 0.44, size = 93, normalized size = 1.09

$$\frac{2\,bcd\left(\frac{\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,d}-\frac{\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{-cd}}\right)}{\sqrt{-cd}\,d}\right)-\frac{b\log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{dx}}-\frac{2\,a}{\sqrt{dx}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] (2*b*c*d*(arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d) - arctan(sqrt(d*x)*c/
sqrt(-c*d))/(sqrt(-c*d)*d)) - b*log(-(c*d*x + d)/(c*d*x - d))/sqrt(d*x) - 2
*a/sqrt(d*x))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{atanh}(c\,x)}{(d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d*x)^(3/2),x)
```

```
[Out] int((a + b*atanh(c*x))/(d*x)^(3/2), x)
```

3.40 $\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{4bc}{3d^2\sqrt{dx}}-\frac{2bc^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}+\frac{2bc^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}$$

```
[Out] -2/3*b*c^(3/2)*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))/d^(5/2)-2/3*(a+b*arctanh
(c*x))/d/(d*x)^(3/2)+2/3*b*c^(3/2)*arctanh(c^(1/2)*(d*x)^(1/2)/d^(1/2))/d^(
5/2)-4/3*b*c/d^2/(d*x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6049, 331, 335, 304, 211, 214}

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}-\frac{2bc^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2bc^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4bc}{3d^2\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d*x)^(5/2),x]
```

```
[Out] (-4*b*c)/(3*d^2*Sqrt[d*x]) - (2*b*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d
]])/(3*d^(5/2)) - (2*(a + b*ArcTanh[c*x]))/(3*d*(d*x)^(3/2)) + (2*b*c^(3/2)
*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{5/2}}\,dx = -\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}+\frac{(2bc)\int \frac{1}{(dx)^{3/2}(1-c^2x^2)}\,dx}{3d}$$

$$= -\frac{4bc}{3d^2\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}+\frac{(2bc^3)\int \frac{\sqrt{dx}}{1-c^2x^2}\,dx}{3d^3}$$

$$= -\frac{4bc}{3d^2\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}+\frac{(4bc^3)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{c^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{3d^4}$$

$$= -\frac{4bc}{3d^2\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}+\frac{(2bc^2)\,\text{Subst}\left(\int \frac{1}{d-cx^2}\,dx,x,\sqrt{dx}\right)}{3d^2}-\frac{(2bc^2)}{}$$

$$= -\frac{4bc}{3d^2\sqrt{dx}}-\frac{2bc^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}+\frac{2bc^{3/2}\tanh^{-1}}{3d^5}$$

Mathematica [A]
time = 0.04, size = 107, normalized size = 1.00

$$-\frac{x\left(2a+4bcx+2bc^{3/2}x^{3/2}\text{ArcTan}\left(\sqrt{c}\sqrt{x}\right)+2b\tanh^{-1}(cx)+bc^{3/2}x^{3/2}\log\left(1-\sqrt{c}\sqrt{x}\right)-bc^{3/2}x^{3/2}\log\left(1+\sqrt{c}\sqrt{x}\right)\right)}{3(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d*x)^(5/2),x]
```

```
[Out] -1/3*(x*(2*a + 4*b*c*x + 2*b*c^(3/2)*x^(3/2)*ArcTan[Sqrt[c]*Sqrt[x]] + 2*b*
ArcTanh[c*x] + b*c^(3/2)*x^(3/2)*Log[1 - Sqrt[c]*Sqrt[x]] - b*c^(3/2)*x^(3/
2)*Log[1 + Sqrt[c]*Sqrt[x]]))/(d*x)^(5/2)
```

Maple [A]
time = 0.05, size = 93, normalized size = 0.87

method	result	size
derivativedivides	$\frac{-\frac{2a}{3(dx)^{\frac{3}{2}}}-\frac{2b\,\mathrm{arctanh}(cx)}{3(dx)^{\frac{3}{2}}}-\frac{2b\,c^{2}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{3d\sqrt{dc}}+\frac{2b\,c^{2}\,\mathrm{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{3d\sqrt{dc}}-\frac{4bc}{3d\sqrt{dx}}}{d}$	93
default	$\frac{-\frac{2a}{3(dx)^{\frac{3}{2}}}-\frac{2b\,\mathrm{arctanh}(cx)}{3(dx)^{\frac{3}{2}}}-\frac{2b\,c^{2}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{3d\sqrt{dc}}+\frac{2b\,c^{2}\,\mathrm{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{3d\sqrt{dc}}-\frac{4bc}{3d\sqrt{dx}}}{d}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/3*a/(d*x)^(3/2)-1/3*b/(d*x)^(3/2)*arctanh(c*x)-1/3*b/d*c^2/(d*c)^(1
/2)*arctan(c*(d*x)^(1/2)/(d*c)^(1/2))+1/3*b/d*c^2/(d*c)^(1/2)*arctanh(c*(d*
x)^(1/2)/(d*c)^(1/2))-2/3*b/d*c/(d*x)^(1/2))
```

Maxima [A]
time = 0.46, size = 101, normalized size = 0.94

$$-\frac{b\left(\frac{\left(\frac{2\,c\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}}+\frac{c\log\left(\frac{\sqrt{dx}\,c-\sqrt{cd}}{\sqrt{dx}\,c+\sqrt{cd}}\right)}{\sqrt{cd}}+\frac{4}{\sqrt{dx}}\right)c}{d}+\frac{2\,\mathrm{artanh}(cx)}{(dx)^{\frac{3}{2}}}\right)+\frac{2\,a}{(dx)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(b*((2*c*arctan(sqrt(d*x)*c/sqrt(c*d))/sqrt(c*d) + c*log((sqrt(d*x)*c
- sqrt(c*d))/(sqrt(d*x)*c + sqrt(c*d)))/sqrt(c*d) + 4/sqrt(d*x))*c/d + 2*ar
ctanh(c*x)/(d*x)^(3/2)) + 2*a/(d*x)^(3/2))/d
```

Fricas [A]
time = 0.37, size = 243, normalized size = 2.27

$$\left[\frac{2\,bcdx^2\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{\frac{c}{d}}}{cx}\right)+bcdx^2\sqrt{\frac{c}{d}}\log\left(\frac{cx+2\sqrt{dx}\sqrt{\frac{c}{d}}+1}{cx-1}\right)-\left(4\,bcx+b\log\left(-\frac{cx+1}{cx-1}\right)+2\,a\right)\sqrt{dx}}{3\,d^3x^2},-\frac{2\,bcdx^2\sqrt{-\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{c}{d}}}{cx}\right)-bcdx^2\sqrt{-\frac{c}{d}}\log\left(\frac{cx-2\sqrt{dx}\sqrt{-\frac{c}{d}}-1}{cx+1}\right)+\left(4\,bcx+b\log\left(-\frac{cx+1}{cx-1}\right)+2\,a\right)\sqrt{dx}}{3\,d^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(2*b*c*d*x^2*sqrt(c/d)*arctan(sqrt(d*x)*sqrt(c/d)/(c*x)) + b*c*d*x^2*s
qrt(c/d)*log((c*x + 2*sqrt(d*x)*sqrt(c/d) + 1)/(c*x - 1)) - (4*b*c*x + b*lo
g(-(c*x + 1)/(c*x - 1)) + 2*a)*sqrt(d*x))/(d^3*x^2), -1/3*(2*b*c*d*x^2*sqrt
(-c/d)*arctan(sqrt(d*x)*sqrt(-c/d)/(c*x)) - b*c*d*x^2*sqrt(-c/d)*log((c*x -
 2*sqrt(d*x)*sqrt(-c/d) - 1)/(c*x + 1)) + (4*b*c*x + b*log(-(c*x + 1)/(c*x
- 1)) + 2*a)*sqrt(d*x))/(d^3*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{atanh}(cx)}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(d*x)**(5/2),x)
```

```
[Out] Integral((a + b*atanh(c*x))/(d*x)**(5/2), x)
```

Giac [A]
time = 0.44, size = 117, normalized size = 1.09

$$-\frac{\frac{2\,bc^2\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,d}+\frac{2\,bc^2\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{-cd}}\right)}{\sqrt{-cd}\,d}+\frac{b\log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{dx}\,dx}+\frac{2\,(2\,bcdx+ad)}{\sqrt{dx}\,d^2x}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(2*b*c^2*arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d) + 2*b*c^2*arctan(
sqrt(d*x)*c/sqrt(-c*d))/(sqrt(-c*d)*d) + b*log(-(c*d*x + d)/(c*d*x - d))/(s
qrt(d*x)*d*x) + 2*(2*b*c*d*x + a*d)/(sqrt(d*x)*d^2*x))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atanh}(c\,x)}{{(d\,x)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d*x)^(5/2),x)
[Out] int((a + b*atanh(c*x))/(d*x)^(5/2), x)
```

3.41 $\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{4bc}{15d^2(dx)^{3/2}}+\frac{2bc^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{2bc^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}$$

```
[Out] -4/15*b*c/d^2/(d*x)^(3/2)+2/5*b*c^(5/2)*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))
/d^(7/2)-2/5*(a+b*arctanh(c*x))/d/(d*x)^(5/2)+2/5*b*c^(5/2)*arctanh(c^(1/2)
*(d*x)^(1/2)/d^(1/2))/d^(7/2)
```

Rubi [A]
time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6049, 331, 335, 218, 214, 211}

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{2bc^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{2bc^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{4bc}{15d^2(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d*x)^(7/2),x]
```

```
[Out] (-4*b*c)/(15*d^2*(d*x)^(3/2)) + (2*b*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqr
t[d]])/(5*d^(7/2)) - (2*(a + b*ArcTanh[c*x]))/(5*d*(d*x)^(5/2)) + (2*b*c^(5
/2)*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(5*d^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{7/2}}\,dx &= -\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{(2bc)\int \frac{1}{(dx)^{5/2}(1-c^2x^2)}\,dx}{5d}\\
&= -\frac{4bc}{15d^2(dx)^{3/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{(2bc^3)\int \frac{1}{\sqrt{dx}\,(1-c^2x^2)}\,dx}{5d^3}\\
&= -\frac{4bc}{15d^2(dx)^{3/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{(4bc^3)\operatorname{Subst}\left(\int \frac{1}{1-\frac{c^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{5d^4}\\
&= -\frac{4bc}{15d^2(dx)^{3/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{(2bc^3)\operatorname{Subst}\left(\int \frac{1}{d-cx^2}\,dx,x,\sqrt{dx}\right)}{5d^3}+\frac{(2bc
\\
&= -\frac{4bc}{15d^2(dx)^{3/2}}+\frac{2bc^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{2bc^{5/2}\tanh^{-1}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 108, normalized size = 1.01

$$
\frac{x\left(-6a-4bcx+6bc^{5/2}x^{5/2}\text{ArcTan}\left(\sqrt{c}\,\sqrt{x}\right)-6b\tanh^{-1}(cx)-3bc^{5/2}x^{5/2}\log\left(1-\sqrt{c}\,\sqrt{x}\right)+3bc^{5/2}x^{5/2}\log\left(1+\sqrt{c}\,\sqrt{x}\right)\right)}{15(dx)^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d*x)^(7/2),x]
```

```
[Out] (x*(-6*a - 4*b*c*x + 6*b*c^(5/2)*x^(5/2)*ArcTan[Sqrt[c]*Sqrt[x]] - 6*b*ArcT
anh[c*x] - 3*b*c^(5/2)*x^(5/2)*Log[1 - Sqrt[c]*Sqrt[x]] + 3*b*c^(5/2)*x^(5/
2)*Log[1 + Sqrt[c]*Sqrt[x]]))/(15*(d*x)^(7/2))
```

Maple [A]
time = 0.05, size = 93, normalized size = 0.87

method	result	size
derivativedivides	$\frac{-\frac{2a}{5(dx)^{\frac{5}{2}}}-\frac{2b\,\mathrm{arctanh}(cx)}{5(dx)^{\frac{5}{2}}}+\frac{2b\,c^{3}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{5d^{2}\sqrt{dc}}+\frac{2b\,c^{3}\,\mathrm{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{5d^{2}\sqrt{dc}}-\frac{4bc}{15d(dx)^{\frac{3}{2}}}}{d}$	93
default	$\frac{-\frac{2a}{5(dx)^{\frac{5}{2}}}-\frac{2b\,\mathrm{arctanh}(cx)}{5(dx)^{\frac{5}{2}}}+\frac{2b\,c^{3}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{5d^{2}\sqrt{dc}}+\frac{2b\,c^{3}\,\mathrm{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{5d^{2}\sqrt{dc}}-\frac{4bc}{15d(dx)^{\frac{3}{2}}}}{d}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/5*a/(d*x)^(5/2)-1/5*b/(d*x)^(5/2)*arctanh(c*x)+1/5*b/d^2*c^3/(d*c)^
(1/2)*arctan(c*(d*x)^(1/2)/(d*c)^(1/2))+1/5*b/d^2*c^3/(d*c)^(1/2)*arctanh(c
*(d*x)^(1/2)/(d*c)^(1/2))-2/15*b/d*c/(d*x)^(3/2))
```

Maxima [A]
time = 0.47, size = 112, normalized size = 1.05

$$\frac{b\left(\frac{\left(\frac{6\,c^{2}\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,d}-\frac{3\,c^{2}\log\left(\frac{\sqrt{dx}\,c-\sqrt{cd}}{\sqrt{dx}\,c+\sqrt{cd}}\right)}{\sqrt{cd}\,d}-\frac{4}{(dx)^{\frac{3}{2}}}\right)c}{d}-\frac{6\,\mathrm{artanh}(cx)}{(dx)^{\frac{5}{2}}}\right)-\frac{6\,a}{(dx)^{\frac{5}{2}}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*(b*((6*c^2*arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d) - 3*c^2*log((sq
rt(d*x)*c - sqrt(c*d))/(sqrt(d*x)*c + sqrt(c*d)))/(sqrt(c*d)*d) - 4/(d*x)^(
3/2))*c/d - 6*arctanh(c*x)/(d*x)^(5/2)) - 6*a/(d*x)^(5/2))/d
```

Fricas [A]
time = 0.37, size = 253, normalized size = 2.36

$$\left[-\frac{6\,bc^{2}dx^{3}\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{\frac{c}{d}}}{cx}\right)-3\,bc^{2}dx^{3}\sqrt{\frac{c}{d}}\log\left(\frac{cx+2\sqrt{dx}\sqrt{\frac{c}{d}}+1}{cx-1}\right)+\left(4\,bcx+3\,b\log\left(-\frac{cx+1}{cx-1}\right)+6\,a\right)\sqrt{dx}}{15\,d^{4}x^{3}},\ -\frac{6\,bc^{2}dx^{3}\sqrt{-\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{c}{d}}}{cx}\right)-3\,bc^{2}dx^{3}\sqrt{-\frac{c}{d}}\log\left(\frac{cx+2\sqrt{dx}\sqrt{-\frac{c}{d}}-1}{cx+1}\right)+\left(4\,bcx+3\,b\log\left(-\frac{cx+1}{cx-1}\right)+6\,a\right)\sqrt{dx}}{15\,d^{4}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/15*(6*b*c^2*d*x^3*sqrt(c/d)*arctan(sqrt(d*x)*sqrt(c/d)/(c*x)) - 3*b*c^2
*d*x^3*sqrt(c/d)*log((c*x + 2*sqrt(d*x)*sqrt(c/d) + 1)/(c*x - 1)) + (4*b*c*
x + 3*b*log(-(c*x + 1)/(c*x - 1)) + 6*a)*sqrt(d*x))/(d^4*x^3), -1/15*(6*b*c
^2*d*x^3*sqrt(-c/d)*arctan(sqrt(d*x)*sqrt(-c/d)/(c*x)) - 3*b*c^2*d*x^3*sqrt
(-c/d)*log((c*x + 2*sqrt(d*x)*sqrt(-c/d) - 1)/(c*x + 1)) + (4*b*c*x + 3*b*l
og(-(c*x + 1)/(c*x - 1)) + 6*a)*sqrt(d*x))/(d^4*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{atanh}\,(cx)}{(dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(d*x)**(7/2),x)
```

```
[Out] Integral((a + b*atanh(c*x))/(d*x)**(7/2), x)
```

Giac [A]
time = 0.44, size = 117, normalized size = 1.09

$$\frac{6\,bc^{3}\left(\frac{\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^{2}}-\frac{\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{-cd}}\right)}{\sqrt{-cd}\,d^{2}}\right)-\frac{3\,b\log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{dx}\,d^{2}x^{2}}-\frac{2\,(2\,bcdx+3\,ad)}{\sqrt{dx}\,d^{3}x^{2}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(6*b*c^3*(arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d^2) - arctan(sqrt(
d*x)*c/sqrt(-c*d))/(sqrt(-c*d)*d^2)) - 3*b*log(-(c*d*x + d)/(c*d*x - d))/(s
qrt(d*x)*d^2*x^2) - 2*(2*b*c*d*x + 3*a*d)/(sqrt(d*x)*d^3*x^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atanh}(c\,x)}{{(d\,x)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d*x)^(7/2),x)
```

```
[Out] int((a + b*atanh(c*x))/(d*x)^(7/2), x)
```

3.42 $\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{9/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{4bc}{35d^2(dx)^{5/2}}-\frac{4bc^3}{7d^4\sqrt{dx}}-\frac{2bc^{7/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}}+\frac{2bc^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}$$

```
[Out] -4/35*b*c/d^2/(d*x)^(5/2)-2/7*b*c^(7/2)*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))
/d^(9/2)-2/7*(a+b*arctanh(c*x))/d/(d*x)^(7/2)+2/7*b*c^(7/2)*arctanh(c^(1/2)
*(d*x)^(1/2)/d^(1/2))/d^(9/2)-4/7*b*c^3/d^4/(d*x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6049, 331, 335, 304, 211, 214}

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}}-\frac{2bc^{7/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}+\frac{2bc^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{4bc^3}{7d^4\sqrt{dx}}-\frac{4bc}{35d^2(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d*x)^(9/2),x]
```

```
[Out] (-4*b*c)/(35*d^2*(d*x)^(5/2)) - (4*b*c^3)/(7*d^4*Sqrt[d*x]) - (2*b*c^(7/2)*
ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) - (2*(a + b*ArcTanh[c*x]))
/(7*d*(d*x)^(7/2)) + (2*b*c^(7/2)*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(7*
d^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{9/2}}\,dx &= -\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}}+\frac{(2bc)\int\frac{1}{(dx)^{7/2}(1-c^2x^2)}\,dx}{7d}\\
&= -\frac{4bc}{35d^2(dx)^{5/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}}+\frac{(2bc^3)\int\frac{1}{(dx)^{3/2}(1-c^2x^2)}\,dx}{7d^3}\\
&= -\frac{4bc}{35d^2(dx)^{5/2}}-\frac{4bc^3}{7d^4\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}}+\frac{(2bc^5)\int\frac{\sqrt{dx}}{1-c^2x^2}\,dx}{7d^5}\\
&= -\frac{4bc}{35d^2(dx)^{5/2}}-\frac{4bc^3}{7d^4\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}}+\frac{(4bc^5)\,\mathrm{Subst}\left(\int\frac{x^2}{1-\frac{c^2x^4}{d^2}}\,dx,x,\right.}{7d^6}\\
&= -\frac{4bc}{35d^2(dx)^{5/2}}-\frac{4bc^3}{7d^4\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}}+\frac{(2bc^4)\,\mathrm{Subst}\left(\int\frac{1}{d-cx^2}\,dx,x,\right.}{7d^4}\\
&= -\frac{4bc}{35d^2(dx)^{5/2}}-\frac{4bc^3}{7d^4\sqrt{dx}}-\frac{2bc^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}}+
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 122, normalized size = 0.98

$$-\frac{\sqrt{dx}\left(10a+4bcx+20bc^3x^3+10bc^{7/2}x^{7/2}\operatorname{ArcTan}\left(\sqrt{c}\sqrt{x}\right)+10b\tanh^{-1}(cx)+5bc^{7/2}x^{7/2}\log\left(1-\sqrt{c}\sqrt{x}\right)-5bc^{7/2}x^{7/2}\log\left(1+\sqrt{c}\sqrt{x}\right)\right)}{35d^5x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d*x)^(9/2),x]
```

```
[Out] -1/35*(Sqrt[d*x]*(10*a + 4*b*c*x + 20*b*c^3*x^3 + 10*b*c^(7/2)*x^(7/2)*ArcT
an[Sqrt[c]*Sqrt[x]] + 10*b*ArcTanh[c*x] + 5*b*c^(7/2)*x^(7/2)*Log[1 - Sqrt[
c]*Sqrt[x]] - 5*b*c^(7/2)*x^(7/2)*Log[1 + Sqrt[c]*Sqrt[x]]))/(d^5*x^4)
```

Maple [A]
time = 0.05, size = 107, normalized size = 0.86

method	result	size
derivativedivides	$\frac{-\frac{2a}{7(dx)^{\frac{7}{2}}}-\frac{2b\,\mathrm{arctanh}(cx)}{7(dx)^{\frac{7}{2}}}-\frac{2b\,c^4\arctan\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{7d^3\sqrt{dc}}+\frac{2b\,c^4\,\mathrm{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{7d^3\sqrt{dc}}-\frac{4bc}{35d(dx)^{\frac{5}{2}}}-\frac{4b\,c^3}{7d^3\sqrt{dx}}}{d}$	107
default	$\frac{-\frac{2a}{7(dx)^{\frac{7}{2}}}-\frac{2b\,\mathrm{arctanh}(cx)}{7(dx)^{\frac{7}{2}}}-\frac{2b\,c^4\arctan\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{7d^3\sqrt{dc}}+\frac{2b\,c^4\,\mathrm{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{dc}}\right)}{7d^3\sqrt{dc}}-\frac{4bc}{35d(dx)^{\frac{5}{2}}}-\frac{4b\,c^3}{7d^3\sqrt{dx}}}{d}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(d*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/7*a/(d*x)^(7/2)-1/7*b/(d*x)^(7/2)*arctanh(c*x)-1/7*b/d^3*c^4/(d*c)^
(1/2)*arctan(c*(d*x)^(1/2)/(d*c)^(1/2))+1/7*b/d^3*c^4/(d*c)^(1/2)*arctanh(c
*(d*x)^(1/2)/(d*c)^(1/2))-2/35*b/d*c/(d*x)^(5/2)-2/7*b/d^3*c^3/(d*x)^(1/2))
```

Maxima [A]
time = 0.47, size = 130, normalized size = 1.04

$$-\frac{b\left(\frac{\left(\frac{10\,c^3\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^2}+\frac{5\,c^3\log\left(\frac{\sqrt{dx}\,c-\sqrt{cd}}{\sqrt{dx}\,c+\sqrt{cd}}\right)}{\sqrt{cd}\,d^2}+\frac{4\left(5\,c^2d^2x^2+d^2\right)}{(dx)^{\frac{5}{2}}d^2}\right)c}{d}+\frac{10\,\mathrm{artanh}(cx)}{(dx)^{\frac{7}{2}}}\right)+\frac{10\,a}{(dx)^{\frac{7}{2}}}}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/35*(b*((10*c^3*arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d^2) + 5*c^3*log
((sqrt(d*x)*c - sqrt(c*d))/(sqrt(d*x)*c + sqrt(c*d)))/(sqrt(c*d)*d^2) + 4*(
5*c^2*d^2*x^2 + d^2)/((d*x)^(5/2)*d^2))*c/d + 10*arctanh(c*x)/(d*x)^(7/2))
+ 10*a/(d*x)^(7/2))/d
```

Fricas [A]
time = 0.36, size = 272, normalized size = 2.18

$$\left[\frac{10\,bc^3dx^4\sqrt{\frac{c}{d}}\,\arctan\left(\frac{\sqrt{dx}\,\sqrt{\frac{c}{d}}}{cx}\right)+5\,bc^3dx^4\sqrt{\frac{c}{d}}\,\log\left(\frac{cx+2\,\sqrt{dx}\,\sqrt{\frac{c}{d}}+1}{cx-1}\right)-\left(20\,bc^3x^3+4\,bcx+5\,b\log\left(-\frac{cx+1}{cx-1}\right)+10\,a\right)\sqrt{dx}}{35\,d^5x^4},-\frac{10\,bc^3dx^4\sqrt{-\frac{c}{d}}\,\arctan\left(\frac{\sqrt{dx}\,\sqrt{-\frac{c}{d}}}{cx}\right)-5\,bc^3dx^4\sqrt{-\frac{c}{d}}\,\log\left(\frac{cx-2\,\sqrt{dx}\,\sqrt{-\frac{c}{d}}-1}{cx+1}\right)+\left(20\,bc^3x^3+4\,bcx+5\,b\log\left(-\frac{cx+1}{cx-1}\right)+10\,a\right)\sqrt{dx}}{35\,d^5x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/35*(10*b*c^3*d*x^4*sqrt(c/d)*arctan(sqrt(d*x)*sqrt(c/d)/(c*x)) + 5*b*c^3
*d*x^4*sqrt(c/d)*log((c*x + 2*sqrt(d*x)*sqrt(c/d) + 1)/(c*x - 1)) - (20*b*c
^3*x^3 + 4*b*c*x + 5*b*log(-(c*x + 1)/(c*x - 1)) + 10*a)*sqrt(d*x))/(d^5*x^
4), -1/35*(10*b*c^3*d*x^4*sqrt(-c/d)*arctan(sqrt(d*x)*sqrt(-c/d)/(c*x)) - 5
*b*c^3*d*x^4*sqrt(-c/d)*log((c*x - 2*sqrt(d*x)*sqrt(-c/d) - 1)/(c*x + 1)) +
 (20*b*c^3*x^3 + 4*b*c*x + 5*b*log(-(c*x + 1)/(c*x - 1)) + 10*a)*sqrt(d*x))
/(d^5*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{atanh}\,(cx)}{(dx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(d*x)**(9/2),x)
```

```
[Out] Integral((a + b*atanh(c*x))/(d*x)**(9/2), x)
```

Giac [A]
time = 0.51, size = 135, normalized size = 1.08

$$-\frac{\frac{10\,bc^4\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^3}+\frac{10\,bc^4\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{-cd}}\right)}{\sqrt{-cd}\,d^3}+\frac{5\,b\log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{dx}\,d^3x^3}+\frac{2\,\left(10\,bc^3d^3x^3+2\,bcd^3x+5\,ad^3\right)}{\sqrt{dx}\,d^6x^3}}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*(10*b*c^4*arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d^3) + 10*b*c^4*ar
ctan(sqrt(d*x)*c/sqrt(-c*d))/(sqrt(-c*d)*d^3) + 5*b*log(-(c*d*x + d)/(c*d*x
```

```
 - d))/(sqrt(d*x)*d^3*x^3) + 2*(10*b*c^3*d^3*x^3 + 2*b*c*d^3*x + 5*a*d^3)/(
sqrt(d*x)*d^6*x^3))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atanh}(c\,x)}{(d\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d*x)^(9/2),x)
```

```
[Out] int((a + b*atanh(c*x))/(d*x)^(9/2), x)
```

3.43 $\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left((dx)^m \left(a+b\tanh^{-1}(cx)\right)^3,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c*x))^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x])^3,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^3\,dx=\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^3\,dx$$

Mathematica [A]
time = 2.69, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^3, x]
```

Maple [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int (dx)^m (a+b\,\mathrm{arctanh}\,(cx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*d^m*x*x^m*log(-c*x + 1)^3/(m + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1))
+ integrate(1/8*((b^3*c*d^m*(m + 1)*x - b^3*d^m*(m + 1))*x^m*log(c*x + 1)^3
 + 6*(a*b^2*c*d^m*(m + 1)*x - a*b^2*d^m*(m + 1))*x^m*log(c*x + 1)^2 + 12*(a
^2*b*c*d^m*(m + 1)*x - a^2*b*d^m*(m + 1))*x^m*log(c*x + 1) + 3*((b^3*c*d^m*
(m + 1)*x - b^3*d^m*(m + 1))*x^m*log(c*x + 1) - (2*a*b^2*d^m*(m + 1) - (2*a
*b^2*c*d^m*(m + 1) + b^3*c*d^m)*x)*x^m)*log(-c*x + 1)^2 - 3*((b^3*c*d^m*(m
+ 1)*x - b^3*d^m*(m + 1))*x^m*log(c*x + 1)^2 + 4*(a*b^2*c*d^m*(m + 1)*x - a
*b^2*d^m*(m + 1))*x^m*log(c*x + 1) + 4*(a^2*b*c*d^m*(m + 1)*x - a^2*b*d^m*(
m + 1))*x^m)*log(-c*x + 1))/(c*(m + 1)*x - m - 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)*(d*x)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{atanh}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c*x))**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*(d*x)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (a+b\,\mathrm{atanh}(c\,x))^3\,(d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3*(d*x)^m,x)
```

```
[Out] int((a + b*atanh(c*x))^3*(d*x)^m, x)
```

3.44 $\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=19

$$\text{Int}\left((dx)^m \left(a+b\tanh^{-1}(cx)\right)^2, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c*x))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x])^2,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$$

Mathematica [A]
time = 1.69, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^2, x]
```

Maple [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\,\mathrm{arctanh}\,(cx)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x))^2,x)
[Out] int((d*x)^m*(a+b*arctanh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
[Out] 1/4*b^2*d^m*x*x^m*log(-c*x + 1)^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1)) -
 integrate(-1/4*((b^2*c*d^m*(m + 1)*x - b^2*d^m*(m + 1))*x^m*log(c*x + 1)^2
 + 4*(a*b*c*d^m*(m + 1)*x - a*b*d^m*(m + 1))*x^m*log(c*x + 1) - 2*((b^2*c*d
^m*(m + 1)*x - b^2*d^m*(m + 1))*x^m*log(c*x + 1) - (2*a*b*d^m*(m + 1) - (2*
a*b*c*d^m*(m + 1) + b^2*c*d^m)*x)*x^m)*log(-c*x + 1))/(c*(m + 1)*x - m - 1)
, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)*(d*x)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b\operatorname{atanh}(cx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x))**2,x)
[Out] Integral((d*x)**m*(a + b*atanh(c*x))**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*(d*x)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (a + b\,\mathrm{atanh}(c\,x))^2\,(d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2*(d*x)^m,x)
[Out] int((a + b*atanh(c*x))^2*(d*x)^m, x)
```

3.45 $\int (dx)^m \left(a+b\tanh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=72

$$\frac{(dx)^{1+m}\left(a+b\tanh^{-1}(cx)\right)}{d(1+m)}-\frac{bc(dx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};c^2x^2\right)}{d^2(1+m)(2+m)}$$

```
[Out] (d*x)^(1+m)*(a+b*arctanh(c*x))/d/(1+m)-b*c*(d*x)^(2+m)*hypergeom([1, 1+1/2*
m],[2+1/2*m],c^2*x^2)/d^2/(1+m)/(2+m)
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6049, 371}

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}(cx)\right)}{d(m+1)}-\frac{bc(dx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{d^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTanh[c*x]))/(d*(1 + m)) - (b*c*(d*x)^(2 + m)*Hyper
geometric2F1[1, (2 + m)/2, (4 + m)/2, c^2*x^2])/(d^2*(1 + m)*(2 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a+b\tanh^{-1}(cx)\right)\,dx &= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}(cx)\right)}{d(1+m)}-\frac{(bc)\int\frac{(dx)^{1+m}}{1-c^2x^2}\,dx}{d(1+m)}\\ &= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}(cx)\right)}{d(1+m)}-\frac{bc(dx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};c^2x^2\right)}{d^2(1+m)(2+m)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 0.82

$$-\frac{x(dx)^m\left(-\left((2+m)\left(a+b\tanh^{-1}(cx)\right)\right)+bcx\,{}_2F_1\left(1,1+\frac{m}{2};2+\frac{m}{2};c^2x^2\right)\right)}{(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x]),x]
```

```
[Out] -((x*(d*x)^m*(-((2 + m)*(a + b*ArcTanh[c*x])) + b*c*x*Hypergeometric2F1[1,
1 + m/2, 2 + m/2, c^2*x^2]))/((1 + m)*(2 + m)))
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\operatorname{arctanh}(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x)),x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*c*d^m*integrate(x*x^m/(c^2*(m + 1)*x^2 - m - 1), x) + (d^m*x*x^m*log
(c*x + 1) - d^m*x*x^m*log(-c*x + 1))/(m + 1))*b + (d*x)^(m + 1)*a/(d*(m + 1
))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)*(d*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{atanh}(cx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x)),x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{atanh}(c\,x)) \, (d\,x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d*x)^m,x)
```

```
[Out] int((a + b*atanh(c*x))*(d*x)^m, x)
```

3.46 $\int \frac{(dx)^m}{a+b\tanh^{-1}(cx)}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{(dx)^m}{a+b\tanh^{-1}(cx)},x\right)$$

[Out] Unintegrable((d*x)^m/(a+b*arctanh(c*x)),x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(dx)^m}{a+b\tanh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

[In] Int[(d*x)^m/(a + b*ArcTanh[c*x]),x]

[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x]), x]

Rubi steps

$$\int \frac{(dx)^m}{a+b\tanh^{-1}(cx)}\,dx=\int \frac{(dx)^m}{a+b\tanh^{-1}(cx)}\,dx$$

Mathematica [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\tanh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x]),x]

[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x]), x]

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\,\mathrm{arctanh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x)),x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b*arctanh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b\,\mathrm{atanh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x)),x)
```

```
[Out] Integral((d*x)**m/(a + b*atanh(c*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a + b\,\mathrm{atanh}\,(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c*x)),x)
```

```
[Out] int((d*x)^m/(a + b*atanh(c*x)), x)
```

3.47 $\int \frac{(dx)^m}{(a+b\tanh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{(dx)^m}{(a+b\tanh^{-1}(cx))^2},x\right)$$

[Out] Unintegrable((d*x)^m/(a+b*arctanh(c*x))^2,x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{(a+b\tanh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

[In] Int[(d*x)^m/(a + b*ArcTanh[c*x])^2,x]

[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x])^2, x]

Rubi steps

$$\int \frac{(dx)^m}{(a+b\tanh^{-1}(cx))^2}\,dx = \int \frac{(dx)^m}{(a+b\tanh^{-1}(cx))^2}\,dx$$

Mathematica [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a+b\tanh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x])^2,x]

[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x])^2, x]

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a+b\,\mathrm{arctanh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 2*(c^2*d^m*x^2 - d^m)*x^m/(b^2*c*log(c*x + 1) - b^2*c*log(-c*x + 1) + 2*a*b
*c) + integrate(-2*(c^2*d^m*(m + 2)*x^2 - d^m*m)*x^m/(b^2*c*x*log(c*x + 1)
- b^2*c*x*log(-c*x + 1) + 2*a*b*c*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + b\,\mathrm{atanh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x))**2,x)
```

```
[Out] Integral((d*x)**m/(a + b*atanh(c*x))**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c*x))^2,x)
[Out] int((d*x)^m/(a + b*atanh(c*x))^2, x)
```

3.48 $\int \left(a+b\tanh^{-1}(cx)\right)^p \, dx$

Optimal. Leaf size=13

$$\text{Int}\left(\left(a+b\tanh^{-1}(cx)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*arctanh(c*x))^p,x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(a+b\tanh^{-1}(cx)\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x])^p,x]
[Out] Defer[Int][(a + b*ArcTanh[c*x])^p, x]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}(cx)\right)^p \, dx=\int \left(a+b\tanh^{-1}(cx)\right)^p \, dx$$

Mathematica [A]
time = 0.33, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^{-1}(cx)\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x])^p,x]
[Out] Integrate[(a + b*ArcTanh[c*x])^p, x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arctanh}\left(cx\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^p,x)
```

```
[Out] int((a+b*arctanh(c*x))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^p,x, algorithm="maxima")
[Out] integrate((b*arctanh(c*x) + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^p,x, algorithm="fricas")
[Out] integral((b*arctanh(c*x) + a)^p, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(cx))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**p,x)
[Out] Integral((a + b*atanh(c*x))**p, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^p,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int (a + b \operatorname{atanh}(c\,x))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^p,x)
[Out] int((a + b*atanh(c*x))^p, x)
```

3.49 $\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^p\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left((dx)^m \left(a+b\tanh^{-1}(cx)\right)^p,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c*x))^p,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x])^p,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x])^p, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^p\,dx = \int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^p\,dx$$

Mathematica [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^p,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^p, x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\,\mathrm{arctanh}\left(cx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x))^p,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^p,x, algorithm="maxima")
[Out] integrate((d*x)^m*(b*arctanh(c*x) + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^p,x, algorithm="fricas")
[Out] integral((d*x)^m*(b*arctanh(c*x) + a)^p, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{atanh}(cx))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x))**p,x)
[Out] Integral((d*x)**m*(a + b*atanh(c*x))**p, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^p,x, algorithm="giac")
[Out] integrate((d*x)^m*(b*arctanh(c*x) + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (a + b \operatorname{atanh}(c\,x))^p \, (d\,x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^p*(d*x)^m,x)
[Out] int((a + b*atanh(c*x))^p*(d*x)^m, x)
```

3.50 $\int x^7 \left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=54

$$\frac{bx^2}{8c^3}+\frac{bx^6}{24c}-\frac{b\tanh^{-1}\left(cx^2\right)}{8c^4}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)$$

```
[Out] 1/8*b*x^2/c^3+1/24*b*x^6/c-1/8*b*arctanh(c*x^2)/c^4+1/8*x^8*(a+b*arctanh(c*
x^2))
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6037, 281, 308, 212}

$$\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\tanh^{-1}\left(cx^2\right)}{8c^4}+\frac{bx^2}{8c^3}+\frac{bx^6}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (b*x^2)/(8*c^3) + (b*x^6)/(24*c) - (b*ArcTanh[c*x^2])/(8*c^4) + (x^8*(a + b
*ArcTanh[c*x^2]))/8
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
```

```
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx &= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{4}(bc)\int\frac{x^9}{1-c^2x^4}\,dx\\
&= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{8}(bc)\text{Subst}\left(\int\frac{x^4}{1-c^2x^2}\,dx,x,x^2\right)\\
&= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{8}(bc)\text{Subst}\left(\int\left(-\frac{1}{c^4}-\frac{x^2}{c^2}+\frac{1}{c^4\left(1-c^2x^2\right)}\right)\,d\right.\\
&= \frac{bx^2}{8c^3}+\frac{bx^6}{24c}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\text{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,x^2\right)}{8c^3}\\
&= \frac{bx^2}{8c^3}+\frac{bx^6}{24c}-\frac{b\tanh^{-1}\left(cx^2\right)}{8c^4}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 78, normalized size = 1.44

$$\frac{bx^2}{8c^3}+\frac{bx^6}{24c}+\frac{ax^8}{8}+\frac{1}{8}bx^8\tanh^{-1}\left(cx^2\right)+\frac{b\log\left(1-cx^2\right)}{16c^4}-\frac{b\log\left(1+cx^2\right)}{16c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (b*x^2)/(8*c^3) + (b*x^6)/(24*c) + (a*x^8)/8 + (b*x^8*ArcTanh[c*x^2])/8 + (
b*Log[1 - c*x^2])/(16*c^4) - (b*Log[1 + c*x^2])/(16*c^4)
```

Maple [A]
time = 0.03, size = 66, normalized size = 1.22

method	result	size
default	$\frac{x^8 a}{8}+\frac{b\,x^8\,\mathrm{arctanh}(c\,x^2)}{8}+\frac{b\,x^6}{24c}+\frac{b\,x^2}{8c^3}+\frac{b\ln(c\,x^2-1)}{16c^4}-\frac{b\ln(c\,x^2+1)}{16c^4}$	66
risch	$\frac{x^8 b\ln(c\,x^2+1)}{16}-\frac{x^8 b\ln(-c\,x^2+1)}{16}+\frac{x^8 a}{8}+\frac{b\,x^6}{24c}+\frac{b\,x^2}{8c^3}-\frac{b\ln(c\,x^2+1)}{16c^4}+\frac{b\ln(c\,x^2-1)}{16c^4}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^8*a+1/8*b*x^8*arctanh(c*x^2)+1/24*b*x^6/c+1/8*b*x^2/c^3+1/16*b/c^4*ln
(c*x^2-1)-1/16*b/c^4*ln(c*x^2+1)
```

Maxima [A]
time = 0.27, size = 69, normalized size = 1.28

$$\frac{1}{8}\,ax^8+\frac{1}{48}\left(6\,x^8\,\mathrm{artanh}\left(cx^2\right)+c\left(\frac{2\,(c^2x^6+3\,x^2)}{c^4}-\frac{3\,\log\left(cx^2+1\right)}{c^5}+\frac{3\,\log\left(cx^2-1\right)}{c^5}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8 + 1/48*(6*x^8*arctanh(c*x^2) + c*(2*(c^2*x^6 + 3*x^2)/c^4 - 3*log
(c*x^2 + 1)/c^5 + 3*log(c*x^2 - 1)/c^5))*b
```

Fricas [A]
time = 0.34, size = 64, normalized size = 1.19

$$\frac{6\,ac^4x^8+2\,bc^3x^6+6\,bcx^2+3\,(bc^4x^8-b)\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{48\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/48*(6*a*c^4*x^8 + 2*b*c^3*x^6 + 6*b*c*x^2 + 3*(b*c^4*x^8 - b)*log(-(c*x^2
 + 1)/(c*x^2 - 1)))/c^4
```

Sympy [A]
time = 7.60, size = 58, normalized size = 1.07

$$\begin{cases}\frac{ax^8}{8}+\frac{bx^8\,\mathrm{atanh}\left(cx^2\right)}{8}+\frac{bx^6}{24c}+\frac{bx^2}{8c^3}-\frac{b\,\mathrm{atanh}\left(cx^2\right)}{8c^4} & \text{for } c\neq 0\\ \frac{ax^8}{8} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((a*x**8/8 + b*x**8*atanh(c*x**2)/8 + b*x**6/(24*c) + b*x**2/(8*c*
*3) - b*atanh(c*x**2)/(8*c**4), Ne(c, 0)), (a*x**8/8, True))
```

Giac [A]
time = 0.43, size = 78, normalized size = 1.44

$$\frac{1}{16}\,bx^8\log\left(-\frac{cx^2+1}{cx^2-1}\right)+\frac{1}{8}\,ax^8+\frac{bx^6}{24\,c}+\frac{bx^2}{8\,c^3}-\frac{b\log\left(cx^2+1\right)}{16\,c^4}+\frac{b\log\left(cx^2-1\right)}{16\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/16*b*x^8*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/8*a*x^8 + 1/24*b*x^6/c + 1/8*b
*x^2/c^3 - 1/16*b*log(c*x^2 + 1)/c^4 + 1/16*b*log(c*x^2 - 1)/c^4
```

Mupad [B]
time = 1.06, size = 69, normalized size = 1.28

$$\frac{a\,x^8}{8}+\frac{b\,x^2}{8\,c^3}+\frac{b\,x^6}{24\,c}+\frac{b\,x^8\,\ln\left(c\,x^2+1\right)}{16}-\frac{b\,x^8\,\ln\left(1-c\,x^2\right)}{16}+\frac{b\,\mathrm{atan}(c\,x^2\,1\mathrm{i})\,1\mathrm{i}}{8\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*atanh(c*x^2)),x)
```

```
[Out] (a*x^8)/8 + (b*x^2)/(8*c^3) + (b*x^6)/(24*c) + (b*atan(c*x^2*1i)*1i)/(8*c^4
) + (b*x^8*log(c*x^2 + 1))/16 - (b*x^8*log(1 - c*x^2))/16
```

3.51 $\int x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=48

$$\frac{bx^4}{12c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1-c^2x^4\right)}{12c^3}$$

```
[Out] 1/12*b*x^4/c+1/6*x^6*(a+b*arctanh(c*x^2))+1/12*b*ln(-c^2*x^4+1)/c^3
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6037, 272, 45}

$$\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1-c^2x^4\right)}{12c^3}+\frac{bx^4}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (b*x^4)/(12*c) + (x^6*(a + b*ArcTanh[c*x^2]))/6 + (b*Log[1 - c^2*x^4])/(12*
c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx &= \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{3}(bc)\int\frac{x^7}{1-c^2x^4}\,dx\\ &= \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{12}(bc)\text{Subst}\left(\int\frac{x}{1-c^2x}\,dx,x,x^4\right)\\ &= \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{12}(bc)\text{Subst}\left(\int\left(-\frac{1}{c^2}-\frac{1}{c^2\left(-1+c^2x\right)}\right)\,dx,x\right.\\ &= \frac{bx^4}{12c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1-c^2x^4\right)}{12c^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 1.10

$$\frac{bx^4}{12c}+\frac{ax^6}{6}+\frac{1}{6}bx^6\tanh^{-1}\left(cx^2\right)+\frac{b\log\left(1-c^2x^4\right)}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x^2]),x]
[Out] (b*x^4)/(12*c) + (a*x^6)/6 + (b*x^6*ArcTanh[c*x^2])/6 + (b*Log[1 - c^2*x^4]
)/(12*c^3)
```

Maple [A]
time = 0.04, size = 45, normalized size = 0.94

method	result	size
default	$\frac{x^6a}{6}+\frac{b\,x^6\,\text{arctanh}\left(c\,x^2\right)}{6}+\frac{b\,x^4}{12c}+\frac{b\ln\left(c^2x^4-1\right)}{12c^3}$	45
risch	$\frac{x^6b\ln\left(c\,x^2+1\right)}{12}-\frac{x^6b\ln\left(-c\,x^2+1\right)}{12}+\frac{x^6a}{6}+\frac{b\,x^4}{12c}+\frac{b\ln\left(c^2x^4-1\right)}{12c^3}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)
[Out] 1/6*x^6*a+1/6*b*x^6*arctanh(c*x^2)+1/12*b*x^4/c+1/12*b/c^3*ln(c^2*x^4-1)
```

Maxima [A]
time = 0.25, size = 46, normalized size = 0.96

$$\frac{1}{6}\,ax^6+\frac{1}{12}\left(2\,x^6\,\text{artanh}\left(cx^2\right)+\left(\frac{x^4}{c^2}+\frac{\log\left(c^2x^4-1\right)}{c^4}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/12*(2*x^6*arctanh(c*x^2) + (x^4/c^2 + log(c^2*x^4 - 1)/c^4)*c
)*b
```

Fricas [A]
time = 0.35, size = 62, normalized size = 1.29

$$\frac{bc^3x^6\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,ac^3x^6+bc^2x^4+b\log\left(c^2x^4-1\right)}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/12*(b*c^3*x^6*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c^3*x^6 + b*c^2*x^4 + b
*log(c^2*x^4 - 1))/c^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(39) = 78.
time = 5.12, size = 85, normalized size = 1.77

$$\begin{cases} \frac{ax^6}{6}+\frac{bx^6\operatorname{atanh}\left(cx^2\right)}{6}+\frac{bx^4}{12c}+\frac{b\log\left(x-\sqrt{-\frac{1}{c}}\right)}{6c^3}+\frac{b\log\left(x+\sqrt{-\frac{1}{c}}\right)}{6c^3}-\frac{b\operatorname{atanh}\left(cx^2\right)}{6c^3} & \text{for}\ c\neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((a*x**6/6 + b*x**6*atanh(c*x**2)/6 + b*x**4/(12*c) + b*log(x - sq
rt(-1/c))/(6*c**3) + b*log(x + sqrt(-1/c))/(6*c**3) - b*atanh(c*x**2)/(6*c*
*3), Ne(c, 0)), (a*x**6/6, True))
```

Giac [A]
time = 0.41, size = 57, normalized size = 1.19

$$\frac{1}{12}\,bx^6\log\left(-\frac{cx^2+1}{cx^2-1}\right)+\frac{1}{6}\,ax^6+\frac{bx^4}{12\,c}+\frac{b\log\left(c^2x^4-1\right)}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/12*b*x^6*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/6*a*x^6 + 1/12*b*x^4/c + 1/12*
b*log(c^2*x^4 - 1)/c^3
```

Mupad [B]
time = 0.79, size = 61, normalized size = 1.27

$$\frac{a\,x^6}{6}+\frac{b\,\ln\left(c^2\,x^4-1\right)}{12\,c^3}+\frac{b\,x^4}{12\,c}+\frac{b\,x^6\,\ln\left(c\,x^2+1\right)}{12}-\frac{b\,x^6\,\ln\left(1-c\,x^2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atanh(c*x^2)),x)
```

```
[Out] (a*x^6)/6 + (b*log(c^2*x^4 - 1))/(12*c^3) + (b*x^4)/(12*c) + (b*x^6*log(c*x
^2 + 1))/12 - (b*x^6*log(1 - c*x^2))/12
```

3.52 $\int x^3 \left(a + b \tanh^{-1}\left(cx^2\right)\right) \, dx$

Optimal. Leaf size=43

$$\frac{bx^2}{4c} - \frac{b \tanh^{-1}\left(cx^2\right)}{4c^2} + \frac{1}{4}x^4\left(a + b \tanh^{-1}\left(cx^2\right)\right)$$

```
[Out] 1/4*b*x^2/c-1/4*b*arctanh(c*x^2)/c^2+1/4*x^4*(a+b*arctanh(c*x^2))
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6037, 281, 327, 212}

$$\frac{1}{4}x^4\left(a + b \tanh^{-1}\left(cx^2\right)\right) - \frac{b \tanh^{-1}\left(cx^2\right)}{4c^2} + \frac{bx^2}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (b*x^2)/(4*c) - (b*ArcTanh[c*x^2])/(4*c^2) + (x^4*(a + b*ArcTanh[c*x^2]))/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
```

```
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx&=\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{2}(bc)\int\frac{x^5}{1-c^2x^4}dx\\&=\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{4}(bc)\text{Subst}\left(\int\frac{x^2}{1-c^2x^2}dx,x,x^2\right)\\&=\frac{bx^2}{4c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\text{Subst}\left(\int\frac{1}{1-c^2x^2}dx,x,x^2\right)}{4c}\\&=\frac{bx^2}{4c}-\frac{b\tanh^{-1}\left(cx^2\right)}{4c^2}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 67, normalized size = 1.56

$$\frac{bx^2}{4c}+\frac{ax^4}{4}+\frac{1}{4}bx^4\tanh^{-1}\left(cx^2\right)+\frac{b\log\left(1-cx^2\right)}{8c^2}-\frac{b\log\left(1+cx^2\right)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x^2]),x]
[Out] (b*x^2)/(4*c) + (a*x^4)/4 + (b*x^4*ArcTanh[c*x^2])/4 + (b*Log[1 - c*x^2])/(
8*c^2) - (b*Log[1 + c*x^2])/(8*c^2)
```

Maple [A]
time = 0.04, size = 57, normalized size = 1.33

method	result	size
default	$\frac{x^4a}{4}+\frac{b\,x^4\,\text{arctanh}(c\,x^2)}{4}+\frac{b\,x^2}{4c}+\frac{b\ln(c\,x^2-1)}{8c^2}-\frac{b\ln(c\,x^2+1)}{8c^2}$	57
risch	$\frac{x^4b\ln(c\,x^2+1)}{8}-\frac{x^4b\ln(-c\,x^2+1)}{8}+\frac{x^4a}{4}+\frac{b\,x^2}{4c}+\frac{b\ln(c\,x^2-1)}{8c^2}-\frac{b\ln(c\,x^2+1)}{8c^2}+\frac{b^2}{16a\,c^2}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4*a+1/4*b*x^4*arctanh(c*x^2)+1/4*b*x^2/c+1/8*b/c^2*ln(c*x^2-1)-1/8*b/
c^2*ln(c*x^2+1)
```

Maxima [A]
time = 0.26, size = 58, normalized size = 1.35

$$\frac{1}{4}\,ax^4+\frac{1}{8}\left(2\,x^4\,\text{artanh}\left(cx^2\right)+c\left(\frac{2\,x^2}{c^2}-\frac{\log\left(cx^2+1\right)}{c^3}+\frac{\log\left(cx^2-1\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/8*(2*x^4*arctanh(c*x^2) + c*(2*x^2/c^2 - log(c*x^2 + 1)/c^3 +
 log(c*x^2 - 1)/c^3))*b
```

Fricas [A]
time = 0.36, size = 54, normalized size = 1.26

$$\frac{2\,ac^2x^4 + 2\,bcx^2 + (bc^2x^4 - b)\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*c^2*x^4 + 2*b*c*x^2 + (b*c^2*x^4 - b)*log(-(c*x^2 + 1)/(c*x^2 - 1)
))/c^2
```

Sympy [A]
time = 3.61, size = 48, normalized size = 1.12

$$\begin{cases} \frac{ax^4}{4} + \frac{bx^4\operatorname{atanh}(cx^2)}{4} + \frac{bx^2}{4c} - \frac{b\operatorname{atanh}(cx^2)}{4c^2} & \text{for } c \neq 0 \\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((a*x**4/4 + b*x**4*atanh(c*x**2)/4 + b*x**2/(4*c) - b*atanh(c*x**
2)/(4*c**2), Ne(c, 0)), (a*x**4/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(37) = 74.
time = 0.41, size = 181, normalized size = 4.21

$$\frac{1}{2}\,c\left(\frac{(cx^2+1)b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{\left(\frac{(cx^2+1)^2c^3}{(cx^2-1)^2} - \frac{2\,(cx^2+1)c^3}{cx^2-1} + c^3\right)(cx^2-1)} + \frac{\frac{2\,(cx^2+1)a}{cx^2-1} + \frac{(cx^2+1)b}{cx^2-1} - b}{\frac{(cx^2+1)^2c^3}{(cx^2-1)^2} - \frac{2\,(cx^2+1)c^3}{cx^2-1} + c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/2*c*((c*x^2 + 1)*b*log(-(c*x^2 + 1)/(c*x^2 - 1))/(((c*x^2 + 1)^2*c^3/(c*x
^2 - 1)^2 - 2*(c*x^2 + 1)*c^3/(c*x^2 - 1) + c^3)*(c*x^2 - 1)) + (2*(c*x^2 +
 1)*a/(c*x^2 - 1) + (c*x^2 + 1)*b/(c*x^2 - 1) - b)/((c*x^2 + 1)^2*c^3/(c*x^
2 - 1)^2 - 2*(c*x^2 + 1)*c^3/(c*x^2 - 1) + c^3))
```

Mupad [B]
time = 0.94, size = 60, normalized size = 1.40

$$\frac{a\,x^4}{4}+\frac{b\,x^2}{4\,c}+\frac{b\,x^4\,\ln\left(c\,x^2+1\right)}{8}-\frac{b\,x^4\,\ln\left(1-c\,x^2\right)}{8}+\frac{b\,\mathrm{atan}(c\,x^2\,1\mathrm{i})\,1\mathrm{i}}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x^2)),x)
```

```
[Out] (a*x^4)/4 + (b*x^2)/(4*c) + (b*atan(c*x^2*1i)*1i)/(4*c^2) + (b*x^4*log(c*x^
2 + 1))/8 - (b*x^4*log(1 - c*x^2))/8
```

3.53 $\int x\left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=37

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1-c^2x^4\right)}{4c}$$

```
[Out] 1/2*x^2*(a+b*arctanh(c*x^2))+1/4*b*ln(-c^2*x^4+1)/c
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6037, 266}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1-c^2x^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (x^2*(a + b*ArcTanh[c*x^2]))/2 + (b*Log[1 - c^2*x^4])/(4*c)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx&=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)-(bc)\int\frac{x^3}{1-c^2x^4}\,dx\\&=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1-c^2x^4\right)}{4c}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.14

$$\frac{ax^2}{2}+\frac{1}{2}bx^2\tanh^{-1}\left(cx^2\right)+\frac{b\log\left(1-c^2x^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^2]),x]
[Out] (a*x^2)/2 + (b*x^2*ArcTanh[c*x^2])/2 + (b*Log[1 - c^2*x^4])/(4*c)
```

Maple [A]
time = 0.02, size = 39, normalized size = 1.05

method	result	size
derivativedivides	$\frac{a c \,x^{2}+b c \,x^{2} \operatorname{arctanh}\left(c \,x^{2}\right)+\frac{b \ln \left(-c^{2} x^{4}+1\right)}{2}}{2 c}$	39
default	$\frac{a c \,x^{2}+b c \,x^{2} \operatorname{arctanh}\left(c \,x^{2}\right)+\frac{b \ln \left(-c^{2} x^{4}+1\right)}{2}}{2 c}$	39
risch	$\frac{b \,x^{2} \ln \left(c \,x^{2}+1\right)}{4}-\frac{b \,x^{2} \ln \left(-c \,x^{2}+1\right)}{4}+\frac{a \,x^{2}}{2}+\frac{b \ln \left(c^{2} x^{4}-1\right)}{4 c}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)
[Out] 1/2/c*(a*c*x^2+b*c*x^2*arctanh(c*x^2)+1/2*b*ln(-c^2*x^4+1))
```

Maxima [A]
time = 0.25, size = 37, normalized size = 1.00

$$\frac{1}{2}\,ax^{2}+\frac{(2\,cx^{2}\operatorname{artanh}\left(cx^{2}\right)+\log\left(-c^{2}x^{4}+1\right))b}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
[Out] 1/2*a*x^2 + 1/4*(2*c*x^2*arctanh(c*x^2) + log(-c^2*x^4 + 1))*b/c
```

Fricas [A]
time = 0.33, size = 50, normalized size = 1.35

$$\frac{bcx^{2}\log\left(-\frac{cx^{2}+1}{cx^{2}-1}\right)+2\,acx^{2}+b\log\left(c^{2}x^{4}-1\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
[Out] 1/4*(b*c*x^2*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*x^2 + b*log(c^2*x^4 - 1)
)/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(29) = 58$.
time = 3.20, size = 71, normalized size = 1.92

$$\begin{cases} \frac{ax^2}{2} + \frac{bx^2 \operatorname{atanh}(cx^2)}{2} + \frac{b\log\left(x - \sqrt{-\frac{1}{c}}\right)}{2c} + \frac{b\log\left(x + \sqrt{-\frac{1}{c}}\right)}{2c} - \frac{b\operatorname{atanh}(cx^2)}{2c} & \text{for } c \neq 0 \\ \frac{ax^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**2)),x)
[Out] Piecewise((a*x**2/2 + b*x**2*atanh(c*x**2)/2 + b*log(x - sqrt(-1/c))/(2*c)
+ b*log(x + sqrt(-1/c))/(2*c) - b*atanh(c*x**2)/(2*c), Ne(c, 0)), (a*x**2/2
, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(33) = 66$.
time = 0.42, size = 188, normalized size = 5.08

$$\frac{1}{2}ax^2 + \frac{1}{2}bc\left(\frac{\log\left(\frac{|-cx^2-1|}{|cx^2-1|}\right)}{c^2} - \frac{\log\left(\left|-\frac{cx^2+1}{cx^2-1}+1\right|\right)}{c^2} + \frac{\log\left(-\frac{\frac{c\left(\frac{cx^2+1}{cx^2-1}+1\right)}{\frac{(cx^2+1)c}{cx^2-1}-c}+1}{\frac{c\left(\frac{cx^2+1}{cx^2-1}+1\right)}{\frac{(cx^2+1)c}{cx^2-1}-c}-1}\right)}{c^2\left(\frac{cx^2+1}{cx^2-1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2)),x, algorithm="giac")
[Out] 1/2*a*x^2 + 1/2*b*c*(log(abs(-c*x^2 - 1)/abs(c*x^2 - 1))/c^2 - log(abs(-(c*
x^2 + 1)/(c*x^2 - 1) + 1))/c^2 + log(-(c*((c*x^2 + 1)/(c*x^2 - 1) + 1)/((c*
x^2 + 1)*c/(c*x^2 - 1) - c) + 1)/(c*((c*x^2 + 1)/(c*x^2 - 1) + 1)/((c*x^2 +
 1)*c/(c*x^2 - 1) - c) - 1))/(c^2*((c*x^2 + 1)/(c*x^2 - 1) - 1)))
```

Mupad [B]
time = 0.77, size = 52, normalized size = 1.41

$$\frac{a\,x^2}{2} + \frac{b\,\ln\left(c^2\,x^4-1\right)}{4\,c} + \frac{b\,x^2\,\ln\left(c\,x^2+1\right)}{4} - \frac{b\,x^2\,\ln\left(1-c\,x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^2)),x)
[Out] (a*x^2)/2 + (b*log(c^2*x^4 - 1))/(4*c) + (b*x^2*log(c*x^2 + 1))/4 - (b*x^2*
log(1 - c*x^2))/4
```

3.54 $\int \frac{a+b\tanh^{-1}(cx^2)}{x}\,dx$

Optimal. Leaf size=30

$$a\log(x) - \frac{1}{4}b\text{PolyLog}(2, -cx^2) + \frac{1}{4}b\text{PolyLog}(2, cx^2)$$

```
[Out] a*ln(x)-1/4*b*polylog(2,-c*x^2)+1/4*b*polylog(2,c*x^2)
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6035, 6031}

$$a\log(x) - \frac{1}{4}b\text{Li}_2(-cx^2) + \frac{1}{4}b\text{Li}_2(cx^2)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x,x]
```

```
[Out] a*Log[x] - (b*PolyLog[2, -(c*x^2)])/4 + (b*PolyLog[2, c*x^2])/4
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a + b\tanh^{-1}(cx^2)}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{a + b\tanh^{-1}(cx)}{x}\,dx, x, x^2\right)\\ &= a\log(x) - \frac{1}{4}b\text{Li}_2(-cx^2) + \frac{1}{4}b\text{Li}_2(cx^2)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.93

$$a\log(x) + \frac{1}{4}b\left(-\text{PolyLog}(2, -cx^2) + \text{PolyLog}(2, cx^2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x,x]
```

```
[Out] a*Log[x] + (b*(-PolyLog[2, -(c*x^2)] + PolyLog[2, c*x^2]))/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(26) = 52$.
time = 0.05, size = 124, normalized size = 4.13

method	result
default	$a\ln(x) + b\ln(x)\operatorname{arctanh}(c\,x^2) + \frac{b\ln(x)\ln\left(1-x\sqrt{c}\right)}{2} + \frac{b\ln(x)\ln\left(1+x\sqrt{c}\right)}{2} + \frac{b\,\mathrm{dilog}\left(1-x\sqrt{c}\right)}{2} + \frac{b\,\mathrm{dilog}\left(1+\right.}{2}$
risch	$a\ln(x) + \frac{b\ln(x)\ln\left(1-x\sqrt{c}\right)}{2} + \frac{b\ln(x)\ln\left(1+x\sqrt{c}\right)}{2} - \frac{\ln(x)\ln\left(-c\,x^2+1\right)b}{2} + \frac{b\,\mathrm{dilog}\left(1-x\sqrt{c}\right)}{2} + \frac{b\,\mathrm{dilog}\left(1+x\sqrt{c}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(x)+b*ln(x)*arctanh(c*x^2)+1/2*b*ln(x)*ln(1-x*c^(1/2))+1/2*b*ln(x)*ln(1
+x*c^(1/2))+1/2*b*dilog(1-x*c^(1/2))+1/2*b*dilog(1+x*c^(1/2))-1/2*b*ln(x)*l
n(1+x*(-c)^(1/2))-1/2*b*ln(x)*ln(1-x*(-c)^(1/2))-1/2*b*dilog(1+x*(-c)^(1/2)
)-1/2*b*dilog(1-x*(-c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(c*x^2 + 1) - log(-c*x^2 + 1))/x, x) + a*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^2) + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{atanh}(cx^2)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a+b\,\mathrm{atanh}\left(c\,x^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/x,x)
```

```
[Out] int((a + b*atanh(c*x^2))/x, x)
```

3.55 $\int \frac{a+b\tanh^{-1}(cx^2)}{x^3}\,dx$

Optimal. Leaf size=40

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{2x^2}+bc\log(x)-\frac{1}{4}bc\log\left(1-c^2x^4\right)$$

```
[Out] 1/2*(-a-b*arctanh(c*x^2))/x^2+b*c*ln(x)-1/4*b*c*ln(-c^2*x^4+1)
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6037, 272, 36, 29, 31}

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{2x^2}-\frac{1}{4}bc\log\left(1-c^2x^4\right)+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x^3,x]
```

```
[Out] -1/2*(a + b*ArcTanh[c*x^2])/x^2 + b*c*Log[x] - (b*c*Log[1 - c^2*x^4])/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx^2)}{x^3}\,dx &= -\frac{a+b\tanh^{-1}(cx^2)}{2x^2}+(bc)\int \frac{1}{x(1-c^2x^4)}\,dx\\ &= -\frac{a+b\tanh^{-1}(cx^2)}{2x^2}+\frac{1}{4}(bc)\text{Subst}\left(\int \frac{1}{x(1-c^2x)}\,dx,x,x^4\right)\\ &= -\frac{a+b\tanh^{-1}(cx^2)}{2x^2}+\frac{1}{4}(bc)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^4\right)+\frac{1}{4}(bc^3)\,\text{Subst}\left(\int \frac{1}{1-c^2x}\right.\\ &= -\frac{a+b\tanh^{-1}(cx^2)}{2x^2}+bc\log(x)-\frac{1}{4}bc\log(1-c^2x^4)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 45, normalized size = 1.12

$$-\frac{a}{2x^2}-\frac{b\tanh^{-1}(cx^2)}{2x^2}+bc\log(x)-\frac{1}{4}bc\log(1-c^2x^4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*ArcTanh[c*x^2])/(2*x^2) + b*c*Log[x] - (b*c*Log[1 - c^2*x^4
])/4
```

Maple [A]
time = 0.03, size = 49, normalized size = 1.22

method	result	size
default	$-\frac{a}{2x^2}-\frac{b\,\text{arctanh}(c\,x^2)}{2x^2}-\frac{bc\ln(c\,x^2-1)}{4}-\frac{bc\ln(c\,x^2+1)}{4}+bc\ln(x)$	49
risch	$-\frac{b\ln(c\,x^2+1)}{4x^2}+\frac{4bc\ln(x)x^2-bc\ln(c^2x^4-1)x^2+b\ln(-c\,x^2+1)-2a}{4x^2}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctanh(c*x^2)-1/4*b*c*ln(c*x^2-1)-1/4*b*c*ln(c*x^2+1)
+b*c*ln(x)
```

Maxima [A]
time = 0.26, size = 41, normalized size = 1.02

$$-\frac{1}{4}\left(c\left(\log\left(c^2x^4-1\right)-\log\left(x^4\right)\right)+\frac{2\,\operatorname{artanh}\left(cx^2\right)}{x^2}\right)b-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(c*(log(c^2*x^4 - 1) - log(x^4)) + 2*arctanh(c*x^2)/x^2)*b - 1/2*a/x^2
```

Fricas [A]
time = 0.39, size = 55, normalized size = 1.38

$$-\frac{bcx^2\log\left(c^2x^4-1\right)-4\,bcx^2\log\left(x\right)+b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*c*x^2*log(c^2*x^4 - 1) - 4*b*c*x^2*log(x) + b*log(-(c*x^2 + 1)/(c*x
^2 - 1)) + 2*a)/x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(39) = 78.
time = 5.46, size = 80, normalized size = 2.00

$$\begin{cases} -\frac{a}{2x^2}+bc\log\left(x\right)-\frac{bc\log\left(x-\sqrt{-\frac{1}{c}}\right)}{2}-\frac{bc\log\left(x+\sqrt{-\frac{1}{c}}\right)}{2}+\frac{bc\operatorname{atanh}\left(cx^2\right)}{2}-\frac{b\operatorname{atanh}\left(cx^2\right)}{2x^2} & \text{for}\ c\neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) + b*c*log(x) - b*c*log(x - sqrt(-1/c))/2 - b*c*log(x
 + sqrt(-1/c))/2 + b*c*atanh(c*x**2)/2 - b*atanh(c*x**2)/(2*x**2), Ne(c, 0)
), (-a/(2*x**2), True))
```

Giac [A]
time = 0.43, size = 51, normalized size = 1.28

$$-\frac{1}{4}\,bc\log\left(c^2x^4-1\right)+bc\log\left(x\right)-\frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{4\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*b*c*log(c^2*x^4 - 1) + b*c*log(x) - 1/4*b*log(-(c*x^2 + 1)/(c*x^2 - 1)
)/x^2 - 1/2*a/x^2
```

Mupad [B]
time = 0.85, size = 55, normalized size = 1.38

$$b\,c\,\ln\left(x\right)-\frac{a}{2\,x^2}-\frac{b\,c\,\ln\left(c^2\,x^4-1\right)}{4}-\frac{b\,\ln\left(c\,x^2+1\right)}{4\,x^2}+\frac{b\,\ln\left(1-c\,x^2\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/x^3,x)
```

```
[Out] b*c*log(x) - a/(2*x^2) - (b*c*log(c^2*x^4 - 1))/4 - (b*log(c*x^2 + 1))/(4*x
^2) + (b*log(1 - c*x^2))/(4*x^2)
```

3.56 $\int \frac{a+b\tanh^{-1}(cx^2)}{x^5}\,dx$

Optimal. Leaf size=41

$$-\frac{bc}{4x^2}+\frac{1}{4}bc^2\tanh^{-1}\left(cx^2\right)-\frac{a+b\tanh^{-1}\left(cx^2\right)}{4x^4}$$

```
[Out] -1/4*b*c/x^2+1/4*b*c^2*arctanh(c*x^2)+1/4*(-a-b*arctanh(c*x^2))/x^4
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6037, 281, 331, 212}

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{4x^4}+\frac{1}{4}bc^2\tanh^{-1}\left(cx^2\right)-\frac{bc}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x^5,x]
```

```
[Out] -1/4*(b*c)/x^2 + (b*c^2*ArcTanh[c*x^2])/4 - (a + b*ArcTanh[c*x^2])/(4*x^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
```

```
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx^2)}{x^5}\,dx &= -\frac{a+b\tanh^{-1}(cx^2)}{4x^4}+\frac{1}{2}(bc)\int \frac{1}{x^3(1-c^2x^4)}\,dx\\ &= -\frac{a+b\tanh^{-1}(cx^2)}{4x^4}+\frac{1}{4}(bc)\mathrm{Subst}\left(\int \frac{1}{x^2(1-c^2x^2)}\,dx,x,x^2\right)\\ &= -\frac{bc}{4x^2}-\frac{a+b\tanh^{-1}(cx^2)}{4x^4}+\frac{1}{4}(bc^3)\,\mathrm{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx,x,x^2\right)\\ &= -\frac{bc}{4x^2}+\frac{1}{4}bc^2\tanh^{-1}(cx^2)-\frac{a+b\tanh^{-1}(cx^2)}{4x^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 65, normalized size = 1.59

$$-\frac{a}{4x^4}-\frac{bc}{4x^2}-\frac{b\tanh^{-1}(cx^2)}{4x^4}-\frac{1}{8}bc^2\log(1-cx^2)+\frac{1}{8}bc^2\log(1+cx^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^5,x]
```

```
[Out] -1/4*a/x^4 - (b*c)/(4*x^2) - (b*ArcTanh[c*x^2])/(4*x^4) - (b*c^2*Log[1 - c*
x^2])/8 + (b*c^2*Log[1 + c*x^2])/8
```

Maple [A]
time = 0.04, size = 55, normalized size = 1.34

method	result	size
default	$-\frac{a}{4x^4}-\frac{b\,\mathrm{arctanh}(c\,x^2)}{4x^4}-\frac{b\,c^2\ln(c\,x^2-1)}{8}+\frac{b\,c^2\ln(c\,x^2+1)}{8}-\frac{bc}{4x^2}$	55
risch	$-\frac{b\ln(c\,x^2+1)}{8x^4}+\frac{b\,c^2\ln(c\,x^2+1)x^4-b\,c^2\ln(c\,x^2-1)x^4-2bc\,x^2+b\ln(-c\,x^2+1)-2a}{8x^4}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctanh(c*x^2)-1/8*b*c^2*ln(c*x^2-1)+1/8*b*c^2*ln(c*x^
2+1)-1/4*b*c/x^2
```

Maxima [A]
time = 0.26, size = 51, normalized size = 1.24

$$\frac{1}{8}\left(\left(c\log\left(cx^2+1\right)-c\log\left(cx^2-1\right)-\frac{2}{x^2}\right)c-\frac{2\,\mathrm{artanh}\left(cx^2\right)}{x^4}\right)b-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*((c*log(c*x^2 + 1) - c*log(c*x^2 - 1) - 2/x^2)*c - 2*arctanh(c*x^2)/x^4
)*b - 1/4*a/x^4
```

Fricas [A]
time = 0.37, size = 49, normalized size = 1.20

$$-\frac{2\,bcx^2-\left(bc^2x^4-b\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*c*x^2 - (b*c^2*x^4 - b)*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/x^4
```

Sympy [A]
time = 3.44, size = 41, normalized size = 1.00

$$-\frac{a}{4x^4}+\frac{bc^2\,\mathrm{atanh}\left(cx^2\right)}{4}-\frac{bc}{4x^2}-\frac{b\,\mathrm{atanh}\left(cx^2\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x**5,x)
```

```
[Out] -a/(4*x**4) + b*c**2*atanh(c*x**2)/4 - b*c/(4*x**2) - b*atanh(c*x**2)/(4*x*
*4)
```

Giac [A]
time = 0.42, size = 67, normalized size = 1.63

$$\frac{1}{8}\,bc^2\log\left(cx^2+1\right)-\frac{1}{8}\,bc^2\log\left(cx^2-1\right)-\frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{8\,x^4}-\frac{bcx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b*c^2*log(c*x^2 + 1) - 1/8*b*c^2*log(c*x^2 - 1) - 1/8*b*log(-(c*x^2 + 1
)/(c*x^2 - 1))/x^4 - 1/4*(b*c*x^2 + a)/x^4
```

Mupad [B]
time = 1.00, size = 52, normalized size = 1.27

$$\frac{b\,c^2\,\mathrm{atanh}\left(c\,x^2\right)}{4}-\frac{\frac{a}{4}+\frac{b\,\ln\left(c\,x^2+1\right)}{8}-\frac{b\,\ln\left(1-c\,x^2\right)}{8}+\frac{b\,c\,x^2}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/x^5,x)
[Out] (b*c^2*atanh(c*x^2))/4 - (a/4 + (b*log(c*x^2 + 1))/8 - (b*log(1 - c*x^2))/8
 + (b*c*x^2)/4)/x^4
```

3.57 $\int \frac{a+b\tanh^{-1}(cx^2)}{x^7}\,dx$

Optimal. Leaf size=56

$$-\frac{bc}{12x^4}-\frac{a+b\tanh^{-1}\left(cx^2\right)}{6x^6}+\frac{1}{3}bc^3\log(x)-\frac{1}{12}bc^3\log\left(1-c^2x^4\right)$$

```
[Out] -1/12*b*c/x^4+1/6*(-a-b*arctanh(c*x^2))/x^6+1/3*b*c^3*ln(x)-1/12*b*c^3*ln(-
c^2*x^4+1)
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {6037, 272, 46}

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{6x^6}+\frac{1}{3}bc^3\log(x)-\frac{1}{12}bc^3\log\left(1-c^2x^4\right)-\frac{bc}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x^7,x]
```

```
[Out] -1/12*(b*c)/x^4 - (a + b*ArcTanh[c*x^2])/(6*x^6) + (b*c^3*Log[x])/3 - (b*c^
3*Log[1 - c^2*x^4])/12
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx^2)}{x^7}\,dx = -\frac{a+b\tanh^{-1}(cx^2)}{6x^6}+\frac{1}{3}(bc)\int \frac{1}{x^5(1-c^2x^4)}\,dx$$
$$= -\frac{a+b\tanh^{-1}(cx^2)}{6x^6}+\frac{1}{12}(bc)\text{Subst}\left(\int \frac{1}{x^2(1-c^2x)}\,dx,x,x^4\right)$$
$$= -\frac{a+b\tanh^{-1}(cx^2)}{6x^6}+\frac{1}{12}(bc)\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{c^2}{x}-\frac{c^4}{-1+c^2x}\right)dx,x,x^4\right)$$
$$= -\frac{bc}{12x^4}-\frac{a+b\tanh^{-1}(cx^2)}{6x^6}+\frac{1}{3}bc^3\log(x)-\frac{1}{12}bc^3\log(1-c^2x^4)$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 1.09

$$-\frac{a}{6x^6}-\frac{bc}{12x^4}-\frac{b\tanh^{-1}(cx^2)}{6x^6}+\frac{1}{3}bc^3\log(x)-\frac{1}{12}bc^3\log(1-c^2x^4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^7,x]
```

```
[Out] -1/6*a/x^6 - (b*c)/(12*x^4) - (b*ArcTanh[c*x^2])/(6*x^6) + (b*c^3*Log[x])/3
 - (b*c^3*Log[1 - c^2*x^4])/12
```

Maple [A]
time = 0.04, size = 63, normalized size = 1.12

method	result	size
default	$-\frac{a}{6x^6}-\frac{b\,\mathrm{arctanh}(c\,x^2)}{6x^6}-\frac{b\,c^3\ln(c\,x^2-1)}{12}-\frac{b\,c^3\ln(c\,x^2+1)}{12}-\frac{bc}{12x^4}+\frac{b\,c^3\ln(x)}{3}$	63
risch	$-\frac{b\ln(c\,x^2+1)}{12x^6}+\frac{4b\,c^3\ln(x)x^6-b\,c^3\ln(c^2x^4-1)x^6-bc\,x^2+b\ln(-c\,x^2+1)-2a}{12x^6}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a/x^6-1/6*b/x^6*arctanh(c*x^2)-1/12*b*c^3*ln(c*x^2-1)-1/12*b*c^3*ln(c*
x^2+1)-1/12*b*c/x^4+1/3*b*c^3*ln(x)
```

Maxima [A]
time = 0.26, size = 51, normalized size = 0.91

$$-\frac{1}{12}\left(\left(c^2\log(c^2x^4-1)-c^2\log(x^4)+\frac{1}{x^4}\right)c+\frac{2\,\mathrm{artanh}(cx^2)}{x^6}\right)b-\frac{a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*((c^2*log(c^2*x^4 - 1) - c^2*log(x^4) + 1/x^4)*c + 2*arctanh(c*x^2)/x
^6)*b - 1/6*a/x^6
```

Fricas [A]
time = 0.34, size = 65, normalized size = 1.16

$$-\frac{bc^3x^6\log\left(c^2x^4-1\right)-4\,bc^3x^6\log\left(x\right)+bcx^2+b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(b*c^3*x^6*log(c^2*x^4 - 1) - 4*b*c^3*x^6*log(x) + b*c*x^2 + b*log(-(
c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/x^6
```

Sympy [A]
time = 8.95, size = 97, normalized size = 1.73

$$\begin{cases} -\frac{a}{6x^6}+\frac{bc^3\log\left(x\right)}{3}-\frac{bc^3\log\left(x-\sqrt{-\frac{1}{c}}\right)}{6}-\frac{bc^3\log\left(x+\sqrt{-\frac{1}{c}}\right)}{6}+\frac{bc^3\operatorname{atanh}\left(cx^2\right)}{6}-\frac{bc}{12x^4}-\frac{b\operatorname{atanh}\left(cx^2\right)}{6x^6} & \text{for}\ c\neq 0 \\ -\frac{a}{6x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x**7,x)
```

```
[Out] Piecewise((-a/(6*x**6) + b*c**3*log(x)/3 - b*c**3*log(x - sqrt(-1/c))/6 - b
*c**3*log(x + sqrt(-1/c))/6 + b*c**3*atanh(c*x**2)/6 - b*c/(12*x**4) - b*at
anh(c*x**2)/(6*x**6), Ne(c, 0)), (-a/(6*x**6), True))
```

Giac [A]
time = 0.43, size = 65, normalized size = 1.16

$$-\frac{1}{12}\,bc^3\log\left(c^2x^4-1\right)+\frac{1}{3}\,bc^3\log\left(x\right)-\frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{12\,x^6}-\frac{bcx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^7,x, algorithm="giac")
```

```
[Out] -1/12*b*c^3*log(c^2*x^4 - 1) + 1/3*b*c^3*log(x) - 1/12*b*log(-(c*x^2 + 1)/(
c*x^2 - 1))/x^6 - 1/12*(b*c*x^2 + 2*a)/x^6
```

Mupad [B]
time = 0.88, size = 67, normalized size = 1.20

$$\frac{b\,c^3\,\ln\left(x\right)}{3}-\frac{b\,c^3\,\ln\left(c^2\,x^4-1\right)}{12}-\frac{a}{6\,x^6}-\frac{b\,c}{12\,x^4}-\frac{b\,\ln\left(c\,x^2+1\right)}{12\,x^6}+\frac{b\,\ln\left(1-c\,x^2\right)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/x^7,x)
```

```
[Out] (b*c^3*log(x))/3 - (b*c^3*log(c^2*x^4 - 1))/12 - a/(6*x^6) - (b*c)/(12*x^4)
 - (b*log(c*x^2 + 1))/(12*x^6) + (b*log(1 - c*x^2))/(12*x^6)
```

3.58 $\int x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=65

$$\frac{2bx^3}{15c}+\frac{b\text{ArcTan}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{b\tanh^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)$$

```
[Out] 2/15*b*x^3/c+1/5*b*arctan(x*c^(1/2))/c^(5/2)+1/5*x^5*(a+b*arctanh(c*x^2))-1
/5*b*arctanh(x*c^(1/2))/c^(5/2)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.357$, Rules used = {6037, 327, 304, 209, 212}

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\text{ArcTan}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{b\tanh^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}+\frac{2bx^3}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (2*b*x^3)/(15*c) + (b*ArcTan[Sqrt[c]*x])/(5*c^(5/2)) - (b*ArcTanh[Sqrt[c]*x
])/(5*c^(5/2)) + (x^5*(a + b*ArcTanh[c*x^2]))/5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx &= \frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{5}(2bc)\int\frac{x^6}{1-c^2x^4}\,dx\\ &= \frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{(2b)\int\frac{x^2}{1-c^2x^4}\,dx}{5c}\\ &= \frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\int\frac{1}{1-cx^2}\,dx}{5c^2}+\frac{b\int\frac{1}{1+cx^2}\,dx}{5c^2}\\ &= \frac{2bx^3}{15c}+\frac{b\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{b\tanh^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 93, normalized size = 1.43

$$\frac{2bx^3}{15c}+\frac{ax^5}{5}+\frac{b\text{ArcTan}\left(\sqrt{c}\,x\right)}{5c^{5/2}}+\frac{1}{5}bx^5\tanh^{-1}\left(cx^2\right)+\frac{b\log\left(1-\sqrt{c}\,x\right)}{10c^{5/2}}-\frac{b\log\left(1+\sqrt{c}\,x\right)}{10c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (2*b*x^3)/(15*c) + (a*x^5)/5 + (b*ArcTan[Sqrt[c]*x])/(5*c^(5/2)) + (b*x^5*A
rcTanh[c*x^2])/5 + (b*Log[1 - Sqrt[c]*x])/(10*c^(5/2)) - (b*Log[1 + Sqrt[c]
*x])/(10*c^(5/2))
```

Maple [A]
time = 0.09, size = 53, normalized size = 0.82

method	result
default	$\frac{a\,x^5}{5}+\frac{x^5 b\,\mathrm{arctanh}\left(c\,x^2\right)}{5}+\frac{2b\,x^3}{15c}-\frac{b\,\mathrm{arctanh}\left(x\sqrt{c}\right)}{5c^{\frac{5}{2}}}+\frac{b\arctan\left(x\sqrt{c}\right)}{5c^{\frac{5}{2}}}$

risch	$\frac{x^5 b \ln(c x^2+1)}{10} - \frac{b x^5 \ln(-c x^2+1)}{10} + \frac{a x^5}{5} + \frac{2b x^3}{15c} + \frac{b \ln\left(1-x\sqrt{c}\right)}{10 c^{\frac{5}{2}}} - \frac{b \ln\left(1+x\sqrt{c}\right)}{10 c^{\frac{5}{2}}} + \frac{\sqrt{-c}\ \ln\left(-\sqrt{-c}\ c - x c^2\right) b}{10 c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*x^5+1/5*x^5*b*arctanh(c*x^2)+2/15*b*x^3/c-1/5*b*arctanh(x*c^(1/2))/c^
(5/2)+1/5*b*arctan(x*c^(1/2))/c^(5/2)
```

Maxima [A]
time = 0.46, size = 69, normalized size = 1.06

$$\frac{1}{5}\,ax^5 + \frac{1}{30}\left(6\,x^5\,\operatorname{artanh}\left(cx^2\right) + c\left(\frac{4\,x^3}{c^2} + \frac{6\,\arctan\left(\sqrt{c}\,x\right)}{c^{\frac{7}{2}}} + \frac{3\,\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{c^{\frac{7}{2}}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/30*(6*x^5*arctanh(c*x^2) + c*(4*x^3/c^2 + 6*arctan(sqrt(c)*x)
/c^(7/2) + 3*log((c*x - sqrt(c))/(c*x + sqrt(c)))/c^(7/2)))*b
```

Fricas [A] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(49) = 98.
time = 0.36, size = 197, normalized size = 3.03

$$\left[\frac{3\,bc^3x^5\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 6\,ac^3x^5 + 4\,bc^2x^3 + 6\,b\sqrt{c}\,\arctan\left(\sqrt{c}\,x\right) + 3\,b\sqrt{c}\,\log\left(\frac{cx^2-2\,\sqrt{c}\,x+1}{cx^2-1}\right)}{30\,c^3},\ \frac{3\,bc^3x^5\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 6\,ac^3x^5 + 4\,bc^2x^3 + 6\,b\sqrt{-c}\,\arctan\left(\sqrt{-c}\,x\right) - 3\,b\sqrt{-c}\,\log\left(\frac{cx^2-2\,\sqrt{-c}\,x-1}{cx^2+1}\right)}{30\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] [1/30*(3*b*c^3*x^5*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a*c^3*x^5 + 4*b*c^2*x^
3 + 6*b*sqrt(c)*arctan(sqrt(c)*x) + 3*b*sqrt(c)*log((c*x^2 - 2*sqrt(c)*x +
1)/(c*x^2 - 1)))/c^3, 1/30*(3*b*c^3*x^5*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a
*c^3*x^5 + 4*b*c^2*x^3 + 6*b*sqrt(-c)*arctan(sqrt(-c)*x) - 3*b*sqrt(-c)*log
((c*x^2 - 2*sqrt(-c)*x - 1)/(c*x^2 + 1)))/c^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(58) = 116.
time = 5.80, size = 185, normalized size = 2.85

$$\begin{cases} \frac{ax^5}{5} + \frac{bx^5\operatorname{atanh}\left(cx^2\right)}{5} + \frac{2bx^3}{15c} - \frac{b\sqrt{-\frac{1}{c}}\log\left(x-\sqrt{-\frac{1}{c}}\right)}{10c^2} + \frac{b\sqrt{-\frac{1}{c}}\log\left(x+\sqrt{-\frac{1}{c}}\right)}{10c^2} - \frac{b\log\left(x-\sqrt{-\frac{1}{c}}\right)}{10c^3\sqrt{\frac{1}{c}}} - \frac{b\log\left(x+\sqrt{-\frac{1}{c}}\right)}{10c^3\sqrt{\frac{1}{c}}} + \frac{b\log\left(x-\sqrt{\frac{1}{c}}\right)}{5c^3\sqrt{\frac{1}{c}}} + \frac{b\operatorname{atanh}\left(cx^2\right)}{5c^3\sqrt{\frac{1}{c}}} & \text{for}\ c \neq 0 \\ \frac{ax^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((a*x**5/5 + b*x**5*atanh(c*x**2)/5 + 2*b*x**3/(15*c) - b*sqrt(-1/
c)*log(x - sqrt(-1/c))/(10*c**2) + b*sqrt(-1/c)*log(x + sqrt(-1/c))/(10*c**
2) - b*log(x - sqrt(-1/c))/(10*c**3*sqrt(1/c)) - b*log(x + sqrt(-1/c))/(10*
c**3*sqrt(1/c)) + b*log(x - sqrt(1/c))/(5*c**3*sqrt(1/c)) + b*atanh(c*x**2)
/(5*c**3*sqrt(1/c)), Ne(c, 0)), (a*x**5/5, True))
```

Giac [A]
time = 0.48, size = 73, normalized size = 1.12

$$\frac{1}{10}\,bx^5\log\left(-\frac{cx^2+1}{cx^2-1}\right)+\frac{1}{5}\,ax^5+\frac{2\,bx^3}{15\,c}+\frac{b\arctan\left(\sqrt{c}\,x\right)}{5\,c^{\frac{5}{2}}}+\frac{b\arctan\left(\frac{cx}{\sqrt{-c}}\right)}{5\,\sqrt{-c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/10*b*x^5*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/5*a*x^5 + 2/15*b*x^3/c + 1/5*b
*arctan(sqrt(c)*x)/c^(5/2) + 1/5*b*arctan(c*x/sqrt(-c))/(sqrt(-c)*c^2)
```

Mupad [B]
time = 0.98, size = 72, normalized size = 1.11

$$\frac{a\,x^5}{5}+\frac{2\,b\,x^3}{15\,c}+\frac{b\,\mathrm{atan}\left(\sqrt{c}\,\,x\right)}{5\,c^{5/2}}+\frac{b\,x^5\,\ln\left(c\,x^2+1\right)}{10}-\frac{b\,x^5\,\ln\left(1-c\,x^2\right)}{10}+\frac{b\,\mathrm{atan}\left(\sqrt{c}\,\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{5\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atanh(c*x^2)),x)
```

```
[Out] (a*x^5)/5 + (2*b*x^3)/(15*c) + (b*atan(c^(1/2)*x))/(5*c^(5/2)) + (b*atan(c^
(1/2)*x*1i)*1i)/(5*c^(5/2)) + (b*x^5*log(c*x^2 + 1))/10 - (b*x^5*log(1 - c*
x^2))/10
```

3.59 $\int x^2 (a + b \tanh^{-1}(cx^2))\ dx$

Optimal. Leaf size=63

$$\frac{2bx}{3c} - \frac{b\text{ArcTan}(\sqrt{c}\,x)}{3c^{3/2}} - \frac{b\tanh^{-1}(\sqrt{c}\,x)}{3c^{3/2}} + \frac{1}{3}x^3(a + b\tanh^{-1}(cx^2))$$

[Out] 2/3*b*x/c-1/3*b*arctan(x*c^(1/2))/c^(3/2)+1/3*x^3*(a+b*arctanh(c*x^2))-1/3*b*arctanh(x*c^(1/2))/c^(3/2)

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6037, 327, 218, 212, 209}

$$\frac{1}{3}x^3(a + b\tanh^{-1}(cx^2)) - \frac{b\text{ArcTan}(\sqrt{c}\,x)}{3c^{3/2}} - \frac{b\tanh^{-1}(\sqrt{c}\,x)}{3c^{3/2}} + \frac{2bx}{3c}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*ArcTanh[c*x^2]),x]

[Out] (2*b*x)/(3*c) - (b*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) - (b*ArcTanh[Sqrt[c]*x])/(3*c^(3/2)) + (x^3*(a + b*ArcTanh[c*x^2]))/3

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx&=\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{3}(2bc)\int\frac{x^4}{1-c^2x^4}\,dx\\&=\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{(2b)\int\frac{1}{1-c^2x^4}\,dx}{3c}\\&=\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\int\frac{1}{1-cx^2}\,dx}{3c}-\frac{b\int\frac{1}{1+cx^2}\,dx}{3c}\\&=\frac{2bx}{3c}-\frac{b\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}-\frac{b\tanh^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 91, normalized size = 1.44

$$\frac{2bx}{3c}+\frac{ax^3}{3}-\frac{b\text{ArcTan}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{1}{3}bx^3\tanh^{-1}\left(cx^2\right)+\frac{b\log\left(1-\sqrt{c}\,x\right)}{6c^{3/2}}-\frac{b\log\left(1+\sqrt{c}\,x\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^2]),x]
[Out] (2*b*x)/(3*c) + (a*x^3)/3 - (b*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) + (b*x^3*ArcT
anh[c*x^2])/3 + (b*Log[1 - Sqrt[c]*x])/(6*c^(3/2)) - (b*Log[1 + Sqrt[c]*x])
/(6*c^(3/2))
```

Maple [A]
time = 0.06, size = 51, normalized size = 0.81

method	result
default	$\frac{x^3 a}{3}+\frac{x^3 b\,\mathrm{arctanh}\left(c\,x^2\right)}{3}+\frac{2bx}{3c}-\frac{b\,\mathrm{arctanh}\left(x\sqrt{c}\right)}{3c^{\frac{3}{2}}}-\frac{b\arctan\left(x\sqrt{c}\right)}{3c^{\frac{3}{2}}}$

risch	$\frac{x^3 b \ln(c x^2+1)}{6} - \frac{b x^3 \ln(-c x^2+1)}{6} + \frac{x^3 a}{3} - \frac{b \ln\left(1+x\sqrt{c}\right)}{6c^{\frac{3}{2}}} + \frac{b \ln\left(x\sqrt{c}-1\right)}{6c^{\frac{3}{2}}} + \frac{2bx}{3c} + \frac{\sqrt{-c}\, \ln\left(1+x\sqrt{-c}\right) b}{6c^2} - \sqrt{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*a+1/3*x^3*b*arctanh(c*x^2)+2/3*b*x/c-1/3*b*arctanh(x*c^(1/2))/c^(3/
2)-1/3*b*arctan(x*c^(1/2))/c^(3/2)
```

Maxima [A]
time = 0.48, size = 66, normalized size = 1.05

$$\frac{1}{3}\,ax^3 + \frac{1}{6}\left(2\,x^3 \operatorname{artanh}\left(cx^2\right) + c\left(\frac{4\,x}{c^2} - \frac{2\,\arctan\left(\sqrt{c}\,x\right)}{c^{\frac{5}{2}}} + \frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctanh(c*x^2) + c*(4*x/c^2 - 2*arctan(sqrt(c)*x)/c^
(5/2) + log((c*x - sqrt(c))/(c*x + sqrt(c)))/c^(5/2)))*b
```

Fricas [A] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(47) = 94.
time = 0.36, size = 186, normalized size = 2.95

$$\left[\frac{bc^2x^3\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2\,ac^2x^3 + 4\,bcx - 2\,b\sqrt{c}\,\arctan\left(\sqrt{c}\,x\right) + b\sqrt{c}\,\log\left(\frac{cx^2-2\,\sqrt{c}\,x+1}{cx^2-1}\right)}{6\,c^2}, \frac{bc^2x^3\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2\,ac^2x^3 + 4\,bcx + 2\,b\sqrt{-c}\,\arctan\left(\sqrt{-c}\,x\right) - b\sqrt{-c}\,\log\left(\frac{cx^2+2\,\sqrt{-c}\,x-1}{cx^2+1}\right)}{6\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] [1/6*(b*c^2*x^3*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c^2*x^3 + 4*b*c*x - 2*b
*sqrt(c)*arctan(sqrt(c)*x) + b*sqrt(c)*log((c*x^2 - 2*sqrt(c)*x + 1)/(c*x^2
 - 1)))/c^2, 1/6*(b*c^2*x^3*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c^2*x^3 + 4
*b*c*x + 2*b*sqrt(-c)*arctan(sqrt(-c)*x) - b*sqrt(-c)*log((c*x^2 + 2*sqrt(-
c)*x - 1)/(c*x^2 + 1)))/c^2]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 670 vs. 2(56) = 112.
time = 3.38, size = 670, normalized size = 10.63

$$\begin{cases} \frac{4ac^2x^3\sqrt{-\frac{1}{c}}}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}} + \frac{4ac^2x^3\sqrt{\frac{1}{c}}}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}} + \frac{4bc^2x^3\sqrt{-\frac{1}{c}}\operatorname{atanh}\left(cx^2\right)}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}} + \frac{4bc^2x^3\sqrt{\frac{1}{c}}\operatorname{atanh}\left(cx^2\right)}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}} - \frac{bc^2\left(-\frac{1}{c}\right)^{\frac{3}{2}}\sqrt{\frac{1}{c}}\log\left(x+\sqrt{-\frac{1}{c}}\right)}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}} + \frac{bc^2\sqrt{-\frac{1}{c}}\left(\frac{1}{c}\right)^{\frac{3}{2}}\log\left(x+\sqrt{-\frac{1}{c}}\right)}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}} + \frac{8bcx\sqrt{-\frac{1}{c}}}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}} + \frac{8bcx\sqrt{\frac{1}{c}}}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}} - \frac{6bc\sqrt{-\frac{1}{c}}\sqrt{\frac{1}{c}}\log\left(x+\sqrt{-\frac{1}{c}}\right)}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}} + \frac{4bc\sqrt{-\frac{1}{c}}\sqrt{\frac{1}{c}}\log\left(x-\sqrt{\frac{1}{c}}\right)}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}} + \frac{4bc\sqrt{-\frac{1}{c}}\sqrt{\frac{1}{c}}\operatorname{atanh}\left(cx^2\right)}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}} - \frac{4b\log\left(x-\sqrt{-\frac{1}{c}}\right)}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}} + \frac{4b\log\left(x-\sqrt{\frac{1}{c}}\right)}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}} + \frac{4b\operatorname{atanh}\left(cx^2\right)}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}} & \text{for}\: c \neq 0 \\ \frac{ax^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((4*a*c**2*x**3*sqrt(-1/c)/(12*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/c)
) + 4*a*c**2*x**3*sqrt(1/c)/(12*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/c)) + 4*b*
c**2*x**3*sqrt(-1/c)*atanh(c*x**2)/(12*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/c))
 + 4*b*c**2*x**3*sqrt(1/c)*atanh(c*x**2)/(12*c**2*sqrt(-1/c) + 12*c**2*sqrt
(1/c)) - b*c**2*(-1/c)**(3/2)*sqrt(1/c)*log(x + sqrt(-1/c))/(12*c**2*sqrt(-
1/c) + 12*c**2*sqrt(1/c)) + b*c**2*sqrt(-1/c)*(1/c)**(3/2)*log(x + sqrt(-1/
c))/(12*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/c)) + 8*b*c*x*sqrt(-1/c)/(12*c**2*
sqrt(-1/c) + 12*c**2*sqrt(1/c)) + 8*b*c*x*sqrt(1/c)/(12*c**2*sqrt(-1/c) + 1
2*c**2*sqrt(1/c)) - 6*b*c*sqrt(-1/c)*sqrt(1/c)*log(x + sqrt(-1/c))/(12*c**2
*sqrt(-1/c) + 12*c**2*sqrt(1/c)) + 4*b*c*sqrt(-1/c)*sqrt(1/c)*log(x - sqrt(
1/c))/(12*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/c)) + 4*b*c*sqrt(-1/c)*sqrt(1/c)
*atanh(c*x**2)/(12*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/c)) - 4*b*log(x - sqrt(
-1/c))/(12*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/c)) + 4*b*log(x - sqrt(1/c))/(1
2*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/c)) + 4*b*atanh(c*x**2)/(12*c**2*sqrt(-1
/c) + 12*c**2*sqrt(1/c)), Ne(c, 0)), (a*x**3/3, True))
```

<u>**Giac**</u> [**A**]
time = 0.46, size = 75, normalized size = 1.19

$$-\frac{1}{3}\,bc^5\left(\frac{\arctan\left(\sqrt{c}\,x\right)}{c^{\frac{13}{2}}}-\frac{\arctan\left(\frac{cx}{\sqrt{-c}}\right)}{\sqrt{-c}\,c^6}\right)+\frac{1}{6}\,bx^3\log\left(-\frac{cx^2+1}{cx^2-1}\right)+\frac{1}{3}\,ax^3+\frac{2\,bx}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] -1/3*b*c^5*(arctan(sqrt(c)*x)/c^(13/2) - arctan(c*x/sqrt(-c))/(sqrt(-c)*c^6
)) + 1/6*b*x^3*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/3*a*x^3 + 2/3*b*x/c
```

<u>**Mupad**</u> [**B**]
time = 0.85, size = 70, normalized size = 1.11

$$\frac{a\,x^3}{3}-\frac{b\,\mathrm{atan}\left(\sqrt{c}\,\,x\right)}{3\,c^{3/2}}+\frac{2\,b\,x}{3\,c}+\frac{b\,x^3\,\ln\left(c\,x^2+1\right)}{6}-\frac{b\,x^3\,\ln\left(1-c\,x^2\right)}{6}+\frac{b\,\mathrm{atan}\left(\sqrt{c}\,\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{3\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^2)),x)
```

```
[Out] (a*x^3)/3 - (b*atan(c^(1/2)*x))/(3*c^(3/2)) + (b*atan(c^(1/2)*x*1i)*1i)/(3*
c^(3/2)) + (2*b*x)/(3*c) + (b*x^3*log(c*x^2 + 1))/6 - (b*x^3*log(1 - c*x^2)
)/6
```

3.60 $\int \left(a+b\tanh^{-1}\left(cx^2\right)\right)\, dx$

Optimal. Leaf size=44

$$ax+\frac{b\text{ArcTan}\left(\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{b\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}+bx\tanh^{-1}\left(cx^2\right)$$

```
[Out] a*x+b*x*arctanh(c*x^2)+b*arctan(x*c^(1/2))/c^(1/2)-b*arctanh(x*c^(1/2))/c^(
1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.400$, Rules used = {6021, 304, 209, 212}

$$ax+\frac{b\text{ArcTan}\left(\sqrt{c}\,x\right)}{\sqrt{c}}+bx\tanh^{-1}\left(cx^2\right)-\frac{b\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c*x^2],x]
```

```
[Out] a*x + (b*ArcTan[Sqrt[c]*x])/Sqrt[c] - (b*ArcTanh[Sqrt[c]*x])/Sqrt[c] + b*x*
ArcTanh[c*x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
```

```
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx &= ax+b\int \tanh^{-1}\left(cx^2\right)\,dx\\
&= ax+bx\tanh^{-1}\left(cx^2\right)-(2bc)\int \frac{x^2}{1-c^2x^4}\,dx\\
&= ax+bx\tanh^{-1}\left(cx^2\right)-b\int \frac{1}{1-cx^2}\,dx+b\int \frac{1}{1+cx^2}\,dx\\
&= ax+\frac{b\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{b\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}+bx\tanh^{-1}\left(cx^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 57, normalized size = 1.30

$$ax+bx\tanh^{-1}\left(cx^2\right)+\frac{b\left(2\text{ArcTan}\left(\sqrt{c}\,x\right)+\log\left(1-\sqrt{c}\,x\right)-\log\left(1+\sqrt{c}\,x\right)\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c*x^2],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x^2] + (b*(2*ArcTan[Sqrt[c]*x] + Log[1 - Sqrt[c]*x] - L
og[1 + Sqrt[c]*x]))/(2*Sqrt[c])
```

Maple [A]
time = 0.05, size = 37, normalized size = 0.84

method	result
default	$ax+bx\,\mathrm{arctanh}\left(c\,x^2\right)+\frac{b\arctan\left(x\sqrt{c}\right)}{\sqrt{c}}-\frac{b\,\mathrm{arctanh}\left(x\sqrt{c}\right)}{\sqrt{c}}$
risch	$ax+\frac{bx\ln\left(c\,x^2+1\right)}{2}-\frac{bx\ln\left(-c\,x^2+1\right)}{2}+\frac{b\sqrt{-c}\,\ln\left(cx+\sqrt{-c}\right)}{2c}-\frac{b\sqrt{-c}\,\ln\left(-cx+\sqrt{-c}\right)}{2c}+\frac{b\ln\left(1-x\sqrt{c}\right)}{2\sqrt{c}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*x*arctanh(c*x^2)+b*arctan(x*c^(1/2))/c^(1/2)-b*arctanh(x*c^(1/2))/c^(
1/2)
```

Maxima [A]
time = 0.49, size = 55, normalized size = 1.25

$$\frac{1}{2}\left(c\left(\frac{2\arctan\left(\sqrt{c}\,x\right)}{c^{\frac{3}{2}}}+\frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)+2\,x\,\mathrm{artanh}\left(cx^{2}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*(c*(2*arctan(sqrt(c)*x)/c^(3/2) + log((c*x - sqrt(c))/(c*x + sqrt(c)))/
c^(3/2)) + 2*x*arctanh(c*x^2))*b + a*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(36) = 72.
time = 0.42, size = 160, normalized size = 3.64

$$\left[\frac{bcx\log\left(-\frac{cx^{2}+1}{cx^{2}-1}\right)+2\,acx+2\,b\sqrt{c}\,\arctan\left(\sqrt{c}\,x\right)+b\sqrt{c}\,\log\left(\frac{cx^{2}-2\,\sqrt{c}\,x+1}{cx^{2}-1}\right)}{2\,c},\ \frac{bcx\log\left(-\frac{cx^{2}+1}{cx^{2}-1}\right)+2\,acx+2\,b\sqrt{-c}\,\arctan\left(\sqrt{-c}\,x\right)-b\sqrt{-c}\,\log\left(\frac{cx^{2}-2\,\sqrt{-c}\,x-1}{cx^{2}+1}\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(b*c*x*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*x + 2*b*sqrt(c)*arctan(sq
rt(c)*x) + b*sqrt(c)*log((c*x^2 - 2*sqrt(c)*x + 1)/(c*x^2 - 1)))/c, 1/2*(b*
c*x*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*x + 2*b*sqrt(-c)*arctan(sqrt(-c)*
x) - b*sqrt(-c)*log((c*x^2 - 2*sqrt(-c)*x - 1)/(c*x^2 + 1)))/c]
```

Sympy [A]
time = 2.54, size = 702, normalized size = 15.95

$$ax+b\left(\begin{cases}\frac{4cx\sqrt{-\frac{1}{c}}\sqrt{\frac{1}{c}}\,\mathrm{atanh}\left(cx^{2}\right)}{c^{2}\left(-\frac{1}{c}\right)^{\frac{3}{2}}\sqrt{\frac{1}{c}}-c^{2}\sqrt{-\frac{1}{c}}\left(\frac{1}{c}\right)^{\frac{3}{2}}+6c\sqrt{-\frac{1}{c}}\sqrt{\frac{1}{c}}}-\frac{c\left(-\frac{1}{c}\right)^{\frac{3}{2}}\log\left(x+\sqrt{-\frac{1}{c}}\right)}{c^{2}\left(-\frac{1}{c}\right)^{\frac{3}{2}}\sqrt{\frac{1}{c}}-c^{2}\sqrt{-\frac{1}{c}}\left(\frac{1}{c}\right)^{\frac{3}{2}}+6c\sqrt{-\frac{1}{c}}\sqrt{\frac{1}{c}}}+\frac{c\left(\frac{1}{c}\right)^{\frac{3}{2}}\log\left(x+\sqrt{-\frac{1}{c}}\right)}{c^{2}\left(-\frac{1}{c}\right)^{\frac{3}{2}}\sqrt{\frac{1}{c}}-c^{2}\sqrt{-\frac{1}{c}}\left(\frac{1}{c}\right)^{\frac{3}{2}}+6c\sqrt{-\frac{1}{c}}\sqrt{\frac{1}{c}}}-\frac{2\sqrt{-\frac{1}{c}}\log\left(x-\sqrt{-\frac{1}{c}}\right)}{c^{2}\left(-\frac{1}{c}\right)^{\frac{3}{2}}\sqrt{\frac{1}{c}}-c^{2}\sqrt{-\frac{1}{c}}\left(\frac{1}{c}\right)^{\frac{3}{2}}+6c\sqrt{-\frac{1}{c}}\sqrt{\frac{1}{c}}}-\frac{3\sqrt{-\frac{1}{c}}\log\left(x+\sqrt{-\frac{1}{c}}\right)}{c^{2}\left(-\frac{1}{c}\right)^{\frac{3}{2}}\sqrt{\frac{1}{c}}-c^{2}\sqrt{-\frac{1}{c}}\left(\frac{1}{c}\right)^{\frac{3}{2}}+6c\sqrt{-\frac{1}{c}}\sqrt{\frac{1}{c}}}+\frac{4\sqrt{-\frac{1}{c}}\log\left(x-\sqrt{\frac{1}{c}}\right)}{c^{2}\left(-\frac{1}{c}\right)^{\frac{3}{2}}\sqrt{\frac{1}{c}}-c^{2}\sqrt{-\frac{1}{c}}\left(\frac{1}{c}\right)^{\frac{3}{2}}+6c\sqrt{-\frac{1}{c}}\sqrt{\frac{1}{c}}}+\frac{4\sqrt{-\frac{1}{c}}\,\mathrm{atanh}\left(cx^{2}\right)}{c^{2}\left(-\frac{1}{c}\right)^{\frac{3}{2}}\sqrt{\frac{1}{c}}-c^{2}\sqrt{-\frac{1}{c}}\left(\frac{1}{c}\right)^{\frac{3}{2}}+6c\sqrt{-\frac{1}{c}}\sqrt{\frac{1}{c}}}+\frac{2\sqrt{\frac{1}{c}}\log\left(x-\sqrt{-\frac{1}{c}}\right)}{c^{2}\left(-\frac{1}{c}\right)^{\frac{3}{2}}\sqrt{\frac{1}{c}}-c^{2}\sqrt{-\frac{1}{c}}\left(\frac{1}{c}\right)^{\frac{3}{2}}+6c\sqrt{-\frac{1}{c}}\sqrt{\frac{1}{c}}}-\frac{3\sqrt{\frac{1}{c}}\log\left(x+\sqrt{-\frac{1}{c}}\right)}{c^{2}\left(-\frac{1}{c}\right)^{\frac{3}{2}}\sqrt{\frac{1}{c}}-c^{2}\sqrt{-\frac{1}{c}}\left(\frac{1}{c}\right)^{\frac{3}{2}}+6c\sqrt{-\frac{1}{c}}\sqrt{\frac{1}{c}}} & \text{for}\ c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x**2),x)
```

```
[Out] a*x + b*Piecewise((4*c*x*sqrt(-1/c)*sqrt(1/c)*atanh(c*x**2)/(c**2*(-1/c)**(
3/2)*sqrt(1/c) - c**2*sqrt(-1/c)*(1/c)**(3/2) + 6*c*sqrt(-1/c)*sqrt(1/c)) -
 c*(-1/c)**(3/2)*log(x + sqrt(-1/c))/(c**2*(-1/c)**(3/2)*sqrt(1/c) - c**2*s
qrt(-1/c)*(1/c)**(3/2) + 6*c*sqrt(-1/c)*sqrt(1/c)) + c*(1/c)**(3/2)*log(x +
 sqrt(-1/c))/(c**2*(-1/c)**(3/2)*sqrt(1/c) - c**2*sqrt(-1/c)*(1/c)**(3/2) +
 6*c*sqrt(-1/c)*sqrt(1/c)) - 2*sqrt(-1/c)*log(x - sqrt(-1/c))/(c**2*(-1/c)*
*(3/2)*sqrt(1/c) - c**2*sqrt(-1/c)*(1/c)**(3/2) + 6*c*sqrt(-1/c)*sqrt(1/c))
 - 3*sqrt(-1/c)*log(x + sqrt(-1/c))/(c**2*(-1/c)**(3/2)*sqrt(1/c) - c**2*sq
```

```
rt(-1/c)*(1/c)**(3/2) + 6*c*sqrt(-1/c)*sqrt(1/c)) + 4*sqrt(-1/c)*log(x - sq
rt(1/c))/(c**2*(-1/c)**(3/2)*sqrt(1/c) - c**2*sqrt(-1/c)*(1/c)**(3/2) + 6*c
*sqrt(-1/c)*sqrt(1/c)) + 4*sqrt(-1/c)*atanh(c*x**2)/(c**2*(-1/c)**(3/2)*sqr
t(1/c) - c**2*sqrt(-1/c)*(1/c)**(3/2) + 6*c*sqrt(-1/c)*sqrt(1/c)) + 2*sqrt(
1/c)*log(x - sqrt(-1/c))/(c**2*(-1/c)**(3/2)*sqrt(1/c) - c**2*sqrt(-1/c)*(1
/c)**(3/2) + 6*c*sqrt(-1/c)*sqrt(1/c)) - 3*sqrt(1/c)*log(x + sqrt(-1/c))/(c
**2*(-1/c)**(3/2)*sqrt(1/c) - c**2*sqrt(-1/c)*(1/c)**(3/2) + 6*c*sqrt(-1/c)
*sqrt(1/c)), Ne(c, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(36) = 72$.
time = 0.40, size = 83, normalized size = 1.89

$$\frac{1}{2}\left(c\left(\frac{2\sqrt{|c|}\arctan\left(x\sqrt{|c|}\right)}{c^2}-\frac{\sqrt{|c|}\log\left(\left|x+\frac{1}{\sqrt{|c|}}\right|\right)}{c^2}+\frac{\sqrt{|c|}\log\left(\left|x-\frac{1}{\sqrt{|c|}}\right|\right)}{c^2}\right)+x\log\left(-\frac{cx^2+1}{cx^2-1}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(c*(2*sqrt(abs(c))*arctan(x*sqrt(abs(c)))/c^2 - sqrt(abs(c))*log(abs(x
+ 1/sqrt(abs(c))))/c^2 + sqrt(abs(c))*log(abs(x - 1/sqrt(abs(c))))/c^2) + x
*log(-(c*x^2 + 1)/(c*x^2 - 1)))*b + a*x
```

Mupad [B]
time = 0.79, size = 55, normalized size = 1.25

$$a\,x+\frac{b\,\mathrm{atan}\left(\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{b\,x\,\ln\left(c\,x^2+1\right)}{2}-\frac{b\,x\,\ln\left(1-c\,x^2\right)}{2}+\frac{b\,\mathrm{atan}\left(\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atanh(c*x^2),x)
```

```
[Out] a*x + (b*atan(c^(1/2)*x))/c^(1/2) + (b*atan(c^(1/2)*x*1i)*1i)/c^(1/2) + (b*
x*log(c*x^2 + 1))/2 - (b*x*log(1 - c*x^2))/2
```

3.61 $\int \frac{a+b\tanh^{-1}(cx^2)}{x^2}\,dx$

Optimal. Leaf size=46

$$b\sqrt{c}\,\mathrm{ArcTan}\left(\sqrt{c}\,x\right)+b\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right)-\frac{a+b\tanh^{-1}\left(cx^2\right)}{x}$$

```
[Out] (-a-b*arctanh(c*x^2))/x+b*arctan(x*c^(1/2))*c^(1/2)+b*arctanh(x*c^(1/2))*c^
(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6037, 218, 212, 209}

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{x}+b\sqrt{c}\,\mathrm{ArcTan}\left(\sqrt{c}\,x\right)+b\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x^2,x]
```

```
[Out] b*Sqrt[c]*ArcTan[Sqrt[c]*x] + b*Sqrt[c]*ArcTanh[Sqrt[c]*x] - (a + b*ArcTanh
[c*x^2])/x
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{x^2}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{x}+(2bc)\int \frac{1}{1-c^2x^4}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{x}+(bc)\int \frac{1}{1-cx^2}\,dx+(bc)\int \frac{1}{1+cx^2}\,dx\\ &= b\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right)+b\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right)-\frac{a+b\tanh^{-1}\left(cx^2\right)}{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 75, normalized size = 1.63

$$-\frac{a}{x}+b\sqrt{c}\,\mathrm{ArcTan}\left(\sqrt{c}\,x\right)-\frac{b\tanh^{-1}\left(cx^2\right)}{x}-\frac{1}{2}b\sqrt{c}\,\log\left(1-\sqrt{c}\,x\right)+\frac{1}{2}b\sqrt{c}\,\log\left(1+\sqrt{c}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^2,x]
```

```
[Out] -(a/x) + b*Sqrt[c]*ArcTan[Sqrt[c]*x] - (b*ArcTanh[c*x^2])/x - (b*Sqrt[c]*Lo
g[1 - Sqrt[c]*x])/2 + (b*Sqrt[c]*Log[1 + Sqrt[c]*x])/2
```

Maple [A]
time = 0.06, size = 42, normalized size = 0.91

method	result	size
default	$-\frac{a}{x}-\frac{b\,\mathrm{arctanh}\left(c\,x^{2}\right)}{x}+b\,\mathrm{arctanh}\left(x\sqrt{c}\right)\sqrt{c}+b\arctan\left(x\sqrt{c}\right)\sqrt{c}$	42
risch	$-\frac{b\ln\left(c\,x^{2}+1\right)}{2x}+b\arctan\left(x\sqrt{c}\right)\sqrt{c}-\frac{a}{x}+\frac{b\ln\left(-c\,x^{2}+1\right)}{2x}+b\,\mathrm{arctanh}\left(x\sqrt{c}\right)\sqrt{c}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/x-b/x*arctanh(c*x^2)+b*arctanh(x*c^(1/2))*c^(1/2)+b*arctan(x*c^(1/2))*c^
(1/2)
```

Maxima [A]
time = 0.48, size = 61, normalized size = 1.33

$$\frac{1}{2}\left(c\left(\frac{2\,\arctan\left(\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{2\,\mathrm{artanh}\left(cx^2\right)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c*(2*arctan(sqrt(c)*x)/sqrt(c) - log((c*x - sqrt(c))/(c*x + sqrt(c)))/
sqrt(c)) - 2*arctanh(c*x^2)/x)*b - a/x
```

Fricas [A] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(38) = 76.
time = 0.38, size = 157, normalized size = 3.41

$$\left[\frac{2\,b\sqrt{c}\,x\arctan\left(\sqrt{c}\,x\right)+b\sqrt{c}\,x\log\left(\frac{cx^2+2\,\sqrt{c}\,x+1}{cx^2-1}\right)-b\log\left(-\frac{cx^2+1}{cx^2-1}\right)-2\,a}{2\,x},-\frac{2\,b\sqrt{-c}\,x\arctan\left(\sqrt{-c}\,x\right)-b\sqrt{-c}\,x\log\left(\frac{cx^2+2\,\sqrt{-c}\,x-1}{cx^2+1}\right)+b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*sqrt(c)*x*arctan(sqrt(c)*x) + b*sqrt(c)*x*log((c*x^2 + 2*sqrt(c)*
x + 1)/(c*x^2 - 1)) - b*log(-(c*x^2 + 1)/(c*x^2 - 1)) - 2*a)/x, -1/2*(2*b*s
qrt(-c)*x*arctan(sqrt(-c)*x) - b*sqrt(-c)*x*log((c*x^2 + 2*sqrt(-c)*x - 1)/
(c*x^2 + 1)) + b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/x]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1374 vs. 2(42) = 84.
time = 4.70, size = 1374, normalized size = 29.87

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x**2,x)
```

```
[Out] Piecewise((-a/x, Eq(c, 0)), (-(a - oo*b)/x, Eq(c, -1/x**2)), (-(a + oo*b)/x
, Eq(c, x**(-2))), (-a*c*x**4*sqrt(-1/c)/(c*x**5*sqrt(-1/c) + c*x**5*sqrt(1
/c) - x*sqrt(-1/c)/c - x*sqrt(1/c)/c) - a*c*x**4*sqrt(1/c)/(c*x**5*sqrt(-1/
c) + c*x**5*sqrt(1/c) - x*sqrt(-1/c)/c - x*sqrt(1/c)/c) + a*sqrt(-1/c)/(c**
2*x**5*sqrt(-1/c) + c**2*x**5*sqrt(1/c) - x*sqrt(-1/c) - x*sqrt(1/c)) + a*s
qrt(1/c)/(c**2*x**5*sqrt(-1/c) + c**2*x**5*sqrt(1/c) - x*sqrt(-1/c) - x*sqr
t(1/c)) + b*c**2*x**5*sqrt(-1/c)*sqrt(1/c)*log(x + sqrt(-1/c))/(c*x**5*sqrt
(-1/c) + c*x**5*sqrt(1/c) - x*sqrt(-1/c)/c - x*sqrt(1/c)/c) - b*c**2*x**5*s
qrt(-1/c)*sqrt(1/c)*log(x - sqrt(1/c))/(c*x**5*sqrt(-1/c) + c*x**5*sqrt(1/c
) - x*sqrt(-1/c)/c - x*sqrt(1/c)/c) - b*c**2*x**5*sqrt(-1/c)*sqrt(1/c)*atan
h(c*x**2)/(c*x**5*sqrt(-1/c) + c*x**5*sqrt(1/c) - x*sqrt(-1/c)/c - x*sqrt(1
/c)/c) + b*c*x**5*log(x - sqrt(-1/c))/(c*x**5*sqrt(-1/c) + c*x**5*sqrt(1/c)
 - x*sqrt(-1/c)/c - x*sqrt(1/c)/c) - b*c*x**5*log(x - sqrt(1/c))/(c*x**5*sq
rt(-1/c) + c*x**5*sqrt(1/c) - x*sqrt(-1/c)/c - x*sqrt(1/c)/c) - b*c*x**5*at
anh(c*x**2)/(c*x**5*sqrt(-1/c) + c*x**5*sqrt(1/c) - x*sqrt(-1/c)/c - x*sqrt
```

```
(1/c)/c) - b*c*x**4*sqrt(-1/c)*atanh(c*x**2)/(c*x**5*sqrt(-1/c) + c*x**5*sq
rt(1/c) - x*sqrt(-1/c)/c - x*sqrt(1/c)/c) - b*c*x**4*sqrt(1/c)*atanh(c*x**2
)/(c*x**5*sqrt(-1/c) + c*x**5*sqrt(1/c) - x*sqrt(-1/c)/c - x*sqrt(1/c)/c) -
 b*x*sqrt(-1/c)*sqrt(1/c)*log(x + sqrt(-1/c))/(c*x**5*sqrt(-1/c) + c*x**5*s
qrt(1/c) - x*sqrt(-1/c)/c - x*sqrt(1/c)/c) + b*x*sqrt(-1/c)*sqrt(1/c)*log(x
 - sqrt(1/c))/(c*x**5*sqrt(-1/c) + c*x**5*sqrt(1/c) - x*sqrt(-1/c)/c - x*sq
rt(1/c)/c) + b*x*sqrt(-1/c)*sqrt(1/c)*atanh(c*x**2)/(c*x**5*sqrt(-1/c) + c*
x**5*sqrt(1/c) - x*sqrt(-1/c)/c - x*sqrt(1/c)/c) - b*x*log(x - sqrt(-1/c))/
(c**2*x**5*sqrt(-1/c) + c**2*x**5*sqrt(1/c) - x*sqrt(-1/c) - x*sqrt(1/c)) +
 b*x*log(x - sqrt(1/c))/(c**2*x**5*sqrt(-1/c) + c**2*x**5*sqrt(1/c) - x*sqr
t(-1/c) - x*sqrt(1/c)) + b*x*atanh(c*x**2)/(c**2*x**5*sqrt(-1/c) + c**2*x**
5*sqrt(1/c) - x*sqrt(-1/c) - x*sqrt(1/c)) + b*sqrt(-1/c)*atanh(c*x**2)/(c**
2*x**5*sqrt(-1/c) + c**2*x**5*sqrt(1/c) - x*sqrt(-1/c) - x*sqrt(1/c)) + b*s
qrt(1/c)*atanh(c*x**2)/(c**2*x**5*sqrt(-1/c) + c**2*x**5*sqrt(1/c) - x*sqrt
(-1/c) - x*sqrt(1/c)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(38) = 76$.
time = 0.44, size = 79, normalized size = 1.72

$$\frac{1}{2}\,bc\left(\frac{2\,\arctan\left(x\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{\log\left(\left|x+\frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}}-\frac{\log\left(\left|x-\frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}}\right)-\frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{2\,x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b*c*(2*arctan(x*sqrt(abs(c)))/sqrt(abs(c)) + log(abs(x + 1/sqrt(abs(c))
))/sqrt(abs(c)) - log(abs(x - 1/sqrt(abs(c))))/sqrt(abs(c))) - 1/2*b*log(-(
c*x^2 + 1)/(c*x^2 - 1))/x - a/x
```

Mupad [B]
time = 0.92, size = 62, normalized size = 1.35

$$b\,\sqrt{c}\,\operatorname{atan}\left(\sqrt{c}\,x\right)-\frac{a}{x}-\frac{b\,\ln\left(c\,x^2+1\right)}{2\,x}+\frac{b\,\ln\left(1-c\,x^2\right)}{2\,x}-b\,\sqrt{c}\,\operatorname{atan}\left(\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/x^2,x)
```

```
[Out] b*c^(1/2)*atan(c^(1/2)*x) - a/x - b*c^(1/2)*atan(c^(1/2)*x*1i)*1i - (b*log(
c*x^2 + 1))/(2*x) + (b*log(1 - c*x^2))/(2*x)
```

3.62 $\int \frac{a+b\tanh^{-1}(cx^2)}{x^4}\,dx$

Optimal. Leaf size=63

$$-\frac{2bc}{3x}-\frac{1}{3}bc^{3/2}\text{ArcTan}\left(\sqrt{c}\,x\right)+\frac{1}{3}bc^{3/2}\tanh^{-1}\left(\sqrt{c}\,x\right)-\frac{a+b\tanh^{-1}\left(cx^2\right)}{3x^3}$$

```
[Out] -2/3*b*c/x-1/3*b*c^(3/2)*arctan(x*c^(1/2))+1/3*(-a-b*arctanh(c*x^2))/x^3+1/
3*b*c^(3/2)*arctanh(x*c^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6037, 331, 304, 209, 212}

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{3x^3}-\frac{1}{3}bc^{3/2}\text{ArcTan}\left(\sqrt{c}\,x\right)+\frac{1}{3}bc^{3/2}\tanh^{-1}\left(\sqrt{c}\,x\right)-\frac{2bc}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x^4,x]
```

```
[Out] (-2*b*c)/(3*x) - (b*c^(3/2)*ArcTan[Sqrt[c]*x])/3 + (b*c^(3/2)*ArcTanh[Sqrt[
c]*x])/3 - (a + b*ArcTanh[c*x^2])/(3*x^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{x^4}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{3x^3}+\frac{1}{3}(2bc)\int \frac{1}{x^2\left(1-c^2x^4\right)}\,dx\\ &= -\frac{2bc}{3x}-\frac{a+b\tanh^{-1}\left(cx^2\right)}{3x^3}+\frac{1}{3}\left(2bc^3\right)\int \frac{x^2}{1-c^2x^4}\,dx\\ &= -\frac{2bc}{3x}-\frac{a+b\tanh^{-1}\left(cx^2\right)}{3x^3}+\frac{1}{3}\left(bc^2\right)\int \frac{1}{1-cx^2}\,dx-\frac{1}{3}\left(bc^2\right)\int \frac{1}{1+cx^2}\,dx\\ &= -\frac{2bc}{3x}-\frac{1}{3}bc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right)+\frac{1}{3}bc^{3/2}\tanh^{-1}\left(\sqrt{c}\,x\right)-\frac{a+b\tanh^{-1}\left(cx^2\right)}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 91, normalized size = 1.44

$$-\frac{a}{3x^3}-\frac{2bc}{3x}-\frac{1}{3}bc^{3/2}\text{ArcTan}\left(\sqrt{c}\,x\right)-\frac{b\tanh^{-1}\left(cx^2\right)}{3x^3}-\frac{1}{6}bc^{3/2}\log\left(1-\sqrt{c}\,x\right)+\frac{1}{6}bc^{3/2}\log\left(1+\sqrt{c}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^4,x]
[Out] -1/3*a/x^3 - (2*b*c)/(3*x) - (b*c^(3/2)*ArcTan[Sqrt[c]*x])/3 - (b*ArcTanh[c
*x^2])/(3*x^3) - (b*c^(3/2)*Log[1 - Sqrt[c]*x])/6 + (b*c^(3/2)*Log[1 + Sqrt
[c]*x])/6
```

Maple [A]
time = 0.06, size = 51, normalized size = 0.81

method	result
default	$-\frac{a}{3x^3}-\frac{b\,\mathrm{arctanh}\left(c\,x^2\right)}{3x^3}+\frac{b\,c^{\frac{3}{2}}\,\mathrm{arctanh}\left(x\sqrt{c}\right)}{3}-\frac{b\,c^{\frac{3}{2}}\,\arctan\left(x\sqrt{c}\right)}{3}-\frac{2bc}{3x}$

risch	$-\frac{b\ln(c\,x^{2}+1)}{6x^{3}}-\frac{-c\sqrt{-c}\;b\ln\left(c^{4}\sqrt{-c}\,-x\,c^{5}\right)x^{3}+c\sqrt{-c}\;b\ln\left(-c^{4}\sqrt{-c}\,-x\,c^{5}\right)x^{3}-c^{\frac{3}{2}}b\ln\left(-c^{\frac{11}{2}}-x\,c^{6}\right)x^{3}+c^{\frac{3}{2}}b\ln\left(c^{\frac{11}{2}}-\right.}{6x^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctanh(c*x^2)+1/3*b*c^(3/2)*arctanh(x*c^(1/2))-1/3*b*
c^(3/2)*arctan(x*c^(1/2))-2/3*b*c/x
```

Maxima [A]
time = 0.46, size = 65, normalized size = 1.03

$$-\frac{1}{6}\left(\left(2\sqrt{c}\,\arctan\left(\sqrt{c}\,x\right)+\sqrt{c}\,\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)+\frac{4}{x}\right)c+\frac{2\,\operatorname{artanh}\left(cx^{2}\right)}{x^{3}}\right)b-\frac{a}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*((2*sqrt(c)*arctan(sqrt(c)*x) + sqrt(c)*log((c*x - sqrt(c))/(c*x + sqr
t(c))) + 4/x)*c + 2*arctanh(c*x^2)/x^3)*b - 1/3*a/x^3
```

Fricas [A] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(47) = 94.
time = 0.35, size = 181, normalized size = 2.87

$$\left[-\frac{2\,bc^{\frac{3}{2}}x^{3}\arctan\left(\sqrt{c}\,x\right)-bc^{\frac{3}{2}}x^{3}\log\left(\frac{cx^{2}+2\,\sqrt{c}\,x+1}{cx^{2}-1}\right)+4\,bcx^{2}+b\log\left(-\frac{cx^{2}+1}{cx^{2}-1}\right)+2\,a}{6\,x^{3}},-\frac{2\,b\sqrt{-c}\,cx^{3}\arctan\left(\sqrt{-c}\,x\right)-b\sqrt{-c}\,cx^{3}\log\left(\frac{cx^{2}-2\,\sqrt{-c}\,x-1}{cx^{2}+1}\right)+4\,bcx^{2}+b\log\left(-\frac{cx^{2}+1}{cx^{2}-1}\right)+2\,a}{6\,x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^4,x, algorithm="fricas")
```

```
[Out] [-1/6*(2*b*c^(3/2)*x^3*arctan(sqrt(c)*x) - b*c^(3/2)*x^3*log((c*x^2 + 2*sqr
t(c)*x + 1)/(c*x^2 - 1)) + 4*b*c*x^2 + b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*
a)/x^3, -1/6*(2*b*sqrt(-c)*c*x^3*arctan(sqrt(-c)*x) - b*sqrt(-c)*c*x^3*log(
(c*x^2 - 2*sqrt(-c)*x - 1)/(c*x^2 + 1)) + 4*b*c*x^2 + b*log(-(c*x^2 + 1)/(c
*x^2 - 1)) + 2*a)/x^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1904 vs. 2(60) = 120.
time = 6.34, size = 1904, normalized size = 30.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x**4,x)
```

```
[Out] Piecewise((-(a - oo*b)/(3*x**3), Eq(c, -1/x**2)), (-(a + oo*b)/(3*x**3), Eq
(c, x**(-2))), (-a/(3*x**3), Eq(c, 0)), (-a*c*x**4*sqrt(-1/c)/(3*c*x**7*sqr
t(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c)/c + 3*x**3*sqrt(1/c)/c) +
a*c*x**4*sqrt(1/c)/(3*c*x**7*sqrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(
-1/c)/c + 3*x**3*sqrt(1/c)/c) + a*sqrt(-1/c)/(3*c**2*x**7*sqrt(-1/c) - 3*c*
*2*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c) + 3*x**3*sqrt(1/c)) - a*sqrt(1/c)/(3*
c**2*x**7*sqrt(-1/c) - 3*c**2*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c) + 3*x**3*s
qrt(1/c)) + b*c**3*x**7*sqrt(-1/c)*sqrt(1/c)*log(x + sqrt(-1/c))/(3*c*x**7*
sqrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c)/c + 3*x**3*sqrt(1/c)/c)
 - b*c**3*x**7*sqrt(-1/c)*sqrt(1/c)*log(x - sqrt(1/c))/(3*c*x**7*sqrt(-1/c)
 - 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c)/c + 3*x**3*sqrt(1/c)/c) - b*c**3*
x**7*sqrt(-1/c)*sqrt(1/c)*atanh(c*x**2)/(3*c*x**7*sqrt(-1/c) - 3*c*x**7*sqr
t(1/c) - 3*x**3*sqrt(-1/c)/c + 3*x**3*sqrt(1/c)/c) - b*c**2*x**7*log(x - sq
rt(-1/c))/(3*c*x**7*sqrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c)/c +
 3*x**3*sqrt(1/c)/c) + b*c**2*x**7*log(x - sqrt(1/c))/(3*c*x**7*sqrt(-1/c)
- 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c)/c + 3*x**3*sqrt(1/c)/c) + b*c**2*x
**7*atanh(c*x**2)/(3*c*x**7*sqrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-
1/c)/c + 3*x**3*sqrt(1/c)/c) - 2*b*c**2*x**6*sqrt(-1/c)/(3*c*x**7*sqrt(-1/c
) - 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c)/c + 3*x**3*sqrt(1/c)/c) + 2*b*c*
*2*x**6*sqrt(1/c)/(3*c*x**7*sqrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-
1/c)/c + 3*x**3*sqrt(1/c)/c) - b*c*x**4*sqrt(-1/c)*atanh(c*x**2)/(3*c*x**7*
sqrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c)/c + 3*x**3*sqrt(1/c)/c)
 + b*c*x**4*sqrt(1/c)*atanh(c*x**2)/(3*c*x**7*sqrt(-1/c) - 3*c*x**7*sqrt(1/
c) - 3*x**3*sqrt(-1/c)/c + 3*x**3*sqrt(1/c)/c) - b*c*x**3*sqrt(-1/c)*sqrt(1
/c)*log(x + sqrt(-1/c))/(3*c*x**7*sqrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*
sqrt(-1/c)/c + 3*x**3*sqrt(1/c)/c) + b*c*x**3*sqrt(-1/c)*sqrt(1/c)*log(x -
sqrt(1/c))/(3*c*x**7*sqrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c)/c
+ 3*x**3*sqrt(1/c)/c) + b*c*x**3*sqrt(-1/c)*sqrt(1/c)*atanh(c*x**2)/(3*c*x*
*7*sqrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c)/c + 3*x**3*sqrt(1/c)
/c) + b*x**3*log(x - sqrt(-1/c))/(3*c*x**7*sqrt(-1/c) - 3*c*x**7*sqrt(1/c)
- 3*x**3*sqrt(-1/c)/c + 3*x**3*sqrt(1/c)/c) - b*x**3*log(x - sqrt(1/c))/(3*
c*x**7*sqrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c)/c + 3*x**3*sqrt(
1/c)/c) - b*x**3*atanh(c*x**2)/(3*c*x**7*sqrt(-1/c) - 3*c*x**7*sqrt(1/c) -
3*x**3*sqrt(-1/c)/c + 3*x**3*sqrt(1/c)/c) + 2*b*x**2*sqrt(-1/c)/(3*c*x**7*s
qrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c)/c + 3*x**3*sqrt(1/c)/c)
- 2*b*x**2*sqrt(1/c)/(3*c*x**7*sqrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*sqr
t(-1/c)/c + 3*x**3*sqrt(1/c)/c) + b*sqrt(-1/c)*atanh(c*x**2)/(3*c**2*x**7*s
qrt(-1/c) - 3*c**2*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c) + 3*x**3*sqrt(1/c)) -
 b*sqrt(1/c)*atanh(c*x**2)/(3*c**2*x**7*sqrt(-1/c) - 3*c**2*x**7*sqrt(1/c)
- 3*x**3*sqrt(-1/c) + 3*x**3*sqrt(1/c)), True))
```

Giac [**A**]

time = 0.42, size = 93, normalized size = 1.48

$$-\frac{bc^3\arctan\left(x\sqrt{|c|}\right)}{3\,|c|^{\frac{3}{2}}}+\frac{1}{6}\,bc\sqrt{|c|}\,\log\left(\left|x+\frac{1}{\sqrt{|c|}}\right|\right)-\frac{bc^3\log\left(\left|x-\frac{1}{\sqrt{|c|}}\right|\right)}{6\,|c|^{\frac{3}{2}}}-\frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{6\,x^3}-\frac{2\,bcx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^4,x, algorithm="giac")
```

```
[Out] -1/3*b*c^3*arctan(x*sqrt(abs(c)))/abs(c)^(3/2) + 1/6*b*c*sqrt(abs(c))*log(a
bs(x + 1/sqrt(abs(c)))) - 1/6*b*c^3*log(abs(x - 1/sqrt(abs(c))))/abs(c)^(3/
2) - 1/6*b*log(-(c*x^2 + 1)/(c*x^2 - 1))/x^3 - 1/3*(2*b*c*x^2 + a)/x^3
```

Mupad [B]
time = 0.99, size = 71, normalized size = 1.13

$$\frac{b\,\ln\left(1-c\,x^2\right)}{6\,x^3}-\frac{b\,c^{3/2}\,\mathrm{atan}\left(\sqrt{c}\,x\right)}{3}-\frac{b\,\ln\left(c\,x^2+1\right)}{6\,x^3}-\frac{2\,b\,c\,x^2+a}{3\,x^3}-\frac{b\,c^{3/2}\,\mathrm{atan}\left(\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/x^4,x)
```

```
[Out] (b*log(1 - c*x^2))/(6*x^3) - (b*c^(3/2)*atan(c^(1/2)*x))/3 - (b*c^(3/2)*ata
n(c^(1/2)*x*1i)*1i)/3 - (b*log(c*x^2 + 1))/(6*x^3) - (a + 2*b*c*x^2)/(3*x^3
)
```

3.63 $\int \frac{a+b\tanh^{-1}(cx^2)}{x^6}\,dx$

Optimal. Leaf size=63

$$-\frac{2bc}{15x^3}+\frac{1}{5}bc^{5/2}\text{ArcTan}\left(\sqrt{c}\,x\right)+\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\sqrt{c}\,x\right)-\frac{a+b\tanh^{-1}\left(cx^2\right)}{5x^5}$$

```
[Out] -2/15*b*c/x^3+1/5*b*c^(5/2)*arctan(x*c^(1/2))+1/5*(-a-b*arctanh(c*x^2))/x^5
+1/5*b*c^(5/2)*arctanh(x*c^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6037, 331, 218, 212, 209}

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{5x^5}+\frac{1}{5}bc^{5/2}\text{ArcTan}\left(\sqrt{c}\,x\right)+\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\sqrt{c}\,x\right)-\frac{2bc}{15x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x^6,x]
```

```
[Out] (-2*b*c)/(15*x^3) + (b*c^(5/2)*ArcTan[Sqrt[c]*x])/5 + (b*c^(5/2)*ArcTanh[Sq
rt[c]*x])/5 - (a + b*ArcTanh[c*x^2])/(5*x^5)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx^2)}{x^6}\,dx &= -\frac{a+b\tanh^{-1}(cx^2)}{5x^5}+\frac{1}{5}(2bc)\int \frac{1}{x^4(1-c^2x^4)}\,dx\\ &= -\frac{2bc}{15x^3}-\frac{a+b\tanh^{-1}(cx^2)}{5x^5}+\frac{1}{5}\left(2bc^3\right)\int \frac{1}{1-c^2x^4}\,dx\\ &= -\frac{2bc}{15x^3}-\frac{a+b\tanh^{-1}(cx^2)}{5x^5}+\frac{1}{5}\left(bc^3\right)\int \frac{1}{1-cx^2}\,dx+\frac{1}{5}\left(bc^3\right)\int \frac{1}{1+cx^2}\,dx\\ &= -\frac{2bc}{15x^3}+\frac{1}{5}bc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)+\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\sqrt{c}\,x\right)-\frac{a+b\tanh^{-1}(cx^2)}{5x^5}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 91, normalized size = 1.44

$$-\frac{a}{5x^5}-\frac{2bc}{15x^3}+\frac{1}{5}bc^{5/2}\mathrm{ArcTan}\left(\sqrt{c}\,x\right)-\frac{b\tanh^{-1}(cx^2)}{5x^5}-\frac{1}{10}bc^{5/2}\log\left(1-\sqrt{c}\,x\right)+\frac{1}{10}bc^{5/2}\log\left(1+\sqrt{c}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^6,x]
```

```
[Out] -1/5*a/x^5 - (2*b*c)/(15*x^3) + (b*c^(5/2)*ArcTan[Sqrt[c]*x])/5 - (b*ArcTan
h[c*x^2])/(5*x^5) - (b*c^(5/2)*Log[1 - Sqrt[c]*x])/10 + (b*c^(5/2)*Log[1 +
Sqrt[c]*x])/10
```

Maple [A]
time = 0.07, size = 51, normalized size = 0.81

method	result
default	$-\frac{a}{5x^5}-\frac{b\,\mathrm{arctanh}(c\,x^2)}{5x^5}+\frac{b\,c^{\frac{5}{2}}\,\mathrm{arctanh}\left(x\sqrt{c}\right)}{5}+\frac{b\,c^{\frac{5}{2}}\,\arctan\left(x\sqrt{c}\right)}{5}-\frac{2bc}{15x^3}$

risch	$-\frac{b\ln(c\,x^2+1)}{10x^5}-\frac{-3c^{\frac{5}{2}}b\ln\left(c^{\frac{19}{2}}+x\,c^{10}\right)x^5+3c^{\frac{5}{2}}b\ln\left(-c^{\frac{19}{2}}+x\,c^{10}\right)x^5-3c^2\sqrt{-c}\;b\ln\left(c^2\sqrt{-c}+x\,c^3\right)x^5+3c^2\sqrt{-c}\;b\ln\left(-c\right.}{30x^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a/x^5-1/5*b/x^5*arctanh(c*x^2)+1/5*b*c^(5/2)*arctanh(x*c^(1/2))+1/5*b*
c^(5/2)*arctan(x*c^(1/2))-2/15*b*c/x^3
```

Maxima [A]
time = 0.46, size = 66, normalized size = 1.05

$$\frac{1}{30}\left(\left(6\,c^{\frac{3}{2}}\arctan\left(\sqrt{c}\,x\right)-3\,c^{\frac{3}{2}}\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)-\frac{4}{x^3}\right)c-\frac{6\,\mathrm{artanh}\left(cx^2\right)}{x^5}\right)b-\frac{a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^6,x, algorithm="maxima")
```

```
[Out] 1/30*((6*c^(3/2)*arctan(sqrt(c)*x) - 3*c^(3/2)*log((c*x - sqrt(c))/(c*x + s
qrt(c))) - 4/x^3)*c - 6*arctanh(c*x^2)/x^5)*b - 1/5*a/x^5
```

Fricas [A] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(47) = 94$.
time = 0.36, size = 187, normalized size = 2.97

$$\left[\frac{6\,bc^{\frac{5}{2}}x^5\arctan\left(\sqrt{c}\,x\right)+3\,bc^{\frac{5}{2}}x^5\log\left(\frac{cx^2+2\,\sqrt{c}\,x+1}{cx^2-1}\right)-4\,bcx^2-3\,b\log\left(-\frac{cx^2+1}{cx^2-1}\right)-6\,a}{30\,x^5},\,-\frac{6\,b\sqrt{-c}\,c^2x^5\arctan\left(\sqrt{-c}\,x\right)-3\,b\sqrt{-c}\,c^2x^5\log\left(\frac{cx^2+2\,\sqrt{-c}\,x-1}{cx^2+1}\right)+4\,bcx^2+3\,b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+6\,a}{30\,x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^6,x, algorithm="fricas")
```

```
[Out] [1/30*(6*b*c^(5/2)*x^5*arctan(sqrt(c)*x) + 3*b*c^(5/2)*x^5*log((c*x^2 + 2*s
qrt(c)*x + 1)/(c*x^2 - 1)) - 4*b*c*x^2 - 3*b*log(-(c*x^2 + 1)/(c*x^2 - 1))
- 6*a)/x^5, -1/30*(6*b*sqrt(-c)*c^2*x^5*arctan(sqrt(-c)*x) - 3*b*sqrt(-c)*c
^2*x^5*log((c*x^2 + 2*sqrt(-c)*x - 1)/(c*x^2 + 1)) + 4*b*c*x^2 + 3*b*log(-(
c*x^2 + 1)/(c*x^2 - 1)) + 6*a)/x^5]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1948 vs. $2(61) = 122$.
time = 8.69, size = 1948, normalized size = 30.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x**6,x)
```

```
[Out] Piecewise((-a/(5*x**5), Eq(c, 0)), (-(a - oo*b)/(5*x**5), Eq(c, -1/x**2)),
(-(a + oo*b)/(5*x**5), Eq(c, x**(-2))), (-3*a*c*x**4*sqrt(-1/c)/(15*c*x**9*
sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c)/c - 15*x**5*sqrt(1/c)
/c) - 3*a*c*x**4*sqrt(1/c)/(15*c*x**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15
*x**5*sqrt(-1/c)/c - 15*x**5*sqrt(1/c)/c) + 3*a*sqrt(-1/c)/(15*c**2*x**9*sq
rt(-1/c) + 15*c**2*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c) - 15*x**5*sqrt(1/c))
 + 3*a*sqrt(1/c)/(15*c**2*x**9*sqrt(-1/c) + 15*c**2*x**9*sqrt(1/c) - 15*x**
5*sqrt(-1/c) - 15*x**5*sqrt(1/c)) + 3*b*c**4*x**9*sqrt(-1/c)*sqrt(1/c)*log(
x + sqrt(-1/c))/(15*c*x**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt(
-1/c)/c - 15*x**5*sqrt(1/c)/c) - 3*b*c**4*x**9*sqrt(-1/c)*sqrt(1/c)*log(x -
 sqrt(1/c))/(15*c*x**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c
)/c - 15*x**5*sqrt(1/c)/c) - 3*b*c**4*x**9*sqrt(-1/c)*sqrt(1/c)*atanh(c*x**
2)/(15*c*x**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c)/c - 15*
x**5*sqrt(1/c)/c) + 3*b*c**3*x**9*log(x - sqrt(-1/c))/(15*c*x**9*sqrt(-1/c)
 + 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c)/c - 15*x**5*sqrt(1/c)/c) - 3*b*
c**3*x**9*log(x - sqrt(1/c))/(15*c*x**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) -
15*x**5*sqrt(-1/c)/c - 15*x**5*sqrt(1/c)/c) - 3*b*c**3*x**9*atanh(c*x**2)/(
15*c*x**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c)/c - 15*x**5
*sqrt(1/c)/c) - 2*b*c**2*x**6*sqrt(-1/c)/(15*c*x**9*sqrt(-1/c) + 15*c*x**9*
sqrt(1/c) - 15*x**5*sqrt(-1/c)/c - 15*x**5*sqrt(1/c)/c) - 2*b*c**2*x**6*sqr
t(1/c)/(15*c*x**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c)/c -
 15*x**5*sqrt(1/c)/c) - 3*b*c**2*x**5*sqrt(-1/c)*sqrt(1/c)*log(x + sqrt(-1/
c))/(15*c*x**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c)/c - 15
*x**5*sqrt(1/c)/c) + 3*b*c**2*x**5*sqrt(-1/c)*sqrt(1/c)*log(x - sqrt(1/c))/
(15*c*x**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c)/c - 15*x**
5*sqrt(1/c)/c) + 3*b*c**2*x**5*sqrt(-1/c)*sqrt(1/c)*atanh(c*x**2)/(15*c*x**
9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c)/c - 15*x**5*sqrt(1/
c)/c) - 3*b*c*x**5*log(x - sqrt(-1/c))/(15*c*x**9*sqrt(-1/c) + 15*c*x**9*sq
rt(1/c) - 15*x**5*sqrt(-1/c)/c - 15*x**5*sqrt(1/c)/c) + 3*b*c*x**5*log(x -
sqrt(1/c))/(15*c*x**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c)
/c - 15*x**5*sqrt(1/c)/c) + 3*b*c*x**5*atanh(c*x**2)/(15*c*x**9*sqrt(-1/c)
+ 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c)/c - 15*x**5*sqrt(1/c)/c) - 3*b*c
*x**4*sqrt(-1/c)*atanh(c*x**2)/(15*c*x**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c)
- 15*x**5*sqrt(-1/c)/c - 15*x**5*sqrt(1/c)/c) - 3*b*c*x**4*sqrt(1/c)*atanh(
c*x**2)/(15*c*x**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c)/c
- 15*x**5*sqrt(1/c)/c) + 2*b*x**2*sqrt(-1/c)/(15*c*x**9*sqrt(-1/c) + 15*c*x
**9*sqrt(1/c) - 15*x**5*sqrt(-1/c)/c - 15*x**5*sqrt(1/c)/c) + 2*b*x**2*sqrt
(1/c)/(15*c*x**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c)/c -
15*x**5*sqrt(1/c)/c) + 3*b*sqrt(-1/c)*atanh(c*x**2)/(15*c**2*x**9*sqrt(-1/c
) + 15*c**2*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c) - 15*x**5*sqrt(1/c)) + 3*b*
sqrt(1/c)*atanh(c*x**2)/(15*c**2*x**9*sqrt(-1/c) + 15*c**2*x**9*sqrt(1/c) -
 15*x**5*sqrt(-1/c) - 15*x**5*sqrt(1/c)), True))
```

Giac [A]

time = 0.46, size = 91, normalized size = 1.44

$$\frac{1}{10}\,bc^3\left(\frac{2\,\arctan\left(x\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{\log\left(\left|x+\frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}}-\frac{\log\left(\left|x-\frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}}\right)-\frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{10\,x^5}-\frac{2\,bcx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^6,x, algorithm="giac")
```

```
[Out] 1/10*b*c^3*(2*arctan(x*sqrt(abs(c)))/sqrt(abs(c)) + log(abs(x + 1/sqrt(abs(
c))))/sqrt(abs(c)) - log(abs(x - 1/sqrt(abs(c))))/sqrt(abs(c))) - 1/10*b*lo
g(-(c*x^2 + 1)/(c*x^2 - 1))/x^5 - 1/15*(2*b*c*x^2 + 3*a)/x^5
```

Mupad [B]
time = 1.03, size = 71, normalized size = 1.13

$$\frac{b\,c^{5/2}\,\mathrm{atan}\left(\sqrt{c}\,\,x\right)}{5}-\frac{\frac{2\,b\,c\,x^2}{3}+a}{5\,x^5}-\frac{b\,\ln\left(c\,x^2+1\right)}{10\,x^5}+\frac{b\,\ln\left(1-c\,x^2\right)}{10\,x^5}-\frac{b\,c^{5/2}\,\mathrm{atan}\left(\sqrt{c}\,\,x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/x^6,x)
```

```
[Out] (b*c^(5/2)*atan(c^(1/2)*x))/5 - (a + (2*b*c*x^2)/3)/(5*x^5) - (b*c^(5/2)*at
an(c^(1/2)*x*1i)*1i)/5 - (b*log(c*x^2 + 1))/(10*x^5) + (b*log(1 - c*x^2))/(
10*x^5)
```

3.64 $\int x^7 \left(a + b\tanh^{-1}\left(cx^2\right)\right)^2 \, dx$

Optimal. Leaf size=125

$$\frac{abx^2}{4c^3}+\frac{b^2x^4}{24c^2}+\frac{b^2x^2\tanh^{-1}\left(cx^2\right)}{4c^3}+\frac{bx^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{12c}-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{8c^4}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2-$$

```
[Out] 1/4*a*b*x^2/c^3+1/24*b^2*x^4/c^2+1/4*b^2*x^2*arctanh(c*x^2)/c^3+1/12*b*x^6*
(a+b*arctanh(c*x^2))/c-1/8*(a+b*arctanh(c*x^2))^2/c^4+1/8*x^8*(a+b*arctanh(
c*x^2))^2+1/6*b^2*ln(-c^2*x^4+1)/c^4
```

Rubi [A]
time = 0.19, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6039, 6037, 6127, 272, 45, 6021, 266, 6095}

$$-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{8c^4}+\frac{abx^2}{4c^3}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2+\frac{bx^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{12c}+\frac{b^2x^2\tanh^{-1}\left(cx^2\right)}{4c^3}+\frac{b^2x^4}{24c^2}+\frac{b^2\log\left(1-c^2x^4\right)}{6c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (a*b*x^2)/(4*c^3) + (b^2*x^4)/(24*c^2) + (b^2*x^2*ArcTanh[c*x^2])/(4*c^3) +
 (b*x^6*(a + b*ArcTanh[c*x^2]))/(12*c) - (a + b*ArcTanh[c*x^2])^2/(8*c^4) +
 (x^8*(a + b*ArcTanh[c*x^2])^2)/8 + (b^2*Log[1 - c^2*x^4])/(6*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
```

```
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$
\begin{aligned}
\int x^7\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx &= \int \left(\frac{1}{4}x^7\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{1}{2}bx^7\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right.\right. \\
&= \frac{1}{4}\int x^7\left(2a-b\log\left(1-cx^2\right)\right)^2\,dx-\frac{1}{2}b\int x^7\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1\right. \\
&= \frac{1}{8}\text{Subst}\left(\int x^3(2a-b\log(1-cx))^2\,dx,x,x^2\right)-\frac{1}{4}b\text{Subst}\left(\int x^3(-2a+b\log\right. \\
&= \frac{1}{32}x^8\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{16}bx^8\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+ \\
&= \frac{1}{32}x^8\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{16}bx^8\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+ \\
&= \frac{1}{32}x^8\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{1}{192}b\left(2a-b\log\left(1-cx^2\right)\right)\left(\frac{48(1-cx^2)}{c^4}-\right. \\
&= \frac{abx^2}{8c^3}-\frac{bx^4(2a-b\log\left(1-cx^2\right))}{32c^2}+\frac{bx^6(2a-b\log\left(1-cx^2\right))}{48c}-\frac{1}{64}bx^8(2a- \\
&= \frac{abx^2}{8c^3}-\frac{bx^4(2a-b\log\left(1-cx^2\right))}{32c^2}+\frac{bx^6(2a-b\log\left(1-cx^2\right))}{48c}-\frac{1}{64}bx^8(2a- \\
&= \frac{abx^2}{8c^3}+\frac{55b^2x^2}{192c^3}-\frac{5b^2x^4}{384c^2}+\frac{b^2x^6}{576c}-\frac{b^2x^8}{256}+\frac{3b^2\left(1-cx^2\right)^2}{32c^4}-\frac{b^2\left(1-cx^2\right)^3}{36c^4}+ \\
&= \frac{abx^2}{8c^3}+\frac{55b^2x^2}{192c^3}-\frac{5b^2x^4}{384c^2}+\frac{b^2x^6}{576c}-\frac{b^2x^8}{256}+\frac{3b^2\left(1-cx^2\right)^2}{32c^4}-\frac{b^2\left(1-cx^2\right)^3}{36c^4}+
\end{aligned}
$$

Mathematica [**A**]
time = 0.05, size = 146, normalized size = 1.17

$$\frac{6abcx^2+b^2c^2x^4+2abc^3x^6+3a^2c^4x^8+2bcx^2(3ac^3x^6+b(3+c^2x^4))\tanh^{-1}\left(cx^2\right)+3b^2(-1+c^4x^8)\tanh^{-1}\left(cx^2\right)^2+b(3a+4b)\log\left(1-cx^2\right)-3ab\log\left(1+cx^2\right)+4b^2\log\left(1+cx^2\right)}{24c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (6*a*b*c*x^2 + b^2*c^2*x^4 + 2*a*b*c^3*x^6 + 3*a^2*c^4*x^8 + 2*b*c*x^2*(3*a
*c^3*x^6 + b*(3 + c^2*x^4))*ArcTanh[c*x^2] + 3*b^2*(-1 + c^4*x^8)*ArcTanh[c
*x^2]^2 + b*(3*a + 4*b)*Log[1 - c*x^2] - 3*a*b*Log[1 + c*x^2] + 4*b^2*Log[1
 + c*x^2])/(24*c^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(111) = 222$.
time = 0.08, size = 298, normalized size = 2.38

method	result
risch	$\frac{b^2(x^8c^4-1)\ln(cx^2+1)^2}{32c^4} + \frac{b(-3x^8b\ln(-cx^2+1)c^4+6ac^4x^8+2bc^3x^6+6bcx^2+3b\ln(-cx^2+1))\ln(cx^2+1)}{48c^4} + \frac{b^2x^8\ln(-cx^2+1)}{32}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctanh(c*x^2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*b^2*(c^4*x^8-1)/c^4*ln(c*x^2+1)^2+1/48*b*(-3*x^8*b*ln(-c*x^2+1)*c^4+6*
a*c^4*x^8+2*b*c^3*x^6+6*b*c*x^2+3*b*ln(-c*x^2+1))/c^4*ln(c*x^2+1)+1/32*b^2*
x^8*ln(-c*x^2+1)^2-1/8*a*b*x^8*ln(-c*x^2+1)+1/8*x^8*a^2-1/24/c*b^2*x^6*ln(-
c*x^2+1)+1/12/c*a*b*x^6+1/24*b^2*x^4/c^2-1/8/c^3*b^2*x^2*ln(-c*x^2+1)+1/4*a
*b*x^2/c^3-1/32/c^4*b^2*ln(-c*x^2+1)^2+1/8/c^4*b*ln(-c*x^2+1)*a+1/6/c^4*b^2
*ln(-c*x^2+1)-1/8/c^4*b*ln(-c*x^2-1)*a+1/6/c^4*b^2*ln(-c*x^2-1)
```

Maxima [A]
time = 0.26, size = 217, normalized size = 1.74

$$\frac{1}{8}b^2x^8\operatorname{artanh}(cx^2)^2 + \frac{1}{8}a^2x^8 + \frac{1}{24}\left(6x^8\operatorname{artanh}(cx^2) + c\left(\frac{2(c^2x^6+3x^2)}{c^4} - \frac{3\log(cx^2+1)}{c^5} + \frac{3\log(cx^2-1)}{c^5}\right)\right)ab + \frac{1}{96}\left(4c\left(\frac{2(c^2x^6+3x^2)}{c^4} - \frac{3\log(cx^2+1)}{c^5} + \frac{3\log(cx^2-1)}{c^5}\right)\operatorname{artanh}(cx^2) + \frac{4c^2x^4 - 2(3\log(cx^2-1)-8)\log(cx^2+1) + 3\log(cx^2+1)^2 + 3\log(cx^2-1)^2 + 16\log(cx^2-1)}{c^4}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8*arctanh(c*x^2)^2 + 1/8*a^2*x^8 + 1/24*(6*x^8*arctanh(c*x^2) + c
*(2*(c^2*x^6 + 3*x^2)/c^4 - 3*log(c*x^2 + 1)/c^5 + 3*log(c*x^2 - 1)/c^5))*a
*b + 1/96*(4*c*(2*(c^2*x^6 + 3*x^2)/c^4 - 3*log(c*x^2 + 1)/c^5 + 3*log(c*x^
2 - 1)/c^5)*arctanh(c*x^2) + (4*c^2*x^4 - 2*(3*log(c*x^2 - 1) - 8)*log(c*x^
2 + 1) + 3*log(c*x^2 + 1)^2 + 3*log(c*x^2 - 1)^2 + 16*log(c*x^2 - 1))/c^4)*
b^2
```

Fricas [A]
time = 0.37, size = 176, normalized size = 1.41

$$\frac{12a^2c^4x^8 + 8abc^3x^6 + 4b^2c^2x^4 + 24abcx^2 + 3(b^2c^4x^8 - b^2)\log\left(-\frac{cx^2+1}{cx^2-1}\right)^2 - 4(3ab-4b^2)\log(cx^2+1) + 4(3ab+4b^2)\log(cx^2-1) + 4(3abc^4x^8 + b^2c^3x^6 + 3b^2cx^2)\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{96c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/96*(12*a^2*c^4*x^8 + 8*a*b*c^3*x^6 + 4*b^2*c^2*x^4 + 24*a*b*c*x^2 + 3*(b^
2*c^4*x^8 - b^2)*log(-(c*x^2 + 1)/(c*x^2 - 1))^2 - 4*(3*a*b - 4*b^2)*log(c*
x^2 + 1) + 4*(3*a*b + 4*b^2)*log(c*x^2 - 1) + 4*(3*a*b*c^4*x^8 + b^2*c^3*x^
6 + 3*b^2*c*x^2)*log(-(c*x^2 + 1)/(c*x^2 - 1)))/c^4
```

Sympy [A]
time = 9.25, size = 206, normalized size = 1.65

$$\begin{cases} \frac{a^2x^8}{8} + \frac{abx^8\operatorname{atanh}(cx^2)}{4} + \frac{abx^6}{12c} + \frac{abx^2}{4c^3} - \frac{ab\operatorname{atanh}(cx^2)}{4c^4} + \frac{b^2x^8\operatorname{atanh}^2(cx^2)}{8} + \frac{b^2x^6\operatorname{atanh}(cx^2)}{12c} + \frac{b^2x^4}{24c^2} + \frac{b^2x^2\operatorname{atanh}(cx^2)}{4c^3} + \frac{b^2\log\left(x-\sqrt{-\frac{1}{c}}\right)}{3c^4} + \frac{b^2\log\left(x+\sqrt{-\frac{1}{c}}\right)}{3c^4} - \frac{b^2\operatorname{atanh}^2(cx^2)}{8c^4} - \frac{b^2\operatorname{atanh}(cx^2)}{3c^4} & \text{for } c \neq 0 \\ \frac{a^2x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Piecewise((a**2*x**8/8 + a*b*x**8*atanh(c*x**2)/4 + a*b*x**6/(12*c) + a*b*x
**2/(4*c**3) - a*b*atanh(c*x**2)/(4*c**4) + b**2*x**8*atanh(c*x**2)**2/8 +
b**2*x**6*atanh(c*x**2)/(12*c) + b**2*x**4/(24*c**2) + b**2*x**2*atanh(c*x*
*2)/(4*c**3) + b**2*log(x - sqrt(-1/c))/(3*c**4) + b**2*log(x + sqrt(-1/c))
/(3*c**4) - b**2*atanh(c*x**2)**2/(8*c**4) - b**2*atanh(c*x**2)/(3*c**4), N
e(c, 0)), (a**2*x**8/8, True))
```

Giac [A]
time = 0.52, size = 175, normalized size = 1.40

$$\frac{1}{8}\,a^2x^8+\frac{abx^6}{12\,c}+\frac{b^2x^4}{24\,c^2}+\frac{1}{32}\left(b^2x^8-\frac{b^2}{c^4}\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)^2+\frac{1}{24}\left(3\,abx^8+\frac{b^2x^6}{c}+\frac{3\,b^2x^2}{c^3}\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)+\frac{abx^2}{4\,c^3}-\frac{(3\,ab-4\,b^2)\log\left(cx^2+1\right)}{24\,c^4}+\frac{(3\,ab+4\,b^2)\log\left(cx^2-1\right)}{24\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*x^8 + 1/12*a*b*x^6/c + 1/24*b^2*x^4/c^2 + 1/32*(b^2*x^8 - b^2/c^4)*
log(-(c*x^2 + 1)/(c*x^2 - 1))^2 + 1/24*(3*a*b*x^8 + b^2*x^6/c + 3*b^2*x^2/c
^3)*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/4*a*b*x^2/c^3 - 1/24*(3*a*b - 4*b^2)*
log(c*x^2 + 1)/c^4 + 1/24*(3*a*b + 4*b^2)*log(c*x^2 - 1)/c^4
```

Mupad [B]
time = 1.73, size = 335, normalized size = 2.68

$$\frac{a^2\,x^8}{8}+\frac{b^2\,\ln\left(c\,x^2-1\right)}{6\,c^4}+\frac{b^2\,\ln\left(c\,x^2+1\right)}{6\,c^4}-\frac{b^2\ln\left(c\,x^2+1\right)^2}{32\,c^4}-\frac{b^2\ln\left(1-c\,x^2\right)^2}{32\,c^4}+\frac{b^2\,x^4}{24\,c^2}+\frac{b^2\,x^8\ln\left(c\,x^2+1\right)^2}{32}+\frac{b^2\,x^8\ln\left(1-c\,x^2\right)^2}{32}+\frac{b^2\,x^2\,\ln\left(c\,x^2+1\right)}{8\,c^3}-\frac{b^2\,x^2\,\ln\left(1-c\,x^2\right)}{8\,c^3}+\frac{b^2\,x^6\,\ln\left(c\,x^2+1\right)}{24\,c}-\frac{b^2\,x^6\,\ln\left(1-c\,x^2\right)}{24\,c}+\frac{a\,b\,\ln\left(c\,x^2-1\right)}{8\,c^4}-\frac{a\,b\,\ln\left(c\,x^2+1\right)}{8\,c^4}+\frac{a\,b\,x^8\,\ln\left(c\,x^2+1\right)}{8}-\frac{a\,b\,x^8\,\ln\left(1-c\,x^2\right)}{8}+\frac{b^2\,\ln\left(c\,x^2+1\right)\,\ln\left(1-c\,x^2\right)}{16\,c^4}+\frac{a\,b\,x^2}{4\,c^3}+\frac{a\,b\,x^6}{12\,c}-\frac{b^2\,x^8\,\ln\left(c\,x^2+1\right)\,\ln\left(1-c\,x^2\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*atanh(c*x^2))^2,x)
```

```
[Out] (a^2*x^8)/8 + (b^2*log(c*x^2 - 1))/(6*c^4) + (b^2*log(c*x^2 + 1))/(6*c^4) -
 (b^2*log(c*x^2 + 1)^2)/(32*c^4) - (b^2*log(1 - c*x^2)^2)/(32*c^4) + (b^2*x
^4)/(24*c^2) + (b^2*x^8*log(c*x^2 + 1)^2)/32 + (b^2*x^8*log(1 - c*x^2)^2)/3
2 + (b^2*x^2*log(c*x^2 + 1))/(8*c^3) - (b^2*x^2*log(1 - c*x^2))/(8*c^3) + (
b^2*x^6*log(c*x^2 + 1))/(24*c) - (b^2*x^6*log(1 - c*x^2))/(24*c) + (a*b*log
(c*x^2 - 1))/(8*c^4) - (a*b*log(c*x^2 + 1))/(8*c^4) + (a*b*x^8*log(c*x^2 +
1))/8 - (a*b*x^8*log(1 - c*x^2))/8 + (b^2*log(c*x^2 + 1)*log(1 - c*x^2))/(1
6*c^4) + (a*b*x^2)/(4*c^3) + (a*b*x^6)/(12*c) - (b^2*x^8*log(c*x^2 + 1)*log
(1 - c*x^2))/16
```

3.65 $\int x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx$

Optimal. Leaf size=146

$$\frac{b^2x^2}{6c^2}-\frac{b^2\tanh^{-1}\left(cx^2\right)}{6c^3}+\frac{bx^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{6c}+\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{6c^3}+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2-\frac{b(a+$$

```
[Out] 1/6*b^2*x^2/c^2-1/6*b^2*arctanh(c*x^2)/c^3+1/6*b*x^4*(a+b*arctanh(c*x^2))/c
+1/6*(a+b*arctanh(c*x^2))^2/c^3+1/6*x^6*(a+b*arctanh(c*x^2))^2-1/3*b*(a+b*a
rctanh(c*x^2))*ln(2/(-c*x^2+1))/c^3-1/6*b^2*polylog(2,1-2/(-c*x^2+1))/c^3
```

Rubi [A]
time = 0.17, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.562$, Rules used = {6039, 6037, 6127, 327, 212, 6131, 6055, 2449, 2352}

$$\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{6c^3}-\frac{b\log\left(\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3c^3}+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2+\frac{bx^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{6c}-\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx^2}\right)}{6c^3}-\frac{b^2\tanh^{-1}\left(cx^2\right)}{6c^3}+\frac{b^2x^2}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (b^2*x^2)/(6*c^2) - (b^2*ArcTanh[c*x^2])/(6*c^3) + (b*x^4*(a + b*ArcTanh[c*
x^2]))/(6*c) + (a + b*ArcTanh[c*x^2])^2/(6*c^3) + (x^6*(a + b*ArcTanh[c*x^2
])^2)/6 - (b*(a + b*ArcTanh[c*x^2])*Log[2/(1 - c*x^2)])/(3*c^3) - (b^2*Poly
Log[2, 1 - 2/(1 - c*x^2)])/(6*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^5 \left(a+b \tanh^{-1}\left(cx^2\right)\right)^2 \, dx &= \int \left(\frac{1}{4} x^5 \left(2a-b\log\left(1-cx^2\right)\right)^2 - \frac{1}{2} b x^5 \left(-2a+b\log\left(1-cx^2\right)\right) \log\left(1+cx\right.\right.\\
&= \frac{1}{4} \int x^5 \left(2a-b\log\left(1-cx^2\right)\right)^2 \, dx - \frac{1}{2} b \int x^5 \left(-2a+b\log\left(1-cx^2\right)\right) \log\left(1\right.\\
&= \frac{1}{8} \text{Subst}\left(\int x^2 (2a-b\log(1-cx))^2 \, dx, x, x^2\right) - \frac{1}{4} b \text{Subst}\left(\int x^2(-2a+b\,l\right.\\
&= \frac{1}{24} x^6 \left(2a-b\log\left(1-cx^2\right)\right)^2 + \frac{1}{12} b x^6 \left(2a-b\log\left(1-cx^2\right)\right) \log\left(1+cx^2\right) -\\
&= \frac{1}{24} x^6 \left(2a-b\log\left(1-cx^2\right)\right)^2 + \frac{1}{12} b x^6 \left(2a-b\log\left(1-cx^2\right)\right) \log\left(1+cx^2\right) -\\
&= \frac{1}{24} x^6 \left(2a-b\log\left(1-cx^2\right)\right)^2 - \frac{1}{72} b \left(2a-b\log\left(1-cx^2\right)\right) \left(\frac{18(1-cx^2)}{c^3} -\right.\\
&= -\frac{abx^2}{6c^2} + \frac{bx^4 (2a-b\log\left(1-cx^2\right))}{24c} - \frac{1}{36} b x^6 \left(2a-b\log\left(1-cx^2\right)\right) + \frac{1}{24} x^6\\
&= -\frac{abx^2}{6c^2} + \frac{bx^4 (2a-b\log\left(1-cx^2\right))}{24c} - \frac{1}{36} b x^6 \left(2a-b\log\left(1-cx^2\right)\right) + \frac{1}{24} x^6\\
&= -\frac{abx^2}{6c^2} + \frac{13b^2x^2}{72c^2} + \frac{b^2x^4}{144c} - \frac{b^2x^6}{108} + \frac{b^2(1-cx^2)^2}{16c^3} - \frac{b^2(1-cx^2)^3}{108c^3} + \frac{b^2\log(1}{72}\\
&= -\frac{abx^2}{6c^2} + \frac{13b^2x^2}{72c^2} + \frac{b^2x^4}{144c} - \frac{b^2x^6}{108} + \frac{b^2(1-cx^2)^2}{16c^3} - \frac{b^2(1-cx^2)^3}{108c^3} + \frac{b^2\log(1}{72}
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 132, normalized size = 0.90

$$
\frac{b^2cx^2 + abc^2x^4 + a^2c^3x^6 + b^2(-1+c^3x^6)\tanh^{-1}\left(cx^2\right)^2 + b\tanh^{-1}\left(cx^2\right)\left(-b+bc^2x^4+2ac^3x^6-2b\log\left(1+e^{-2\tanh^{-1}\left(cx^2\right)}\right)\right) + ab\log\left(-1+c^2x^4\right) + b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(cx^2\right)}\right)}{6c^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (b^2*c*x^2 + a*b*c^2*x^4 + a^2*c^3*x^6 + b^2*(-1 + c^3*x^6)*ArcTanh[c*x^2]^
2 + b*ArcTanh[c*x^2]*(-b + b*c^2*x^4 + 2*a*c^3*x^6 - 2*b*Log[1 + E^(-2*ArcT
anh[c*x^2])]) + a*b*Log[-1 + c^2*x^4] + b^2*PolyLog[2, -E^(-2*ArcTanh[c*x^2
])])/(6*c^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(132) = 264$.
time = 0.28, size = 380, normalized size = 2.60

method	result
risch	$\frac{b^2x^2}{6c^2}+\frac{ba\,x^6\ln(c\,x^2+1)}{6}+\frac{ba\ln(c\,x^2+1)}{6c^3}-\frac{b^2\ln(-c\,x^2+1)\ln(c\,x^2+1)x^6}{12}-\frac{b^2\ln(-c\,x^2+1)\ln(c\,x^2+1)}{12c^3}+\frac{b^2\ln\left(\frac{1}{2}-\frac{c\,x^2}{2}\right)\ln(c}{6c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x^2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b^2*x^2/c^2+1/6*b*a*x^6*ln(c*x^2+1)+1/6*b*a/c^3*ln(c*x^2+1)-1/12*b^2*ln
(-c*x^2+1)*ln(c*x^2+1)*x^6-1/12*b^2/c^3*ln(-c*x^2+1)*ln(c*x^2+1)+1/6*b^2/c^
3*ln(1/2-1/2*c*x^2)*ln(c*x^2+1)-1/6*b^2/c^3*ln(1/2-1/2*c*x^2)*ln(1/2*c*x^2+
1/2)+1/6/c*a*b*x^4-17/108/c^3*b^2-1/12*b^2/c*x^4*ln(-c*x^2+1)-1/6*a*b*x^6*l
n(-c*x^2+1)+1/6*a*b/c^3*ln(c*x^2-1)+1/24*b^2*x^6*ln(-c*x^2+1)^2+11/36/c^3*b
^2*ln(-c*x^2+1)-1/24/c^3*b^2*ln(-c*x^2+1)^2+1/24*b^2*x^6*ln(c*x^2+1)^2-1/12
/c^3*b^2*ln(c*x^2+1)+1/24/c^3*b^2*ln(c*x^2+1)^2+1/6*x^6*a^2-1/6*b^2/c^3*dil
og(1/2*c*x^2+1/2)-2/9*b^2/c^3*ln(c*x^2-1)+1/12*b^2/c*x^4*ln(c*x^2+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*x^6 + 1/6*(2*x^6*arctanh(c*x^2) + (x^4/c^2 + log(c^2*x^4 - 1)/c^4)*
c)*a*b + 1/432*(18*x^6*log(-c*x^2 + 1)^2 - 2*c^4*(2*(c^2*x^6 + 3*x^2)/c^6 -
 3*log(c*x^2 + 1)/c^7 + 3*log(c*x^2 - 1)/c^7) + 3*c^3*(x^4/c^4 + log(c^2*x^
4 - 1)/c^6) + 1296*c^3*integrate(1/9*x^7*log(c*x^2 + 1)/(c^4*x^4 - c^2), x)
 - 9*c^2*(2*x^2/c^4 - log(c*x^2 + 1)/c^5 + log(c*x^2 - 1)/c^5) - 6*c*((2*c^
2*x^6 + 3*c*x^4 + 6*x^2)/c^3 + 6*log(c*x^2 - 1)/c^4)*log(-c*x^2 + 1) + 648*
c*integrate(1/9*x^3*log(c*x^2 + 1)/(c^4*x^4 - c^2), x) + 6*(3*c^3*x^6*log(c
*x^2 + 1)^2 + (2*c^3*x^6 - 3*c^2*x^4 + 6*c*x^2 - 6*(c^3*x^6 + 1)*log(c*x^2
+ 1))*log(-c*x^2 + 1))/c^3 + (4*c^3*x^6 + 15*c^2*x^4 + 66*c*x^2 + 18*log(c*
x^2 - 1)^2 + 66*log(c*x^2 - 1))/c^3 - 18*log(9*c^4*x^4 - 9*c^2)/c^3 + 648*i
ntegrate(1/9*x*log(c*x^2 + 1)/(c^4*x^4 - c^2), x))*b^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^5*arctanh(c*x^2)^2 + 2*a*b*x^5*arctanh(c*x^2) + a^2*x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left(a + b \operatorname{atanh}\left(cx^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x**2))**2,x)
[Out] Integral(x**5*(a + b*atanh(c*x**2))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^2) + a)^2*x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left(a + b \operatorname{atanh}\left(c\, x^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atanh(c*x^2))^2,x)
[Out] int(x^5*(a + b*atanh(c*x^2))^2, x)
```

3.66 $\int x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx$

Optimal. Leaf size=91

$$\frac{abx^2}{2c}+\frac{b^2x^2\tanh^{-1}\left(cx^2\right)}{2c}-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{4c^2}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2+\frac{b^2\log\left(1-c^2x^4\right)}{4c^2}$$

```
[Out] 1/2*a*b*x^2/c+1/2*b^2*x^2*arctanh(c*x^2)/c-1/4*(a+b*arctanh(c*x^2))^2/c^2+1
/4*x^4*(a+b*arctanh(c*x^2))^2+1/4*b^2*ln(-c^2*x^4+1)/c^2
```

Rubi [A]
time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6039, 6037, 6127, 6021, 266, 6095}

$$-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{4c^2}+\frac{abx^2}{2c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2+\frac{b^2\log\left(1-c^2x^4\right)}{4c^2}+\frac{b^2x^2\tanh^{-1}\left(cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (a*b*x^2)/(2*c) + (b^2*x^2*ArcTanh[c*x^2])/(2*c) - (a + b*ArcTanh[c*x^2])^2
/(4*c^2) + (x^4*(a + b*ArcTanh[c*x^2])^2)/4 + (b^2*Log[1 - c^2*x^4])/(4*c^2
)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx &= \int \left(\frac{1}{4}x^3\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{1}{2}bx^3\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx\right.\right.\\
&= \frac{1}{4}\int x^3\left(2a-b\log\left(1-cx^2\right)\right)^2\,dx-\frac{1}{2}b\int x^3\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1\right.\\
&= \frac{1}{8}\text{Subst}\left(\int x(2a-b\log(1-cx))^2\,dx,x,x^2\right)-\frac{1}{4}b\text{Subst}\left(\int x(-2a+b\log\right.\\
&= \frac{1}{8}bx^4\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{1}{8}\text{Subst}\left(\int\left(\frac{(2a-b\log(1-c}{c}\right.\right.\\
&= \frac{1}{8}bx^4\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{\text{Subst}\left(\int(2a-b\log(1-cx))^2\,c\right.}{8c}\\
&= \frac{1}{8}bx^4\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{1}{8}b\text{Subst}\left(\int x(-2a+b\log(1-\right.\\
&= \frac{abx^2}{4c}-\frac{1}{16}bx^4\left(2a-b\log\left(1-cx^2\right)\right)-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8c^2}+\\
&= \frac{3abx^2}{4c}-\frac{3b^2x^2}{8c}+\frac{b^2\left(1-cx^2\right)^2}{32c^2}+\frac{b^2\left(1+cx^2\right)^2}{32c^2}-\frac{1}{16}bx^4\left(2a-b\log\left(1-cx\right.\right.\\
&= \frac{3abx^2}{4c}-\frac{b^2x^4}{16}+\frac{b^2\left(1-cx^2\right)^2}{32c^2}+\frac{b^2\left(1+cx^2\right)^2}{32c^2}-\frac{b^2\log\left(1-cx^2\right)}{16c^2}+\frac{3b^2(1-}{}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 106, normalized size = 1.16

$$\frac{2abcx^2+a^2c^2x^4+2bcx^2(b+acx^2)\tanh^{-1}(cx^2)+b^2(-1+c^2x^4)\tanh^{-1}(cx^2)^2+b(a+b)\log(1-cx^2)-ab\log(1+cx^2)+b^2\log(1+cx^2)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (2*a*b*c*x^2 + a^2*c^2*x^4 + 2*b*c*x^2*(b + a*c*x^2)*ArcTanh[c*x^2] + b^2*(
-1 + c^2*x^4)*ArcTanh[c*x^2]^2 + b*(a + b)*Log[1 - c*x^2] - a*b*Log[1 + c*x
^2] + b^2*Log[1 + c*x^2])/(4*c^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(81) = 162$.
time = 0.17, size = 287, normalized size = 3.15

method	result
risch	$\frac{b^2(c^2x^4-1)\ln(cx^2+1)^2}{16c^2}+\frac{b(-2x^4b\ln(-cx^2+1)ac^2+4a^2c^2x^4+4abcx^2+2b\ln(-cx^2+1)a+b^2)\ln(cx^2+1)}{16c^2a}+\frac{b^2x^4\ln(-cx^2+1)^2}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*b^2*(c^2*x^4-1)/c^2*ln(c*x^2+1)^2+1/16*b*(-2*x^4*b*ln(-c*x^2+1)*a*c^2+
4*a^2*c^2*x^4+4*a*b*c*x^2+2*b*ln(-c*x^2+1)*a+b^2)/c^2/a*ln(c*x^2+1)+1/16*b^
2*x^4*ln(-c*x^2+1)^2-1/4*a*b*x^4*ln(-c*x^2+1)+1/4*a^2*x^4-1/4/c*b^2*x^2*ln(
-c*x^2+1)+1/2*a*b*x^2/c-1/16/c^2*b^2*ln(-c*x^2+1)^2+1/4/c^2*a*ln(-c*x^2+1)*
b+1/4/c^2*b^2*ln(-c*x^2+1)-1/4/c^2*a*ln(-c*x^2-1)*b+1/4/c^2*ln(-c*x^2-1)*b^
2-1/16/c^2/a*ln(-c*x^2-1)*b^3+1/4*b^2/c^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(81) = 162$.
time = 0.26, size = 186, normalized size = 2.04

$$\frac{1}{4}b^2x^4\operatorname{artanh}(cx^2)^2+\frac{1}{4}a^2x^4+\frac{1}{4}\left(2x^4\operatorname{artanh}(cx^2)+c\left(\frac{2x^2}{c^2}-\frac{\log(cx^2+1)}{c^3}+\frac{\log(cx^2-1)}{c^3}\right)\right)ab+\frac{1}{16}\left(4c\left(\frac{2x^2}{c^2}-\frac{\log(cx^2+1)}{c^3}+\frac{\log(cx^2-1)}{c^3}\right)\operatorname{artanh}(cx^2)-\frac{2(\log(cx^2-1)-2)\log(cx^2+1)-\log(cx^2+1)^2-\log(cx^2-1)^2-4\log(cx^2-1)}{c^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctanh(c*x^2)^2 + 1/4*a^2*x^4 + 1/4*(2*x^4*arctanh(c*x^2) + c*
(2*x^2/c^2 - log(c*x^2 + 1)/c^3 + log(c*x^2 - 1)/c^3))*a*b + 1/16*(4*c*(2*x
^2/c^2 - log(c*x^2 + 1)/c^3 + log(c*x^2 - 1)/c^3)*arctanh(c*x^2) - (2*(log(
c*x^2 - 1) - 2)*log(c*x^2 + 1) - log(c*x^2 + 1)^2 - log(c*x^2 - 1)^2 - 4*lo
g(c*x^2 - 1))/c^2)*b^2
```

Fricas [A]
time = 0.34, size = 138, normalized size = 1.52

$$\frac{4\,a^2c^2x^4+8\,abcx^2+\left(b^2c^2x^4-b^2\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)^2-4\left(ab-b^2\right)\log\left(cx^2+1\right)+4\left(ab+b^2\right)\log\left(cx^2-1\right)+4\left(abc^2x^4+b^2cx^2\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{16\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*a^2*c^2*x^4 + 8*a*b*c*x^2 + (b^2*c^2*x^4 - b^2)*log(-(c*x^2 + 1)/(c
*x^2 - 1))^2 - 4*(a*b - b^2)*log(c*x^2 + 1) + 4*(a*b + b^2)*log(c*x^2 - 1)
+ 4*(a*b*c^2*x^4 + b^2*c*x^2)*log(-(c*x^2 + 1)/(c*x^2 - 1)))/c^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(78) = 156.
time = 5.29, size = 163, normalized size = 1.79

$$\begin{cases}\frac{a^2x^4}{4}+\frac{abx^4\operatorname{atanh}\left(cx^2\right)}{2}+\frac{abx^2}{2c}-\frac{ab\operatorname{atanh}\left(cx^2\right)}{2c^2}+\frac{b^2x^4\operatorname{atanh}^2\left(cx^2\right)}{4}+\frac{b^2x^2\operatorname{atanh}\left(cx^2\right)}{2c}+\frac{b^2\log\left(x-\sqrt{-\frac{1}{c}}\right)}{2c^2}+\frac{b^2\log\left(x+\sqrt{-\frac{1}{c}}\right)}{2c^2}-\frac{b^2\operatorname{atanh}^2\left(cx^2\right)}{4c^2}-\frac{b^2\operatorname{atanh}\left(cx^2\right)}{2c^2} & \text{for}\ c\neq 0\\ \frac{a^2x^4}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Piecewise((a**2*x**4/4 + a*b*x**4*atanh(c*x**2)/2 + a*b*x**2/(2*c) - a*b*at
anh(c*x**2)/(2*c**2) + b**2*x**4*atanh(c*x**2)**2/4 + b**2*x**2*atanh(c*x**
2)/(2*c) + b**2*log(x - sqrt(-1/c))/(2*c**2) + b**2*log(x + sqrt(-1/c))/(2*
c**2) - b**2*atanh(c*x**2)**2/(4*c**2) - b**2*atanh(c*x**2)/(2*c**2), Ne(c,
 0)), (a**2*x**4/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(81) = 162.
time = 0.45, size = 361, normalized size = 3.97

$$\frac{1}{4}\left(\frac{\left(cx^2+1\right)b^2\log\left(-\frac{cx^2+1}{cx^2-1}\right)^2}{\left(\frac{\left(cx^2+1\right)^2c^3}{\left(cx^2-1\right)^2}-\frac{2\left(cx^2+1\right)c^3}{cx^2-1}+c^3\right)\left(cx^2-1\right)}+\frac{2\left(\frac{2\left(cx^2+1\right)ab}{cx^2-1}+\frac{\left(cx^2+1\right)b^2}{cx^2-1}-b^2\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{\frac{\left(cx^2+1\right)^2c^3}{\left(cx^2-1\right)^2}-\frac{2\left(cx^2+1\right)c^3}{cx^2-1}+c^3}+\frac{4\left(\frac{\left(cx^2+1\right)a^2}{cx^2-1}+\frac{\left(cx^2+1\right)ab}{cx^2-1}-ab\right)}{\frac{\left(cx^2+1\right)^2c^3}{\left(cx^2-1\right)^2}-\frac{2\left(cx^2+1\right)c^3}{cx^2-1}+c^3}-\frac{2\,b^2\log\left(-\frac{cx^2+1}{cx^2-1}+1\right)}{c^3}+\frac{2\,b^2\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{c^3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] 1/4*((c*x^2 + 1)*b^2*log(-(c*x^2 + 1)/(c*x^2 - 1))^2/(((c*x^2 + 1)^2*c^3/(c
*x^2 - 1)^2 - 2*(c*x^2 + 1)*c^3/(c*x^2 - 1) + c^3)*(c*x^2 - 1)) + 2*(2*(c*x
^2 + 1)*a*b/(c*x^2 - 1) + (c*x^2 + 1)*b^2/(c*x^2 - 1) - b^2)*log(-(c*x^2 +
1)/(c*x^2 - 1))/((c*x^2 + 1)^2*c^3/(c*x^2 - 1)^2 - 2*(c*x^2 + 1)*c^3/(c*x^2
 - 1) + c^3) + 4*((c*x^2 + 1)*a^2/(c*x^2 - 1) + (c*x^2 + 1)*a*b/(c*x^2 - 1)
 - a*b)/((c*x^2 + 1)^2*c^3/(c*x^2 - 1)^2 - 2*(c*x^2 + 1)*c^3/(c*x^2 - 1) +
```

```
c^3) - 2*b^2*log(-(c*x^2 + 1)/(c*x^2 - 1) + 1)/c^3 + 2*b^2*log(-(c*x^2 + 1)
/(c*x^2 - 1))/c^3)*c
```

Mupad [B]
time = 1.26, size = 275, normalized size = 3.02

$$\frac{a^2\,x^4}{4}+\frac{b^2\,\ln\left(c\,x^2-1\right)}{4\,c^2}+\frac{b^2\,\ln\left(c\,x^2+1\right)}{4\,c^2}-\frac{b^2\ln{\left(c\,x^2+1\right)}^2}{16\,c^2}-\frac{b^2\ln{\left(1-c\,x^2\right)}^2}{16\,c^2}+\frac{b^2\,x^4\ln{\left(c\,x^2+1\right)}^2}{16}+\frac{b^2\,x^4\ln{\left(1-c\,x^2\right)}^2}{16}+\frac{b^2\,x^2\,\ln\left(c\,x^2+1\right)}{4\,c}-\frac{b^2\,x^2\,\ln\left(1-c\,x^2\right)}{4\,c}+\frac{a\,b\,\ln\left(c\,x^2-1\right)}{4\,c^2}-\frac{a\,b\,\ln\left(c\,x^2+1\right)}{4\,c^2}+\frac{a\,b\,x^4\,\ln\left(c\,x^2+1\right)}{4}-\frac{a\,b\,x^4\,\ln\left(1-c\,x^2\right)}{4}+\frac{b^2\,\ln\left(c\,x^2+1\right)\,\ln\left(1-c\,x^2\right)}{8\,c^2}+\frac{a\,b\,x^2}{2\,c}-\frac{b^2\,x^4\,\ln\left(c\,x^2+1\right)\,\ln\left(1-c\,x^2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x^2))^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*log(c*x^2 - 1))/(4*c^2) + (b^2*log(c*x^2 + 1))/(4*c^2) -
 (b^2*log(c*x^2 + 1)^2)/(16*c^2) - (b^2*log(1 - c*x^2)^2)/(16*c^2) + (b^2*x
^4*log(c*x^2 + 1)^2)/16 + (b^2*x^4*log(1 - c*x^2)^2)/16 + (b^2*x^2*log(c*x^
2 + 1))/(4*c) - (b^2*x^2*log(1 - c*x^2))/(4*c) + (a*b*log(c*x^2 - 1))/(4*c^
2) - (a*b*log(c*x^2 + 1))/(4*c^2) + (a*b*x^4*log(c*x^2 + 1))/4 - (a*b*x^4*l
og(1 - c*x^2))/4 + (b^2*log(c*x^2 + 1)*log(1 - c*x^2))/(8*c^2) + (a*b*x^2)/
(2*c) - (b^2*x^4*log(c*x^2 + 1)*log(1 - c*x^2))/8
```

3.67 $\int x\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx$

Optimal. Leaf size=94

$$\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2-\frac{b\left(a+b\tanh^{-1}\left(cx^2\right)\right)\log\left(\frac{2}{1-cx^2}\right)}{c}-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)}{2c}$$

```
[Out] 1/2*(a+b*arctanh(c*x^2))^2/c+1/2*x^2*(a+b*arctanh(c*x^2))^2-b*(a+b*arctanh(
c*x^2))*ln(2/(-c*x^2+1))/c-1/2*b^2*polylog(2,1-2/(-c*x^2+1))/c
```

Rubi [A]
time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.429$, Rules used = {6039, 6021, 6131, 6055, 2449, 2352}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2+\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{2c}-\frac{b\log\left(\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{c}-\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx^2}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (a + b*ArcTanh[c*x^2])^2/(2*c) + (x^2*(a + b*ArcTanh[c*x^2])^2)/2 - (b*(a +
 b*ArcTanh[c*x^2])*Log[2/(1 - c*x^2)])/c - (b^2*PolyLog[2, 1 - 2/(1 - c*x^2
)])/(2*c)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
```

```
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx = \int \left(\frac{1}{4}x\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{1}{2}bx\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\right.$$
$$=\frac{1}{4}\int x\left(2a-b\log\left(1-cx^2\right)\right)^2\,dx-\frac{1}{2}b\int x\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+c\right.$$
$$=\frac{1}{8}\text{Subst}\left(\int(2a-b\log(1-cx))^2\,dx,x,x^2\right)-\frac{1}{4}b\text{Subst}\left(\int(-2a+b\log(1-\right.\right.$$
$$=\frac{1}{4}bx^2\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)-\frac{\text{Subst}\left(\int(2a-b\log(x))^2\,dx,x,1\right.}{8c}$$
$$=-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8c}+\frac{1}{4}bx^2\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right.$$
$$=\frac{1}{2}abx^2+\frac{b^2x^2}{4}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8c}-\frac{b^2\left(1+cx^2\right)\log\left(1+cx^2\right)}{4c}$$
$$=\frac{b^2x^2}{2}+\frac{b^2\left(1-cx^2\right)\log\left(1-cx^2\right)}{4c}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8c}+\frac{b(2a-}{}$$
$$=\frac{b^2x^2}{4}+\frac{b^2\left(1-cx^2\right)\log\left(1-cx^2\right)}{4c}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8c}+\frac{b(2a-}{}$$
$$=-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8c}+\frac{b\left(2a-b\log\left(1-cx^2\right)\right)\log\left(\frac{1}{2}\left(1+cx^2\right)\right)}{4c}$$

Mathematica [A]

time = 0.11, size = 99, normalized size = 1.05

$$\frac{b^2(-1+cx^2)\tanh^{-1}(cx^2)^2+2b\tanh^{-1}(cx^2)\left(acx^2-b\log\left(1+e^{-2\tanh^{-1}(cx^2)}\right)\right)+a(acx^2+b\log(1-c^2x^4))+b^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx^2)}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (b^2*(-1 + c*x^2)*ArcTanh[c*x^2]^2 + 2*b*ArcTanh[c*x^2]*(a*c*x^2 - b*Log[1
+ E^(-2*ArcTanh[c*x^2])]) + a*(a*c*x^2 + b*Log[1 - c^2*x^4]) + b^2*PolyLog[
2, -E^(-2*ArcTanh[c*x^2])])/(2*c)
```

Maple [A]
time = 0.17, size = 137, normalized size = 1.46

method	result
derivativedivides	$\frac{c\,x^2a^2+\operatorname{arctanh}(c\,x^2)^2b^2c\,x^2+b^2\operatorname{arctanh}(c\,x^2)^2-2\operatorname{arctanh}(c\,x^2)\ln\left(1+\frac{(c\,x^2+1)^2}{-c^2x^4+1}\right)b^2-\operatorname{polylog}\left(2,-\frac{(c\,x^2+1)^2}{-c^2x^4+1}\right)b^2+2}{2c}$
default	$\frac{c\,x^2a^2+\operatorname{arctanh}(c\,x^2)^2b^2c\,x^2+b^2\operatorname{arctanh}(c\,x^2)^2-2\operatorname{arctanh}(c\,x^2)\ln\left(1+\frac{(c\,x^2+1)^2}{-c^2x^4+1}\right)b^2-\operatorname{polylog}\left(2,-\frac{(c\,x^2+1)^2}{-c^2x^4+1}\right)b^2+2}{2c}$
risch	$-\frac{a^2}{2c}-\frac{b^2}{2c}+\frac{a^2x^2}{2}+\frac{ba\ln(c\,x^2+1)x^2}{2}+\frac{ba\ln(c\,x^2+1)}{2c}-\frac{b^2\ln(-c\,x^2+1)\ln(c\,x^2+1)x^2}{4}-\frac{b^2\ln(-c\,x^2+1)\ln(c}{4c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c*(c*x^2*a^2+arctanh(c*x^2)^2*b^2*c*x^2+b^2*arctanh(c*x^2)^2-2*arctanh(
c*x^2)*ln(1+(c*x^2+1)^2/(-c^2*x^4+1))*b^2-polylog(2,-(c*x^2+1)^2/(-c^2*x^4+
1))*b^2+2*a*b*c*x^2*arctanh(c*x^2)+a*b*ln(-c^2*x^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/8*(x^2*log(-c*x^2 + 1)^2 - c^2*(2*x^2/c^2 - log(c*x^2 + 1)/
c^3 + log(c*x^2 - 1)/c^3) - 2*c*(x^2/c + log(c*x^2 - 1)/c^2)*log(-c*x^2 + 1
) + 12*c*integrate(x^3*log(c*x^2 + 1)/(c^2*x^4 - 1), x) + (c*x^2*log(c*x^2
+ 1)^2 + 2*(c*x^2 - (c*x^2 + 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/c + (2*c*x
^2 + log(c*x^2 - 1)^2 + 2*log(c*x^2 - 1))/c - log(c^2*x^4 - 1)/c + 4*integr
ate(x*log(c*x^2 + 1)/(c^2*x^4 - 1), x))*b^2 + 1/2*(2*c*x^2*arctanh(c*x^2) +
 log(-c^2*x^4 + 1))*a*b/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
[Out] integral(b^2*x*arctanh(c*x^2)^2 + 2*a*b*x*arctanh(c*x^2) + a^2*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a + b\operatorname{atanh}\left(cx^{2}\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**2))**2,x)
[Out] Integral(x*(a + b*atanh(c*x**2))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^2) + a)^2*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^2))^2,x)
[Out] int(x*(a + b*atanh(c*x^2))^2, x)
```

3.68 $\int \frac{(a+b\tanh^{-1}(cx^2))^2}{x}\,dx$

Optimal. Leaf size=137

$$(a+b\tanh^{-1}(cx^2))^2\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)-\frac{1}{2}b(a+b\tanh^{-1}(cx^2))\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)+\frac{1}{2}b(a+b$$

```
[Out] -(a+b*arctanh(c*x^2))^2*arctanh(-1+2/(-c*x^2+1))-1/2*b*(a+b*arctanh(c*x^2))
*polylog(2,1-2/(-c*x^2+1))+1/2*b*(a+b*arctanh(c*x^2))*polylog(2,-1+2/(-c*x^
2+1))+1/4*b^2*polylog(3,1-2/(-c*x^2+1))-1/4*b^2*polylog(3,-1+2/(-c*x^2+1))
```

Rubi [A]
time = 0.23, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6035, 6033, 6199, 6095, 6205, 6745}

$$-\frac{1}{2}b\text{Li}_2\left(1-\frac{2}{1-cx^2}\right)(a+b\tanh^{-1}(cx^2))+\frac{1}{2}b\text{Li}_2\left(\frac{2}{1-cx^2}-1\right)(a+b\tanh^{-1}(cx^2))+\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)(a+b\tanh^{-1}(cx^2))^2+\frac{1}{4}b^2\text{Li}_3\left(1-\frac{2}{1-cx^2}\right)-\frac{1}{4}b^2\text{Li}_3\left(\frac{2}{1-cx^2}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/x,x]
```

```
[Out] (a + b*ArcTanh[c*x^2])^2*ArcTanh[1 - 2/(1 - c*x^2)] - (b*(a + b*ArcTanh[c*x
^2])*PolyLog[2, 1 - 2/(1 - c*x^2)])/2 + (b*(a + b*ArcTanh[c*x^2])*PolyLog[2
, -1 + 2/(1 - c*x^2)])/2 + (b^2*PolyLog[3, 1 - 2/(1 - c*x^2)])/4 - (b^2*Pol
yLog[3, -1 + 2/(1 - c*x^2)])/4
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx,x,x^2\right)$$
$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)-(2bc)\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(\right.}{}\right.$$
$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)+(bc)\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(c\right.}{1-c}\right.$$
$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)-\frac{1}{2}b\left(a+b\tanh^{-1}\left(cx^2\right)\right)\text{Li}_2\left(1\right.$$
$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)-\frac{1}{2}b\left(a+b\tanh^{-1}\left(cx^2\right)\right)\text{Li}_2\left(1\right.$$

Mathematica [A]
time = 0.05, size = 183, normalized size = 1.34

$$\frac{1}{2}\left(2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)-4bc\left(\frac{1}{2}\left(\frac{\left(-a-b\tanh^{-1}\left(cx^2\right)\right)\text{PolyLog}\left(2,\frac{-1-cx^2}{-1+cx^2}\right)}{2c}+\frac{b\text{PolyLog}\left(3,\frac{-1-cx^2}{-1+cx^2}\right)}{4c}\right)+\frac{1}{2}\left(-\frac{\left(-a-b\tanh^{-1}\left(cx^2\right)\right)\text{PolyLog}\left(2,\frac{1+cx^2}{-1+cx^2}\right)}{2c}-\frac{b\text{PolyLog}\left(3,\frac{1+cx^2}{-1+cx^2}\right)}{4c}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x,x]
```

```
[Out] (2*(a + b*ArcTanh[c*x^2])^2*ArcTanh[1 - 2/(1 - c*x^2)] - 4*b*c*((((-a - b*A
rcTanh[c*x^2])*PolyLog[2, (-1 - c*x^2)/(-1 + c*x^2)])/(2*c) + (b*PolyLog[3,
 (-1 - c*x^2)/(-1 + c*x^2)])/(4*c))/2 + (-1/2*((-a - b*ArcTanh[c*x^2])*Poly
Log[2, (1 + c*x^2)/(-1 + c*x^2)])/c - (b*PolyLog[3, (1 + c*x^2)/(-1 + c*x^2
)])/(4*c))/2))/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\,x^{2}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + integrate(1/4*b^2*(log(c*x^2 + 1) - log(-c*x^2 + 1))^2/x + a*b
*(log(c*x^2 + 1) - log(-c*x^2 + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\operatorname{atanh}\left(cx^{2}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^2) + a)^2/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)}^2}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/x,x)
[Out] int((a + b*atanh(c*x^2))^2/x, x)
```

3.69 $\int \frac{(a+b\tanh^{-1}(cx^2))^2}{x^3} dx$

Optimal. Leaf size=87

$$\frac{1}{2}c\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{2x^2}+bc\left(a+b\tanh^{-1}\left(cx^2\right)\right)\log\left(2-\frac{2}{1+cx^2}\right)-\frac{1}{2}b^2c\text{PolyLog}($$

```
[Out] 1/2*c*(a+b*arctanh(c*x^2))^2-1/2*(a+b*arctanh(c*x^2))^2/x^2+b*c*(a+b*arctan
h(c*x^2))*ln(2-2/(c*x^2+1))-1/2*b^2*c*polylog(2,-1+2/(c*x^2+1))
```

Rubi [A]
time = 0.13, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6039, 6037, 6135, 6079, 2497}

$$\frac{1}{2}c\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{2x^2}+bc\log\left(2-\frac{2}{cx^2+1}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{2}b^2c\text{Li}_2\left(\frac{2}{cx^2+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/x^3,x]
```

```
[Out] (c*(a + b*ArcTanh[c*x^2])^2)/2 - (a + b*ArcTanh[c*x^2])^2/(2*x^2) + b*c*(a
+ b*ArcTanh[c*x^2])*Log[2 - 2/(1 + c*x^2)] - (b^2*c*PolyLog[2, -1 + 2/(1 +
c*x^2)])/2
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
```

```
fy[(m + 1)/n]]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^3}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x^3} - \frac{b\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{2x^3} + \frac{b^2\log^2}{4}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{x^3}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{x^3}\,dx$$

$$= \frac{1}{8}\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^2}{x^2}\,dx, x, x^2\right) - \frac{1}{4}b\text{Subst}\left(\int \frac{(-2a+b\log(1-}{x^2}\right.$$

$$= -\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8x^2} - \frac{b\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{4x^2} - \frac{b^2}{}$$

$$= abc\log(x) - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8x^2} - \frac{b\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+\right.}{4x^2}$$

$$= abc\log(x) - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8x^2} - \frac{b\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+\right.}{4x^2}$$

$$= 2abc\log(x) - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8x^2} - \frac{1}{4}bc\left(2a-b\log\left(1-cx^2\right)\right)\log$$

$$= 2abc\log(x) - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8x^2} - \frac{1}{4}bc\left(2a-b\log\left(1-cx^2\right)\right)\log$$

$$= 2abc\log(x) - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8x^2} - \frac{1}{4}bc\left(2a-b\log\left(1-cx^2\right)\right)\log$$

Mathematica [A]
time = 0.10, size = 119, normalized size = 1.37

$$-\frac{a^2}{2x^2}+abc\left(-\frac{\tanh^{-1}(cx^2)}{cx^2}+\log(cx^2)-\frac{1}{2}\log(1-c^2x^4)\right)+\frac{1}{2}b^2c\left(\tanh^{-1}(cx^2)\left(\tanh^{-1}(cx^2)-\frac{\tanh^{-1}(cx^2)}{cx^2}+2\log\left(1-e^{-2\tanh^{-1}(cx^2)}\right)\right)-\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx^2)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 + a*b*c*(-(ArcTanh[c*x^2]/(c*x^2)) + Log[c*x^2] - Log[1 - c^2*
x^4]/2) + (b^2*c*(ArcTanh[c*x^2]*(ArcTanh[c*x^2] - ArcTanh[c*x^2]/(c*x^2) +
 2*Log[1 - E^(-2*ArcTanh[c*x^2])]) - PolyLog[2, E^(-2*ArcTanh[c*x^2])]))/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arctanh}\,(c\,x^2))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x^3,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^4 - 1) - log(x^4)) + 2*arctanh(c*x^2)/x^2)*a*b - 1/8*b^2
*(log(-c*x^2 + 1)^2/x^2 + 2*integrate(-((c*x^2 - 1)*log(c*x^2 + 1)^2 + 2*(c
*x^2 - (c*x^2 - 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^5 - x^3), x)) - 1/
2*a^2/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,\mathrm{atanh}\left(cx^2\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x**3,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\mathrm{atanh}(c\,x^2)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/x^3,x)
```

```
[Out] int((a + b*atanh(c*x^2))^2/x^3, x)
```

3.70 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^5}\,dx$

Optimal. Leaf size=88

$$-\frac{bc\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{2x^2}+\frac{1}{4}c^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{4x^4}+b^2c^2\log(x)-\frac{1}{4}b^2c^2\log\left(1-c^2x\right.$$

```
[Out] -1/2*b*c*(a+b*arctanh(c*x^2))/x^2+1/4*c^2*(a+b*arctanh(c*x^2))^2-1/4*(a+b*a
rctanh(c*x^2))^2/x^4+b^2*c^2*ln(x)-1/4*b^2*c^2*ln(-c^2*x^4+1)
```

Rubi [A]
time = 0.13, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6039, 6037, 6129, 272, 36, 29, 31, 6095}

$$\frac{1}{4}c^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2-\frac{bc\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{2x^2}-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{4x^4}-\frac{1}{4}b^2c^2\log\left(1-c^2x^4\right)+b^2c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/x^5,x]
```

```
[Out] -1/2*(b*c*(a + b*ArcTanh[c*x^2]))/x^2 + (c^2*(a + b*ArcTanh[c*x^2])^2)/4 -
(a + b*ArcTanh[c*x^2])^2/(4*x^4) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 - c^2*x^
4])/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^5}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x^5} - \frac{b(-2a+b\log\left(1-cx^2\right))\log\left(1+cx^2\right)}{2x^5} + \frac{b^2\log\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{x^5}\,dx - \frac{1}{2}b\int \frac{(-2a+b\log\left(1-cx^2\right))\log\left(1+cx^2\right)}{x^5}$$

$$= \frac{1}{8}\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^2}{x^3}\,dx, x, x^2\right) - \frac{1}{4}b\text{Subst}\left(\int \frac{(-2a+b\log(1-\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{16x^4} - \frac{b(2a-b\log\left(1-cx^2\right))\log\left(1+cx^2\right)}{8x^4} - \frac{b^2\log^2(1-}{16x^4}$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{16x^4} - \frac{b(2a-b\log\left(1-cx^2\right))\log\left(1+cx^2\right)}{8x^4} - \frac{b^2\log^2(1-}{16x^4}$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{16x^4} - \frac{b(2a-b\log\left(1-cx^2\right))\log\left(1+cx^2\right)}{8x^4} - \frac{b^2\log^2(1-}{16x^4}$$

$$= -\frac{1}{2}abc^2\log(x) - \frac{bc(2a-b\log\left(1-cx^2\right))}{8x^2} - \frac{bc\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)}{8x^2}$$

$$= \frac{1}{4}b^2c^2\log(x) - \frac{bc(2a-b\log\left(1-cx^2\right))}{8x^2} - \frac{bc\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)}{8x^2} -$$

$$= \frac{1}{2}b^2c^2\log(x) - \frac{1}{8}b^2c^2\log\left(1-cx^2\right) - \frac{bc(2a-b\log\left(1-cx^2\right))}{8x^2} - \frac{bc\left(1-cx^2\right)(}{}$$

Mathematica [A]
time = 0.05, size = 111, normalized size = 1.26

$$\frac{1}{4}\left(-\frac{a^2}{x^4} - \frac{2abc}{x^2} - \frac{2b(a+bcx^2)\tanh^{-1}\left(cx^2\right)}{x^4} + \frac{b^2(-1+c^2x^4)\tanh^{-1}\left(cx^2\right)^2}{x^4} + 4b^2c^2\log(x) - b(a+b)c^2\log\left(1-cx^2\right) + (a-b)bc^2\log\left(1+cx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x^5,x]
```

```
[Out] (-(a^2/x^4) - (2*a*b*c)/x^2 - (2*b*(a + b*c*x^2)*ArcTanh[c*x^2])/x^4 + (b^2
*(-1 + c^2*x^4)*ArcTanh[c*x^2]^2)/x^4 + 4*b^2*c^2*Log[x] - b*(a + b)*c^2*Lo
g[1 - c*x^2] + (a - b)*b*c^2*Log[1 + c*x^2])/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(80) = 160$.
time = 0.15, size = 257, normalized size = 2.92

method	result

risch	$\frac{b^2(c^2x^4-1)\ln(cx^2+1)^2}{16x^4} - \frac{b(bc^2\ln(-cx^2+1)x^4+2bcx^2-b\ln(-cx^2+1)+2a)\ln(cx^2+1)}{8x^4} + \frac{b^2c^2x^4\ln(-cx^2+1)^2+16b^2c^2\ln(x)x^4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*b^2*(c^2*x^4-1)/x^4*ln(c*x^2+1)^2-1/8*b*(b*c^2*ln(-c*x^2+1)*x^4+2*b*c*
x^2-b*ln(-c*x^2+1)+2*a)/x^4*ln(c*x^2+1)+1/16*(b^2*c^2*x^4*ln(-c*x^2+1)^2+16
*b^2*c^2*ln(x)*x^4-4*b*c^2*ln(c*x^2-1)*x^4*a-4*b^2*c^2*ln(c*x^2-1)*x^4+4*b*
c^2*ln(c*x^2+1)*x^4*a-4*b^2*c^2*ln(c*x^2+1)*x^4+4*b^2*c*x^2*ln(-c*x^2+1)-8*
a*b*c*x^2-b^2*ln(-c*x^2+1)^2+4*b*ln(-c*x^2+1)*a-4*a^2)/x^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(80) = 160$.
time = 0.26, size = 175, normalized size = 1.99

$$\frac{1}{4}\left(\left(c\log\left(cx^2+1\right)-c\log\left(cx^2-1\right)-\frac{2}{x^2}\right)c-\frac{2\,\operatorname{artanh}\left(cx^2\right)}{x^4}\right)ab+\frac{1}{16}\left(\left(2\left(\log\left(cx^2-1\right)-2\right)\log\left(cx^2+1\right)-\log\left(cx^2+1\right)^2-\log\left(cx^2-1\right)^2-4\log\left(cx^2-1\right)+16\log\left(x\right)\right)c^2+4\left(c\log\left(cx^2+1\right)-c\log\left(cx^2-1\right)-\frac{2}{x^2}\right)c\,\operatorname{artanh}\left(cx^2\right)\right)b^2-\frac{b^2\operatorname{artanh}\left(cx^2\right)^2}{4\,x^4}-\frac{a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x^2 + 1) - c*log(c*x^2 - 1) - 2/x^2)*c - 2*arctanh(c*x^2)/x^4
)*a*b + 1/16*((2*(log(c*x^2 - 1) - 2)*log(c*x^2 + 1) - log(c*x^2 + 1)^2 - l
og(c*x^2 - 1)^2 - 4*log(c*x^2 - 1) + 16*log(x))*c^2 + 4*(c*log(c*x^2 + 1) -
 c*log(c*x^2 - 1) - 2/x^2)*c*arctanh(c*x^2))*b^2 - 1/4*b^2*arctanh(c*x^2)^2
/x^4 - 1/4*a^2/x^4
```

Fricas [A]
time = 0.35, size = 151, normalized size = 1.72

$$\frac{16\,b^2c^2x^4\log\left(x\right)+4\,(ab-b^2)c^2x^4\log\left(cx^2+1\right)-4\,(ab+b^2)c^2x^4\log\left(cx^2-1\right)-8\,abcx^2+(b^2c^2x^4-b^2)\log\left(-\frac{cx^2+1}{cx^2-1}\right)^2-4\,a^2-4\,(b^2cx^2+ab)\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/16*(16*b^2*c^2*x^4*log(x) + 4*(a*b - b^2)*c^2*x^4*log(c*x^2 + 1) - 4*(a*b
 + b^2)*c^2*x^4*log(c*x^2 - 1) - 8*a*b*c*x^2 + (b^2*c^2*x^4 - b^2)*log(-(c*
x^2 + 1)/(c*x^2 - 1))^2 - 4*a^2 - 4*(b^2*c*x^2 + a*b)*log(-(c*x^2 + 1)/(c*x
^2 - 1)))/x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(80) = 160$.
time = 7.11, size = 175, normalized size = 1.99

$$\begin{cases} -\frac{a^2}{4x^4}+\frac{abc^2\operatorname{atanh}\left(cx^2\right)}{2}-\frac{abc}{2x^2}-\frac{ab\operatorname{atanh}\left(cx^2\right)}{2x^4}+b^2c^2\log\left(x\right)-\frac{b^2c^2\log\left(x-\sqrt{-\frac{1}{c}}\right)}{2}-\frac{b^2c^2\log\left(x+\sqrt{-\frac{1}{c}}\right)}{2}+\frac{b^2c^2\operatorname{atanh}^2\left(cx^2\right)}{4}+\frac{b^2c^2\operatorname{atanh}\left(cx^2\right)}{2}-\frac{b^2c\operatorname{atanh}\left(cx^2\right)}{2x^2}-\frac{b^2\operatorname{atanh}^2\left(cx^2\right)}{4x^4} & \text{for}\: c \neq 0 \\ -\frac{a^2}{4x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x**5,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) + a*b*c**2*atanh(c*x**2)/2 - a*b*c/(2*x**2) - a*b
*atanh(c*x**2)/(2*x**4) + b**2*c**2*log(x) - b**2*c**2*log(x - sqrt(-1/c))/
2 - b**2*c**2*log(x + sqrt(-1/c))/2 + b**2*c**2*atanh(c*x**2)**2/4 + b**2*c
**2*atanh(c*x**2)/2 - b**2*c*atanh(c*x**2)/(2*x**2) - b**2*atanh(c*x**2)**2
/(4*x**4), Ne(c, 0)), (-a**2/(4*x**4), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/x^5, x)
```

Mupad [B]
time = 1.50, size = 278, normalized size = 3.16

$$\frac{b^2\,c^2\,\ln\left(c\,x^2+1\right)^2}{16}-\frac{b^2\,c^2\,\ln\left(c\,x^2-1\right)}{4}-\frac{b^2\,c^2\,\ln\left(c\,x^2+1\right)}{4}-\frac{a^2}{4\,x^4}+\frac{b^2\,c^2\,\ln\left(1-c\,x^2\right)^2}{16}-\frac{b^2\,\ln\left(c\,x^2+1\right)^2}{16\,x^4}-\frac{b^2\,\ln\left(1-c\,x^2\right)^2}{16\,x^4}+b^2\,c^2\,\ln\left(x\right)-\frac{a\,b\,c^2\,\ln\left(c\,x^2-1\right)}{4}+\frac{a\,b\,c^2\,\ln\left(c\,x^2+1\right)}{4}-\frac{a\,b\,c}{2\,x^2}-\frac{a\,b\,\ln\left(c\,x^2+1\right)}{4\,x^4}+\frac{a\,b\,\ln\left(1-c\,x^2\right)}{4\,x^4}-\frac{b^2\,c^2\,\ln\left(c\,x^2+1\right)\,\ln\left(1-c\,x^2\right)}{8}-\frac{b^2\,c\,\ln\left(c\,x^2+1\right)}{4\,x^2}+\frac{b^2\,c\,\ln\left(1-c\,x^2\right)}{4\,x^2}+\frac{b^2\,\ln\left(c\,x^2+1\right)\,\ln\left(1-c\,x^2\right)}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/x^5,x)
```

```
[Out] (b^2*c^2*log(c*x^2 + 1)^2)/16 - (b^2*c^2*log(c*x^2 - 1))/4 - (b^2*c^2*log(c
*x^2 + 1))/4 - a^2/(4*x^4) + (b^2*c^2*log(1 - c*x^2)^2)/16 - (b^2*log(c*x^2
 + 1)^2)/(16*x^4) - (b^2*log(1 - c*x^2)^2)/(16*x^4) + b^2*c^2*log(x) - (a*b
*c^2*log(c*x^2 - 1))/4 + (a*b*c^2*log(c*x^2 + 1))/4 - (a*b*c)/(2*x^2) - (a*
b*log(c*x^2 + 1))/(4*x^4) + (a*b*log(1 - c*x^2))/(4*x^4) - (b^2*c^2*log(c*x
^2 + 1)*log(1 - c*x^2))/8 - (b^2*c*log(c*x^2 + 1))/(4*x^2) + (b^2*c*log(1 -
 c*x^2))/(4*x^2) + (b^2*log(c*x^2 + 1)*log(1 - c*x^2))/(8*x^4)
```

3.71 $\int x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx$

Optimal. Leaf size=1173

$$\frac{8b^2x}{15c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5+\frac{2ab\text{ArcTan}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{4b^2\text{ArcTan}\left(\sqrt{c}\,x\right)}{15c^{5/2}}+\frac{ib^2\text{ArcTan}\left(\sqrt{c}\,x\right)^2}{5c^{5/2}}-\frac{4b^2\tanh^{-1}\left(\sqrt{c}\,x\right)}{15c^{5/2}}-$$

```
[Out] 1/5*I*b^2*polylog(2,1-2/(1-I*x*c^(1/2)))/c^(5/2)+1/5*I*b^2*polylog(2,1-2/(1
+I*x*c^(1/2)))/c^(5/2)+1/5*I*b^2*arctan(x*c^(1/2))^2/c^(5/2)+2/5*a*b*arctan
(x*c^(1/2))/c^(5/2)+8/15*b^2*x/c^2-2/25*a*b*x^5+1/20*x^5*(2*a-b*ln(-c*x^2+1
))^2-1/15*b^2*x^3*ln(-c*x^2+1)/c-1/5*b^2*arctan(x*c^(1/2))*ln(-c*x^2+1)/c^(
5/2)+1/15*b*x^3*(2*a-b*ln(-c*x^2+1))/c-1/5*b*arctanh(x*c^(1/2))*(2*a-b*ln(-
c*x^2+1))/c^(5/2)+2/15*b^2*x^3*ln(c*x^2+1)/c+1/5*a*b*x^5*ln(c*x^2+1)+1/5*b^
2*arctan(x*c^(1/2))*ln(c*x^2+1)/c^(5/2)-1/5*b^2*arctanh(x*c^(1/2))*ln(c*x^2
+1)/c^(5/2)-1/10*b^2*x^5*ln(-c*x^2+1)*ln(c*x^2+1)+2/5*b^2*arctanh(x*c^(1/2)
)*ln(2/(1-x*c^(1/2)))/c^(5/2)-2/5*b^2*arctan(x*c^(1/2))*ln(2/(1-I*x*c^(1/2)
))/c^(5/2)+1/5*b^2*arctan(x*c^(1/2))*ln((1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2))
)/c^(5/2)+2/5*b^2*arctan(x*c^(1/2))*ln(2/(1+I*x*c^(1/2)))/c^(5/2)-2/5*b^2*a
rctanh(x*c^(1/2))*ln(2/(1+x*c^(1/2)))/c^(5/2)+1/5*b^2*arctanh(x*c^(1/2))*ln
(-2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(1/2)))/c^(5/2)+1/
5*b^2*arctanh(x*c^(1/2))*ln(2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))
/(1+x*c^(1/2)))/c^(5/2)+1/5*b^2*arctan(x*c^(1/2))*ln((1-I)*(1+x*c^(1/2))/(1
-I*x*c^(1/2)))/c^(5/2)-1/10*I*b^2*polylog(2,1-(1+I)*(1-x*c^(1/2))/(1-I*x*c^
(1/2)))/c^(5/2)-1/10*I*b^2*polylog(2,1+(-1+I)*(1+x*c^(1/2))/(1-I*x*c^(1/2))
)/c^(5/2)-4/15*b^2*arctan(x*c^(1/2))/c^(5/2)-4/15*b^2*arctanh(x*c^(1/2))/c^
(5/2)-1/5*b^2*arctanh(x*c^(1/2))^2/c^(5/2)+1/25*b^2*x^5*ln(-c*x^2+1)+1/25*b
*x^5*(2*a-b*ln(-c*x^2+1))+1/20*b^2*x^5*ln(c*x^2+1)^2+1/5*b^2*polylog(2,1-2/
(1-x*c^(1/2)))/c^(5/2)+1/5*b^2*polylog(2,1-2/(1+x*c^(1/2)))/c^(5/2)-1/10*b^
2*polylog(2,1+2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(1/2))
)/c^(5/2)-1/10*b^2*polylog(2,1-2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/
2))/(1+x*c^(1/2)))/c^(5/2)+2/15*a*b*x^3/c
```

Rubi [A]
time = 1.64, antiderivative size = 1173, normalized size of antiderivative = 1.00, number of steps used = 102, number of rules used = 26, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.625$, Rules used = {6041, 2507, 2526, 2498, 327, 212, 2505, 308, 2520, 12, 6131, 6055, 2449, 2352, 6874, 209, 30, 2637, 213, 6139, 6057, 2497, 5048, 4966, 5040, 4964}

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (8*b^2*x)/(15*c^2) + (2*a*b*x^3)/(15*c) - (2*a*b*x^5)/25 + (2*a*b*ArcTan[Sq
rt[c]*x])/(5*c^(5/2)) - (4*b^2*ArcTan[Sqrt[c]*x])/(15*c^(5/2)) + ((I/5)*b^2
*ArcTan[Sqrt[c]*x]^2)/c^(5/2) - (4*b^2*ArcTanh[Sqrt[c]*x])/(15*c^(5/2)) - (
b^2*ArcTanh[Sqrt[c]*x]^2)/(5*c^(5/2)) + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1
- Sqrt[c]*x)])/(5*c^(5/2)) - (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*
x)])/(5*c^(5/2)) + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1
- I*Sqrt[c]*x)])/(5*c^(5/2)) + (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c
]*x)])/(5*c^(5/2)) - (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)])/(5*c
^(5/2)) + (b^2*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[
-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/(5*c^(5/2)) + (b^2*ArcTanh[Sqrt[c]*x]*Log
[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/(5*c
^(5/2)) + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[
c]*x)])/(5*c^(5/2)) - (b^2*x^3*Log[1 - c*x^2])/(15*c) + (b^2*x^5*Log[1 - c*
x^2])/25 - (b^2*ArcTan[Sqrt[c]*x]*Log[1 - c*x^2])/(5*c^(5/2)) + (b*x^3*(2*a
 - b*Log[1 - c*x^2]))/(15*c) + (b*x^5*(2*a - b*Log[1 - c*x^2]))/25 - (b*Arc
Tanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/(5*c^(5/2)) + (x^5*(2*a - b*Log[1
 - c*x^2])^2)/20 + (2*b^2*x^3*Log[1 + c*x^2])/(15*c) + (a*b*x^5*Log[1 + c*x
^2])/5 + (b^2*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/(5*c^(5/2)) - (b^2*ArcTanh[
Sqrt[c]*x]*Log[1 + c*x^2])/(5*c^(5/2)) - (b^2*x^5*Log[1 - c*x^2]*Log[1 + c*
x^2])/10 + (b^2*x^5*Log[1 + c*x^2]^2)/20 + (b^2*PolyLog[2, 1 - 2/(1 - Sqrt[
c]*x)])/(5*c^(5/2)) + ((I/5)*b^2*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)])/c^(5/
2) - ((I/10)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)
])/c^(5/2) + ((I/5)*b^2*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)])/c^(5/2) + (b^2
*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/(5*c^(5/2)) - (b^2*PolyLog[2, 1 + (2*Sq
rt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/(10*c^(5/2
)) - (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])
*(1 + Sqrt[c]*x))])/(10*c^(5/2)) - ((I/10)*b^2*PolyLog[2, 1 - ((1 - I)*(1 +
 Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/c^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
```

```
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6041

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p
, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx=\int\left(\frac{1}{4}x^4\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{1}{2}bx^4\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right.\right.$$

$$=\frac{1}{4}\int x^4\left(2a-b\log\left(1-cx^2\right)\right)^2\,dx-\frac{1}{2}b\int x^4\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1\right.$$

$$=\frac{1}{20}x^5\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{20}b^2x^5\log^2\left(1+cx^2\right)-\frac{1}{2}b\int\left(-2ax^4\log\left(1\right.\right.$$

$$=\frac{1}{20}x^5\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{20}b^2x^5\log^2\left(1+cx^2\right)+(ab)\int x^4\log\left(1+c\right.$$

$$=\frac{1}{20}x^5\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{5}abx^5\log\left(1+cx^2\right)-\frac{1}{10}b^2x^5\log\left(1-cx^2\right)\mathrm{l}$$

$$=\frac{2abx}{5c^2}+\frac{bx^3(2a-b\log\left(1-cx^2\right))}{15c}+\frac{1}{25}bx^5\left(2a-b\log\left(1-cx^2\right)\right)-\frac{b\tanh^{-1}}{}$$

$$=\frac{2b^2x}{5c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5-\frac{b^2x\log\left(1-cx^2\right)}{5c^2}+\frac{bx^3(2a-b\log\left(1-cx^2\right))}{15c}+$$

$$=\frac{92b^2x}{75c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5+\frac{4b^2x^5}{125}+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{2b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}$$

$$=\frac{92b^2x}{75c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5+\frac{4b^2x^5}{125}+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{46b^2\tan^{-1}\left(\sqrt{c}\,x\right.}{75c^{5/2}}$$

$$=\frac{32b^2x}{75c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5+\frac{4b^2x^5}{125}+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{46b^2\tan^{-1}\left(\sqrt{c}\,x\right.}{75c^{5/2}}$$

$$=\frac{8b^2x}{15c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{16b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{75c^{5/2}}+\frac{ib^2\,\mathrm{ta}}{}$$

$$=\frac{8b^2x}{15c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{4b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{15c^{5/2}}+\frac{ib^2\,\mathrm{tan}}{}$$

$$=\frac{8b^2x}{15c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{4b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{15c^{5/2}}+\frac{ib^2\,\mathrm{tan}}{}$$

$$=\frac{8b^2x}{15c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{4b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{15c^{5/2}}+\frac{ib^2\,\mathrm{tan}}{}$$

$$=\frac{8b^2x}{15c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{4b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{15c^{5/2}}+\frac{ib^2\,\mathrm{tan}}{}$$

Mathematica [F]
time = 8.45, size = 0, normalized size = 0.00

$$\int x^4 \left(a + b \tanh^{-1}\left(cx^2\right)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x^2])^2,x]
[Out] Integrate[x^4*(a + b*ArcTanh[c*x^2])^2, x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^4 \left(a + b \operatorname{arctanh}\left(c\,x^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x^2))^2,x)
[Out] int(x^4*(a+b*arctanh(c*x^2))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
[Out] 1/5*a^2*x^5 + 1/15*(6*x^5*arctanh(c*x^2) + c*(4*x^3/c^2 + 6*arctan(sqrt(c)*
x)/c^(7/2) + 3*log((c*x - sqrt(c))/(c*x + sqrt(c)))/c^(7/2)))*a*b + 1/20*(x
^5*log(-c*x^2 + 1)^2 - 5*integrate(-1/5*(5*(c*x^6 - x^4)*log(c*x^2 + 1)^2 -
 2*(2*c*x^6 + 5*(c*x^6 - x^4)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^2 - 1),
 x))*b^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
[Out] integral(b^2*x^4*arctanh(c*x^2)^2 + 2*a*b*x^4*arctanh(c*x^2) + a^2*x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 (a + b\,\mathrm{atanh}\left(cx^2\right))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Integral(x**4*(a + b*atanh(c*x**2))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atanh(c*x^2))^2,x)
```

```
[Out] int(x^4*(a + b*atanh(c*x^2))^2, x)
```

3.72 $\int x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx$

Optimal. Leaf size=1129

$$\frac{4abx}{3c}-\frac{2}{9}abx^3-\frac{2ab\text{ArcTan}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{4b^2\text{ArcTan}\left(\sqrt{c}\,x\right)}{3c^{3/2}}-\frac{ib^2\text{ArcTan}\left(\sqrt{c}\,x\right)^2}{3c^{3/2}}-\frac{4b^2\tanh^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}-\frac{b^2\tanh^{-}}{3}$$

```
[Out] -2/9*a*b*x^3-1/3*b^2*arctan(x*c^(1/2))*ln((1-I)*(1+x*c^(1/2))/(1-I*x*c^(1/2
)))/c^(3/2)-1/3*I*b^2*arctan(x*c^(1/2))^2/c^(3/2)-1/3*I*b^2*polylog(2,1-2/(
1-I*x*c^(1/2)))/c^(3/2)-1/3*I*b^2*polylog(2,1-2/(1+I*x*c^(1/2)))/c^(3/2)-2/
3*a*b*arctan(x*c^(1/2))/c^(3/2)-2/3*b^2*x*ln(-c*x^2+1)/c+1/3*b^2*arctan(x*c
^(1/2))*ln(-c*x^2+1)/c^(3/2)-1/3*b*arctanh(x*c^(1/2))*(2*a-b*ln(-c*x^2+1))/
c^(3/2)+2/3*b^2*x*ln(c*x^2+1)/c+1/3*a*b*x^3*ln(c*x^2+1)-1/3*b^2*arctan(x*c^
(1/2))*ln(c*x^2+1)/c^(3/2)-1/3*b^2*arctanh(x*c^(1/2))*ln(c*x^2+1)/c^(3/2)-1
/6*b^2*x^3*ln(-c*x^2+1)*ln(c*x^2+1)+2/3*b^2*arctanh(x*c^(1/2))*ln(2/(1-x*c^
(1/2)))/c^(3/2)+2/3*b^2*arctan(x*c^(1/2))*ln(2/(1-I*x*c^(1/2)))/c^(3/2)-1/3
*b^2*arctan(x*c^(1/2))*ln((1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2)))/c^(3/2)-2/3*
b^2*arctan(x*c^(1/2))*ln(2/(1+I*x*c^(1/2)))/c^(3/2)-2/3*b^2*arctanh(x*c^(1/
2))*ln(2/(1+x*c^(1/2)))/c^(3/2)+1/3*b^2*arctanh(x*c^(1/2))*ln(-2*(1-x*(-c)^
(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(1/2)))/c^(3/2)+1/3*b^2*arctanh(
x*c^(1/2))*ln(2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2))
)/c^(3/2)+1/6*I*b^2*polylog(2,1-(1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2)))/c^(3/2
)+1/6*I*b^2*polylog(2,1+(-1+I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))/c^(3/2)+4/3*a
*b*x/c+1/12*x^3*(2*a-b*ln(-c*x^2+1))^2+1/3*b^2*polylog(2,1-2/(1+x*c^(1/2)))
/c^(3/2)-1/6*b^2*polylog(2,1+2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2)
)/(1+x*c^(1/2)))/c^(3/2)-1/6*b^2*polylog(2,1-2*(1+x*(-c)^(1/2))*c^(1/2)/((-
c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))/c^(3/2)+4/3*b^2*arctan(x*c^(1/2))/c^(3/2)-
4/3*b^2*arctanh(x*c^(1/2))/c^(3/2)-1/3*b^2*arctanh(x*c^(1/2))^2/c^(3/2)+1/9
*b^2*x^3*ln(-c*x^2+1)+1/9*b*x^3*(2*a-b*ln(-c*x^2+1))+1/12*b^2*x^3*ln(c*x^2+
1)^2+1/3*b^2*polylog(2,1-2/(1-x*c^(1/2)))/c^(3/2)
```

Rubi [A]
time = 1.46, antiderivative size = 1129, normalized size of antiderivative = 1.00, number of steps used = 86, number of rules used = 26, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.625$, Rules used = {6041, 2507, 2526, 2498, 327, 212, 2505, 308, 2520, 12, 6131, 6055, 2449, 2352, 6874, 209, 30, 2637, 213, 6139, 6057, 2497, 5048, 4966, 5040, 4964}

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (4*a*b*x)/(3*c) - (2*a*b*x^3)/9 - (2*a*b*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) + (
4*b^2*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) - ((I/3)*b^2*ArcTan[Sqrt[c]*x]^2)/c^(3
```

```
/2) - (4*b^2*ArcTanh[Sqrt[c]*x])/(3*c^(3/2)) - (b^2*ArcTanh[Sqrt[c]*x]^2)/(
3*c^(3/2)) + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/(3*c^(3/2))
+ (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/(3*c^(3/2)) - (b^2*Arc
Tan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/(3*c^(3/2)
) - (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/(3*c^(3/2)) - (2*b^2
*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)])/(3*c^(3/2)) + (b^2*ArcTanh[Sqrt
[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]
*x))])/(3*c^(3/2)) + (b^2*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x
))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/(3*c^(3/2)) - (b^2*ArcTan[Sqrt[
c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/(3*c^(3/2)) - (2*b^
2*x*Log[1 - c*x^2])/(3*c) + (b^2*x^3*Log[1 - c*x^2])/9 + (b^2*ArcTan[Sqrt[c
]*x]*Log[1 - c*x^2])/(3*c^(3/2)) + (b*x^3*(2*a - b*Log[1 - c*x^2]))/9 - (b*
ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/(3*c^(3/2)) + (x^3*(2*a - b*Lo
g[1 - c*x^2])^2)/12 + (2*b^2*x*Log[1 + c*x^2])/(3*c) + (a*b*x^3*Log[1 + c*x
^2])/3 - (b^2*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/(3*c^(3/2)) - (b^2*ArcTanh[
Sqrt[c]*x]*Log[1 + c*x^2])/(3*c^(3/2)) - (b^2*x^3*Log[1 - c*x^2]*Log[1 + c*
x^2])/6 + (b^2*x^3*Log[1 + c*x^2]^2)/12 + (b^2*PolyLog[2, 1 - 2/(1 - Sqrt[c
]*x)])/(3*c^(3/2)) - ((I/3)*b^2*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)])/c^(3/2
) + ((I/6)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])
/c^(3/2) - ((I/3)*b^2*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)])/c^(3/2) + (b^2*P
olyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/(3*c^(3/2)) - (b^2*PolyLog[2, 1 + (2*Sqrt
[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/(6*c^(3/2))
- (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1
 + Sqrt[c]*x))])/(6*c^(3/2)) + ((I/6)*b^2*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt
[c]*x))/(1 - I*Sqrt[c]*x)])/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6041

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p
, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
```

```
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx=\int\left(\frac{1}{4}x^2\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{1}{2}bx^2\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx\right.\right.$$

$$=\frac{1}{4}\int x^2\left(2a-b\log\left(1-cx^2\right)\right)^2\,dx-\frac{1}{2}b\int x^2\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1\right.$$

$$=\frac{1}{12}x^3\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{12}b^2x^3\log^2\left(1+cx^2\right)-\frac{1}{2}b\int\left(-2ax^2\log\left(\right.\right.$$

$$=\frac{1}{12}x^3\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{12}b^2x^3\log^2\left(1+cx^2\right)+(ab)\int x^2\log\left(1+\right.$$

$$=\frac{1}{12}x^3\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{3}abx^3\log\left(1+cx^2\right)-\frac{1}{6}b^2x^3\log\left(1-cx^2\right)\mathrm{l}$$

$$=\frac{2abx}{3c}+\frac{1}{9}bx^3\left(2a-b\log\left(1-cx^2\right)\right)-\frac{b\tanh^{-1}\left(\sqrt{c}\,x\right)\left(2a-b\log\left(1-cx^2\right.\right.}{3c^{3/2}}$$

$$=\frac{4abx}{3c}-\frac{2b^2x}{3c}-\frac{2}{9}abx^3-\frac{b^2x\log\left(1-cx^2\right)}{3c}+\frac{1}{9}bx^3\left(2a-b\log\left(1-cx^2\right)\right)-$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3+\frac{4b^2x^3}{27}-\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{2b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}-\frac{ib^2\tan}{}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3+\frac{4b^2x^3}{27}-\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{8b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{9c^{3/2}}-\frac{ib^2\tan}{}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3+\frac{4b^2x^3}{27}-\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{8b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{9c^{3/2}}-\frac{ib^2\tan}{}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3-\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{14b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{9c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}\right.}{3c^{3/2}}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3-\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{4b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right.}{3c^{3/2}}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3-\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{4b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right.}{3c^{3/2}}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3-\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{4b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right.}{3c^{3/2}}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3-\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{4b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right.}{3c^{3/2}}$$

Mathematica [F]
time = 8.32, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b\tanh^{-1}\left(cx^2\right)\right)^2\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] Integrate[x^2*(a + b*ArcTanh[c*x^2])^2, x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b\,\mathrm{arctanh}\left(c\,x^2\right)\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int(x^2*(a+b*arctanh(c*x^2))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arctanh(c*x^2) + c*(4*x/c^2 - 2*arctan(sqrt(c)*x)/
c^(5/2) + log((c*x - sqrt(c))/(c*x + sqrt(c)))/c^(5/2)))*a*b + 1/12*(x^3*lo
g(-c*x^2 + 1)^2 - 3*integrate(-1/3*(3*(c*x^4 - x^2)*log(c*x^2 + 1)^2 - 2*(2
*c*x^4 + 3*(c*x^4 - x^2)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^2 - 1), x))*
b^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctanh(c*x^2)^2 + 2*a*b*x^2*arctanh(c*x^2) + a^2*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\operatorname{atanh}\left(cx^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*x**2))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^2))^2,x)
```

```
[Out] int(x^2*(a + b*atanh(c*x^2))^2, x)
```

3.73 $\int \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx$

Optimal. Leaf size=958

$$a^2x+\frac{2ab\text{ArcTan}\left(\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{ib^2\text{ArcTan}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}}-\frac{2ab\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{b^2\tanh^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}}+\frac{2b^2\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}$$

```
[Out] 2*b^2*arctanh(x*c^(1/2))*ln(2/(1-x*c^(1/2)))/c^(1/2)-2*b^2*arctan(x*c^(1/2)
)*ln(2/(1-I*x*c^(1/2)))/c^(1/2)+2*b^2*arctan(x*c^(1/2))*ln(2/(1+I*x*c^(1/2)
))/c^(1/2)-2*b^2*arctanh(x*c^(1/2))*ln(2/(1+x*c^(1/2)))/c^(1/2)-1/2*I*b^2*p
olylog(2,1-(1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2)))/c^(1/2)-1/2*I*b^2*polylog(2
,1+(-1+I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))/c^(1/2)-a*b*x*ln(-c*x^2+1)+I*b^2*a
rctan(x*c^(1/2))^2/c^(1/2)+a*b*x*ln(c*x^2+1)-b^2*arctan(x*c^(1/2))*ln(-c*x^
2+1)/c^(1/2)+b^2*arctanh(x*c^(1/2))*ln(-c*x^2+1)/c^(1/2)+b^2*arctan(x*c^(1/
2))*ln(c*x^2+1)/c^(1/2)-b^2*arctanh(x*c^(1/2))*ln(c*x^2+1)/c^(1/2)+b^2*arct
an(x*c^(1/2))*ln((1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2)))/c^(1/2)+b^2*arctanh(x
*c^(1/2))*ln(-2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(1/2))
)/c^(1/2)+b^2*arctanh(x*c^(1/2))*ln(2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+
c^(1/2))/(1+x*c^(1/2)))/c^(1/2)+b^2*arctan(x*c^(1/2))*ln((1-I)*(1+x*c^(1/2)
)/(1-I*x*c^(1/2)))/c^(1/2)+I*b^2*polylog(2,1-2/(1-I*x*c^(1/2)))/c^(1/2)+I*b
^2*polylog(2,1-2/(1+I*x*c^(1/2)))/c^(1/2)-1/2*b^2*x*ln(-c*x^2+1)*ln(c*x^2+1
)+2*a*b*arctan(x*c^(1/2))/c^(1/2)-2*a*b*arctanh(x*c^(1/2))/c^(1/2)+1/4*b^2*
x*ln(-c*x^2+1)^2+1/4*b^2*x*ln(c*x^2+1)^2-1/2*b^2*polylog(2,1+2*(1-x*(-c)^(1
/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(1/2)))/c^(1/2)-1/2*b^2*polylog(2,
1-2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))/c^(1/2)-b^
2*arctanh(x*c^(1/2))^2/c^(1/2)+b^2*polylog(2,1-2/(1-x*c^(1/2)))/c^(1/2)+b^2
*polylog(2,1-2/(1+x*c^(1/2)))/c^(1/2)+a^2*x
```

Rubi [A]
time = 0.99, antiderivative size = 958, normalized size of antiderivative = 1.00, number of steps used = 69, number of rules used = 21, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.750$, Rules used = {6023, 2498, 327, 212, 2500, 2526, 2520, 12, 6131, 6055, 2449, 2352, 209, 2636, 6139, 6057, 2497, 5048, 4966, 5040, 4964}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] a^2*x + (2*a*b*ArcTan[Sqrt[c]*x])/Sqrt[c] + (I*b^2*ArcTan[Sqrt[c]*x]^2)/Sqr
t[c] - (2*a*b*ArcTanh[Sqrt[c]*x])/Sqrt[c] - (b^2*ArcTanh[Sqrt[c]*x]^2)/Sqrt
[c] + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/Sqrt[c] - (2*b^2*Ar
cTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (b^2*ArcTan[Sqrt[c]*x]*
```

```
Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (2*b^2*ArcTan[S
qrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/Sqrt[c] - (2*b^2*ArcTanh[Sqrt[c]*x]*Log
[2/(1 + Sqrt[c]*x)])/Sqrt[c] + (b^2*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 -
 Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/Sqrt[c] + (b^2*ArcTa
nh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + S
qrt[c]*x))])/Sqrt[c] + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))
/(1 - I*Sqrt[c]*x)])/Sqrt[c] - a*b*x*Log[1 - c*x^2] - (b^2*ArcTan[Sqrt[c]*x
]*Log[1 - c*x^2])/Sqrt[c] + (b^2*ArcTanh[Sqrt[c]*x]*Log[1 - c*x^2])/Sqrt[c]
 + (b^2*x*Log[1 - c*x^2]^2)/4 + a*b*x*Log[1 + c*x^2] + (b^2*ArcTan[Sqrt[c]*
x]*Log[1 + c*x^2])/Sqrt[c] - (b^2*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2])/Sqrt[c
] - (b^2*x*Log[1 - c*x^2]*Log[1 + c*x^2])/2 + (b^2*x*Log[1 + c*x^2]^2)/4 +
(b^2*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)])/Sqrt[c] + (I*b^2*PolyLog[2, 1 - 2/(
1 - I*Sqrt[c]*x)])/Sqrt[c] - ((I/2)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c
]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (I*b^2*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*
x)])/Sqrt[c] + (b^2*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/Sqrt[c] - (b^2*PolyL
og[2, 1 + (2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x
))])/(2*Sqrt[c]) - (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[
-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/(2*Sqrt[c]) - ((I/2)*b^2*PolyLog[2, 1 - (
(1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2636

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[x*Log[w]*(D[v, x]/v), x], x] - Int[SimplifyIntegrand[x*Log[v]*(D[
w, x]/w), x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6023

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[ExpandI
ntegrand[(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p, x], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
```

```
0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx = \int \left(a^2 - ab\log\left(1-cx^2\right) + \frac{1}{4}b^2\log^2\left(1-cx^2\right) + ab\log\left(1+cx^2\right) - \frac{1}{2}b^2\log\left(\right.\right.$$

$$= a^2x - (ab)\int \log\left(1-cx^2\right)\,dx + (ab)\int \log\left(1+cx^2\right)\,dx + \frac{1}{4}b^2\int \log^2\left(1-\right.$$

$$= a^2x - abx\log\left(1-cx^2\right) + \frac{1}{4}b^2x\log^2\left(1-cx^2\right) + abx\log\left(1+cx^2\right) - \frac{1}{2}b^2x\log$$

$$= a^2x - abx\log\left(1-cx^2\right) + \frac{1}{4}b^2x\log^2\left(1-cx^2\right) + abx\log\left(1+cx^2\right) - \frac{1}{2}b^2x\log$$

$$= a^2x + \frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{2ab\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - abx\log\left(1-cx^2\right) + \frac{1}{4}b^2x\log$$

$$= a^2x + \frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{2ab\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - abx\log\left(1-cx^2\right) - b^2x\log$$

$$= a^2x + 4b^2x + \frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{2ab\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - abx\log\left(1-cx^2\right) -$$

$$= a^2x + \frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}} - \frac{2ab\tan}{}$$

$$= a^2x + \frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}} - \frac{2ab\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{b^2\tan}{}$$

$$= a^2x + \frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}} - \frac{2ab\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{b^2\tan}{}$$

$$= a^2x + \frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}} - \frac{2ab\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{b^2\tan}{}$$

$$= a^2x + \frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}} - \frac{2ab\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{b^2\tan}{}$$

$$= a^2x + \frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}} - \frac{2ab\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{b^2\tan}{}$$

<u>**Mathematica**</u> **[A]**

time = 1.79, size = 566, normalized size = 0.59

$$\frac{1}{2}x\left(2a^2+4ab\tanh^{-1}(cx^2)+\frac{4ab\left(\text{ArcTan}\left(\sqrt{cx^2}\right)-\tanh^{-1}\left(\sqrt{cx^2}\right)\right)}{\sqrt{cx^2}}+\frac{b^2\left(-2i\text{ArcTan}\left(\sqrt{cx^2}\right)^2+4\text{ArcTan}\left(\sqrt{cx^2}\right)\tanh^{-1}(cx^2)+2\sqrt{cx^2}\tanh^{-1}(cx^2)^2+2\text{ArcTan}\left(\sqrt{cx^2}\right)\log\left(1+e^{4i\text{ArcTan}\left(\sqrt{cx^2}\right)}\right)+2\tanh^{-1}(cx^2)\log\left(1-\sqrt{cx^2}\right)-\log(2)\log\left(1-\sqrt{cx^2}\right)+\frac{1}{2}\log^2\left(1-\sqrt{cx^2}\right)-\log\left(1-\sqrt{cx^2}\right)\log\left(\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i+\sqrt{cx^2}\right)\right)-2\tanh^{-1}(cx^2)\log\left(1+\sqrt{cx^2}\right)+\log(2)\log\left(1+\sqrt{cx^2}\right)+\log\left(\frac{1}{2}\left((1+i)-(1-i)\sqrt{cx^2}\right)\right)\log\left(1+\sqrt{cx^2}\right)+\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)\left(i+\sqrt{cx^2}\right)\right)\log\left(1+\sqrt{cx^2}\right)-\frac{1}{2}\log^2\left(1+\sqrt{cx^2}\right)-\log\left(1-\sqrt{cx^2}\right)\log\left(\frac{1}{2}\left((1+i)+(1-i)\sqrt{cx^2}\right)\right)-\frac{1}{2}i\text{PolyLog}\left(2,-e^{4i\text{ArcTan}\left(\sqrt{cx^2}\right)}\right)+\text{PolyLog}\left(2,\frac{1}{2}\left(1-\sqrt{cx^2}\right)\right)-\text{PolyLog}\left(2,\left(-\frac{1}{2}-\frac{i}{2}\right)\left(-1+\sqrt{cx^2}\right)\right)-\text{PolyLog}\left(2,\left(-\frac{1}{2}+\frac{i}{2}\right)\left(-1+\sqrt{cx^2}\right)\right)-\text{PolyLog}\left(2,\frac{1}{2}\left(1+\sqrt{cx^2}\right)\right)+\text{PolyLog}\left(2,\left(\frac{1}{2}-\frac{i}{2}\right)\left(1+\sqrt{cx^2}\right)\right)+\text{PolyLog}\left(2,\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+\sqrt{cx^2}\right)\right)\right)}{\sqrt{cx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (x*(2*a^2 + 4*a*b*ArcTanh[c*x^2] + (4*a*b*(ArcTan[Sqrt[c*x^2]] - ArcTanh[Sq
rt[c*x^2]]))/Sqrt[c*x^2] + (b^2*((-2*I)*ArcTan[Sqrt[c*x^2]]^2 + 4*ArcTan[Sq
rt[c*x^2]]*ArcTanh[c*x^2] + 2*Sqrt[c*x^2]*ArcTanh[c*x^2]^2 + 2*ArcTan[Sqrt[
c*x^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c*x^2]])] + 2*ArcTanh[c*x^2]*Log[1 - S
qrt[c*x^2]] - Log[2]*Log[1 - Sqrt[c*x^2]] + Log[1 - Sqrt[c*x^2]]^2/2 - Log[
1 - Sqrt[c*x^2]]*Log[(1/2 + I/2)*(-I + Sqrt[c*x^2])] - 2*ArcTanh[c*x^2]*Log
[1 + Sqrt[c*x^2]] + Log[2]*Log[1 + Sqrt[c*x^2]] + Log[((1 + I) - (1 - I)*Sq
rt[c*x^2])/2]*Log[1 + Sqrt[c*x^2]] + Log[(-1/2 - I/2)*(I + Sqrt[c*x^2])]*Lo
g[1 + Sqrt[c*x^2]] - Log[1 + Sqrt[c*x^2]]^2/2 - Log[1 - Sqrt[c*x^2]]*Log[((
1 + I) + (1 - I)*Sqrt[c*x^2])/2] - (I/2)*PolyLog[2, -E^((4*I)*ArcTan[Sqrt[c
*x^2]])] + PolyLog[2, (1 - Sqrt[c*x^2])/2] - PolyLog[2, (-1/2 - I/2)*(-1 +
Sqrt[c*x^2])] - PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[c*x^2])] - PolyLog[2, (1
 + Sqrt[c*x^2])/2] + PolyLog[2, (1/2 - I/2)*(1 + Sqrt[c*x^2])] + PolyLog[2,
 (1/2 + I/2)*(1 + Sqrt[c*x^2])]))/Sqrt[c*x^2]))/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arctanh}\left(c\,x^{2}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] (c*(2*arctan(sqrt(c)*x)/c^(3/2) + log((c*x - sqrt(c))/(c*x + sqrt(c)))/c^(3
/2)) + 2*x*arctanh(c*x^2))*a*b + 1/4*(x*log(-c*x^2 + 1)^2 - integrate(-((c*
x^2 - 1)*log(c*x^2 + 1)^2 - 2*(2*c*x^2 + (c*x^2 - 1)*log(c*x^2 + 1))*log(-c
*x^2 + 1))/(c*x^2 - 1), x))*b^2 + a^2*x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
[Out] integral(b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{atanh}\left(cx^{2}\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2,x)
[Out] Integral((a + b*atanh(c*x**2))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^2) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2,x)
[Out] int((a + b*atanh(c*x^2))^2, x)
```

3.74 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=942

$$2ab\sqrt{c}\,\text{ArcTan}\left(\sqrt{c}\,x\right)+ib^2\sqrt{c}\,\text{ArcTan}\left(\sqrt{c}\,x\right)^2+b^2\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right)^2-2b^2\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right)\log\left(\frac{2}{1-\sqrt{c}\right.$$

```
[Out] 1/2*b^2*ln(-c*x^2+1)*ln(c*x^2+1)/x+2*a*b*arctan(x*c^(1/2))*c^(1/2)-2*b^2*ar
ctanh(x*c^(1/2))*ln(2/(1-x*c^(1/2)))*c^(1/2)-2*b^2*arctan(x*c^(1/2))*ln(2/(
1-I*x*c^(1/2)))*c^(1/2)+2*b^2*arctan(x*c^(1/2))*ln(2/(1+I*x*c^(1/2)))*c^(1/
2)+2*b^2*arctanh(x*c^(1/2))*ln(2/(1+x*c^(1/2)))*c^(1/2)-1/2*I*b^2*polylog(2
,1-(1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2)))*c^(1/2)-1/2*I*b^2*polylog(2,1+(-1+I
)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))*c^(1/2)-a*b*ln(c*x^2+1)/x+I*b^2*arctan(x*c
^(1/2))^2*c^(1/2)-b^2*arctan(x*c^(1/2))*ln(-c*x^2+1)*c^(1/2)+b*arctanh(x*c^
(1/2))*(2*a-b*ln(-c*x^2+1))*c^(1/2)+b^2*arctan(x*c^(1/2))*ln(c*x^2+1)*c^(1/
2)+b^2*arctanh(x*c^(1/2))*ln(c*x^2+1)*c^(1/2)+b^2*arctan(x*c^(1/2))*ln((1+I
)*(1-x*c^(1/2))/(1-I*x*c^(1/2)))*c^(1/2)-b^2*arctanh(x*c^(1/2))*ln(-2*(1-x*
(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(1/2)))*c^(1/2)-b^2*arctanh
(x*c^(1/2))*ln(2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)
))*c^(1/2)+b^2*arctan(x*c^(1/2))*ln((1-I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))*c^
(1/2)+I*b^2*polylog(2,1-2/(1-I*x*c^(1/2)))*c^(1/2)+I*b^2*polylog(2,1-2/(1+I
*x*c^(1/2)))*c^(1/2)-1/4*b^2*ln(c*x^2+1)^2/x+1/2*b^2*polylog(2,1+2*(1-x*(-c
)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(1/2)))*c^(1/2)+1/2*b^2*polylo
g(2,1-2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))*c^(1/2
)+b^2*arctanh(x*c^(1/2))^2*c^(1/2)-b^2*polylog(2,1-2/(1-x*c^(1/2)))*c^(1/2)
-b^2*polylog(2,1-2/(1+x*c^(1/2)))*c^(1/2)-1/4*(2*a-b*ln(-c*x^2+1))^2/x
```

Rubi [A]
time = 0.95, antiderivative size = 942, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 21, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.313$, Rules used = {6041, 2507, 212, 2520, 12, 6131, 6055, 2449, 2352, 2505, 6874, 209, 30, 2637, 6139, 6057, 2497, 5048, 4966, 5040, 4964}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/x^2,x]
```

```
[Out] 2*a*b*Sqrt[c]*ArcTan[Sqrt[c]*x] + I*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]^2 + b^2*S
qrt[c]*ArcTanh[Sqrt[c]*x]^2 - 2*b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[2/(1 - S
qrt[c]*x)] - 2*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)] + b^2
*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)]
 + 2*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)] + 2*b^2*Sqrt[c]
*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)] - b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]
```

```
*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))]
- b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c
] + Sqrt[c])*(1 + Sqrt[c]*x))] + b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[((1 - I)
*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)] - b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[1
- c*x^2] + b*Sqrt[c]*ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]) - (2*a - b
*Log[1 - c*x^2])^2/(4*x) - (a*b*Log[1 + c*x^2])/x + b^2*Sqrt[c]*ArcTan[Sqrt
[c]*x]*Log[1 + c*x^2] + b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2] + (b^
2*Log[1 - c*x^2]*Log[1 + c*x^2])/(2*x) - (b^2*Log[1 + c*x^2]^2)/(4*x) - b^2
*Sqrt[c]*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)] + I*b^2*Sqrt[c]*PolyLog[2, 1 - 2
/(1 - I*Sqrt[c]*x)] - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c
]*x))/(1 - I*Sqrt[c]*x)] + I*b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)
] - b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)] + (b^2*Sqrt[c]*PolyLog[2,
 1 + (2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/
2 + (b^2*Sqrt[c]*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + S
qrt[c])*(1 + Sqrt[c]*x))])/2 - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 - I)*(1
 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6041

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p
, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
```

```
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^2}\,dx = \int \left(\frac{(2a-b\log\left(1-cx^2\right))^2}{4x^2} - \frac{b(-2a+b\log\left(1-cx^2\right))\log\left(1+cx^2\right)}{2x^2} + \frac{b^2\log\cdots}{\cdots}\right.$$

$$= \frac{1}{4}\int \frac{(2a-b\log\left(1-cx^2\right))^2}{x^2}\,dx - \frac{1}{2}b\int \frac{(-2a+b\log\left(1-cx^2\right))\log\left(1+cx^2\right)}{x^2}\cdots$$

$$= -\frac{(2a-b\log\left(1-cx^2\right))^2}{4x} - \frac{b^2\log^2\left(1+cx^2\right)}{4x} - \frac{1}{2}b\int \left(-\frac{2a\log\left(1+cx^2\right)}{x^2} + \frac{b\cdots}{\cdots}\right.$$

$$= b\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right)\left(2a-b\log\left(1-cx^2\right)\right) - \frac{(2a-b\log\left(1-cx^2\right))^2}{4x} + b^2\sqrt{c}\cdots$$

$$= b\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right)\left(2a-b\log\left(1-cx^2\right)\right) - \frac{(2a-b\log\left(1-cx^2\right))^2}{4x} - \frac{ab\log\cdots}{\cdots}$$

$$= 2ab\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right) + ib^2\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right)^2 + b^2\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right)^2 + b\sqrt{\cdots}$$

$$= 2ab\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right) + ib^2\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right)^2 + b^2\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right)^2 - 2b\cdots$$

$$= 2ab\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right) + ib^2\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right)^2 + b^2\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right)^2 - 2b\cdots$$

$$= 2ab\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right) + ib^2\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right)^2 + b^2\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right)^2 - 2b\cdots$$

$$= 2ab\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right) + ib^2\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right)^2 + b^2\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right)^2 - 2b\cdots$$

$$= 2ab\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right) + ib^2\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right)^2 + b^2\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right)^2 - 2b\cdots$$

$$= 2ab\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right) + ib^2\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right)^2 + b^2\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right)^2 - 2b\cdots$$

$$= 2ab\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right) + ib^2\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right)^2 + b^2\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right)^2 - 2b\cdots$$

Mathematica [A]
time = 2.32, size = 566, normalized size = 0.60

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x^2,x]
```

```
[Out] (-2*a^2 - 4*a*b*ArcTanh[c*x^2] + 4*a*b*Sqrt[c*x^2]*(ArcTan[Sqrt[c*x^2]] + A
rcTanh[Sqrt[c*x^2]]) + b^2*Sqrt[c*x^2]*((-2*I)*ArcTan[Sqrt[c*x^2]]^2 + 4*Ar
cTan[Sqrt[c*x^2]]*ArcTanh[c*x^2] - (2*ArcTanh[c*x^2]^2)/Sqrt[c*x^2] + 2*Arc
Tan[Sqrt[c*x^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c*x^2]])] - 2*ArcTanh[c*x^2]*
Log[1 - Sqrt[c*x^2]] + Log[2]*Log[1 - Sqrt[c*x^2]] - Log[1 - Sqrt[c*x^2]]^2
/2 + Log[1 - Sqrt[c*x^2]]*Log[(1/2 + I/2)*(-I + Sqrt[c*x^2])] + 2*ArcTanh[c
*x^2]*Log[1 + Sqrt[c*x^2]] - Log[2]*Log[1 + Sqrt[c*x^2]] - Log[((1 + I) - (
1 - I)*Sqrt[c*x^2])/2]*Log[1 + Sqrt[c*x^2]] - Log[(-1/2 - I/2)*(I + Sqrt[c*
x^2])]*Log[1 + Sqrt[c*x^2]] + Log[1 + Sqrt[c*x^2]]^2/2 + Log[1 - Sqrt[c*x^2
]]*Log[((1 + I) + (1 - I)*Sqrt[c*x^2])/2] - (I/2)*PolyLog[2, -E^((4*I)*ArcT
an[Sqrt[c*x^2]])] - PolyLog[2, (1 - Sqrt[c*x^2])/2] + PolyLog[2, (-1/2 - I/
2)*(-1 + Sqrt[c*x^2])] + PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[c*x^2])] + Poly
Log[2, (1 + Sqrt[c*x^2])/2] - PolyLog[2, (1/2 - I/2)*(1 + Sqrt[c*x^2])] - P
olyLog[2, (1/2 + I/2)*(1 + Sqrt[c*x^2])]))/(2*x)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\,x^{2}\right)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x^2,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^2,x, algorithm="maxima")
```

```
[Out] (c*(2*arctan(sqrt(c)*x)/sqrt(c) - log((c*x - sqrt(c))/(c*x + sqrt(c)))/sqrt
(c)) - 2*arctanh(c*x^2)/x)*a*b - 1/4*b^2*(log(-c*x^2 + 1)^2/x + integrate(-
((c*x^2 - 1)*log(c*x^2 + 1)^2 + 2*(2*c*x^2 - (c*x^2 - 1)*log(c*x^2 + 1))*lo
g(-c*x^2 + 1))/(c*x^4 - x^2), x)) - a^2/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\operatorname{atanh}\left(cx^{2}\right)\right)^{2}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)}^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/x^2,x)
```

```
[Out] int((a + b*atanh(c*x^2))^2/x^2, x)
```

3.75 $\int \frac{(a+b\tanh^{-1}(cx^2))^2}{x^4}\,dx$

Optimal. Leaf size=1102

$$-\frac{2abc}{3x}-\frac{2}{3}abc^{3/2}\text{ArcTan}\left(\sqrt{c}\,x\right)+\frac{4}{3}b^2c^{3/2}\text{ArcTan}\left(\sqrt{c}\,x\right)-\frac{1}{3}ib^2c^{3/2}\text{ArcTan}\left(\sqrt{c}\,x\right)^2+\frac{4}{3}b^2c^{3/2}\tanh^{-1}\left(\sqrt{c}\,x\right)+$$

```
[Out] -2/3*a*b*c/x+4/3*b^2*c^(3/2)*arctan(x*c^(1/2))+4/3*b^2*c^(3/2)*arctanh(x*c^
(1/2))+1/3*b^2*c^(3/2)*arctanh(x*c^(1/2))^2-1/12*b^2*ln(c*x^2+1)^2/x^3-1/3*
b^2*c^(3/2)*polylog(2,1-2/(1-x*c^(1/2)))-1/3*b^2*c^(3/2)*polylog(2,1-2/(1+x
*c^(1/2)))+1/6*b^2*c^(3/2)*polylog(2,1+2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/
2)-c^(1/2))/(1+x*c^(1/2)))+1/6*b^2*c^(3/2)*polylog(2,1-2*(1+x*(-c)^(1/2))*c
^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))-1/12*(2*a-b*ln(-c*x^2+1))^2/x^3-
2/3*a*b*c^(3/2)*arctan(x*c^(1/2))+1/3*b^2*c*ln(-c*x^2+1)/x+1/3*b^2*c^(3/2)*
arctan(x*c^(1/2))*ln(-c*x^2+1)-1/3*b*c*(2*a-b*ln(-c*x^2+1))/x+1/3*b*c^(3/2)
*arctanh(x*c^(1/2))*(2*a-b*ln(-c*x^2+1))-1/3*a*b*ln(c*x^2+1)/x^3-2/3*b^2*c*
ln(c*x^2+1)/x-1/3*b^2*c^(3/2)*arctan(x*c^(1/2))*ln(c*x^2+1)+1/3*b^2*c^(3/2)
*arctanh(x*c^(1/2))*ln(c*x^2+1)+1/6*b^2*ln(-c*x^2+1)*ln(c*x^2+1)/x^3-2/3*b^
2*c^(3/2)*arctanh(x*c^(1/2))*ln(2/(1-x*c^(1/2)))+2/3*b^2*c^(3/2)*arctan(x*c
^(1/2))*ln(2/(1-I*x*c^(1/2)))-1/3*b^2*c^(3/2)*arctan(x*c^(1/2))*ln((1+I)*(1
-x*c^(1/2))/(1-I*x*c^(1/2)))-2/3*b^2*c^(3/2)*arctan(x*c^(1/2))*ln(2/(1+I*x*
c^(1/2)))+2/3*b^2*c^(3/2)*arctanh(x*c^(1/2))*ln(2/(1+x*c^(1/2)))-1/3*b^2*c^
(3/2)*arctanh(x*c^(1/2))*ln(-2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2)
)/(1+x*c^(1/2)))-1/3*b^2*c^(3/2)*arctanh(x*c^(1/2))*ln(2*(1+x*(-c)^(1/2))*c
^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))-1/3*b^2*c^(3/2)*arctan(x*c^(1/2)
)*ln((1-I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))-1/3*I*b^2*c^(3/2)*arctan(x*c^(1/2
))^2-1/3*I*b^2*c^(3/2)*polylog(2,1-2/(1-I*x*c^(1/2)))-1/3*I*b^2*c^(3/2)*pol
ylog(2,1-2/(1+I*x*c^(1/2)))+1/6*I*b^2*c^(3/2)*polylog(2,1-(1+I)*(1-x*c^(1/2
))/(1-I*x*c^(1/2)))+1/6*I*b^2*c^(3/2)*polylog(2,1+(-1+I)*(1+x*c^(1/2))/(1-I
*x*c^(1/2)))
```

Rubi [A]
time = 1.27, antiderivative size = 1102, normalized size of antiderivative = 1.00, number of steps used = 64, number of rules used = 24, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {6041, 2507, 2526, 2505, 212, 213, 2520, 12, 6131, 6055, 2449, 2352, 331, 6874, 209, 30, 2637, 6139, 6057, 2497, 5048, 4966, 5040, 4964}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/x^4,x]
```

```
[Out] (-2*a*b*c)/(3*x) - (2*a*b*c^(3/2)*ArcTan[Sqrt[c]*x])/3 + (4*b^2*c^(3/2)*Arc
Tan[Sqrt[c]*x])/3 - (I/3)*b^2*c^(3/2)*ArcTan[Sqrt[c]*x]^2 + (4*b^2*c^(3/2)*
```

```
ArcTanh[Sqrt[c]*x])/3 + (b^2*c^(3/2)*ArcTanh[Sqrt[c]*x]^2)/3 - (2*b^2*c^(3/
2)*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/3 + (2*b^2*c^(3/2)*ArcTan[Sqr
t[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/3 - (b^2*c^(3/2)*ArcTan[Sqrt[c]*x]*Log[((
1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/3 - (2*b^2*c^(3/2)*ArcTan[Sqrt[
c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/3 + (2*b^2*c^(3/2)*ArcTanh[Sqrt[c]*x]*Log[2
/(1 + Sqrt[c]*x)])/3 - (b^2*c^(3/2)*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 -
 Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/3 - (b^2*c^(3/2)*Arc
Tanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 +
 Sqrt[c]*x))])/3 - (b^2*c^(3/2)*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]
*x))/(1 - I*Sqrt[c]*x)])/3 + (b^2*c*Log[1 - c*x^2])/(3*x) + (b^2*c^(3/2)*Ar
cTan[Sqrt[c]*x]*Log[1 - c*x^2])/3 - (b*c*(2*a - b*Log[1 - c*x^2]))/(3*x) +
(b*c^(3/2)*ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/3 - (2*a - b*Log[1
- c*x^2])^2/(12*x^3) - (a*b*Log[1 + c*x^2])/(3*x^3) - (2*b^2*c*Log[1 + c*x^
2])/(3*x) - (b^2*c^(3/2)*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/3 + (b^2*c^(3/2)
*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2])/3 + (b^2*Log[1 - c*x^2]*Log[1 + c*x^2])
/(6*x^3) - (b^2*Log[1 + c*x^2]^2)/(12*x^3) - (b^2*c^(3/2)*PolyLog[2, 1 - 2/
(1 - Sqrt[c]*x)])/3 - (I/3)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)]
 + (I/6)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c
]*x)] - (I/3)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)] - (b^2*c^(3/2
)*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/3 + (b^2*c^(3/2)*PolyLog[2, 1 + (2*Sqr
t[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/6 + (b^2*c^
(3/2)*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1
+ Sqrt[c]*x))])/6 + (I/6)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt[c]*
x))/(1 - I*Sqrt[c]*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6041

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p
, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^4}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x^4} - \frac{b(-2a+b\log\left(1-cx^2\right))\log\left(1+cx^2\right)}{2x^4} + \frac{b^2\log^2}{4}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{x^4}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{x^4}\,dx$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{12x^3} - \frac{b^2\log^2\left(1+cx^2\right)}{12x^3} - \frac{1}{2}b\int \left(-\frac{2a\log\left(1+cx^2\right)}{x^4} + \frac{b\log}{}\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{12x^3} - \frac{b^2\log^2\left(1+cx^2\right)}{12x^3} + (ab)\int \frac{\log\left(1+cx^2\right)}{x^4}\,dx - \frac{1}{2}b^2$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{12x^3} - \frac{ab\log\left(1+cx^2\right)}{3x^3} + \frac{b^2\log\left(1-cx^2\right)\log\left(1+cx^2\right)}{6x^3} -$$

$$= -\frac{2abc}{3x} - \frac{bc(2a-b\log\left(1-cx^2\right))}{3x} + \frac{1}{3}bc^{3/2}\tanh^{-1}\left(\sqrt{c}\,x\right)\left(2a-b\log\left(1-cx^2\right.\right.$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{2}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{2}{3}b^2c^{3/2}\tanh^{-1}\left(\sqrt{c}\right.$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{2}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}\right.$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{2}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}\right.$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}\right.$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}\right.$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}\right.$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}\right.$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}\right.$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}\right.$$

Mathematica [F]
time = 2.30, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x^4,x]
```

```
[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/x^4, x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\,x^2\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x^4,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*((2*sqrt(c)*arctan(sqrt(c)*x) + sqrt(c)*log((c*x - sqrt(c))/(c*x + sqr
t(c))) + 4/x)*c + 2*arctanh(c*x^2)/x^3)*a*b - 1/12*b^2*(log(-c*x^2 + 1)^2/x
^3 + 3*integrate(-1/3*(3*(c*x^2 - 1)*log(c*x^2 + 1)^2 + 2*(2*c*x^2 - 3*(c*x
^2 - 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^6 - x^4), x)) - 1/3*a^2/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(cx^{2}\right)\right)^{2}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x**4,x)
[Out] Integral((a + b*atanh(c*x**2))**2/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^4,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^2) + a)^2/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)}^2}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/x^4,x)
[Out] int((a + b*atanh(c*x^2))^2/x^4, x)
```

3.76 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^6}\,dx$

Optimal. Leaf size=1176

$$-\frac{2abc}{15x^3}+\frac{2abc^2}{5x}-\frac{8b^2c^2}{15x}+\frac{2}{5}abc^{5/2}\text{ArcTan}\left(\sqrt{c}\,x\right)-\frac{4}{15}b^2c^{5/2}\text{ArcTan}\left(\sqrt{c}\,x\right)+\frac{1}{5}ib^2c^{5/2}\text{ArcTan}\left(\sqrt{c}\,x\right)^2+\frac{4}{15}b^2c^{5}$$

[Out] -4/15*b^2*c^(5/2)*arctan(x*c^(1/2))+4/15*b^2*c^(5/2)*arctanh(x*c^(1/2))+1/5*b^2*c^(5/2)*arctanh(x*c^(1/2))^2-1/20*b^2*ln(c*x^2+1)^2/x^5-1/5*b^2*c^(5/2)*polylog(2,1-2/(1-x*c^(1/2)))-1/5*b^2*c^(5/2)*polylog(2,1-2/(1+x*c^(1/2)))+1/10*b^2*c^(5/2)*polylog(2,1+2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(1/2)))+1/10*b^2*c^(5/2)*polylog(2,1-2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))-8/15*b^2*c^2/x+2/5*a*b*c^(5/2)*arctan(x*c^(1/2))+1/15*b^2*c*ln(-c*x^2+1)/x^3-1/5*b^2*c^2*ln(-c*x^2+1)/x-1/5*b^2*c^(5/2)*arctan(x*c^(1/2))*ln(-c*x^2+1)-1/15*b*c*(2*a-b*ln(-c*x^2+1))/x^3-1/5*b*c^2*(2*a-b*ln(-c*x^2+1))/x+1/5*b*c^(5/2)*arctanh(x*c^(1/2))*(2*a-b*ln(-c*x^2+1))-1/5*a*b*ln(c*x^2+1)/x^5-2/15*b^2*c*ln(c*x^2+1)/x^3+1/5*b^2*c^(5/2)*arctan(x*c^(1/2))*ln(c*x^2+1)+1/5*b^2*c^(5/2)*arctanh(x*c^(1/2))*ln(c*x^2+1)+1/10*b^2*ln(-c*x^2+1)*ln(c*x^2+1)/x^5-2/5*b^2*c^(5/2)*arctanh(x*c^(1/2))*ln(2/(1-x*c^(1/2)))-2/5*b^2*c^(5/2)*arctan(x*c^(1/2))*ln(2/(1-I*x*c^(1/2)))+1/5*b^2*c^(5/2)*arctan(x*c^(1/2))*ln((1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2)))+2/5*b^2*c^(5/2)*arctan(x*c^(1/2))*ln(2/(1+I*x*c^(1/2)))+2/5*b^2*c^(5/2)*arctanh(x*c^(1/2))*ln(2/(1+x*c^(1/2)))-1/5*b^2*c^(5/2)*arctanh(x*c^(1/2))*ln(-2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(1/2)))-1/5*b^2*c^(5/2)*arctanh(x*c^(1/2))*ln(2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))+1/5*b^2*c^(5/2)*arctan(x*c^(1/2))*ln((1-I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))-1/10*I*b^2*c^(5/2)*polylog(2,1-(1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2)))-1/10*I*b^2*c^(5/2)*polylog(2,1+(-1+I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))+1/5*I*b^2*c^(5/2)*polylog(2,1-2/(1-I*x*c^(1/2)))+1/5*I*b^2*c^(5/2)*polylog(2,1-2/(1+I*x*c^(1/2)))+1/5*I*b^2*c^(5/2)*arctan(x*c^(1/2))^2-1/20*(2*a-b*ln(-c*x^2+1))^2/x^5-2/15*a*b*c/x^3+2/5*a*b*c^2/x

Rubi [A]
time = 1.39, antiderivative size = 1176, normalized size of antiderivative = 1.00, number of steps used = 77, number of rules used = 24, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {6041, 2507, 2526, 2505, 331, 212, 213, 2520, 12, 6131, 6055, 2449, 2352, 6874, 209, 30, 2637, 6139, 6057, 2497, 5048, 4966, 5040, 4964}

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x^2])^2/x^6,x]

[Out] (-2*a*b*c)/(15*x^3) + (2*a*b*c^2)/(5*x) - (8*b^2*c^2)/(15*x) + (2*a*b*c^(5/2)*ArcTan[Sqrt[c]*x])/5 - (4*b^2*c^(5/2)*ArcTan[Sqrt[c]*x])/15 + (I/5)*b^2*

```
c^(5/2)*ArcTan[Sqrt[c]*x]^2 + (4*b^2*c^(5/2)*ArcTanh[Sqrt[c]*x])/15 + (b^2*
c^(5/2)*ArcTanh[Sqrt[c]*x]^2)/5 - (2*b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[2/(
1 - Sqrt[c]*x)])/5 - (2*b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*
x)])/5 + (b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 -
I*Sqrt[c]*x)])/5 + (2*b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)
])/5 + (2*b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)])/5 - (b^2*c
^(5/2)*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sq
rt[c])*(1 + Sqrt[c]*x))])/5 - (b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c
]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/5 + (b^2*c^(5/
2)*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/5 +
(b^2*c*Log[1 - c*x^2])/(15*x^3) - (b^2*c^2*Log[1 - c*x^2])/(5*x) - (b^2*c^(
5/2)*ArcTan[Sqrt[c]*x]*Log[1 - c*x^2])/5 - (b*c*(2*a - b*Log[1 - c*x^2]))/(
15*x^3) - (b*c^2*(2*a - b*Log[1 - c*x^2]))/(5*x) + (b*c^(5/2)*ArcTanh[Sqrt[
c]*x]*(2*a - b*Log[1 - c*x^2]))/5 - (2*a - b*Log[1 - c*x^2])^2/(20*x^5) - (
a*b*Log[1 + c*x^2])/(5*x^5) - (2*b^2*c*Log[1 + c*x^2])/(15*x^3) + (b^2*c^(5
/2)*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/5 + (b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*L
og[1 + c*x^2])/5 + (b^2*Log[1 - c*x^2]*Log[1 + c*x^2])/(10*x^5) - (b^2*Log[
1 + c*x^2]^2)/(20*x^5) - (b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)])/5
+ (I/5)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)] - (I/10)*b^2*c^(5/2
)*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)] + (I/5)*b^2*c
^(5/2)*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)] - (b^2*c^(5/2)*PolyLog[2, 1 - 2/
(1 + Sqrt[c]*x)])/5 + (b^2*c^(5/2)*PolyLog[2, 1 + (2*Sqrt[c]*(1 - Sqrt[-c]*
x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/10 + (b^2*c^(5/2)*PolyLog[2, 1
 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/10
 - (I/10)*b^2*c^(5/2)*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[
c]*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
```

```
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6041

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p
, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^6}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x^6} - \frac{b(-2a+b\log\left(1-cx^2\right))\log\left(1+cx^2\right)}{2x^6} + \frac{b^2\log$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{x^6}\,dx - \frac{1}{2}b\int \frac{(-2a+b\log\left(1-cx^2\right))\log\left(1+cx^2\right)}{x^6}$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{20x^5} - \frac{b^2\log^2\left(1+cx^2\right)}{20x^5} - \frac{1}{2}b\int \left(-\frac{2a\log\left(1+cx^2\right)}{x^6} + \frac{b$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{20x^5} - \frac{b^2\log^2\left(1+cx^2\right)}{20x^5} + (ab)\int \frac{\log\left(1+cx^2\right)}{x^6}\,dx - \frac{1}{2}b^2$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{20x^5} - \frac{ab\log\left(1+cx^2\right)}{5x^5} + \frac{b^2\log\left(1-cx^2\right)\log\left(1+cx^2\right)}{10x^5} -$$

$$= -\frac{2abc}{15x^3} - \frac{bc(2a-b\log\left(1-cx^2\right))}{15x^3} - \frac{bc^2(2a-b\log\left(1-cx^2\right))}{5x} + \frac{1}{5}bc^{5/2}\tanh^{-}$$

$$= -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{4b^2c^2}{15x} - \frac{2}{5}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{2}{5}b^2c^{5/2}\tanh^{-1}\left(\sqrt{c}\,x\right) -$$

$$= -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{4b^2c^2}{15x} + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{8}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right) +$$

$$= -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{4b^2c^2}{15x} + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{8}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right) +$$

$$= -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{8b^2c^2}{15x} + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{2}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right) +$$

$$= -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{8b^2c^2}{15x} + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right) +$$

$$= -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{8b^2c^2}{15x} + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right) +$$

$$= -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{8b^2c^2}{15x} + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right) +$$

$$= -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{8b^2c^2}{15x} + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right) +$$

$$= -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{8b^2c^2}{15x} + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right) +$$

Mathematica [F]
time = 2.32, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x^6,x]
```

```
[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/x^6, x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\,x^2\right)\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x^6,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/15*((6*c^(3/2)*arctan(sqrt(c)*x) - 3*c^(3/2)*log((c*x - sqrt(c))/(c*x + s
qrt(c))) - 4/x^3)*c - 6*arctanh(c*x^2)/x^5)*a*b - 1/20*b^2*(log(-c*x^2 + 1)
^2/x^5 + 5*integrate(-1/5*(5*(c*x^2 - 1)*log(c*x^2 + 1)^2 + 2*(2*c*x^2 - 5*
(c*x^2 - 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^8 - x^6), x)) - 1/5*a^2/x
^5
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^6,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x^6, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}{\left(c x^{2} \right)}\right)^{2}}{x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x**6,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/x**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)}^2}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/x^6,x)
```

```
[Out] int((a + b*atanh(c*x^2))^2/x^6, x)
```

3.77 $\int x^3 \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 dx$

Optimal. Leaf size=141

$$\frac{3b\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{4c^2}+\frac{3bx^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{4c}-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{4c^2}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3-\frac{3b^2(}{}$$

```
[Out] 3/4*b*(a+b*arctanh(c*x^2))^2/c^2+3/4*b*x^2*(a+b*arctanh(c*x^2))^2/c-1/4*(a+
b*arctanh(c*x^2))^3/c^2+1/4*x^4*(a+b*arctanh(c*x^2))^3-3/2*b^2*(a+b*arctanh
(c*x^2))*ln(2/(-c*x^2+1))/c^2-3/4*b^3*polylog(2,1-2/(-c*x^2+1))/c^2
```

Rubi [A]

time = 0.22, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {6039, 6037, 6127, 6021, 6131, 6055, 2449, 2352, 6095}

$$-\frac{3b^2\log\left(\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{2c^2}-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{4c^2}+\frac{3b\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{4c^2}+\frac{3bx^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{4c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3-\frac{3b^3\text{Li}_2\left(1-\frac{2}{1-cx^2}\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x^2])^3,x]
```

```
[Out] (3*b*(a + b*ArcTanh[c*x^2])^2)/(4*c^2) + (3*b*x^2*(a + b*ArcTanh[c*x^2])^2)
/(4*c) - (a + b*ArcTanh[c*x^2])^3/(4*c^2) + (x^4*(a + b*ArcTanh[c*x^2])^3)/
4 - (3*b^2*(a + b*ArcTanh[c*x^2])*Log[2/(1 - c*x^2)])/(2*c^2) - (3*b^3*Poly
Log[2, 1 - 2/(1 - c*x^2)])/(4*c^2)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3 \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 \,dx = \int \left(\frac{1}{8}x^3\left(2a-b\log\left(1-cx^2\right)\right)^3+\frac{3}{8}bx^3\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+cx\right.\right.$$

$$= \frac{1}{8}\int x^3\left(2a-b\log\left(1-cx^2\right)\right)^3\,dx+\frac{1}{8}(3b)\int x^3\left(-2a+b\log\left(1-cx^2\right)\right)^2\log$$

$$= \frac{1}{16}\text{Subst}\left(\int x(2a-b\log(1-cx))^3\,dx,x,x^2\right)+\frac{1}{16}(3b)\text{Subst}\left(\int x(-2a+\right.$$

$$= \frac{3}{32}bx^4\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)+\frac{3}{32}b^2x^4\left(2a-b\log\left(1-cx^2\right)\right)\text{l}$$

$$= \frac{3}{32}bx^4\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)+\frac{3}{32}b^2x^4\left(2a-b\log\left(1-cx^2\right)\right)\text{l}$$

$$= \frac{3}{32}bx^4\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)+\frac{3}{32}b^2x^4\left(2a-b\log\left(1-cx^2\right)\right)\text{l}$$

$$= -\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c^2}+\frac{\left(1-cx^2\right)^2\left(2a-b\log\left(1-cx^2\right)\right)^3}{32c^2}-\frac{3b}{}$$

$$= -\frac{9b(1-cx^2)\left(2a-b\log\left(1-cx^2\right)\right)^2}{32c^2}+\frac{3b(1-cx^2)^2\left(2a-b\log\left(1-cx^2\right)\right)^2}{64c^2}$$

$$= \frac{9ab^2x^2}{8c}+\frac{9b^3x^2}{16c}+\frac{3b^3(1-cx^2)^2}{128c^2}-\frac{3b^3(1+cx^2)^2}{128c^2}+\frac{3b^2(1-cx^2)^2\left(2a-b\,\text{lo}\right.}{64c^2}$$

$$= \frac{9ab^2x^2}{8c}+\frac{9b^3x^2}{8c}+\frac{3b^3(1-cx^2)^2}{128c^2}-\frac{3b^3(1+cx^2)^2}{128c^2}+\frac{9b^3(1-cx^2)\log\left(1-cx\right.}{16c^2}$$

$$= \frac{3ab^2x^2}{4c}+\frac{15b^3x^2}{16c}+\frac{9b^3(1-cx^2)\log\left(1-cx^2\right)}{16c^2}-\frac{3b(1-cx^2)\left(2a-b\log\left(1-\right.\right.}{16c^2}$$

$$= \frac{3ab^2x^2}{4c}+\frac{3b^3x^2}{4c}+\frac{3b^3(1-cx^2)\log\left(1-cx^2\right)}{8c^2}-\frac{3b(1-cx^2)\left(2a-b\log\left(1-\right.\right.}{16c^2}$$

Mathematica [A]
time = 0.34, size = 185, normalized size = 1.31

$$\frac{6b^2(-1+cx^2)\left(a+b+acx^2\right)\tanh^{-1}\left(cx^2\right)^2+2b^3(-1+c^2x^4)\tanh^{-1}\left(cx^2\right)^3+6b\tanh^{-1}\left(cx^2\right)\left(acx^2(2b+acx^2)-2b^2\log\left(1+e^{-2\tanh^{-1}(cx^2)}\right)\right)+a(6abcx^2+2a^2c^2x^4+3ab\log\left(1-cx^2\right)-3ab\log\left(1+cx^2\right)+6b^2\log\left(1-c^2x^4\right))+6b^3\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx^2)}\right)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x^2])^3,x]
```

```
[Out] (6*b^2*(-1 + c*x^2)*(a + b + a*c*x^2)*ArcTanh[c*x^2]^2 + 2*b^3*(-1 + c^2*x^
4)*ArcTanh[c*x^2]^3 + 6*b*ArcTanh[c*x^2]*(a*c*x^2*(2*b + a*c*x^2) - 2*b^2*L
og[1 + E^(-2*ArcTanh[c*x^2])]) + a*(6*a*b*c*x^2 + 2*a^2*c^2*x^4 + 3*a*b*Log
[1 - c*x^2] - 3*a*b*Log[1 + c*x^2] + 6*b^2*Log[1 - c^2*x^4]) + 6*b^3*PolyLo
g[2, -E^(-2*ArcTanh[c*x^2])])/(8*c^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 798, normalized size = 5.66

method	result	size
risch	Expression too large to display	798

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*b^3*(c^2*x^4-1)/c^2*ln(c*x^2+1)^3+3/32*b^2*(-b*c^2*ln(-c*x^2+1)*x^4+2*
a*c^2*x^4+2*b*c*x^2+b*ln(-c*x^2+1)-2*a+2*b)/c^2*ln(c*x^2+1)^2+(3/32*b^3*(c^
2*x^4-1)/c^2*ln(-c*x^2+1)^2-3/32*b^2*(2*a*c*x^2+b)^2/c^2/a*ln(-c*x^2+1)-3/3
2*b*(-4*a^3*c^2*x^4-8*a^2*b*c*x^2-4*ln(-c*x^2+1)*a^2*b-4*ln(-c*x^2+1)*a*b^2
-ln(-c*x^2+1)*b^3-4*a*b^2)/c^2/a)*ln(c*x^2+1)-3/4*a*b^2/c*x^2*ln(-c*x^2+1)+
1/4*a^3*x^4+3/4*b^2/c^2*ln(c*x^2+1)*a+3/16*b^2/c^2*a*ln(c*x^2-1)-3/8*b/c^2*
ln(c*x^2+1)*a^2-1/32*b^3*x^4*ln(-c*x^2+1)^3-3/16*b^3/c^2*ln(-c*x^2+1)^2+1/3
2*b^3/c^2*ln(-c*x^2+1)^3-3/8/c^2*b^3*ln(c*x^2-1)-3/8/c^2*b^3*ln(c*x^2+1)-3/
16*b^3/c^2+3/8/c^2*b^3*ln(-c*x^2+1)+3/16*b^3/c*x^2*ln(-c*x^2+1)^2-3/8*a^2*b
*x^4*ln(-c*x^2+1)+3/4*a^2*b/c*x^2+3/8*a^2*b/c^2*ln(c*x^2-1)+3/16*a*b^2*x^4*
ln(-c*x^2+1)^2+9/16*a*b^2/c^2*ln(-c*x^2+1)-3/16*a*b^2/c^2*ln(-c*x^2+1)^2+3/
4*b^2/c*Sum(-(ln(x-_alpha)*ln(-c*x^2+1)+2*c*(-1/2*ln(x-_alpha)*(ln((RootOf(
_Z^2*c+2*_Z*_alpha*c-2,index=1)-x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c-2,ind
ex=1))+ln((RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=2)-x+_alpha)/RootOf(_Z^2*c+2
*_Z*_alpha*c-2,index=2)))/c-1/2*(dilog((RootOf(_Z^2*c+2*_Z*_alpha*c-2,index
=1)-x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=1))+dilog((RootOf(_Z^2*c+
2*_Z*_alpha*c-2,index=2)-x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=2)))
/c))*b/c,_alpha=RootOf(_Z^2*c+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arctanh(c*x^2)^2 + 1/4*a^3*x^4 + 3/8*(2*x^4*arctanh(c*x^2) +
c*(2*x^2/c^2 - log(c*x^2 + 1)/c^3 + log(c*x^2 - 1)/c^3))*a^2*b + 3/16*(4*c*
(2*x^2/c^2 - log(c*x^2 + 1)/c^3 + log(c*x^2 - 1)/c^3)*arctanh(c*x^2) - (2*(
```

```
log(c*x^2 - 1) - 2)*log(c*x^2 + 1) - log(c*x^2 + 1)^2 - log(c*x^2 - 1)^2 -
4*log(c*x^2 - 1))/c^2)*a*b^2 - 1/128*(4*x^4*log(-c*x^2 + 1)^3 + 3*c^3*(x^4/
c^3 + log(c^2*x^4 - 1)/c^5) - 6*c*((c*x^4 + 2*x^2)/c^2 + 2*log(c*x^2 - 1)/c
^3)*log(-c*x^2 + 1)^2 + 21*c^2*(2*x^2/c^3 - log(c*x^2 + 1)/c^4 + log(c*x^2
- 1)/c^4) + c*(6*(c^2*x^4 + 6*c*x^2 + 2*log(c*x^2 - 1)^2 + 6*log(c*x^2 - 1)
)*log(-c*x^2 + 1)/c^3 - (3*c^2*x^4 + 42*c*x^2 + 4*log(c*x^2 - 1)^3 + 18*log
(c*x^2 - 1)^2 + 42*log(c*x^2 - 1))/c^3) - 1152*c*integrate(1/4*x^3*log(c*x^
2 + 1)/(c^3*x^4 - c), x) - 2*(12*c*x^2*log(c*x^2 + 1)^2 + 2*(c^2*x^4 - 1)*l
og(c*x^2 + 1)^3 - 3*(c^2*x^4 - 2*c*x^2 - 2*(c^2*x^4 - 1)*log(c*x^2 + 1) + 1
)*log(-c*x^2 + 1)^2 + 3*(c^2*x^4 + 6*c*x^2 - 2*(c^2*x^4 - 1)*log(c*x^2 + 1)
^2 - 8*(c*x^2 + 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/c^2 + 18*log(4*c^3*x^4
- 4*c)/c^2 - 384*integrate(1/4*x*log(c*x^2 + 1)/(c^3*x^4 - c), x))*b^3
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2))^3,x, algorithm="fricas")
[Out] integral(b^3*x^3*arctanh(c*x^2)^3 + 3*a*b^2*x^3*arctanh(c*x^2)^2 + 3*a^2*b*
x^3*arctanh(c*x^2) + a^3*x^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + b \operatorname{atanh}\left(cx^2\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**2))**3,x)
[Out] Integral(x**3*(a + b*atanh(c*x**2))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2))^3,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^2) + a)^3*x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(a + b \operatorname{atanh}\left(c x^2\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x^2))^3,x)
[Out] int(x^3*(a + b*atanh(c*x^2))^3, x)
```

3.78 $\int x\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\,dx$

Optimal. Leaf size=134

$$\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{2c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3-\frac{3b\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\log\left(\frac{2}{1-cx^2}\right)}{2c}-\frac{3b^2\left(a+b\tanh^{-1}\left(cx\right.\right.}{}$$

```
[Out] 1/2*(a+b*arctanh(c*x^2))^3/c+1/2*x^2*(a+b*arctanh(c*x^2))^3-3/2*b*(a+b*arct
anh(c*x^2))^2*ln(2/(-c*x^2+1))/c-3/2*b^2*(a+b*arctanh(c*x^2))*polylog(2,1-2
/(-c*x^2+1))/c+3/4*b^3*polylog(3,1-2/(-c*x^2+1))/c
```

Rubi [A]
time = 0.19, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6039, 6021, 6131, 6055, 6095, 6205, 6745}

$$-\frac{3b^2\text{Li}_2\left(1-\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{2c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3+\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{2c}-\frac{3b\log\left(\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{2c}+\frac{3b^3\text{Li}_3\left(1-\frac{2}{1-cx^2}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x^2])^3,x]
```

```
[Out] (a + b*ArcTanh[c*x^2])^3/(2*c) + (x^2*(a + b*ArcTanh[c*x^2])^3)/2 - (3*b*(a
 + b*ArcTanh[c*x^2])^2*Log[2/(1 - c*x^2)])/(2*c) - (3*b^2*(a + b*ArcTanh[c*
x^2])*PolyLog[2, 1 - 2/(1 - c*x^2)])/(2*c) + (3*b^3*PolyLog[3, 1 - 2/(1 - c
*x^2)])/(4*c)
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
```

```
0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int x\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\,dx &= \int\left(\frac{1}{8}x\left(2a-b\log\left(1-cx^2\right)\right)^3+\frac{3}{8}bx\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)\right. \\
&= \frac{1}{8}\int x\left(2a-b\log\left(1-cx^2\right)\right)^3\,dx+\frac{1}{8}(3b)\int x\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1\right. \\
&= \frac{1}{16}\text{Subst}\left(\int(2a-b\log(1-cx))^3\,dx,x,x^2\right)+\frac{1}{16}(3b)\text{Subst}\left(\int\left(-2a+b\log\right.\right. \\
&= \frac{3}{16}bx^2\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)+\frac{3}{16}b^2x^2\left(2a-b\log\left(1-cx^2\right)\right)\text{lo} \\
&= -\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}+\frac{3}{16}bx^2\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(1+c\right. \\
&= -\frac{3b(1-cx^2)\left(2a-b\log\left(1-cx^2\right)\right)^2}{16c}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}+\frac{3}{16} \\
&= \frac{3}{4}ab^2x^2-\frac{3b^3x^2}{8}-\frac{3b(1-cx^2)\left(2a-b\log\left(1-cx^2\right)\right)^2}{16c}-\frac{\left(1-cx^2\right)\left(2a-b\log\right.}{16c} \\
&= \frac{3}{4}ab^2x^2+\frac{3b^3(1-cx^2)\log\left(1-cx^2\right)}{8c}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}+\frac{3b}{} \\
&= \frac{3b^3x^2}{8}+\frac{3b^3(1-cx^2)\log\left(1-cx^2\right)}{8c}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}+\frac{3b(}{} \\
&= -\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}+\frac{3b(2a-b\log\left(1-cx^2\right))^2\log\left(\frac{1}{2}(1+cx^2\right.}{8c} \\
&= -\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}+\frac{3b(2a-b\log\left(1-cx^2\right))^2\log\left(\frac{1}{2}(1+cx^2\right.}{8c}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 211, normalized size = 1.57

$$\frac{2a^3cx^2+6a^2bcx^2\tanh^{-1}\left(cx^2\right)-6ab^2\tanh^{-1}\left(cx^2\right)^2+6ab^2cx^2\tanh^{-1}\left(cx^2\right)^2-2b^3\tanh^{-1}\left(cx^2\right)^3+2b^3cx^2\tanh^{-1}\left(cx^2\right)^3-12ab^2\tanh^{-1}\left(cx^2\right)\log\left(1+e^{-2\tanh^{-1}(cx^2)}\right)-6b^3\tanh^{-1}\left(cx^2\right)^2\log\left(1+e^{-2\tanh^{-1}(cx^2)}\right)+3a^2b\log\left(1-c^2x^4\right)+6b^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx^2)}\right)+3b^3\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx^2)}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^2])^3,x]
```

```
[Out] (2*a^3*c*x^2 + 6*a^2*b*c*x^2*ArcTanh[c*x^2] - 6*a*b^2*ArcTanh[c*x^2]^2 + 6*
a*b^2*c*x^2*ArcTanh[c*x^2]^2 - 2*b^3*ArcTanh[c*x^2]^3 + 2*b^3*c*x^2*ArcTanh
[c*x^2]^3 - 12*a*b^2*ArcTanh[c*x^2]*Log[1 + E^(-2*ArcTanh[c*x^2])] - 6*b^3*
ArcTanh[c*x^2]^2*Log[1 + E^(-2*ArcTanh[c*x^2])] + 3*a^2*b*Log[1 - c^2*x^4]
```

```
+ 6*b^2*(a + b*ArcTanh[c*x^2])*PolyLog[2, -E^(-2*ArcTanh[c*x^2])] + 3*b^3*P
olyLog[3, -E^(-2*ArcTanh[c*x^2])])/(4*c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(124) = 248$.
time = 0.32, size = 280, normalized size = 2.09

method	result
derivativedivides	$\frac{a^3 c x^2+b^3 \operatorname{arctanh}(c x^2)^3 c x^2+b^3 \operatorname{arctanh}(c x^2)^3-3b^3 \operatorname{arctanh}(c x^2)^2 \ln\left(1+\frac{(c x^2+1)^2}{-c^2 x^4+1}\right)-3b^3 \operatorname{arctanh}(c x^2) \operatorname{polylog}\left(2\right.}{}$
default	$\frac{a^3 c x^2+b^3 \operatorname{arctanh}(c x^2)^3 c x^2+b^3 \operatorname{arctanh}(c x^2)^3-3b^3 \operatorname{arctanh}(c x^2)^2 \ln\left(1+\frac{(c x^2+1)^2}{-c^2 x^4+1}\right)-3b^3 \operatorname{arctanh}(c x^2) \operatorname{polylog}\left(2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c*(a^3*c*x^2+b^3*arctanh(c*x^2)^3*c*x^2+b^3*arctanh(c*x^2)^3-3*b^3*arct
anh(c*x^2)^2*ln(1+(c*x^2+1)^2/(-c^2*x^4+1))-3*b^3*arctanh(c*x^2)*polylog(2,
-(c*x^2+1)^2/(-c^2*x^4+1))+3/2*b^3*polylog(3,-(c*x^2+1)^2/(-c^2*x^4+1))+3*a
rctanh(c*x^2)^2*a*b^2*c*x^2+3*a*b^2*arctanh(c*x^2)^2-6*arctanh(c*x^2)*ln(1+
(c*x^2+1)^2/(-c^2*x^4+1))*a*b^2-3*polylog(2,-(c*x^2+1)^2/(-c^2*x^4+1))*a*b^
2+3*a^2*b*c*x^2*arctanh(c*x^2)+3/2*a^2*b*ln(-c^2*x^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*x^2 + 3/4*(2*c*x^2*arctanh(c*x^2) + log(-c^2*x^4 + 1))*a^2*b/c - 1/
16*((b^3*c*x^2 - b^3)*log(-c*x^2 + 1)^3 - 3*(2*a*b^2*c*x^2 + (b^3*c*x^2 + b
^3)*log(c*x^2 + 1))*log(-c*x^2 + 1)^2)/c - integrate(-1/8*((b^3*c*x^3 - b^3
*x)*log(c*x^2 + 1)^3 + 6*(a*b^2*c*x^3 - a*b^2*x)*log(c*x^2 + 1)^2 - 3*(4*a*
b^2*c*x^3 + (b^3*c*x^3 - b^3*x)*log(c*x^2 + 1)^2 + 2*((2*a*b^2*c + b^3*c)*x
^3 - (2*a*b^2 - b^3)*x)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^2 - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctanh(c*x^2)^3 + 3*a*b^2*x*arctanh(c*x^2)^2 + 3*a^2*b*x*ar
ctanh(c*x^2) + a^3*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\operatorname{atanh}\left(cx^2\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**2))**3,x)
```

```
[Out] Integral(x*(a + b*atanh(c*x**2))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^3*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^2))^3,x)
```

```
[Out] int(x*(a + b*atanh(c*x^2))^3, x)
```

3.79 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x}\,dx$

Optimal. Leaf size=207

$$\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)-\frac{3}{4}b\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\text{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)+\frac{3}{4}b(a+$$

```
[Out] -(a+b*arctanh(c*x^2))^3*arctanh(-1+2/(-c*x^2+1))-3/4*b*(a+b*arctanh(c*x^2))
^2*polylog(2,1-2/(-c*x^2+1))+3/4*b*(a+b*arctanh(c*x^2))^2*polylog(2,-1+2/(-
c*x^2+1))+3/4*b^2*(a+b*arctanh(c*x^2))*polylog(3,1-2/(-c*x^2+1))-3/4*b^2*(a
+b*arctanh(c*x^2))*polylog(3,-1+2/(-c*x^2+1))-3/8*b^3*polylog(4,1-2/(-c*x^2
+1))+3/8*b^3*polylog(4,-1+2/(-c*x^2+1))
```

Rubi [A]
time = 0.37, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6035, 6033, 6199, 6095, 6205, 6209, 6745}

$$\frac{3}{4}b^2\text{Li}_3\left(1-\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{3}{4}b^2\text{Li}_3\left(\frac{2}{1-cx^2}-1\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{3}{4}b\text{Li}_2\left(1-\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2+\frac{3}{4}b\text{Li}_2\left(\frac{2}{1-cx^2}-1\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2+\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3-\frac{3}{8}b^3\text{Li}_4\left(1-\frac{2}{1-cx^2}\right)+\frac{3}{8}b^3\text{Li}_4\left(\frac{2}{1-cx^2}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^3/x,x]
```

```
[Out] (a + b*ArcTanh[c*x^2])^3*ArcTanh[1 - 2/(1 - c*x^2)] - (3*b*(a + b*ArcTanh[c
*x^2])^2*PolyLog[2, 1 - 2/(1 - c*x^2)])/4 + (3*b*(a + b*ArcTanh[c*x^2])^2*P
olyLog[2, -1 + 2/(1 - c*x^2)])/4 + (3*b^2*(a + b*ArcTanh[c*x^2])*PolyLog[3,
 1 - 2/(1 - c*x^2)])/4 - (3*b^2*(a + b*ArcTanh[c*x^2])*PolyLog[3, -1 + 2/(1
 - c*x^2)])/4 - (3*b^3*PolyLog[4, 1 - 2/(1 - c*x^2)])/8 + (3*b^3*PolyLog[4,
 -1 + 2/(1 - c*x^2)])/8
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6209

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}\,dx, x, x^2\right)$$

$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right) - (3bc)\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}\right.}{}\right.$$

$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right) + \frac{1}{2}(3bc)\text{Subst}\left(\int \frac{\left(a+b\tanh\right.}{}\right.$$

$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right) - \frac{3}{4}b\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\text{Li}_2\left(\right.$$

$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right) - \frac{3}{4}b\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\text{Li}_2\left(\right.$$

$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right) - \frac{3}{4}b\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\text{Li}_2\left(\right.$$

Mathematica [**A**]
time = 0.13, size = 211, normalized size = 1.02

$$\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1+\frac{2}{-1+cx^2}\right)+\frac{3}{8}b\left(2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\text{PolyLog}\left(2,\frac{1+cx^2}{1-cx^2}\right)-2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\text{PolyLog}\left(2,\frac{1+cx^2}{-1+cx^2}\right)+b\left(-2\left(a+b\tanh^{-1}\left(cx^2\right)\right)\text{PolyLog}\left(3,\frac{1+cx^2}{1-cx^2}\right)+2\left(a+b\tanh^{-1}\left(cx^2\right)\right)\text{PolyLog}\left(3,\frac{1+cx^2}{-1+cx^2}\right)+b\left(\text{PolyLog}\left(4,\frac{1+cx^2}{1-cx^2}\right)-\text{PolyLog}\left(4,\frac{1+cx^2}{-1+cx^2}\right)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^3/x,x]
```

```
[Out] (a + b*ArcTanh[c*x^2])^3*ArcTanh[1 + 2/(-1 + c*x^2)] + (3*b*(2*(a + b*ArcTa
nh[c*x^2])^2*PolyLog[2, (1 + c*x^2)/(1 - c*x^2)] - 2*(a + b*ArcTanh[c*x^2])
^2*PolyLog[2, (1 + c*x^2)/(-1 + c*x^2)] + b*(-2*(a + b*ArcTanh[c*x^2])*Poly
Log[3, (1 + c*x^2)/(1 - c*x^2)] + 2*(a + b*ArcTanh[c*x^2])*PolyLog[3, (1 +
c*x^2)/(-1 + c*x^2)] + b*(PolyLog[4, (1 + c*x^2)/(1 - c*x^2)] - PolyLog[4,
(1 + c*x^2)/(-1 + c*x^2)]))))/8
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\,x^2\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^3/x,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^3/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + integrate(1/8*b^3*(log(c*x^2 + 1) - log(-c*x^2 + 1))^3/x + 3/4
*a*b^2*(log(c*x^2 + 1) - log(-c*x^2 + 1))^2/x + 3/2*a^2*b*(log(c*x^2 + 1) -
 log(-c*x^2 + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^2)^3 + 3*a*b^2*arctanh(c*x^2)^2 + 3*a^2*b*arctanh
(c*x^2) + a^3)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(cx^{2}\right)\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**3/x,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)}^3}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^3/x,x)
[Out] int((a + b*atanh(c*x^2))^3/x, x)
```

3.80 $\int \frac{(a+b\tanh^{-1}(cx^2))^3}{x^3} dx$

Optimal. Leaf size=125

$$\frac{1}{2}c(a+b\tanh^{-1}(cx^2))^3-\frac{(a+b\tanh^{-1}(cx^2))^3}{2x^2}+\frac{3}{2}bc(a+b\tanh^{-1}(cx^2))^2\log\left(2-\frac{2}{1+cx^2}\right)-\frac{3}{2}b^2c(a+b\,\mathrm{t}$$

```
[Out] 1/2*c*(a+b*arctanh(c*x^2))^3-1/2*(a+b*arctanh(c*x^2))^3/x^2+3/2*b*c*(a+b*ar
ctanh(c*x^2))^2*ln(2-2/(c*x^2+1))-3/2*b^2*c*(a+b*arctanh(c*x^2))*polylog(2,
-1+2/(c*x^2+1))-3/4*b^3*c*polylog(3,-1+2/(c*x^2+1))
```

Rubi [A]
time = 0.23, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6039, 6037, 6135, 6079, 6095, 6203, 6745}

$$-\frac{3}{2}b^2c\,\mathrm{Li}_2\left(\frac{2}{cx^2+1}-1\right)(a+b\tanh^{-1}(cx^2))+\frac{1}{2}c(a+b\tanh^{-1}(cx^2))^3-\frac{(a+b\tanh^{-1}(cx^2))^3}{2x^2}+\frac{3}{2}bc\log\left(2-\frac{2}{cx^2+1}\right)(a+b\tanh^{-1}(cx^2))^2-\frac{3}{4}b^3c\,\mathrm{Li}_3\left(\frac{2}{cx^2+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^3/x^3,x]
```

```
[Out] (c*(a + b*ArcTanh[c*x^2])^3)/2 - (a + b*ArcTanh[c*x^2])^3/(2*x^2) + (3*b*c*
(a + b*ArcTanh[c*x^2])^2*Log[2 - 2/(1 + c*x^2)])/2 - (3*b^2*c*(a + b*ArcTan
h[c*x^2])*PolyLog[2, -1 + 2/(1 + c*x^2)])/2 - (3*b^3*c*PolyLog[3, -1 + 2/(1
 + c*x^2)])/4
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
```

```
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x^3}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{8x^3} + \frac{3b(-2a+b\log\left(1-cx^2\right))^2\log\left(1+cx^2\right)}{8x^3} - \frac{3b^2(-$$

$$= \frac{1}{8}\int \frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{x^3}\,dx + \frac{1}{8}(3b)\int \frac{\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+cx\right.}{x^3}$$

$$= \frac{1}{16}\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^3}{x^2}\,dx, x, x^2\right) + \frac{1}{16}(3b)\text{Subst}\left(\int \frac{(-2a+b\log}{}\right.$$

$$= -\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16x^2} - \frac{b^3(1+cx^2)\log^3\left(1+cx^2\right)}{16x^2} + \frac{1}{16}(3b)\text{Subst}$$

$$= \frac{3}{16}bc\log\left(cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2 - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16x^2} + \frac{3}{16}$$

$$= \frac{3}{16}bc\log\left(cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2 - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16x^2} + \frac{3}{16}$$

$$= \frac{3}{16}bc\log\left(cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2 - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16x^2} + \frac{3}{16}$$

$$= \frac{3}{16}bc\log\left(cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2 - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16x^2} + \frac{3}{16}$$

<u>**Mathematica**</u> **[C]** Result contains complex when optimal does not.
time = 0.29, size = 222, normalized size = 1.78

$$\frac{1}{4}\left(-\frac{2a^3}{x^2} - \frac{6a^2b\tanh^{-1}\left(cx^2\right)}{x^2} + 12a^2bc\log(x) - 3a^2bc\log\left(1-c^2x^4\right) + 6ab^2c\left(\tanh^{-1}\left(cx^2\right)\left(\left(1-\frac{1}{cx^2}\right)\tanh^{-1}\left(cx^2\right) + 2\log\left(1-e^{-2\tanh^{-1}(cx^2)}\right)\right) - \text{PolyLog}\left(2, e^{-2\tanh^{-1}(cx^2)}\right)\right) + 2b^3c\left(\frac{i\pi^3}{8} - \tanh^{-1}\left(cx^2\right)^3 - \frac{\tanh^{-1}\left(cx^2\right)^3}{cx^2} + 3\tanh^{-1}\left(cx^2\right)^2\log\left(1-e^{2\tanh^{-1}(cx^2)}\right) + 3\tanh^{-1}\left(cx^2\right)\text{PolyLog}\left(2, e^{2\tanh^{-1}(cx^2)}\right) - \frac{3}{2}\text{PolyLog}\left(3, e^{2\tanh^{-1}(cx^2)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^3/x^3,x]
```

```
[Out] ((-2*a^3)/x^2 - (6*a^2*b*ArcTanh[c*x^2])/x^2 + 12*a^2*b*c*Log[x] - 3*a^2*b*
c*Log[1 - c^2*x^4] + 6*a*b^2*c*(ArcTanh[c*x^2]*((1 - 1/(c*x^2))*ArcTanh[c*x
^2] + 2*Log[1 - E^(-2*ArcTanh[c*x^2])]) - PolyLog[2, E^(-2*ArcTanh[c*x^2])]
) + 2*b^3*c*((I/8)*Pi^3 - ArcTanh[c*x^2]^3 - ArcTanh[c*x^2]^3/(c*x^2) + 3*A
rcTanh[c*x^2]^2*Log[1 - E^(2*ArcTanh[c*x^2])] + 3*ArcTanh[c*x^2]*PolyLog[2,
 E^(2*ArcTanh[c*x^2])] - (3*PolyLog[3, E^(2*ArcTanh[c*x^2])])/2))/4
```

<u>**Maple**</u> **[F]**
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\,x^2\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^3/x^3,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^3/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^3,x, algorithm="maxima")
```

```
[Out] -3/4*(c*(log(c^2*x^4 - 1) - log(x^4)) + 2*arctanh(c*x^2)/x^2)*a^2*b - 1/2*a
^3/x^2 - 1/16*((b^3*c*x^2 - b^3)*log(-c*x^2 + 1)^3 + 3*(2*a*b^2 + (b^3*c*x^
2 + b^3)*log(c*x^2 + 1))*log(-c*x^2 + 1)^2)/x^2 - integrate(-1/8*((b^3*c*x^
2 - b^3)*log(c*x^2 + 1)^3 + 6*(a*b^2*c*x^2 - a*b^2)*log(c*x^2 + 1)^2 + 3*(4
*a*b^2*c*x^2 - (b^3*c*x^2 - b^3)*log(c*x^2 + 1)^2 + 2*(b^3*c^2*x^4 + 2*a*b^
2 - (2*a*b^2*c - b^3*c)*x^2)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^5 - x^3)
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^2)^3 + 3*a*b^2*arctanh(c*x^2)^2 + 3*a^2*b*arctanh
(c*x^2) + a^3)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(cx^2\right)\right)^3}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**3/x**3,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**3/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^3/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^3/x^3,x)
```

```
[Out] int((a + b*atanh(c*x^2))^3/x^3, x)
```

3.81 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x^5}\,dx$

Optimal. Leaf size=139

$$\frac{3}{4}bc^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2-\frac{3bc\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{4x^2}+\frac{1}{4}c^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{4x^4}+\frac{3}{2}$$

[Out] 3/4*b*c^2*(a+b*arctanh(c*x^2))^2-3/4*b*c*(a+b*arctanh(c*x^2))^2/x^2+1/4*c^2*(a+b*arctanh(c*x^2))^3-1/4*(a+b*arctanh(c*x^2))^3/x^4+3/2*b^2*c^2*(a+b*arctanh(c*x^2))*ln(2-2/(c*x^2+1))-3/4*b^3*c^2*polylog(2,-1+2/(c*x^2+1))

Rubi [A]
time = 0.25, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6039, 6037, 6129, 6135, 6079, 2497, 6095}

$$\frac{3}{2}b^2c^2\log\left(2-\frac{2}{cx^2+1}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{3}{4}bc^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2+\frac{1}{4}c^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3-\frac{3bc\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{4x^2}-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{4x^4}-\frac{3}{4}b^3c^2\text{Li}_2\left(\frac{2}{cx^2+1}-1\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x^2])^3/x^5,x]

[Out] (3*b*c^2*(a + b*ArcTanh[c*x^2])^2)/4 - (3*b*c*(a + b*ArcTanh[c*x^2])^2)/(4*x^2) + (c^2*(a + b*ArcTanh[c*x^2])^3)/4 - (a + b*ArcTanh[c*x^2])^3/(4*x^4) + (3*b^2*c^2*(a + b*ArcTanh[c*x^2])*Log[2 - 2/(1 + c*x^2)])/2 - (3*b^3*c^2*PolyLog[2, -1 + 2/(1 + c*x^2)])/4

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
```

```
fy[(m + 1)/n]]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x^5}\,dx = \int \Bigg(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{8x^5}+\frac{3b\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)}{8x^5}-\frac{3b^2}{\cdots}$$

$$=\frac{1}{8}\int\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{x^5}\,dx+\frac{1}{8}(3b)\int\frac{\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+c\cdots\right.}{x^5}$$

$$=\frac{1}{16}\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^3}{x^3}\,dx,x,x^2\right)+\frac{1}{16}(3b)\text{Subst}\Bigg(\int\frac{(-2a+b\log\cdots}{\cdots}$$

$$=-\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{32x^4}-\frac{b^3\log^3\left(1+cx^2\right)}{32x^4}+\frac{1}{16}(3b)\text{Subst}\Bigg(\int\frac{(-2a+b\log\cdots}{\cdots}$$

$$=-\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{32x^4}-\frac{b^3\log^3\left(1+cx^2\right)}{32x^4}-\frac{1}{32}(3b)\text{Subst}\Bigg(\int\frac{(2a-b\log(\cdots}{x\left(\frac{1}{c}-\frac{x}{c}\right)\cdots}$$

$$=-\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{32x^4}-\frac{b^3\log^3\left(1+cx^2\right)}{32x^4}-\frac{1}{32}(3b)\text{Subst}\Bigg(\int\frac{(2a-b\log(\cdots}{\left(\frac{1}{c}-\frac{x}{c}\right)^2\cdots}$$

$$=-\frac{3bc(1-cx^2)\left(2a-b\log\left(1-cx^2\right)\right)^2}{32x^2}-\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{32x^4}-\frac{3b^3c(1+c\cdots}{\cdots}$$

$$=\frac{3}{4}ab^2c^2\log(x)-\frac{3bc(1-cx^2)\left(2a-b\log\left(1-cx^2\right)\right)^2}{32x^2}+\frac{3}{32}bc^2\log\left(cx^2\right)(2a-\cdots$$

$$=\frac{3}{4}ab^2c^2\log(x)-\frac{3bc(1-cx^2)\left(2a-b\log\left(1-cx^2\right)\right)^2}{32x^2}+\frac{3}{32}bc^2\log\left(cx^2\right)(2a-\cdots$$

$$=\frac{3}{4}ab^2c^2\log(x)-\frac{3bc(1-cx^2)\left(2a-b\log\left(1-cx^2\right)\right)^2}{32x^2}+\frac{3}{32}bc^2\log\left(cx^2\right)(2a-\cdots$$

Mathematica [A]
time = 0.20, size = 218, normalized size = 1.57

$$\frac{6b^2(-1+cx^2)(a+acx^2+bcx^2)\tanh^{-1}(cx^2)^2+2b^3(-1+c^2x^4)\tanh^{-1}(cx^2)^3-6b\tanh^{-1}(cx^2)\left(a^2+2abcx^2-2b^2c^2x^4\log\left(1-e^{-2\tanh^{-1}(cx^2)}\right)\right)+a\left(-2a^2-6abcx^2-3abc^2x^4\log(1-cx^2)+3abc^2x^4\log(1+cx^2)+12b^2c^2x^4\log\left(\frac{cx^2}{\sqrt{1-c^2x^4}}\right)\right)-6b^3c^2x^4\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx^2)}\right)}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^3/x^5,x]
```

```
[Out] (6*b^2*(-1 + c*x^2)*(a + a*c*x^2 + b*c*x^2)*ArcTanh[c*x^2]^2 + 2*b^3*(-1 +
c^2*x^4)*ArcTanh[c*x^2]^3 - 6*b*ArcTanh[c*x^2]*(a^2 + 2*a*b*c*x^2 - 2*b^2*c
^2*x^4*Log[1 - E^(-2*ArcTanh[c*x^2])]) + a*(-2*a^2 - 6*a*b*c*x^2 - 3*a*b*c^
2*x^4*Log[1 - c*x^2] + 3*a*b*c^2*x^4*Log[1 + c*x^2] + 12*b^2*c^2*x^4*Log[(c
*x^2)/Sqrt[1 - c^2*x^4]]) - 6*b^3*c^2*x^4*PolyLog[2, E^(-2*ArcTanh[c*x^2])]
)/(8*x^4)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \,\mathrm{arctanh}\left(c \,x^{2}\right)\right)^{3}}{x^{5}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^3/x^5,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^3/x^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^5,x, algorithm="maxima")
```

```
[Out] 3/8*((c*log(c*x^2 + 1) - c*log(c*x^2 - 1) - 2/x^2)*c - 2*arctanh(c*x^2)/x^4
)*a^2*b + 3/16*((2*(log(c*x^2 - 1) - 2)*log(c*x^2 + 1) - log(c*x^2 + 1)^2 -
 log(c*x^2 - 1)^2 - 4*log(c*x^2 - 1) + 16*log(x))*c^2 + 4*(c*log(c*x^2 + 1)
 - c*log(c*x^2 - 1) - 2/x^2)*c*arctanh(c*x^2))*a*b^2 - 1/32*b^3*(((c^2*x^4
- 1)*log(-c*x^2 + 1)^3 + 3*(2*c*x^2 - (c^2*x^4 - 1)*log(c*x^2 + 1))*log(-c*
x^2 + 1)^2)/x^4 + 4*integrate(-((c*x^2 - 1)*log(c*x^2 + 1)^3 + 3*(2*c^2*x^4
 - (c*x^2 - 1)*log(c*x^2 + 1)^2 - (c^3*x^6 - c*x^2)*log(c*x^2 + 1))*log(-c*
x^2 + 1))/(c*x^7 - x^5), x)) - 3/4*a*b^2*arctanh(c*x^2)^2/x^4 - 1/4*a^3/x^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^2)^3 + 3*a*b^2*arctanh(c*x^2)^2 + 3*a^2*b*arctanh
(c*x^2) + a^3)/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}{\left(c x^{2} \right)}\right)^{3}}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**3/x**5,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**3/x**5, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^3/x^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^3/x^5,x)
```

```
[Out] int((a + b*atanh(c*x^2))^3/x^5, x)
```

3.82 $\int (dx)^{5/2} \left(a+b\tanh^{-1}\left(cx^2\right)\right) \, dx$

Optimal. Leaf size=317

$$\frac{8bd(dx)^{3/2}}{21c}+\frac{2bd^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}}+\frac{\sqrt{2}\,bd^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}}-\frac{\sqrt{2}\,bd^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}}$$

```
[Out] 8/21*b*d*(d*x)^(3/2)/c+2/7*b*d^(5/2)*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))/c^
(7/4)+2/7*(d*x)^(7/2)*(a+b*arctanh(c*x^2))/d-2/7*b*d^(5/2)*arctanh(c^(1/4)*
(d*x)^(1/2)/d^(1/2))/c^(7/4)-1/14*b*d^(5/2)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)-c^
(1/4)*2^(1/2)*(d*x)^(1/2))/c^(7/4)*2^(1/2)+1/14*b*d^(5/2)*ln(d^(1/2)+x*c^(1
/2)*d^(1/2)+c^(1/4)*2^(1/2)*(d*x)^(1/2))/c^(7/4)*2^(1/2)-1/7*b*d^(5/2)*arct
an(-1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/c^(7/4)-1/7*b*d^(5/2)*ar
ctan(1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/c^(7/4)
```

Rubi [A]
time = 0.21, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {6049, 327, 335, 306, 303, 1176, 631, 210, 1179, 642, 304, 211, 214}

$$\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}+\frac{2bd^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}}+\frac{\sqrt{2}\,bd^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}}-\frac{\sqrt{2}\,bd^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}+1\right)}{7c^{7/4}}-\frac{bd^{5/2}\log\left(\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,\sqrt[4]{c}\,\sqrt{dx}+\sqrt{d}\right)}{7\sqrt{2}\,c^{7/4}}+\frac{bd^{5/2}\log\left(\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,\sqrt[4]{c}\,\sqrt{dx}+\sqrt{d}\right)}{7\sqrt{2}\,c^{7/4}}-\frac{2bd^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}}+\frac{8bd(dx)^{3/2}}{21c}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (8*b*d*(d*x)^(3/2))/(21*c) + (2*b*d^(5/2)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d
]])/(7*c^(7/4)) + (Sqrt[2]*b*d^(5/2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])
/Sqrt[d]])/(7*c^(7/4)) - (Sqrt[2]*b*d^(5/2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[d*x])/Sqrt[d]])/(7*c^(7/4)) + (2*(d*x)^(7/2)*(a + b*ArcTanh[c*x^2]))/(7*d
) - (2*b*d^(5/2)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/4)) - (b*d^(
5/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(7*Sqrt[
2]*c^(7/4)) + (b*d^(5/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*
Sqrt[d*x]])/(7*Sqrt[2]*c^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 306

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[x^m/(r + s*x^(
n/2)), x], x] + Dist[r/(2*a), Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ[{a,
 b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int (dx)^{5/2} \left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx &= \frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d} - \frac{(4bc)\int \frac{x(dx)^{7/2}}{1-c^2x^4}\,dx}{7d} \\
&= \frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d} - \frac{(4bc)\int \frac{(dx)^{9/2}}{1-c^2x^4}\,dx}{7d^2} \\
&= \frac{8bd(dx)^{3/2}}{21c} + \frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d} - \frac{(4bd^2)\int \frac{\sqrt{dx}}{1-c^2x^4}\,dx}{7c} \\
&= \frac{8bd(dx)^{3/2}}{21c} + \frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d} - \frac{(8bd)\text{Subst}\left(\int \frac{x^2}{1-\frac{c^2x^8}{d^4}}\,dx \right.}{7c} \\
&= \frac{8bd(dx)^{3/2}}{21c} + \frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d} - \frac{(4bd^3)\,\text{Subst}\left(\int \frac{x^2}{d^2-cx^4}\,d\right.}{7c} \\
&= \frac{8bd(dx)^{3/2}}{21c} + \frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d} - \frac{(2bd^3)\,\text{Subst}\left(\int \frac{1}{d-\sqrt{c}\,x^2}\right.}{7c^{3/2}} \\
&= \frac{8bd(dx)^{3/2}}{21c} + \frac{2bd^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}} + \frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right.\right.}{7d} \\
&= \frac{8bd(dx)^{3/2}}{21c} + \frac{2bd^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}} + \frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right.\right.}{7d} \\
&= \frac{8bd(dx)^{3/2}}{21c} + \frac{2bd^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}} + \frac{\sqrt{2}\,bd^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{}}{}\right.}{7c^{7/4}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 241, normalized size = 0.76

$$
\frac{(dx)^{5/2}\left(16bc^{3/4}x^{3/2}+12ac^{7/4}x^{7/2}+6\sqrt{2}\,b\text{ArcTan}\left(1-\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}\right)-6\sqrt{2}\,b\text{ArcTan}\left(1+\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}\right)+12b\text{ArcTan}\left(\sqrt[4]{c}\,\sqrt{x}\right)+12bc^{7/4}x^{7/2}\tanh^{-1}\left(cx^2\right)+6b\log\left(1-\sqrt[4]{c}\,\sqrt{x}\right)-6b\log\left(1+\sqrt[4]{c}\,\sqrt{x}\right)-3\sqrt{2}\,b\log\left(1-\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{c}\,x\right)+3\sqrt{2}\,b\log\left(1+\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{c}\,x\right)\right)}{42c^{7/4}x^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] ((d*x)^(5/2)*(16*b*c^(3/4)*x^(3/2) + 12*a*c^(7/4)*x^(7/2) + 6*Sqrt[2]*b*Arc
Tan[1 - Sqrt[2]*c^(1/4)*Sqrt[x]] - 6*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*c^(1/4)*S
qrt[x]] + 12*b*ArcTan[c^(1/4)*Sqrt[x]] + 12*b*c^(7/4)*x^(7/2)*ArcTanh[c*x^2
] + 6*b*Log[1 - c^(1/4)*Sqrt[x]] - 6*b*Log[1 + c^(1/4)*Sqrt[x]] - 3*Sqrt[2]
```

```
*b*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 3*Sqrt[2]*b*Log[1 + Sqrt[
2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))/(42*c^(7/4)*x^(5/2))
```

Maple [A]
time = 0.08, size = 303, normalized size = 0.96

method	result
derivativedivides	$\frac{\frac{2(dx)^{\frac{7}{2}}a}{7}+\frac{2b(dx)^{\frac{7}{2}}\operatorname{arctanh}\left(cx^{2}\right)}{7}+\frac{8bd^{2}(dx)^{\frac{3}{2}}}{21c}-\frac{bd^{4}\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)}{14c^{2}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}-\frac{bd^{4}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}}{(}\right.}{7c^{2}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}}{d}$
default	$\frac{\frac{2(dx)^{\frac{7}{2}}a}{7}+\frac{2b(dx)^{\frac{7}{2}}\operatorname{arctanh}\left(cx^{2}\right)}{7}+\frac{8bd^{2}(dx)^{\frac{3}{2}}}{21c}-\frac{bd^{4}\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)}{14c^{2}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}-\frac{bd^{4}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}}{(}\right.}{7c^{2}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/7*(d*x)^(7/2)*a+1/7*b*(d*x)^(7/2)*arctanh(c*x^2)+4/21*b/c*d^2*(d*x)^
(3/2)-1/28*b/c^2*d^4/(d^2/c)^(1/4)*2^(1/2)*ln((d*x-(d^2/c)^(1/4)*(d*x)^(1/2
)*2^(1/2)+(d^2/c)^(1/2))/(d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/
2)))-1/14*b/c^2*d^4/(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x
)^(1/2)+1)-1/14*b/c^2*d^4/(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4
)*(d*x)^(1/2)-1)+1/7*b/c^2*d^4/(d^2/c)^(1/4)*arctan((d*x)^(1/2)/(d^2/c)^(1/
4))-1/14*b/c^2*d^4/(d^2/c)^(1/4)*ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2
)-(d^2/c)^(1/4))))
```

Maxima [A]
time = 0.47, size = 316, normalized size = 1.00

$$\frac{12\,(dx)^{\frac{7}{2}}a+\left(12\,(dx)^{\frac{7}{2}}\operatorname{artanh}\left(cx^{2}\right)-\frac{\left(\frac{3\,d^{6}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\,\sqrt{c}\right)}{2\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\,\sqrt{c}\right)}{2\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{c}\,dx+\sqrt{2}\sqrt{dx}\,c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{3}{4}}\sqrt{d}}+\frac{\sqrt{2}\log\left(\sqrt{c}\,dx-\sqrt{2}\sqrt{dx}\,c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{3}{4}}\sqrt{d}}\right)}{c^{2}}-\frac{6\,d^{6}\left(\frac{2\arctan\left(\frac{\sqrt{dx}\,\sqrt{c}}{\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}\,\sqrt{c}}+\frac{\log\left(\frac{\sqrt{dx}\,\sqrt{c}-\sqrt{\sqrt{c}\,d}}{\sqrt{dx}\,\sqrt{c}+\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}\,\sqrt{c}}\right)}{c^{2}}-\frac{16\,(dx)^{\frac{3}{2}}d^{4}}{c^{2}}\right)c}{d^{2}}\right)b}{42\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/42*(12*(d*x)^(7/2)*a + (12*(d*x)^(7/2)*arctanh(c*x^2) - (3*d^6*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) + 2*sqrt(d*x)*sqrt(c))/sqrt(sq
rt(c)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2
)*c^(1/4)*sqrt(d) - 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*
sqrt(c)) - sqrt(2)*log(sqrt(c)*d*x + sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)
/(c^(3/4)*sqrt(d)) + sqrt(2)*log(sqrt(c)*d*x - sqrt(2)*sqrt(d*x)*c^(1/4)*sq
rt(d) + d)/(c^(3/4)*sqrt(d)))/c^2 - 6*d^6*(2*arctan(sqrt(d*x)*sqrt(c)/sqrt(
sqrt(c)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) + log((sqrt(d*x)*sqrt(c) - sqrt(sqrt(
c)*d))/(sqrt(d*x)*sqrt(c) + sqrt(sqrt(c)*d)))/(sqrt(sqrt(c)*d)*sqrt(c)))/c^
2 - 16*(d*x)^(3/2)*d^4/c^2)*c/d^2)*b)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(212) = 424$.
time = 0.40, size = 455, normalized size = 1.44

$$-\frac{12\left(\frac{b^4d^{10}}{c^7}\right)^{\frac{1}{4}}c\arctan\left(-\frac{\left(\frac{b^4d^{10}}{c^7}\right)^{\frac{1}{4}}\sqrt{dx}\,b^3c^2d^7-\sqrt{b^6d^{15}x+\sqrt{\frac{b^4d^{10}}{c^7}}\,b^4c^3d^{10}}\left(\frac{b^4d^{10}}{c^7}\right)^{\frac{1}{4}}c^2}{b^4d^{10}}\right)-12\left(-\frac{b^4d^{10}}{c^7}\right)^{\frac{1}{4}}c\arctan\left(-\frac{\left(-\frac{b^4d^{10}}{c^7}\right)^{\frac{1}{4}}\sqrt{dx}\,b^3c^2d^7-\sqrt{b^6d^{15}x-\sqrt{-\frac{b^4d^{10}}{c^7}}\,b^4c^3d^{10}}\left(-\frac{b^4d^{10}}{c^7}\right)^{\frac{1}{4}}c^2}{b^4d^{10}}\right)+3\left(\frac{b^4d^{10}}{c^7}\right)^{\frac{1}{4}}c\log\left(\sqrt{dx}\,b^3d^7+\left(\frac{b^4d^{10}}{c^7}\right)^{\frac{3}{4}}c^5\right)-3\left(\frac{b^4d^{10}}{c^7}\right)^{\frac{1}{4}}c\log\left(\sqrt{dx}\,b^3d^7-\left(\frac{b^4d^{10}}{c^7}\right)^{\frac{3}{4}}c^5\right)+3\left(-\frac{b^4d^{10}}{c^7}\right)^{\frac{1}{4}}c\log\left(\sqrt{dx}\,b^3d^7+\left(-\frac{b^4d^{10}}{c^7}\right)^{\frac{3}{4}}c^5\right)-3\left(-\frac{b^4d^{10}}{c^7}\right)^{\frac{1}{4}}c\log\left(\sqrt{dx}\,b^3d^7-\left(-\frac{b^4d^{10}}{c^7}\right)^{\frac{3}{4}}c^5\right)-\left(3\,bcd^2x^3\log\left(-\frac{cx^2+1}{cx^2-1}\right)+6\,acd^2x^3+8\,bd^2x\right)\sqrt{dx}}{21\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] -1/21*(12*(b^4*d^10/c^7)^(1/4)*c*arctan(-((b^4*d^10/c^7)^(1/4)*sqrt(d*x)*b^
3*c^2*d^7 - sqrt(b^6*d^15*x + sqrt(b^4*d^10/c^7)*b^4*c^3*d^10)*(b^4*d^10/c^
7)^(1/4)*c^2)/(b^4*d^10)) - 12*(-b^4*d^10/c^7)^(1/4)*c*arctan(-((-b^4*d^10/
c^7)^(1/4)*sqrt(d*x)*b^3*c^2*d^7 - sqrt(b^6*d^15*x - sqrt(-b^4*d^10/c^7)*b^
4*c^3*d^10)*(-b^4*d^10/c^7)^(1/4)*c^2)/(b^4*d^10)) + 3*(b^4*d^10/c^7)^(1/4)
*c*log(sqrt(d*x)*b^3*d^7 + (b^4*d^10/c^7)^(3/4)*c^5) - 3*(b^4*d^10/c^7)^(1/
4)*c*log(sqrt(d*x)*b^3*d^7 - (b^4*d^10/c^7)^(3/4)*c^5) + 3*(-b^4*d^10/c^7)^
(1/4)*c*log(sqrt(d*x)*b^3*d^7 + (-b^4*d^10/c^7)^(3/4)*c^5) - 3*(-b^4*d^10/c
^7)^(1/4)*c*log(sqrt(d*x)*b^3*d^7 - (-b^4*d^10/c^7)^(3/4)*c^5) - (3*b*c*d^2
*x^3*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a*c*d^2*x^3 + 8*b*d^2*x)*sqrt(d*x))/
c
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{5}{2}}\left(a+b\,\mathrm{atanh}\left(cx^2\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(a+b*atanh(c*x**2)),x)
```

```
[Out] Integral((d*x)**(5/2)*(a + b*atanh(c*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(5/2)*(b*arctanh(c*x^2) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^{5/2}\,\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a + b*atanh(c*x^2)),x)
```

```
[Out] int((d*x)^(5/2)*(a + b*atanh(c*x^2)), x)
```

3.83 $\int (dx)^{3/2} \left(a+b\tanh^{-1}\left(cx^2\right)\right) \, dx$

Optimal. Leaf size=317

$$\frac{8bd\sqrt{dx}}{5c}-\frac{2bd^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}}+\frac{\sqrt{2}\,bd^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}}-\frac{\sqrt{2}\,bd^{3/2}\text{ArcTan}\left(1+\sqrt{2}\ldots\right.}{5c^{5/4}}$$

```
[Out] -2/5*b*d^(3/2)*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))/c^(5/4)+2/5*(d*x)^(5/2)*
(a+b*arctanh(c*x^2))/d-2/5*b*d^(3/2)*arctanh(c^(1/4)*(d*x)^(1/2)/d^(1/2))/c
^(5/4)+1/10*b*d^(3/2)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)-c^(1/4)*2^(1/2)*(d*x)^(1
/2))/c^(5/4)*2^(1/2)-1/10*b*d^(3/2)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)+c^(1/4)*2^
(1/2)*(d*x)^(1/2))/c^(5/4)*2^(1/2)-1/5*b*d^(3/2)*arctan(-1+c^(1/4)*2^(1/2)*
(d*x)^(1/2)/d^(1/2))*2^(1/2)/c^(5/4)-1/5*b*d^(3/2)*arctan(1+c^(1/4)*2^(1/2)
*(d*x)^(1/2)/d^(1/2))*2^(1/2)/c^(5/4)+8/5*b*d*(d*x)^(1/2)/c
```

Rubi [A]
time = 0.19, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {6049, 327, 335, 220, 218, 214, 211, 217, 1179, 642, 1176, 631, 210}

$$\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d}-\frac{2bd^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}}+\frac{\sqrt{2}\,bd^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}}-\frac{\sqrt{2}\,bd^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{5c^{5/4}}+\frac{bd^{3/2}\log\left(\sqrt{c}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}\,c^{5/4}}-\frac{bd^{3/2}\log\left(\sqrt{c}\sqrt{d}\,x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}\,c^{5/4}}-\frac{2bd^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}}+\frac{8bd\sqrt{dx}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (8*b*d*Sqrt[d*x])/(5*c) - (2*b*d^(3/2)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])
/(5*c^(5/4)) + (Sqrt[2]*b*d^(3/2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sq
rt[d]])/(5*c^(5/4)) - (Sqrt[2]*b*d^(3/2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d
*x])/Sqrt[d]])/(5*c^(5/4)) + (2*(d*x)^(5/2)*(a + b*ArcTanh[c*x^2]))/(5*d) -
 (2*b*d^(3/2)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(5*c^(5/4)) + (b*d^(3/2
)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(5*Sqrt[2]*
c^(5/4)) - (b*d^(3/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqr
t[d*x]])/(5*Sqrt[2]*c^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int (dx)^{3/2} \left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx &= \frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d} - \frac{(4bc)\int \frac{x(dx)^{5/2}}{1-c^2x^4}\,dx}{5d} \\
&= \frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d} - \frac{(4bc)\int \frac{(dx)^{7/2}}{1-c^2x^4}\,dx}{5d^2} \\
&= \frac{8bd\sqrt{dx}}{5c} + \frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d} - \frac{(4bd^2)\int \frac{1}{\sqrt{dx}\,(1-c^2x^4)}\,dx}{5c} \\
&= \frac{8bd\sqrt{dx}}{5c} + \frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d} - \frac{(8bd)\text{Subst}\left(\int \frac{1}{1-\frac{c^2x^8}{d^4}}\,dx, x\ldots\right.}{5c} \\
&= \frac{8bd\sqrt{dx}}{5c} + \frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d} - \frac{(4bd^3)\,\text{Subst}\left(\int \frac{1}{d^2-cx^4}\,dx, x\ldots\right.}{5c} \\
&= \frac{8bd\sqrt{dx}}{5c} + \frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d} - \frac{(2bd^2)\,\text{Subst}\left(\int \frac{1}{d-\sqrt{c}\,x^2}\,dx\ldots\right.}{5c} \\
&= \frac{8bd\sqrt{dx}}{5c} - \frac{2bd^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}} + \frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d} \\
&= \frac{8bd\sqrt{dx}}{5c} - \frac{2bd^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}} + \frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d} \\
&= \frac{8bd\sqrt{dx}}{5c} - \frac{2bd^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}} + \frac{\sqrt{2}\,bd^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\ldots}{\sqrt{d}\ldots}\right.}{5c^{5/4}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 240, normalized size = 0.76

$$
\frac{(dx)^{3/2}\left(16b\sqrt[4]{c}\,\sqrt{x}+4ac^{5/4}x^{5/2}+2\sqrt{2}\,b\text{ArcTan}\left(1-\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}\right)-2\sqrt{2}\,b\text{ArcTan}\left(1+\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}\right)-4b\text{ArcTan}\left(\sqrt[4]{c}\,\sqrt{x}\right)+4bc^{5/4}x^{5/2}\tanh^{-1}\left(cx^2\right)+2b\log\left(1-\sqrt[4]{c}\,\sqrt{x}\right)-2b\log\left(1+\sqrt[4]{c}\,\sqrt{x}\right)+\sqrt{2}\,b\log\left(1-\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{c}\,x\right)-\sqrt{2}\,b\log\left(1+\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{c}\,x\right)\right)}{10c^{5/4}x^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] ((d*x)^(3/2)*(16*b*c^(1/4)*Sqrt[x] + 4*a*c^(5/4)*x^(5/2) + 2*Sqrt[2]*b*ArcT
an[1 - Sqrt[2]*c^(1/4)*Sqrt[x]] - 2*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*c^(1/4)*Sq
rt[x]] - 4*b*ArcTan[c^(1/4)*Sqrt[x]] + 4*b*c^(5/4)*x^(5/2)*ArcTanh[c*x^2] +
 2*b*Log[1 - c^(1/4)*Sqrt[x]] - 2*b*Log[1 + c^(1/4)*Sqrt[x]] + Sqrt[2]*b*Lo
```

```
g[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - Sqrt[2]*b*Log[1 + Sqrt[2]*c^(1
/4)*Sqrt[x] + Sqrt[c]*x]))/(10*c^(5/4)*x^(3/2))
```

Maple [A]
time = 0.04, size = 303, normalized size = 0.96

method	result
derivativedivides	$\frac{2(dx)^{\frac{5}{2}} a}{5}+\frac{2b(dx)^{\frac{5}{2}}\operatorname{arctanh}\left(c x^{2}\right)}{5}+\frac{8b d^{2}\sqrt{dx}}{5c}-\frac{b d^{2}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)}{10c}-\frac{b d^{2}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}{}$
default	$\frac{2(dx)^{\frac{5}{2}} a}{5}+\frac{2b(dx)^{\frac{5}{2}}\operatorname{arctanh}\left(c x^{2}\right)}{5}+\frac{8b d^{2}\sqrt{dx}}{5c}-\frac{b d^{2}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)}{10c}-\frac{b d^{2}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/5*(d*x)^(5/2)*a+1/5*b*(d*x)^(5/2)*arctanh(c*x^2)+4/5*b/c*d^2*(d*x)^(
1/2)-1/20*b/c*d^2*(d^2/c)^(1/4)*2^(1/2)*ln((d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(d^2/c)^(1/2))/(d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))
)-1/10*b/c*d^2*(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/
2)+1)-1/10*b/c*d^2*(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)
^(1/2)-1)-1/10*b/c*d^2*(d^2/c)^(1/4)*ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^
(1/2)-(d^2/c)^(1/4)))-1/5*b/c*d^2*(d^2/c)^(1/4)*arctan((d*x)^(1/2)/(d^2/c)^
(1/4)))
```

Maxima [A]
time = 0.47, size = 310, normalized size = 0.98

$$\frac{4\,(dx)^{\frac{5}{2}}\,a+\left(4\,(dx)^{\frac{5}{2}}\operatorname{artanh}\left(cx^{2}\right)+\frac{\left(\frac{16\sqrt{dx}\,d^{4}}{c^{2}}-\frac{\frac{2\sqrt{2}\,d^{5}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\,\sqrt{c}\right)}{2\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}}+\frac{2\sqrt{2}\,d^{5}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\,\sqrt{c}\right)}{2\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}}+\frac{\sqrt{2}\,d^{\frac{9}{2}}\log\left(\sqrt{c}\,dx+\sqrt{2}\sqrt{dx}\,c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{\frac{9}{2}}\log\left(\sqrt{c}\,dx-\sqrt{2}\sqrt{dx}\,c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{1}{4}}}}{c^{2}}-\frac{2\left(\frac{2\,d^{5}\arctan\left(\frac{\sqrt{dx}\,\sqrt{c}}{\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}}-\frac{d^{5}\log\left(\frac{\sqrt{dx}\,\sqrt{c}-\sqrt{\sqrt{c}\,d}}{\sqrt{dx}\,\sqrt{c}+\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}}\right)}{c^{2}}\right)c}{d^{2}}\right)b}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/10*(4*(d*x)^(5/2)*a + (4*(d*x)^(5/2)*arctanh(c*x^2) + (16*sqrt(d*x)*d^4/c
^2 - (2*sqrt(2)*d^5*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) + 2*sqrt(d*
x)*sqrt(c))/sqrt(sqrt(c)*d))/sqrt(sqrt(c)*d) + 2*sqrt(2)*d^5*arctan(-1/2*sq
rt(2)*(sqrt(2)*c^(1/4)*sqrt(d) - 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/sqrt
(sqrt(c)*d) + sqrt(2)*d^(9/2)*log(sqrt(c)*d*x + sqrt(2)*sqrt(d*x)*c^(1/4)*s
qrt(d) + d)/c^(1/4) - sqrt(2)*d^(9/2)*log(sqrt(c)*d*x - sqrt(2)*sqrt(d*x)*c
^(1/4)*sqrt(d) + d)/c^(1/4))/c^2 - 2*(2*d^5*arctan(sqrt(d*x)*sqrt(c)/sqrt(s
qrt(c)*d))/sqrt(sqrt(c)*d) - d^5*log((sqrt(d*x)*sqrt(c) - sqrt(sqrt(c)*d))/
(sqrt(d*x)*sqrt(c) + sqrt(sqrt(c)*d)))/sqrt(sqrt(c)*d))/c^2)*c/d^2)*b)/d
```

Fricas [A]
time = 0.38, size = 399, normalized size = 1.26

$$\frac{4\left(\frac{b^4d^6}{c^5}\right)^{\frac{1}{4}}c\arctan\left(-\frac{\left(\frac{b^4d^6}{c^5}\right)^{\frac{3}{4}}\sqrt{dx}\,bc^4d-\sqrt{b^2d^3x+\sqrt{\frac{b^4d^6}{c^5}}\,c^2}\,\left(\frac{b^4d^6}{c^5}\right)^{\frac{3}{4}}c^4}{b^4d^6}\right)-4\left(-\frac{b^4d^6}{c^5}\right)^{\frac{1}{4}}c\arctan\left(-\frac{\left(-\frac{b^4d^6}{c^5}\right)^{\frac{3}{4}}\sqrt{dx}\,bc^4d-\sqrt{b^2d^3x+\sqrt{-\frac{b^4d^6}{c^5}}\,c^2}\,\left(-\frac{b^4d^6}{c^5}\right)^{\frac{3}{4}}c^4}{b^4d^6}\right)-\left(\frac{b^4d^6}{c^5}\right)^{\frac{1}{4}}c\log\left(\sqrt{dx}\,bd+\left(\frac{b^4d^6}{c^5}\right)^{\frac{1}{4}}c\right)+\left(\frac{b^4d^6}{c^5}\right)^{\frac{1}{4}}c\log\left(\sqrt{dx}\,bd-\left(\frac{b^4d^6}{c^5}\right)^{\frac{1}{4}}c\right)-\left(-\frac{b^4d^6}{c^5}\right)^{\frac{1}{4}}c\log\left(\sqrt{dx}\,bd+\left(-\frac{b^4d^6}{c^5}\right)^{\frac{1}{4}}c\right)+\left(-\frac{b^4d^6}{c^5}\right)^{\frac{1}{4}}c\log\left(\sqrt{dx}\,bd-\left(-\frac{b^4d^6}{c^5}\right)^{\frac{1}{4}}c\right)+\left(bcdx^2\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,acdx^2+8\,bd\right)\sqrt{dx}}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/5*(4*(b^4*d^6/c^5)^(1/4)*c*arctan(-((b^4*d^6/c^5)^(3/4)*sqrt(d*x)*b*c^4*d
 - sqrt(b^2*d^3*x + sqrt(b^4*d^6/c^5)*c^2)*(b^4*d^6/c^5)^(3/4)*c^4)/(b^4*d^
6)) - 4*(-b^4*d^6/c^5)^(1/4)*c*arctan(-((-b^4*d^6/c^5)^(3/4)*sqrt(d*x)*b*c^
4*d - sqrt(b^2*d^3*x + sqrt(-b^4*d^6/c^5)*c^2)*(-b^4*d^6/c^5)^(3/4)*c^4)/(b
^4*d^6)) - (b^4*d^6/c^5)^(1/4)*c*log(sqrt(d*x)*b*d + (b^4*d^6/c^5)^(1/4)*c)
 + (b^4*d^6/c^5)^(1/4)*c*log(sqrt(d*x)*b*d - (b^4*d^6/c^5)^(1/4)*c) - (-b^4
*d^6/c^5)^(1/4)*c*log(sqrt(d*x)*b*d + (-b^4*d^6/c^5)^(1/4)*c) + (-b^4*d^6/c
^5)^(1/4)*c*log(sqrt(d*x)*b*d - (-b^4*d^6/c^5)^(1/4)*c) + (b*c*d*x^2*log(-(
c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*d*x^2 + 8*b*d)*sqrt(d*x))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}}\left(a+b\,\mathrm{atanh}\left(cx^2\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(a+b*atanh(c*x**2)),x)
```

```
[Out] Integral((d*x)**(3/2)*(a + b*atanh(c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*(b*arctanh(c*x^2) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^{3/2}\,\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a + b*atanh(c*x^2)),x)
[Out] int((d*x)^(3/2)*(a + b*atanh(c*x^2)), x)
```

3.84 $\int \sqrt{dx} \left(a + b\tanh^{-1}\left(cx^2\right)\right) \, dx$

Optimal. Leaf size=301

$$\frac{2b\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}}-\frac{\sqrt{2}\,b\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}}+\frac{\sqrt{2}\,b\sqrt{d}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}}$$

```
[Out] 2/3*(d*x)^(3/2)*(a+b*arctanh(c*x^2))/d+2/3*b*arctan(c^(1/4)*(d*x)^(1/2)/d^(
1/2))*d^(1/2)/c^(3/4)-2/3*b*arctanh(c^(1/4)*(d*x)^(1/2)/d^(1/2))*d^(1/2)/c^
(3/4)+1/6*b*ln(d^(1/2)+x*c^(1/2)*d^(1/2)-c^(1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/
2)/c^(3/4)*2^(1/2)-1/6*b*ln(d^(1/2)+x*c^(1/2)*d^(1/2)+c^(1/4)*2^(1/2)*(d*x)
^(1/2))*d^(1/2)/c^(3/4)*2^(1/2)+1/3*b*arctan(-1+c^(1/4)*2^(1/2)*(d*x)^(1/2)
/d^(1/2))*2^(1/2)*d^(1/2)/c^(3/4)+1/3*b*arctan(1+c^(1/4)*2^(1/2)*(d*x)^(1/2
)/d^(1/2))*2^(1/2)*d^(1/2)/c^(3/4)
```

Rubi [A]
time = 0.16, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6049, 335, 307, 303, 1176, 631, 210, 1179, 642, 304, 211, 214}

$$\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d}+\frac{2b\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}}-\frac{\sqrt{2}\,b\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}}+\frac{\sqrt{2}\,b\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{3c^{3/4}}+\frac{b\sqrt{d}\,\log\left(\sqrt{c}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}\,c^{3/4}}-\frac{b\sqrt{d}\,\log\left(\sqrt{c}\sqrt{d}\,x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}\,c^{3/4}}-\frac{2b\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (2*b*Sqrt[d]*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/4)) - (Sqrt[2]*b*
Sqrt[d]*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/4)) + (Sqr
t[2]*b*Sqrt[d]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/4))
 + (2*(d*x)^(3/2)*(a + b*ArcTanh[c*x^2]))/(3*d) - (2*b*Sqrt[d]*ArcTanh[(c^(
1/4)*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/4)) + (b*Sqrt[d]*Log[Sqrt[d] + Sqrt[c]*Sq
rt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*c^(3/4)) - (b*Sqrt[d]*Log[
Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*c^(3/4
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 307

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[x^(m - n/2)/(r
 + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && LtQ
[m, n] &&  !GtQ[a/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx = \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{(4bc)\int \frac{x(dx)^{3/2}}{1-c^2x^4}\,dx}{3d}$$

$$= \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{(4bc)\int \frac{(dx)^{5/2}}{1-c^2x^4}\,dx}{3d^2}$$

$$= \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{(8bc)\text{Subst}\left(\int \frac{x^6}{1-\frac{c^2x^8}{d^4}}\,dx, x, \sqrt{dx}\right)}{3d^3}$$

$$= \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{1}{3}(4bd)\text{Subst}\left(\int \frac{x^2}{d^2-cx^4}\,dx, x, \sqrt{dx}\right)$$

$$= \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{(2bd)\text{Subst}\left(\int \frac{1}{d-\sqrt{c}\,x^2}\,dx, x, \sqrt{dx}\right)}{3\sqrt{c}} +$$

$$= \frac{2b\sqrt{d}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} + \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{2b\sqrt{d}\tan}{}$$

$$= \frac{2b\sqrt{d}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} + \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{2b\sqrt{d}\tan}{}$$

$$= \frac{2b\sqrt{d}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} - \frac{\sqrt{2}\,b\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} +$$

Mathematica [A]
time = 0.05, size = 227, normalized size = 0.75

$$\frac{\sqrt{dx}\left(4ac^{3/4}x^{3/2}-2\sqrt{2}\,b\text{ArcTan}\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)+2\sqrt{2}\,b\text{ArcTan}\left(1+\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)+4b\text{ArcTan}\left(\sqrt[4]{c}\sqrt{x}\right)+4bc^{3/4}x^{3/2}\tanh^{-1}\left(cx^2\right)+2b\log\left(1-\sqrt[4]{c}\sqrt{x}\right)-2b\log\left(1+\sqrt[4]{c}\sqrt{x}\right)+\sqrt{2}\,b\log\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)-\sqrt{2}\,b\log\left(1+\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)\right)}{6c^{3/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (Sqrt[d*x]*(4*a*c^(3/4)*x^(3/2) - 2*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*c^(1/4)*Sq
rt[x]] + 2*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]] + 4*b*ArcTan[c^(1/
4)*Sqrt[x]] + 4*b*c^(3/4)*x^(3/2)*ArcTanh[c*x^2] + 2*b*Log[1 - c^(1/4)*Sqrt
[x]] - 2*b*Log[1 + c^(1/4)*Sqrt[x]] + Sqrt[2]*b*Log[1 - Sqrt[2]*c^(1/4)*Sqr
t[x] + Sqrt[c]*x] - Sqrt[2]*b*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x])
)/(6*c^(3/4)*Sqrt[x])
```

Maple [A]
time = 0.04, size = 289, normalized size = 0.96

method	result
derivativedivides	$\frac{\frac{2(dx)^{\frac{3}{2}}a}{3}+\frac{2b(dx)^{\frac{3}{2}}\operatorname{arctanh}\left(c\,x^{2}\right)}{3}+\frac{b\,d^{2}\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)}{6c\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+\frac{b\,d^{2}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+1\right)}{3c\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}}{d}$
default	$\frac{\frac{2(dx)^{\frac{3}{2}}a}{3}+\frac{2b(dx)^{\frac{3}{2}}\operatorname{arctanh}\left(c\,x^{2}\right)}{3}+\frac{b\,d^{2}\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)}{6c\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+\frac{b\,d^{2}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+1\right)}{3c\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/3*(d*x)^(3/2)*a+1/3*b*(d*x)^(3/2)*arctanh(c*x^2)+1/12*b/c*d^2/(d^2/c
)^(1/4)*2^(1/2)*ln((d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(d
*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))+1/6*b/c*d^2/(d^2/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)+1/6*b/c*d^2/(d^2/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)+1/3*b/c*d^2/(d^2
/c)^(1/4)*arctan((d*x)^(1/2)/(d^2/c)^(1/4))-1/6*b/c*d^2/(d^2/c)^(1/4)*ln(((
d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4))))
```

Maxima [A]

time = 0.47, size = 301, normalized size = 1.00

$$\frac{4\,(dx)^{\frac{3}{2}}a+\left(4\,(dx)^{\frac{3}{2}}\operatorname{artanh}\left(cx^{2}\right)+\frac{\left(\frac{d^{4}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{c}\,dx+\sqrt{2}\sqrt{dx}\,c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{3}{4}}\sqrt{d}}+\frac{\sqrt{2}\log\left(\sqrt{c}\,dx-\sqrt{2}\sqrt{dx}\,c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{3}{4}}\sqrt{d}}\right)}{c}+\frac{2\,d^{4}\left(\frac{2\arctan\left(\frac{\sqrt{dx}\sqrt{c}}{\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}\,\sqrt{c}}+\frac{\log\left(\frac{\sqrt{dx}\sqrt{c}-\sqrt{\sqrt{c}\,d}}{\sqrt{dx}\sqrt{c}+\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}\,\sqrt{c}}\right)}{c}\right)c}{d^{2}}\right)b}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/6*(4*(d*x)^(3/2)*a + (4*(d*x)^(3/2)*arctanh(c*x^2) + (d^4*(2*sqrt(2)*arct
an(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) + 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)
*d))/(sqrt(sqrt(c)*d)*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(
1/4)*sqrt(d) - 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*sqrt(
```

```
c)) - sqrt(2)*log(sqrt(c)*d*x + sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/(c^(
3/4)*sqrt(d)) + sqrt(2)*log(sqrt(c)*d*x - sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d)
 + d)/(c^(3/4)*sqrt(d)))/c + 2*d^4*(2*arctan(sqrt(d*x)*sqrt(c)/sqrt(sqrt(c)
*d))/(sqrt(sqrt(c)*d)*sqrt(c)) + log((sqrt(d*x)*sqrt(c) - sqrt(sqrt(c)*d))/
(sqrt(d*x)*sqrt(c) + sqrt(sqrt(c)*d)))/(sqrt(sqrt(c)*d)*sqrt(c)))/c)*c/d^2)
*b)/d
```

Fricas [**A**]
time = 0.43, size = 400, normalized size = 1.33

$$\frac{1}{3}\left(bx\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,ax\right)\sqrt{dx}-\frac{4}{3}\left(\frac{b^4d^2}{c^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(\frac{b^4d^2}{c^3}\right)^{\frac{1}{4}}\sqrt{dx}\,b^3cd-\sqrt{b^6d^3x+\sqrt{\frac{b^4d^2}{c^3}}\,b^4cd^2}\left(\frac{b^4d^2}{c^3}\right)^{\frac{1}{4}}c}{b^4d^2}\right)-\frac{4}{3}\left(-\frac{b^4d^2}{c^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{b^4d^2}{c^3}\right)^{\frac{1}{4}}\sqrt{dx}\,b^3cd-\sqrt{b^6d^3x-\sqrt{-\frac{b^4d^2}{c^3}}\,b^4cd^2}\left(-\frac{b^4d^2}{c^3}\right)^{\frac{1}{4}}c}{b^4d^2}\right)-\frac{1}{3}\left(\frac{b^4d^2}{c^3}\right)^{\frac{1}{4}}\log\left(\sqrt{dx}\,b^3d+\left(\frac{b^4d^2}{c^3}\right)^{\frac{3}{4}}c^2\right)+\frac{1}{3}\left(\frac{b^4d^2}{c^3}\right)^{\frac{1}{4}}\log\left(\sqrt{dx}\,b^3d-\left(\frac{b^4d^2}{c^3}\right)^{\frac{3}{4}}c^2\right)+\frac{1}{3}\left(-\frac{b^4d^2}{c^3}\right)^{\frac{1}{4}}\log\left(\sqrt{dx}\,b^3d+\left(-\frac{b^4d^2}{c^3}\right)^{\frac{3}{4}}c^2\right)-\frac{1}{3}\left(-\frac{b^4d^2}{c^3}\right)^{\frac{1}{4}}\log\left(\sqrt{dx}\,b^3d-\left(-\frac{b^4d^2}{c^3}\right)^{\frac{3}{4}}c^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/3*(b*x*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*x)*sqrt(d*x) - 4/3*(b^4*d^2/c^
3)^(1/4)*arctan(-((b^4*d^2/c^3)^(1/4)*sqrt(d*x)*b^3*c*d - sqrt(b^6*d^3*x +
sqrt(b^4*d^2/c^3)*b^4*c*d^2)*(b^4*d^2/c^3)^(1/4)*c)/(b^4*d^2)) - 4/3*(-b^4*
d^2/c^3)^(1/4)*arctan(-((-b^4*d^2/c^3)^(1/4)*sqrt(d*x)*b^3*c*d - sqrt(b^6*d
^3*x - sqrt(-b^4*d^2/c^3)*b^4*c*d^2)*(-b^4*d^2/c^3)^(1/4)*c)/(b^4*d^2)) - 1
/3*(b^4*d^2/c^3)^(1/4)*log(sqrt(d*x)*b^3*d + (b^4*d^2/c^3)^(3/4)*c^2) + 1/3
*(b^4*d^2/c^3)^(1/4)*log(sqrt(d*x)*b^3*d - (b^4*d^2/c^3)^(3/4)*c^2) + 1/3*(
-b^4*d^2/c^3)^(1/4)*log(sqrt(d*x)*b^3*d + (-b^4*d^2/c^3)^(3/4)*c^2) - 1/3*(
-b^4*d^2/c^3)^(1/4)*log(sqrt(d*x)*b^3*d - (-b^4*d^2/c^3)^(3/4)*c^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(a+b*atanh(c*x**2)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*(b*arctanh(c*x^2) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d\,x}\,\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a + b*atanh(c*x^2)),x)
[Out] int((d*x)^(1/2)*(a + b*atanh(c*x^2)), x)
```

3.85 $\int \frac{a+b\tanh^{-1}(cx^2)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=285

$$-\frac{2b\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}}-\frac{\sqrt{2}\,b\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}}+\frac{\sqrt{2}\,b\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}}+\frac{2\sqrt{dx}}{}$$

```
[Out] -2*b*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))/c^(1/4)/d^(1/2)-2*b*arctanh(c^(1/4
)*(d*x)^(1/2)/d^(1/2))/c^(1/4)/d^(1/2)-1/2*b*ln(d^(1/2)+x*c^(1/2)*d^(1/2)-c
^(1/4)*2^(1/2)*(d*x)^(1/2))/c^(1/4)*2^(1/2)/d^(1/2)+1/2*b*ln(d^(1/2)+x*c^(1
/2)*d^(1/2)+c^(1/4)*2^(1/2)*(d*x)^(1/2))/c^(1/4)*2^(1/2)/d^(1/2)+b*arctan(-
1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/c^(1/4)/d^(1/2)+b*arctan(1+c
^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/c^(1/4)/d^(1/2)+2*(a+b*arctanh(
c*x^2))*(d*x)^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6049, 335, 307, 217, 1179, 642, 1176, 631, 210, 218, 214, 211}

$$\frac{2\sqrt{dx}\left(a+b\tanh^{-1}(cx^2)\right)}{d}-\frac{2b\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}}-\frac{\sqrt{2}\,b\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}}+\frac{\sqrt{2}\,b\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt[4]{c}\sqrt{d}}-\frac{b\log\left(\sqrt{c}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{b\log\left(\sqrt{c}\sqrt{d}\,x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{2b\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/Sqrt[d*x],x]
```

```
[Out] (-2*b*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(c^(1/4)*Sqrt[d]) - (Sqrt[2]*b*A
rcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(c^(1/4)*Sqrt[d]) + (Sqrt[2
]*b*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(c^(1/4)*Sqrt[d]) + (2
*Sqrt[d*x]*(a + b*ArcTanh[c*x^2]))/d - (2*b*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqr
t[d]])/(c^(1/4)*Sqrt[d]) - (b*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(
1/4)*Sqrt[d*x]])/(Sqrt[2]*c^(1/4)*Sqrt[d]) + (b*Log[Sqrt[d] + Sqrt[c]*Sqrt[
d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(Sqrt[2]*c^(1/4)*Sqrt[d])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 307

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[x^(m - n/2)/(r
 + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && LtQ
[m, n] &&  !GtQ[a/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx^2)}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}\,(a+b\tanh^{-1}(cx^2))}{d} - \frac{(4bc)\int \frac{x\sqrt{dx}}{1-c^2x^4}\,dx}{d}$$

$$= \frac{2\sqrt{dx}\,(a+b\tanh^{-1}(cx^2))}{d} - \frac{(4bc)\int \frac{(dx)^{3/2}}{1-c^2x^4}\,dx}{d^2}$$

$$= \frac{2\sqrt{dx}\,(a+b\tanh^{-1}(cx^2))}{d} - \frac{(8bc)\text{Subst}\left(\int \frac{x^4}{1-\frac{c^2x^8}{d^4}}\,dx, x, \sqrt{dx}\right)}{d^3}$$

$$= \frac{2\sqrt{dx}\,(a+b\tanh^{-1}(cx^2))}{d} - (4bd)\text{Subst}\left(\int \frac{1}{d^2-cx^4}\,dx, x, \sqrt{dx}\right) + (4bd)\text{Subs}$$

$$= \frac{2\sqrt{dx}\,(a+b\tanh^{-1}(cx^2))}{d} - (2b)\text{Subst}\left(\int \frac{1}{d-\sqrt{c}\,x^2}\,dx, x, \sqrt{dx}\right) - (2b)\text{Subs}$$

$$= -\frac{2b\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{2\sqrt{dx}\,(a+b\tanh^{-1}(cx^2))}{d} - \frac{2b\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}}$$

$$= -\frac{2b\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{2\sqrt{dx}\,(a+b\tanh^{-1}(cx^2))}{d} - \frac{2b\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}}$$

$$= -\frac{2b\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} - \frac{\sqrt{2}\,b\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{\sqrt{2}\,b\tan^{-1}\left(1+\right.}{\sqrt[4]{c}}$$

Mathematica [A]
time = 0.04, size = 227, normalized size = 0.80

$$\frac{\sqrt{x}\left(4a\sqrt[4]{c}\sqrt{x}-2\sqrt{2}\,b\text{ArcTan}\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)+2\sqrt{2}\,b\text{ArcTan}\left(1+\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)-4b\text{ArcTan}\left(\sqrt[4]{c}\sqrt{x}\right)+4b\sqrt[4]{c}\sqrt{x}\tanh^{-1}(cx^2)+2b\log\left(1-\sqrt[4]{c}\sqrt{x}\right)-2b\log\left(1+\sqrt[4]{c}\sqrt{x}\right)-\sqrt{2}\,b\log\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)+\sqrt{2}\,b\log\left(1+\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)\right)}{2\sqrt[4]{c}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/Sqrt[d*x],x]
```

```
[Out] (Sqrt[x]*(4*a*c^(1/4)*Sqrt[x] - 2*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt
[x]] + 2*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]] - 4*b*ArcTan[c^(1/4)
*Sqrt[x]] + 4*b*c^(1/4)*Sqrt[x]*ArcTanh[c*x^2] + 2*b*Log[1 - c^(1/4)*Sqrt[x
]] - 2*b*Log[1 + c^(1/4)*Sqrt[x]] - Sqrt[2]*b*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[
x] + Sqrt[c]*x] + Sqrt[2]*b*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))/
(2*c^(1/4)*Sqrt[d*x])
```

Maple [A]
time = 0.04, size = 257, normalized size = 0.90

method	result
derivativedivides	$\frac{2\sqrt{dx}\,a+2b\sqrt{dx}\,\operatorname{arctanh}(c\,x^2)+\frac{b\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2}{c}}}{dx-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2}{c}}}\right)}{2}+b\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\Big(}{}$
default	$\frac{2\sqrt{dx}\,a+2b\sqrt{dx}\,\operatorname{arctanh}(c\,x^2)+\frac{b\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2}{c}}}{dx-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2}{c}}}\right)}{2}+b\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\Big(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*((d*x)^(1/2)*a+b*(d*x)^(1/2)*arctanh(c*x^2)+1/4*b*(d^2/c)^(1/4)*2^(1/2)
*ln((d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(d*x-(d^2/c)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))+1/2*b*(d^2/c)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)+1/2*b*(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)-1/2*b*(d^2/c)^(1/4)*ln(((d*x)^(1/2)+(d^2/c)
^(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4)))-b*(d^2/c)^(1/4)*arctan((d*x)^(1/2)/(d^
2/c)^(1/4)))
```

Maxima [A]

time = 0.47, size = 296, normalized size = 1.04

$$\frac{\left(4\sqrt{dx}\,\operatorname{artanh}\left(cx^2\right)+\frac{c\left(\frac{\frac{2\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}}+\frac{2\sqrt{2}\,d^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}}+\frac{\sqrt{2}\,d^{\frac{5}{2}}\log\left(\sqrt{c}\,dx+\sqrt{2}\sqrt{dx}\,c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}\,d^{\frac{5}{2}}\log\left(\sqrt{c}\,dx-\sqrt{2}\sqrt{dx}\,c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{3}{4}}}}{c}-\frac{2\left(\frac{2d^3\arctan\left(\frac{\sqrt{dx}\sqrt{c}}{\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}}-\frac{d^3\log\left(\frac{\sqrt{dx}\sqrt{c}-\sqrt{\sqrt{c}\,d}}{\sqrt{dx}\sqrt{c}+\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}}\right)}{c}\right)}{d^2}\right)b+4\sqrt{dx}\,a}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*((4*sqrt(d*x)*arctanh(c*x^2) + c*((2*sqrt(2)*d^3*arctan(1/2*sqrt(2)*(sq
rt(2)*c^(1/4)*sqrt(d) + 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/sqrt(sqrt(c)*
d) + 2*sqrt(2)*d^3*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) - 2*sqrt(d*
x)*sqrt(c))/sqrt(sqrt(c)*d))/sqrt(sqrt(c)*d) + sqrt(2)*d^(5/2)*log(sqrt(c)*
```

```
d*x + sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/c^(1/4) - sqrt(2)*d^(5/2)*log(
sqrt(c)*d*x - sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/c^(1/4))/c - 2*(2*d^3*
arctan(sqrt(d*x)*sqrt(c)/sqrt(sqrt(c)*d))/sqrt(sqrt(c)*d) - d^3*log((sqrt(d
*x)*sqrt(c) - sqrt(sqrt(c)*d))/(sqrt(d*x)*sqrt(c) + sqrt(sqrt(c)*d)))/sqrt(
sqrt(c)*d))/c)/d^2)*b + 4*sqrt(d*x)*a)/d
```

Fricas [A]
time = 0.39, size = 364, normalized size = 1.28

$$\frac{4\left(\frac{b^4}{cd^2}\right)^{\frac{1}{4}} d\arctan\left(-\frac{\sqrt{dx}\,bc\left(\frac{b^4}{cd^2}\right)^{\frac{3}{4}}d-\sqrt{b^2dx+\sqrt{\frac{b^4}{cd^2}}\,d^2}\,c\left(\frac{b^4}{cd^2}\right)^{\frac{3}{4}}d}{b^4}\right)+4\left(-\frac{b^4}{cd^2}\right)^{\frac{1}{4}} d\arctan\left(-\frac{\sqrt{dx}\,bc\left(-\frac{b^4}{cd^2}\right)^{\frac{3}{4}}d-\sqrt{b^2dx+\sqrt{-\frac{b^4}{cd^2}}\,d^2}\,c\left(-\frac{b^4}{cd^2}\right)^{\frac{3}{4}}d}{b^4}\right)-\left(\frac{b^4}{cd^2}\right)^{\frac{1}{4}} d\log\left(\sqrt{dx}\,b+\left(\frac{b^4}{cd^2}\right)^{\frac{1}{4}}d\right)+\left(\frac{b^4}{cd^2}\right)^{\frac{1}{4}} d\log\left(\sqrt{dx}\,b-\left(\frac{b^4}{cd^2}\right)^{\frac{1}{4}}d\right)+\left(-\frac{b^4}{cd^2}\right)^{\frac{1}{4}} d\log\left(\sqrt{dx}\,b+\left(-\frac{b^4}{cd^2}\right)^{\frac{1}{4}}d\right)-\left(-\frac{b^4}{cd^2}\right)^{\frac{1}{4}} d\log\left(\sqrt{dx}\,b-\left(-\frac{b^4}{cd^2}\right)^{\frac{1}{4}}d\right)+\sqrt{dx}\left(b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2a\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] (4*(b^4/(c*d^2))^(1/4)*d*arctan(-(sqrt(d*x)*b*c*(b^4/(c*d^2))^(3/4)*d - sqr
t(b^2*d*x + sqrt(b^4/(c*d^2))*d^2)*c*(b^4/(c*d^2))^(3/4)*d)/b^4) + 4*(-b^4/
(c*d^2))^(1/4)*d*arctan(-(sqrt(d*x)*b*c*(-b^4/(c*d^2))^(3/4)*d - sqrt(b^2*d
*x + sqrt(-b^4/(c*d^2))*d^2)*c*(-b^4/(c*d^2))^(3/4)*d)/b^4) - (b^4/(c*d^2))
^(1/4)*d*log(sqrt(d*x)*b + (b^4/(c*d^2))^(1/4)*d) + (b^4/(c*d^2))^(1/4)*d*l
og(sqrt(d*x)*b - (b^4/(c*d^2))^(1/4)*d) + (-b^4/(c*d^2))^(1/4)*d*log(sqrt(d
*x)*b + (-b^4/(c*d^2))^(1/4)*d) - (-b^4/(c*d^2))^(1/4)*d*log(sqrt(d*x)*b -
(-b^4/(c*d^2))^(1/4)*d) + sqrt(d*x)*(b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)
)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{atanh}\left(cx^2\right)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/(d*x)**(1/2),x)
```

```
[Out] Integral((a + b*atanh(c*x**2))/sqrt(d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(198) = 396.
time = 0.42, size = 493, normalized size = 1.73

$$\frac{\left(cd^2\left(\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2}+\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2}-\frac{2\sqrt{2}\left(-c^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2}-\frac{2\sqrt{2}\left(-c^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2}+\frac{\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\sqrt{dx}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{d^2}{c}}\right)}{c^2d^2}-\frac{\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\sqrt{dx}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{d^2}{c}}\right)}{c^2d^2}-\frac{\sqrt{2}\left(-c^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\sqrt{dx}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}+\sqrt{-\frac{d^2}{c}}\right)}{c^2d^2}+\frac{\sqrt{2}\left(-c^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\sqrt{dx}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}+\sqrt{-\frac{d^2}{c}}\right)}{c^2d^2}\right)+2\sqrt{dx}\log\left(-\frac{cx^2+1}{cx^2-1}\right)\right)b+4\sqrt{dx}\,a}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((c*d^2*(2*sqrt(2)*(c^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)^
(1/4) + 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^2*d^2) + 2*sqrt(2)*(c^3*d^2)^(1/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^
2*d^2) - 2*sqrt(2)*(-c^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1
/4) + 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c^2*d^2) - 2*sqrt(2)*(-c^3*d^2)^(1/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4))/(
c^2*d^2) + sqrt(2)*(c^3*d^2)^(1/4)*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(1/4
) + sqrt(d^2/c))/(c^2*d^2) - sqrt(2)*(c^3*d^2)^(1/4)*log(d*x - sqrt(2)*sqrt
(d*x)*(d^2/c)^(1/4) + sqrt(d^2/c))/(c^2*d^2) - sqrt(2)*(-c^3*d^2)^(1/4)*log
(d*x + sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c^2*d^2) + sqrt(2)
*(-c^3*d^2)^(1/4)*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c)
)/(c^2*d^2)) + 2*sqrt(d*x)*log(-(c*x^2 + 1)/(c*x^2 - 1)))*b + 4*sqrt(d*x)*a
)/d
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atanh}(c\,x^2)}{\sqrt{d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/(d*x)^(1/2),x)
```

```
[Out] int((a + b*atanh(c*x^2))/(d*x)^(1/2), x)
```

3.86 $\int \frac{a+b\tanh^{-1}(cx^2)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=285

$$-\frac{2b\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{2}\,b\sqrt[4]{c}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{2}\,b\sqrt[4]{c}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

```
[Out] -2*b*c^(1/4)*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(3/2)+2*b*c^(1/4)*arctan
h(c^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(3/2)+1/2*b*c^(1/4)*ln(d^(1/2)+x*c^(1/2)*d
^(1/2)-c^(1/4)*2^(1/2)*(d*x)^(1/2))/d^(3/2)*2^(1/2)-1/2*b*c^(1/4)*ln(d^(1/2
)+x*c^(1/2)*d^(1/2)+c^(1/4)*2^(1/2)*(d*x)^(1/2))/d^(3/2)*2^(1/2)+b*c^(1/4)*
arctan(-1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)+b*c^(1/4)*ar
ctan(1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)-2*(a+b*arctanh(
c*x^2))/d/(d*x)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6049, 335, 306, 303, 1176, 631, 210, 1179, 642, 304, 211, 214}

$$-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}-\frac{2b\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{2}\,b\sqrt[4]{c}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{2}\,b\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}+1\right)}{d^{3/2}}+\frac{b\sqrt[4]{c}\,\log\left(\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,\sqrt[4]{c}\,\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}\,d^{3/2}}-\frac{b\sqrt[4]{c}\,\log\left(\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,\sqrt[4]{c}\,\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}\,d^{3/2}}+\frac{2b\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/(d*x)^(3/2),x]
```

```
[Out] (-2*b*c^(1/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) - (Sqrt[2]*b*c^(
1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (Sqrt[2]*b*
c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) - (2*(a +
b*ArcTanh[c*x^2]))/(d*Sqrt[d*x]) + (2*b*c^(1/4)*ArcTanh[(c^(1/4)*Sqrt[d*x])
/Sqrt[d]])/d^(3/2) + (b*c^(1/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c
^(1/4)*Sqrt[d*x]])/(Sqrt[2]*d^(3/2)) - (b*c^(1/4)*Log[Sqrt[d] + Sqrt[c]*Sqr
t[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(Sqrt[2]*d^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 306

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[x^m/(r + s*x^(
n/2)), x], x] + Dist[r/(2*a), Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ[{a,
 b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx^2)}{(dx)^{3/2}}\,dx = -\frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{d\sqrt{dx}} + \frac{(4bc)\int \frac{x}{\sqrt{dx}\,(1-c^2x^4)}\,dx}{d}$$

$$= -\frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{d\sqrt{dx}} + \frac{(4bc)\int \frac{\sqrt{dx}}{1-c^2x^4}\,dx}{d^2}$$

$$= -\frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{d\sqrt{dx}} + \frac{(8bc)\text{Subst}\left(\int \frac{x^2}{1-\frac{c^2x^8}{d^4}}\,dx, x, \sqrt{dx}\right)}{d^3}$$

$$= -\frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{d\sqrt{dx}} + \frac{(4bc)\text{Subst}\left(\int \frac{x^2}{d^2-cx^4}\,dx, x, \sqrt{dx}\right)}{d} + \frac{(4bc)\text{Subst}\left(\int \frac{}{d^{\cdots}}\right.}{}$$

$$= -\frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{d\sqrt{dx}} + \frac{\left(2b\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{d-\sqrt{c}\,x^2}\,dx, x, \sqrt{dx}\right)}{d} - \frac{\left(2b\sqrt{c}\right)\cdots}{}$$

$$= -\frac{2b\sqrt[4]{c}\,\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{d\sqrt{dx}} + \frac{2b\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{d\cdots}}{\sqrt{d}}\right.}{d^{3/2}}$$

$$= -\frac{2b\sqrt[4]{c}\,\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{d\sqrt{dx}} + \frac{2b\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{d\cdots}}{\sqrt{d}}\right.}{d^{3/2}}$$

$$= -\frac{2b\sqrt[4]{c}\,\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{\sqrt{2}\,b\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{2}\,b\sqrt[4]{\cdots}}{}$$

Mathematica [A]
time = 0.07, size = 268, normalized size = 0.94

$$-\frac{x\left(4a+2\sqrt{2}\,b\sqrt[4]{c}\sqrt{x}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)-2\sqrt{2}\,b\sqrt[4]{c}\sqrt{x}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)+4b\sqrt[4]{c}\sqrt{x}\,\text{ArcTan}\left(\sqrt[4]{c}\sqrt{x}\right)+4b\tanh^{-1}(cx^2)+2b\sqrt[4]{c}\sqrt{x}\log\left(1-\sqrt[4]{c}\sqrt{x}\right)-2b\sqrt[4]{c}\sqrt{x}\log\left(1+\sqrt[4]{c}\sqrt{x}\right)-\sqrt{2}\,b\sqrt[4]{c}\sqrt{x}\log\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)+\sqrt{2}\,b\sqrt[4]{c}\sqrt{x}\log\left(1+\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)\right)}{2(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/(d*x)^(3/2),x]
```

```
[Out] -1/2*(x*(4*a + 2*Sqrt[2]*b*c^(1/4)*Sqrt[x]*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt[
x]] - 2*Sqrt[2]*b*c^(1/4)*Sqrt[x]*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]] + 4*b
*c^(1/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]] + 4*b*ArcTanh[c*x^2] + 2*b*c^(1/4)
*Sqrt[x]*Log[1 - c^(1/4)*Sqrt[x]] - 2*b*c^(1/4)*Sqrt[x]*Log[1 + c^(1/4)*Sqr
t[x]] - Sqrt[2]*b*c^(1/4)*Sqrt[x]*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]
```

```
*x] + Sqrt[2]*b*c^(1/4)*Sqrt[x]*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x
]))/(d*x)^(3/2)
```

Maple [A]
time = 0.05, size = 259, normalized size = 0.91

method	result
derivativedivides	$\frac{-\frac{2a}{\sqrt{dx}}-\frac{2b\,\mathrm{arctanh}\left(c\,x^{2}\right)}{\sqrt{dx}}+\frac{b\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)}{2\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+\frac{b\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+1\right)}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+\frac{b\sqrt{2}}{}}{d}$
default	$\frac{-\frac{2a}{\sqrt{dx}}-\frac{2b\,\mathrm{arctanh}\left(c\,x^{2}\right)}{\sqrt{dx}}+\frac{b\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)}{2\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+\frac{b\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+1\right)}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+\frac{b\sqrt{2}}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-a/(d*x)^(1/2)-b/(d*x)^(1/2)*arctanh(c*x^2)+1/4*b/(d^2/c)^(1/4)*2^(1/2
)*ln((d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(d*x+(d^2/c)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))+1/2*b/(d^2/c)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)+1/2*b/(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)-b/(d^2/c)^(1/4)*arctan((d*x)^(1/2)/(d^2/c)
^(1/4))+1/2*b/(d^2/c)^(1/4)*ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^
2/c)^(1/4))))
```

Maxima [A]
time = 0.47, size = 296, normalized size = 1.04

$$-\frac{b\left(\frac{4\,\mathrm{artanh}\left(cx^{2}\right)}{\sqrt{dx}}-\frac{\left(d^{2}\left(\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\,\sqrt{c}\right)}{2\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}\,\sqrt{c}}+\frac{2\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\,\sqrt{c}\right)}{2\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{c}\,dx+\sqrt{2}\,\sqrt{dx}\,c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{3}{4}}\sqrt{d}}+\frac{\sqrt{2}\,\log\left(\sqrt{c}\,dx-\sqrt{2}\,\sqrt{dx}\,c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{3}{4}}\sqrt{d}}\right)-2d^{2}\left(\frac{2\arctan\left(\frac{\sqrt{dx}\,\sqrt{c}}{\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}\,\sqrt{c}}+\frac{\log\left(\frac{\sqrt{dx}\,\sqrt{c}-\sqrt{\sqrt{c}\,d}}{\sqrt{dx}\,\sqrt{c}+\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}\,\sqrt{c}}\right)\right)c}{d^{2}}\right)+\frac{4a}{\sqrt{dx}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(b*(4*arctanh(c*x^2)/sqrt(d*x) - (d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(s
qrt(2)*c^(1/4)*sqrt(d) + 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/(sqrt(sqrt(c
)*d)*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) - 2*
sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) - sqrt(2)*log
```

```
(sqrt(c)*d*x + sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/(c^(3/4)*sqrt(d)) + s
qrt(2)*log(sqrt(c)*d*x - sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/(c^(3/4)*sq
rt(d))) - 2*d^2*(2*arctan(sqrt(d*x)*sqrt(c)/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*
d)*sqrt(c)) + log((sqrt(d*x)*sqrt(c) - sqrt(sqrt(c)*d))/(sqrt(d*x)*sqrt(c)
+ sqrt(sqrt(c)*d)))/(sqrt(sqrt(c)*d)*sqrt(c))))*c/d^2) + 4*a/sqrt(d*x))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(198) = 396$.
time = 0.39, size = 397, normalized size = 1.39

$$\frac{4\,d^2x\left(\frac{b^4c}{d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,b^3cd\left(\frac{b^4c}{d^6}\right)^{\frac{1}{4}}-\sqrt{b^6c^2dx+b^4cd^4\sqrt{\frac{b^4c}{d^6}}}\,d\left(\frac{b^4c}{d^6}\right)^{\frac{1}{4}}}{b^4c}\right)-4\,d^2x\left(-\frac{b^4c}{d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,b^3cd\left(-\frac{b^4c}{d^6}\right)^{\frac{1}{4}}-\sqrt{b^6c^2dx-b^4cd^4\sqrt{-\frac{b^4c}{d^6}}}\,d\left(-\frac{b^4c}{d^6}\right)^{\frac{1}{4}}}{b^4c}\right)+d^2x\left(\frac{b^4c}{d^6}\right)^{\frac{1}{4}}\log\left(d^5\left(\frac{b^4c}{d^6}\right)^{\frac{3}{4}}+\sqrt{dx}\,b^3c\right)-d^2x\left(\frac{b^4c}{d^6}\right)^{\frac{1}{4}}\log\left(-d^5\left(\frac{b^4c}{d^6}\right)^{\frac{3}{4}}+\sqrt{dx}\,b^3c\right)+d^2x\left(-\frac{b^4c}{d^6}\right)^{\frac{1}{4}}\log\left(d^5\left(-\frac{b^4c}{d^6}\right)^{\frac{3}{4}}+\sqrt{dx}\,b^3c\right)-d^2x\left(-\frac{b^4c}{d^6}\right)^{\frac{1}{4}}\log\left(-d^5\left(-\frac{b^4c}{d^6}\right)^{\frac{3}{4}}+\sqrt{dx}\,b^3c\right)-\sqrt{dx}\left(b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a\right)}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] (4*d^2*x*(b^4*c/d^6)^(1/4)*arctan(-(sqrt(d*x)*b^3*c*d*(b^4*c/d^6)^(1/4) - s
qrt(b^6*c^2*d*x + b^4*c*d^4*sqrt(b^4*c/d^6))*d*(b^4*c/d^6)^(1/4))/(b^4*c))
- 4*d^2*x*(-b^4*c/d^6)^(1/4)*arctan(-(sqrt(d*x)*b^3*c*d*(-b^4*c/d^6)^(1/4)
- sqrt(b^6*c^2*d*x - b^4*c*d^4*sqrt(-b^4*c/d^6))*d*(-b^4*c/d^6)^(1/4))/(b^4
*c)) + d^2*x*(b^4*c/d^6)^(1/4)*log(d^5*(b^4*c/d^6)^(3/4) + sqrt(d*x)*b^3*c)
 - d^2*x*(b^4*c/d^6)^(1/4)*log(-d^5*(b^4*c/d^6)^(3/4) + sqrt(d*x)*b^3*c) +
d^2*x*(-b^4*c/d^6)^(1/4)*log(d^5*(-b^4*c/d^6)^(3/4) + sqrt(d*x)*b^3*c) - d^
2*x*(-b^4*c/d^6)^(1/4)*log(-d^5*(-b^4*c/d^6)^(3/4) + sqrt(d*x)*b^3*c) - sqr
t(d*x)*(b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a))/(d^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\operatorname{atanh}\left(cx^2\right)}{\left(dx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/(d*x)**(3/2),x)
```

```
[Out] Integral((a + b*atanh(c*x**2))/(d*x)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(198) = 396$.
time = 0.58, size = 505, normalized size = 1.77

$$-\frac{\frac{2\,b\log\left(-\frac{cd^2x^2+d^2}{cd^2x^2-d^2}\right)}{\sqrt{dx}}+\frac{4\,a}{\sqrt{dx}}-\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac{3}{4}}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2}-\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac{3}{4}}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2}-\frac{2\sqrt{2}\left(-c^3d^2\right)^{\frac{3}{4}}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2}-\frac{2\sqrt{2}\left(-c^3d^2\right)^{\frac{3}{4}}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2}+\frac{\sqrt{2}\left(c^3d^2\right)^{\frac{3}{4}}b\log\left(dx+\sqrt{2}\sqrt{dx}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{d^2}{c}}\right)}{c^2d^2}-\frac{\sqrt{2}\left(c^3d^2\right)^{\frac{3}{4}}b\log\left(dx-\sqrt{2}\sqrt{dx}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{d^2}{c}}\right)}{c^2d^2}+\frac{\sqrt{2}\left(-c^3d^2\right)^{\frac{3}{4}}b\log\left(dx+\sqrt{2}\sqrt{dx}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}+\sqrt{-\frac{d^2}{c}}\right)}{c^2d^2}-\frac{\sqrt{2}\left(-c^3d^2\right)^{\frac{3}{4}}b\log\left(dx-\sqrt{2}\sqrt{dx}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}+\sqrt{-\frac{d^2}{c}}\right)}{c^2d^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*log(-(c*d^2*x^2 + d^2)/(c*d^2*x^2 - d^2))/sqrt(d*x) + 4*a/sqrt(d*
x) - 2*sqrt(2)*(c^3*d^2)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4)
+ 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^2*d^2) - 2*sqrt(2)*(c^3*d^2)^(3/4)*b*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^2*d^
2) - 2*sqrt(2)*(-c^3*d^2)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4
) + 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c^2*d^2) - 2*sqrt(2)*(-c^3*d^2)^(3/4)*b*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4))/(
c^2*d^2) + sqrt(2)*(c^3*d^2)^(3/4)*b*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(1
/4) + sqrt(d^2/c))/(c^2*d^2) - sqrt(2)*(c^3*d^2)^(3/4)*b*log(d*x - sqrt(2)*
sqrt(d*x)*(d^2/c)^(1/4) + sqrt(d^2/c))/(c^2*d^2) + sqrt(2)*(-c^3*d^2)^(3/4)
*b*log(d*x + sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c^2*d^2) - s
qrt(2)*(-c^3*d^2)^(3/4)*b*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt
(-d^2/c))/(c^2*d^2))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{atanh}(c\,x^2)}{(d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/(d*x)^(3/2),x)
```

```
[Out] int((a + b*atanh(c*x^2))/(d*x)^(3/2), x)
```

3.87 $\int \frac{a+b\tanh^{-1}(cx^2)}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=301

$$\frac{2bc^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{\sqrt{2}\,bc^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{\sqrt{2}\,bc^{3/4}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}$$

```
[Out] 2/3*b*c^(3/4)*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(5/2)-2/3*(a+b*arctanh(
c*x^2))/d/(d*x)^(3/2)+2/3*b*c^(3/4)*arctanh(c^(1/4)*(d*x)^(1/2)/d^(1/2))/d^
(5/2)-1/6*b*c^(3/4)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)-c^(1/4)*2^(1/2)*(d*x)^(1/2
))/d^(5/2)*2^(1/2)+1/6*b*c^(3/4)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)+c^(1/4)*2^(1/
2)*(d*x)^(1/2))/d^(5/2)*2^(1/2)+1/3*b*c^(3/4)*arctan(-1+c^(1/4)*2^(1/2)*(d*
x)^(1/2)/d^(1/2))*2^(1/2)/d^(5/2)+1/3*b*c^(3/4)*arctan(1+c^(1/4)*2^(1/2)*(d
*x)^(1/2)/d^(1/2))*2^(1/2)/d^(5/2)
```

Rubi [A]

time = 0.18, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6049, 335, 220, 218, 214, 211, 217, 1179, 642, 1176, 631, 210}

$$-\frac{2(a+b\tanh^{-1}(cx^2))}{3d(dx)^{3/2}}+\frac{2bc^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{\sqrt{2}\,bc^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{\sqrt{2}\,bc^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{3d^{5/2}}-\frac{bc^{3/4}\log\left(\sqrt{c}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}\,d^{5/2}}+\frac{bc^{3/4}\log\left(\sqrt{c}\sqrt{d}\,x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}\,d^{5/2}}+\frac{2bc^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/(d*x)^(5/2),x]
```

```
[Out] (2*b*c^(3/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2)) - (Sqrt[2]*b*
c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2)) + (Sqr
t[2]*b*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2))
 - (2*(a + b*ArcTanh[c*x^2]))/(3*d*(d*x)^(3/2)) + (2*b*c^(3/4)*ArcTanh[(c^(
1/4)*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2)) - (b*c^(3/4)*Log[Sqrt[d] + Sqrt[c]*Sq
rt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*d^(5/2)) + (b*c^(3/4)*Log[
Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*d^(5/2
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx^2)}{(dx)^{5/2}}\,dx = -\frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{3d(dx)^{3/2}} + \frac{(4bc)\int \frac{x}{(dx)^{3/2}(1-c^2x^4)}\,dx}{3d}$$

$$= -\frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{3d(dx)^{3/2}} + \frac{(4bc)\int \frac{1}{\sqrt{dx}\,(1-c^2x^4)}\,dx}{3d^2}$$

$$= -\frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{3d(dx)^{3/2}} + \frac{(8bc)\text{Subst}\left(\int \frac{1}{1-\frac{c^2x^8}{d^4}}\,dx, x, \sqrt{dx}\right)}{3d^3}$$

$$= -\frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{3d(dx)^{3/2}} + \frac{(4bc)\text{Subst}\left(\int \frac{1}{d^2-cx^4}\,dx, x, \sqrt{dx}\right)}{3d} + \frac{(4bc)\text{Subst}\left(\int \frac{1}{d^2+}\right.}{3d}$$

$$= -\frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{3d(dx)^{3/2}} + \frac{(2bc)\text{Subst}\left(\int \frac{1}{d-\sqrt{c}\,x^2}\,dx, x, \sqrt{dx}\right)}{3d^2} + \frac{(2bc)\text{Subst}\left(\int\right.}{}$$

$$= \frac{2bc^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{3d(dx)^{3/2}} + \frac{2bc^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}$$

$$= \frac{2bc^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{3d(dx)^{3/2}} + \frac{2bc^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}$$

$$= \frac{2bc^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{\sqrt{2}\,bc^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{\sqrt{2}\,bc^{3/4}\tan}{}$$

Mathematica [A]

time = 0.07, size = 268, normalized size = 0.89

$$-\frac{x\left(4a+2\sqrt{2}\,bc^{3/4}x^{3/2}\text{ArcTan}\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)-2\sqrt{2}\,bc^{3/4}x^{3/2}\text{ArcTan}\left(1+\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)-4bc^{3/4}x^{3/2}\text{ArcTan}\left(\sqrt[4]{c}\sqrt{x}\right)+4b\tanh^{-1}(cx^2)+2bc^{3/4}x^{3/2}\log\left(1-\sqrt[4]{c}\sqrt{x}\right)-2bc^{3/4}x^{3/2}\log\left(1+\sqrt[4]{c}\sqrt{x}\right)+\sqrt{2}\,bc^{3/4}x^{3/2}\log\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)-\sqrt{2}\,bc^{3/4}x^{3/2}\log\left(1+\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)\right)}{6(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/(d*x)^(5/2),x]
```

```
[Out] -1/6*(x*(4*a + 2*Sqrt[2]*b*c^(3/4)*x^(3/2)*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt[
x]] - 2*Sqrt[2]*b*c^(3/4)*x^(3/2)*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]] - 4*b
*c^(3/4)*x^(3/2)*ArcTan[c^(1/4)*Sqrt[x]] + 4*b*ArcTanh[c*x^2] + 2*b*c^(3/4)
*x^(3/2)*Log[1 - c^(1/4)*Sqrt[x]] - 2*b*c^(3/4)*x^(3/2)*Log[1 + c^(1/4)*Sqr
t[x]] + Sqrt[2]*b*c^(3/4)*x^(3/2)*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]
*x] - Sqrt[2]*b*c^(3/4)*x^(3/2)*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x
]))/(d*x)^(5/2)
```

Maple [A]
time = 0.04, size = 279, normalized size = 0.93

method	result
derivativedivides	$\dfrac{-\frac{2a}{3(dx)^{\frac{3}{2}}}-\frac{2b\,\mathrm{arctanh}\left(c\,x^{2}\right)}{3(dx)^{\frac{3}{2}}}+\frac{bc\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)}{6d^{2}}+\frac{bc\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{d^{2}}{c}\right)}\right.}{3d^{2}}}{d}$
default	$\dfrac{-\frac{2a}{3(dx)^{\frac{3}{2}}}-\frac{2b\,\mathrm{arctanh}\left(c\,x^{2}\right)}{3(dx)^{\frac{3}{2}}}+\frac{bc\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)}{6d^{2}}+\frac{bc\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{d^{2}}{c}\right)}\right.}{3d^{2}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/3*a/(d*x)^(3/2)-1/3*b/(d*x)^(3/2)*arctanh(c*x^2)+1/12*b*c/d^2*(d^2/
c)^(1/4)*2^(1/2)*ln((d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(
d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))+1/6*b*c/d^2*(d^2/c)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)+1/6*b*c/d^2*(d^2/c
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)+1/6*b*c/d^2*(d^
2/c)^(1/4)*ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4)))+1/3*
b*c/d^2*(d^2/c)^(1/4)*arctan((d*x)^(1/2)/(d^2/c)^(1/4)))
```

Maxima [A]
time = 0.47, size = 277, normalized size = 0.92

$$\frac{b\left(\frac{\left(\frac{2\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\,\sqrt{c}\right)}{2\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}}+\frac{2\sqrt{2}\,d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\,\sqrt{c}\right)}{2\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}}+\frac{\sqrt{2}\,\sqrt{d}\,\log\left(\sqrt{c}\,dx+\sqrt{2}\,\sqrt{dx}\,c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{1}{4}}}-\frac{\sqrt{2}\,\sqrt{d}\,\log\left(\sqrt{c}\,dx-\sqrt{2}\,\sqrt{dx}\,c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{1}{4}}}+\frac{4\,d\arctan\left(\frac{\sqrt{dx}\,\sqrt{c}}{\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}}-\frac{2\,d\log\left(\frac{\sqrt{dx}\,\sqrt{c}-\sqrt{\sqrt{c}\,d}}{\sqrt{dx}\,\sqrt{c}+\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}}\right)c}{d^{2}}-\frac{4\,\mathrm{artanh}\left(cx^{2}\right)}{(dx)^{\frac{3}{2}}}\right)-\frac{4\,a}{(dx)^{\frac{3}{2}}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*((2*sqrt(2)*d*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) + 2*sqrt(d
*x)*sqrt(c))/sqrt(sqrt(c)*d))/sqrt(sqrt(c)*d) + 2*sqrt(2)*d*arctan(-1/2*sqr
t(2)*(sqrt(2)*c^(1/4)*sqrt(d) - 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/sqrt(
sqrt(c)*d) + sqrt(2)*sqrt(d)*log(sqrt(c)*d*x + sqrt(2)*sqrt(d*x)*c^(1/4)*sq
rt(d) + d)/c^(1/4) - sqrt(2)*sqrt(d)*log(sqrt(c)*d*x - sqrt(2)*sqrt(d*x)*c^
(1/4)*sqrt(d) + d)/c^(1/4) + 4*d*arctan(sqrt(d*x)*sqrt(c)/sqrt(sqrt(c)*d))/
sqrt(sqrt(c)*d) - 2*d*log((sqrt(d*x)*sqrt(c) - sqrt(sqrt(c)*d))/(sqrt(d*x)*
```

```
sqrt(c) + sqrt(sqrt(c)*d)))/sqrt(sqrt(c)*d))*c/d^2 - 4*arctanh(c*x^2)/(d*x)
^(3/2)) - 4*a/(d*x)^(3/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(200) = 400.
time = 0.40, size = 428, normalized size = 1.42

$$-\frac{4\,d^{3}x^{2}\left(\frac{b^{4}c^{3}}{d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,bcd^{7}\left(\frac{b^{4}c^{3}}{d^{10}}\right)^{\frac{3}{4}}-\sqrt{d^{6}\sqrt{\frac{b^{4}c^{3}}{d^{10}}}+b^{2}c^{2}dx}\,d^{7}\left(\frac{b^{4}c^{3}}{d^{10}}\right)^{\frac{3}{4}}}{b^{4}c^{3}}\right)-4\,d^{3}x^{2}\left(-\frac{b^{4}c^{3}}{d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,bcd^{7}\left(-\frac{b^{4}c^{3}}{d^{10}}\right)^{\frac{3}{4}}-\sqrt{d^{6}\sqrt{-\frac{b^{4}c^{3}}{d^{10}}}+b^{2}c^{2}dx}\,d^{7}\left(-\frac{b^{4}c^{3}}{d^{10}}\right)^{\frac{3}{4}}}{b^{4}c^{3}}\right)-d^{3}x^{2}\left(\frac{b^{4}c^{3}}{d^{10}}\right)^{\frac{1}{4}}\log\left(d^{3}\left(\frac{b^{4}c^{3}}{d^{10}}\right)^{\frac{1}{4}}+\sqrt{dx}\,bc\right)+d^{3}x^{2}\left(\frac{b^{4}c^{3}}{d^{10}}\right)^{\frac{1}{4}}\log\left(-d^{3}\left(\frac{b^{4}c^{3}}{d^{10}}\right)^{\frac{1}{4}}+\sqrt{dx}\,bc\right)-d^{3}x^{2}\left(-\frac{b^{4}c^{3}}{d^{10}}\right)^{\frac{1}{4}}\log\left(d^{3}\left(-\frac{b^{4}c^{3}}{d^{10}}\right)^{\frac{1}{4}}+\sqrt{dx}\,bc\right)+d^{3}x^{2}\left(-\frac{b^{4}c^{3}}{d^{10}}\right)^{\frac{1}{4}}\log\left(-d^{3}\left(-\frac{b^{4}c^{3}}{d^{10}}\right)^{\frac{1}{4}}+\sqrt{dx}\,bc\right)+\sqrt{dx}\left(b\log\left(-\frac{cx^{2}+1}{cx^{2}-1}\right)+2\,a\right)}{3\,d^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(4*d^3*x^2*(b^4*c^3/d^10)^(1/4)*arctan(-(sqrt(d*x)*b*c*d^7*(b^4*c^3/d^
10)^(3/4) - sqrt(d^6*sqrt(b^4*c^3/d^10) + b^2*c^2*d*x)*d^7*(b^4*c^3/d^10)^(
3/4))/(b^4*c^3)) - 4*d^3*x^2*(-b^4*c^3/d^10)^(1/4)*arctan(-(sqrt(d*x)*b*c*d
^7*(-b^4*c^3/d^10)^(3/4) - sqrt(d^6*sqrt(-b^4*c^3/d^10) + b^2*c^2*d*x)*d^7*
(-b^4*c^3/d^10)^(3/4))/(b^4*c^3)) - d^3*x^2*(b^4*c^3/d^10)^(1/4)*log(d^3*(b
^4*c^3/d^10)^(1/4) + sqrt(d*x)*b*c) + d^3*x^2*(b^4*c^3/d^10)^(1/4)*log(-d^3
*(b^4*c^3/d^10)^(1/4) + sqrt(d*x)*b*c) - d^3*x^2*(-b^4*c^3/d^10)^(1/4)*log(
d^3*(-b^4*c^3/d^10)^(1/4) + sqrt(d*x)*b*c) + d^3*x^2*(-b^4*c^3/d^10)^(1/4)*
log(-d^3*(-b^4*c^3/d^10)^(1/4) + sqrt(d*x)*b*c) + sqrt(d*x)*(b*log(-(c*x^2
+ 1)/(c*x^2 - 1)) + 2*a))/(d^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atanh}{\left(c x^{2} \right)}}{\left(d x\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/(d*x)**(5/2),x)
```

```
[Out] Integral((a + b*atanh(c*x**2))/(d*x)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 516 vs. 2(200) = 400.
time = 0.76, size = 516, normalized size = 1.71

$$\frac{bcd^{2}\left(\frac{2\sqrt{2}\left(c^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)}{cd^{4}}+\frac{2\sqrt{2}\left(c^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)}{cd^{4}}+\frac{2\sqrt{2}\left(-c^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{d^{2}}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(-\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)}{cd^{4}}+\frac{2\sqrt{2}\left(-c^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{d^{2}}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(-\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)}{cd^{4}}+\frac{\sqrt{2}\left(c^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\sqrt{dx}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{d^{2}}{c}}\right)}{cd^{4}}-\frac{\sqrt{2}\left(c^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\sqrt{dx}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{d^{2}}{c}}\right)}{cd^{4}}+\frac{\sqrt{2}\left(-c^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\sqrt{dx}\left(-\frac{d^{2}}{c}\right)^{\frac{1}{4}}+\sqrt{-\frac{d^{2}}{c}}\right)}{cd^{4}}-\frac{\sqrt{2}\left(-c^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\sqrt{dx}\left(-\frac{d^{2}}{c}\right)^{\frac{1}{4}}+\sqrt{-\frac{d^{2}}{c}}\right)}{cd^{4}}\right)-\frac{2\,b\log\left(-\frac{cd^{2}x^{2}+d^{2}}{cd^{2}x^{2}-d^{2}}\right)}{\sqrt{dx}\,dx}-\frac{4\,a}{\sqrt{dx}\,dx}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(b*c*d^2*(2*sqrt(2)*(c^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)
^(1/4) + 2*sqrt(d*x))/(d^2/c)^(1/4))/(c*d^4) + 2*sqrt(2)*(c^3*d^2)^(1/4)*ar
```

```
ctan(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/(c*d
^4) + 2*sqrt(2)*(-c^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4)
 + 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c*d^4) + 2*sqrt(2)*(-c^3*d^2)^(1/4)*arctan
(-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c*d^4
) + sqrt(2)*(c^3*d^2)^(1/4)*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(1/4) + sqr
t(d^2/c))/(c*d^4) - sqrt(2)*(c^3*d^2)^(1/4)*log(d*x - sqrt(2)*sqrt(d*x)*(d^
2/c)^(1/4) + sqrt(d^2/c))/(c*d^4) + sqrt(2)*(-c^3*d^2)^(1/4)*log(d*x + sqrt
(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c*d^4) - sqrt(2)*(-c^3*d^2)^(
1/4)*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c*d^4)) -
2*b*log(-(c*d^2*x^2 + d^2)/(c*d^2*x^2 - d^2))/(sqrt(d*x)*d*x) - 4*a/(sqrt(d
*x)*d*x))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atanh}(c\,x^2)}{(d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/(d*x)^(5/2),x)
[Out] int((a + b*atanh(c*x^2))/(d*x)^(5/2), x)
```

3.88 $\int \frac{a+b\tanh^{-1}(cx^2)}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=317

$$-\frac{8bc}{5d^3\sqrt{dx}}-\frac{2bc^{5/4}\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{\sqrt{2}\,bc^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{\sqrt{2}\,bc^{5/4}\text{ArcTan}\left(1+\frac{\sqrt{2}}{}\right.}{5d^{7/2}}$$

```
[Out] -2/5*b*c^(5/4)*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(7/2)-2/5*(a+b*arctanh
(c*x^2))/d/(d*x)^(5/2)+2/5*b*c^(5/4)*arctanh(c^(1/4)*(d*x)^(1/2)/d^(1/2))/d
^(7/2)-1/10*b*c^(5/4)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)-c^(1/4)*2^(1/2)*(d*x)^(1
/2))/d^(7/2)*2^(1/2)+1/10*b*c^(5/4)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)+c^(1/4)*2^
(1/2)*(d*x)^(1/2))/d^(7/2)*2^(1/2)-1/5*b*c^(5/4)*arctan(-1+c^(1/4)*2^(1/2)*
(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(7/2)-1/5*b*c^(5/4)*arctan(1+c^(1/4)*2^(1/2)
*(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(7/2)-8/5*b*c/d^3/(d*x)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {6049, 331, 335, 307, 303, 1176, 631, 210, 1179, 642, 304, 211, 214}

$$-\frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{5d(dx)^{5/2}}-\frac{2bc^{5/4}\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{\sqrt{2}\,bc^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{\sqrt{2}\,bc^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{5d^{7/2}}-\frac{bc^{5/4}\log\left(\sqrt{c}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}\,d^{7/2}}+\frac{bc^{5/4}\log\left(\sqrt{c}\sqrt{d}\,x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}\,d^{7/2}}+\frac{2bc^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{8bc}{5d^3\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/(d*x)^(7/2),x]
```

```
[Out] (-8*b*c)/(5*d^3*Sqrt[d*x]) - (2*b*c^(5/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d
]])/(5*d^(7/2)) + (Sqrt[2]*b*c^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])
/Sqrt[d]])/(5*d^(7/2)) - (Sqrt[2]*b*c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[d*x])/Sqrt[d]])/(5*d^(7/2)) - (2*(a + b*ArcTanh[c*x^2]))/(5*d*(d*x)^(5/2)
) + (2*b*c^(5/4)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(5*d^(7/2)) - (b*c^(
5/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(5*Sqrt[
2]*d^(7/2)) + (b*c^(5/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*
Sqrt[d*x]])/(5*Sqrt[2]*d^(7/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 307

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[x^(m - n/2)/(r
 + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && LtQ
[m, n] &&  !GtQ[a/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx^2)}{(dx)^{7/2}}\,dx = -\frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{5d(dx)^{5/2}} + \frac{(4bc)\int \frac{x}{(dx)^{5/2}(1-c^2x^4)}\,dx}{5d}$$

$$= -\frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{5d(dx)^{5/2}} + \frac{(4bc)\int \frac{1}{(dx)^{3/2}(1-c^2x^4)}\,dx}{5d^2}$$

$$= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{5d(dx)^{5/2}} + \frac{(4bc^3)\int \frac{(dx)^{5/2}}{1-c^2x^4}\,dx}{5d^6}$$

$$= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{5d(dx)^{5/2}} + \frac{(8bc^3)\,\text{Subst}\left(\int \frac{x^6}{1-\frac{c^2x^8}{d^4}}\,dx, x, \sqrt{dx}\right)}{5d^7}$$

$$= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{5d(dx)^{5/2}} + \frac{(4bc^2)\,\text{Subst}\left(\int \frac{x^2}{d^2-cx^4}\,dx, x, \sqrt{dx}\right)}{5d^3} - (4$$

$$= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{5d(dx)^{5/2}} + \frac{(2bc^{3/2})\,\text{Subst}\left(\int \frac{1}{d-\sqrt{c}\,x^2}\,dx, x, \sqrt{dx}\right)}{5d^3}$$

$$= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2bc^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{5d(dx)^{5/2}} + \frac{2bc^{5/4}\tanh}{5}$$

$$= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2bc^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{5d(dx)^{5/2}} + \frac{2bc^{5/4}\tanh}{5}$$

$$= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2bc^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{\sqrt{2}\,bc^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}$$

Mathematica [A]
time = 0.06, size = 275, normalized size = 0.87

$$\frac{x\left(-4a-16bcx^2+2\sqrt{2}\,bc^{5/4}x^{5/2}\text{ArcTan}\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)-2\sqrt{2}\,bc^{5/4}x^{5/2}\text{ArcTan}\left(1+\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)-4bc^{5/4}x^{5/2}\text{ArcTan}\left(\sqrt[4]{c}\sqrt{x}\right)-4b\tanh^{-1}(cx^2)-2bc^{5/4}x^{5/2}\log\left(1-\sqrt[4]{c}\sqrt{x}\right)+2bc^{5/4}x^{5/2}\log\left(1+\sqrt[4]{c}\sqrt{x}\right)-\sqrt{2}\,bc^{5/4}x^{5/2}\log\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)+\sqrt{2}\,bc^{5/4}x^{5/2}\log\left(1+\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)\right)}{10(dx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/(d*x)^(7/2),x]
```

```
[Out] (x*(-4*a - 16*b*c*x^2 + 2*Sqrt[2]*b*c^(5/4)*x^(5/2)*ArcTan[1 - Sqrt[2]*c^(1
/4)*Sqrt[x]] - 2*Sqrt[2]*b*c^(5/4)*x^(5/2)*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[
x]] - 4*b*c^(5/4)*x^(5/2)*ArcTan[c^(1/4)*Sqrt[x]] - 4*b*ArcTanh[c*x^2] - 2*
b*c^(5/4)*x^(5/2)*Log[1 - c^(1/4)*Sqrt[x]] + 2*b*c^(5/4)*x^(5/2)*Log[1 + c^
```

```
(1/4)*Sqrt[x]] - Sqrt[2]*b*c^(5/4)*x^(5/2)*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x]
+ Sqrt[c]*x] + Sqrt[2]*b*c^(5/4)*x^(5/2)*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] +
Sqrt[c]*x]))/(10*(d*x)^(7/2))
```

Maple [A]
time = 0.05, size = 291, normalized size = 0.92

method	result
derivativedivides	$\frac{-\frac{2a}{5(dx)^{\frac{5}{2}}}-\frac{2b\,\mathrm{arctanh}\left(c\,x^2\right)}{5(dx)^{\frac{5}{2}}}-\frac{bc\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2}{c}}}{dx+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2}{c}}}\right)}{10d^2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}-\frac{bc\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}+1\right)}{5d^2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}-\frac{bc\sqrt{2}}{}}{d}$
default	$\frac{-\frac{2a}{5(dx)^{\frac{5}{2}}}-\frac{2b\,\mathrm{arctanh}\left(c\,x^2\right)}{5(dx)^{\frac{5}{2}}}-\frac{bc\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2}{c}}}{dx+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2}{c}}}\right)}{10d^2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}-\frac{bc\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}+1\right)}{5d^2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}-\frac{bc\sqrt{2}}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/5*a/(d*x)^(5/2)-1/5*b/(d*x)^(5/2)*arctanh(c*x^2)-1/20*b/d^2*c/(d^2/
c)^(1/4)*2^(1/2)*ln((d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(
d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))-1/10*b/d^2*c/(d^2/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)-1/10*b/d^2*c/(d^2
/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)-4/5*b*c/d^2/(
d*x)^(1/2)-1/5*b/d^2*c/(d^2/c)^(1/4)*arctan((d*x)^(1/2)/(d^2/c)^(1/4))+1/10
*b/d^2*c/(d^2/c)^(1/4)*ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^
(1/4)))
```

Maxima [A]
time = 0.47, size = 298, normalized size = 0.94

$$-\frac{b\left(\frac{\left(c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{c}dx+\sqrt{2}\sqrt{dx}c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{3}{4}}\sqrt{d}}+\frac{\sqrt{2}\log\left(\sqrt{c}dx-\sqrt{2}\sqrt{dx}c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{3}{4}}\sqrt{d}}\right)+2c\left(\frac{2\arctan\left(\frac{\sqrt{dx}\sqrt{c}}{\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\sqrt{c}}+\frac{\log\left(\frac{\sqrt{dx}\sqrt{c}-\sqrt{\sqrt{c}d}}{\sqrt{dx}\sqrt{c}+\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\sqrt{c}}\right)+\frac{16}{\sqrt{dx}}\right)c}{d^2}+\frac{4\,\mathrm{artanh}\left(cx^2\right)}{(dx)^{\frac{5}{2}}}\right)+\frac{4a}{(dx)^{\frac{5}{2}}}}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/10*(b*((c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) + 2*sqr
t(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) + 2*sqrt(2)*arct
```

```
an(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) - 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c
)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) - sqrt(2)*log(sqrt(c)*d*x + sqrt(2)*sqrt(d*
x)*c^(1/4)*sqrt(d) + d)/(c^(3/4)*sqrt(d)) + sqrt(2)*log(sqrt(c)*d*x - sqrt(
2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/(c^(3/4)*sqrt(d))) + 2*c*(2*arctan(sqrt(d
*x)*sqrt(c)/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) + log((sqrt(d*x)*sqr
t(c) - sqrt(sqrt(c)*d))/(sqrt(d*x)*sqrt(c) + sqrt(sqrt(c)*d)))/(sqrt(sqrt(c
)*d)*sqrt(c))) + 16/sqrt(d*x))*c/d^2 + 4*arctanh(c*x^2)/(d*x)^(5/2)) + 4*a/
(d*x)^(5/2))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(212) = 424$.
time = 0.38, size = 473, normalized size = 1.49

$$\frac{4\,d^4x^3\left(\frac{b^4c^5}{d^{14}}\right)^{\frac14}\arctan\left(-\frac{\sqrt{dx}\,b^3c^4d^3\left(\frac{b^4c^5}{d^{14}}\right)^{\frac14}-\sqrt{b^4c^5d^8\sqrt{\frac{b^4c^5}{d^{14}}}+b^6c^8dx}\,d^3\left(\frac{b^4c^5}{d^{14}}\right)^{\frac14}}{b^4c^5}\right)+4\,d^4x^3\left(-\frac{b^4c^5}{d^{14}}\right)^{\frac14}\arctan\left(-\frac{\sqrt{dx}\,b^3c^4d^3\left(-\frac{b^4c^5}{d^{14}}\right)^{\frac14}-\sqrt{-b^4c^5d^8\sqrt{-\frac{b^4c^5}{d^{14}}}+b^6c^8dx}\,d^3\left(-\frac{b^4c^5}{d^{14}}\right)^{\frac14}}{b^4c^5}\right)+d^4x^3\left(\frac{b^4c^5}{d^{14}}\right)^{\frac14}\log\left(d^{11}\left(\frac{b^4c^5}{d^{14}}\right)^{\frac34}+\sqrt{dx}\,b^3c^4\right)-d^4x^3\left(\frac{b^4c^5}{d^{14}}\right)^{\frac14}\log\left(-d^{11}\left(\frac{b^4c^5}{d^{14}}\right)^{\frac34}+\sqrt{dx}\,b^3c^4\right)-d^4x^3\left(-\frac{b^4c^5}{d^{14}}\right)^{\frac14}\log\left(d^{11}\left(-\frac{b^4c^5}{d^{14}}\right)^{\frac34}+\sqrt{dx}\,b^3c^4\right)+d^4x^3\left(-\frac{b^4c^5}{d^{14}}\right)^{\frac14}\log\left(-d^{11}\left(-\frac{b^4c^5}{d^{14}}\right)^{\frac34}+\sqrt{dx}\,b^3c^4\right)-\left(8\,bcx^2+b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a\right)\sqrt{dx}}{5\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(4*d^4*x^3*(b^4*c^5/d^14)^(1/4)*arctan(-(sqrt(d*x)*b^3*c^4*d^3*(b^4*c^5
/d^14)^(1/4) - sqrt(b^4*c^5*d^8*sqrt(b^4*c^5/d^14) + b^6*c^8*d*x)*d^3*(b^4*
c^5/d^14)^(1/4))/(b^4*c^5)) + 4*d^4*x^3*(-b^4*c^5/d^14)^(1/4)*arctan(-(sqrt
(d*x)*b^3*c^4*d^3*(-b^4*c^5/d^14)^(1/4) - sqrt(-b^4*c^5*d^8*sqrt(-b^4*c^5/d
^14) + b^6*c^8*d*x)*d^3*(-b^4*c^5/d^14)^(1/4))/(b^4*c^5)) + d^4*x^3*(b^4*c^
5/d^14)^(1/4)*log(d^11*(b^4*c^5/d^14)^(3/4) + sqrt(d*x)*b^3*c^4) - d^4*x^3*
(b^4*c^5/d^14)^(1/4)*log(-d^11*(b^4*c^5/d^14)^(3/4) + sqrt(d*x)*b^3*c^4) -
d^4*x^3*(-b^4*c^5/d^14)^(1/4)*log(d^11*(-b^4*c^5/d^14)^(3/4) + sqrt(d*x)*b^
3*c^4) + d^4*x^3*(-b^4*c^5/d^14)^(1/4)*log(-d^11*(-b^4*c^5/d^14)^(3/4) + sq
rt(d*x)*b^3*c^4) - (8*b*c*x^2 + b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)*sqrt
(d*x))/(d^4*x^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{atanh}\left(cx^{2}\right)}{\left(dx\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/(d*x)**(7/2),x)
```

```
[Out] Integral((a + b*atanh(c*x**2))/(d*x)**(7/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(212) = 424$.
time = 1.87, size = 532, normalized size = 1.68

$$-\frac{\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac34}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac14}+2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac14}}\right)}{cd^4}+\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac34}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac14}-2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac14}}\right)}{cd^4}-\frac{2\sqrt{2}\left(-c^3d^2\right)^{\frac34}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{d^2}{c}\right)^{\frac14}+2\sqrt{dx}\right)}{2\left(-\frac{d^2}{c}\right)^{\frac14}}\right)}{cd^4}-\frac{2\sqrt{2}\left(-c^3d^2\right)^{\frac34}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{d^2}{c}\right)^{\frac14}-2\sqrt{dx}\right)}{2\left(-\frac{d^2}{c}\right)^{\frac14}}\right)}{cd^4}-\frac{\sqrt{2}\left(c^3d^2\right)^{\frac34}b\log\left(dx+\sqrt{2}\sqrt{dx}\left(\frac{d^2}{c}\right)^{\frac14}+\sqrt{\frac{d^2}{c}}\right)}{cd^4}+\frac{\sqrt{2}\left(c^3d^2\right)^{\frac34}b\log\left(dx-\sqrt{2}\sqrt{dx}\left(\frac{d^2}{c}\right)^{\frac14}+\sqrt{\frac{d^2}{c}}\right)}{cd^4}+\frac{\sqrt{2}\left(-c^3d^2\right)^{\frac34}b\log\left(dx+\sqrt{2}\sqrt{dx}\left(-\frac{d^2}{c}\right)^{\frac14}+\sqrt{-\frac{d^2}{c}}\right)}{cd^4}-\frac{\sqrt{2}\left(-c^3d^2\right)^{\frac34}b\log\left(dx-\sqrt{2}\sqrt{dx}\left(-\frac{d^2}{c}\right)^{\frac14}+\sqrt{-\frac{d^2}{c}}\right)}{cd^4}+\frac{2\,b\log\left(-\frac{cd^2x^2+d^2}{cd^2x^2-d^2}\right)}{\sqrt{dx}\,d^2x^2}+\frac{4\left(4\,bcd^2x^2+ad^2\right)}{\sqrt{dx}\,d^4x^2}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -1/10*(2*sqrt(2)*(c^3*d^2)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4
) + 2*sqrt(d*x))/(d^2/c)^(1/4))/(c*d^4) + 2*sqrt(2)*(c^3*d^2)^(3/4)*b*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/(c*d^4)
 - 2*sqrt(2)*(-c^3*d^2)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4)
+ 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c*d^4) - 2*sqrt(2)*(-c^3*d^2)^(3/4)*b*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c*d^
4) - sqrt(2)*(c^3*d^2)^(3/4)*b*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(1/4) +
sqrt(d^2/c))/(c*d^4) + sqrt(2)*(c^3*d^2)^(3/4)*b*log(d*x - sqrt(2)*sqrt(d*x
)*(d^2/c)^(1/4) + sqrt(d^2/c))/(c*d^4) + sqrt(2)*(-c^3*d^2)^(3/4)*b*log(d*x
 + sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c*d^4) - sqrt(2)*(-c^3
*d^2)^(3/4)*b*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c
*d^4) + 2*b*log(-(c*d^2*x^2 + d^2)/(c*d^2*x^2 - d^2))/(sqrt(d*x)*d^2*x^2) +
 4*(4*b*c*d^2*x^2 + a*d^2)/(sqrt(d*x)*d^4*x^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{atanh}(c\,x^2)}{(d\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/(d*x)^(7/2),x)
```

```
[Out] int((a + b*atanh(c*x^2))/(d*x)^(7/2), x)
```

3.89 $\int \frac{a+b\tanh^{-1}(cx^2)}{(dx)^{9/2}}\,dx$

Optimal. Leaf size=317

$$-\frac{8bc}{21d^3(dx)^{3/2}}+\frac{2bc^{7/4}\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}+\frac{\sqrt{2}\,bc^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{\sqrt{2}\,bc^{7/4}\text{ArcTan}\left(1+\right.}{7d^{9/2}}$$

```
[Out] -8/21*b*c/d^3/(d*x)^(3/2)+2/7*b*c^(7/4)*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))
/d^(9/2)-2/7*(a+b*arctanh(c*x^2))/d/(d*x)^(7/2)+2/7*b*c^(7/4)*arctanh(c^(1/
4)*(d*x)^(1/2)/d^(1/2))/d^(9/2)+1/14*b*c^(7/4)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)
-c^(1/4)*2^(1/2)*(d*x)^(1/2))/d^(9/2)*2^(1/2)-1/14*b*c^(7/4)*ln(d^(1/2)+x*c
^(1/2)*d^(1/2)+c^(1/4)*2^(1/2)*(d*x)^(1/2))/d^(9/2)*2^(1/2)-1/7*b*c^(7/4)*a
rctan(-1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(9/2)-1/7*b*c^(7/4)
*arctan(1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(9/2)
```

Rubi [A]
time = 0.18, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {6049, 331, 335, 307, 217, 1179, 642, 1176, 631, 210, 218, 214, 211}

$$-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{2bc^{7/4}\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}+\frac{\sqrt{2}\,bc^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{\sqrt{2}\,bc^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{7d^{9/2}}+\frac{bc^{7/4}\log\left(\sqrt{c}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{7\sqrt{2}\,d^{9/2}}-\frac{bc^{7/4}\log\left(\sqrt{c}\sqrt{d}\,x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{7\sqrt{2}\,d^{9/2}}+\frac{2bc^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{8bc}{21d^3(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/(d*x)^(9/2),x]
```

```
[Out] (-8*b*c)/(21*d^3*(d*x)^(3/2)) + (2*b*c^(7/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqr
t[d]])/(7*d^(9/2)) + (Sqrt[2]*b*c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*
x])/Sqrt[d]])/(7*d^(9/2)) - (Sqrt[2]*b*c^(7/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*
Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) - (2*(a + b*ArcTanh[c*x^2]))/(7*d*(d*x)^(7
/2)) + (2*b*c^(7/4)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) + (b*
c^(7/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(7*Sq
rt[2]*d^(9/2)) - (b*c^(7/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/
4)*Sqrt[d*x]])/(7*Sqrt[2]*d^(9/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 307

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[x^(m - n/2)/(r
 + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && LtQ
[m, n] &&  !GtQ[a/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx^2)}{(dx)^{9/2}}\,dx = -\frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{7d(dx)^{7/2}} + \frac{(4bc)\int \frac{x}{(dx)^{7/2}(1-c^2x^4)}\,dx}{7d}$$

$$= -\frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{7d(dx)^{7/2}} + \frac{(4bc)\int \frac{1}{(dx)^{5/2}(1-c^2x^4)}\,dx}{7d^2}$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}} - \frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{7d(dx)^{7/2}} + \frac{(4bc^3)\int \frac{(dx)^{3/2}}{1-c^2x^4}\,dx}{7d^6}$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}} - \frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{7d(dx)^{7/2}} + \frac{(8bc^3)\,\text{Subst}\left(\int \frac{x^4}{1-\frac{c^2x^8}{d^4}}\,dx, x, \sqrt{dx}\right)}{7d^7}$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}} - \frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{7d(dx)^{7/2}} + \frac{(4bc^2)\,\text{Subst}\left(\int \frac{1}{d^2-cx^4}\,dx, x, \sqrt{dx}\right)}{7d^3} - \cdots$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}} - \frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{7d(dx)^{7/2}} + \frac{(2bc^2)\,\text{Subst}\left(\int \frac{1}{d-\sqrt{c}\,x^2}\,dx, x, \sqrt{dx}\right)}{7d^4} \cdots$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}} + \frac{2bc^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}} - \frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{7d(dx)^{7/2}} + \frac{2bc^{7/4}\tanh \cdots}{\cdots}$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}} + \frac{2bc^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}} - \frac{2\left(a+b\tanh^{-1}(cx^2)\right)}{7d(dx)^{7/2}} + \frac{2bc^{7/4}\tanh \cdots}{\cdots}$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}} + \frac{2bc^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}} + \frac{\sqrt{2}\,bc^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \cdots\right.}{7d^{9/2}} \cdots$$

Mathematica [A]
time = 0.06, size = 281, normalized size = 0.89

$$\frac{\sqrt{dx}\left(-12a-16bcx^2+6\sqrt{2}\,bc^{7/4}x^{7/2}\text{ArcTan}\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)-6\sqrt{2}\,bc^{7/4}x^{7/2}\text{ArcTan}\left(1+\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)+12bc^{7/4}x^{7/2}\text{ArcTan}\left(\sqrt[4]{c}\sqrt{x}\right)-12b\tanh^{-1}(cx^2)-6bc^{7/4}x^{7/2}\log\left(1-\sqrt[4]{c}\sqrt{x}\right)+6bc^{7/4}x^{7/2}\log\left(1+\sqrt[4]{c}\sqrt{x}\right)+3\sqrt{2}\,bc^{7/4}x^{7/2}\log\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)-3\sqrt{2}\,bc^{7/4}x^{7/2}\log\left(1+\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)\right)}{42d^5x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/(d*x)^(9/2),x]
```

```
[Out] (Sqrt[d*x]*(-12*a - 16*b*c*x^2 + 6*Sqrt[2]*b*c^(7/4)*x^(7/2)*ArcTan[1 - Sqr
t[2]*c^(1/4)*Sqrt[x]] - 6*Sqrt[2]*b*c^(7/4)*x^(7/2)*ArcTan[1 + Sqrt[2]*c^(1
/4)*Sqrt[x]] + 12*b*c^(7/4)*x^(7/2)*ArcTan[c^(1/4)*Sqrt[x]] - 12*b*ArcTanh[
c*x^2] - 6*b*c^(7/4)*x^(7/2)*Log[1 - c^(1/4)*Sqrt[x]] + 6*b*c^(7/4)*x^(7/2)
```

```
*Log[1 + c^(1/4)*Sqrt[x]] + 3*Sqrt[2]*b*c^(7/4)*x^(7/2)*Log[1 - Sqrt[2]*c^(
1/4)*Sqrt[x] + Sqrt[c]*x] - 3*Sqrt[2]*b*c^(7/4)*x^(7/2)*Log[1 + Sqrt[2]*c^(
1/4)*Sqrt[x] + Sqrt[c]*x]))/(42*d^5*x^4)
```

Maple [A]

time = 0.06, size = 301, normalized size = 0.95

method	result
derivativedivides	$-\frac{2a}{7(dx)^{\frac{7}{2}}}-\frac{2b\,\mathrm{arctanh}\left(c\,x^{2}\right)}{7(dx)^{\frac{7}{2}}}-\frac{b\,c^{2}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)}{14d^{4}}-\frac{b\,c^{2}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\cdots}{\left(\frac{d^{2}}{c}\right)\cdots}\right.}{7d^{4}}$
default	$-\frac{2a}{7(dx)^{\frac{7}{2}}}-\frac{2b\,\mathrm{arctanh}\left(c\,x^{2}\right)}{7(dx)^{\frac{7}{2}}}-\frac{b\,c^{2}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)}{14d^{4}}-\frac{b\,c^{2}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\cdots}{\left(\frac{d^{2}}{c}\right)\cdots}\right.}{7d^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(d*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/7*a/(d*x)^(7/2)-1/7*b/(d*x)^(7/2)*arctanh(c*x^2)-1/28*b/d^4*c^2*(d^
2/c)^(1/4)*2^(1/2)*ln((d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))
/(d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))-1/14*b/d^4*c^2*(d^2
/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)-1/14*b/d^4*c^
2*(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)-4/21*b*
c/d^2/(d*x)^(3/2)+1/14*b/d^4*c^2*(d^2/c)^(1/4)*ln(((d*x)^(1/2)+(d^2/c)^(1/4
))/((d*x)^(1/2)-(d^2/c)^(1/4)))+1/7*b/d^4*c^2*(d^2/c)^(1/4)*arctan((d*x)^(1
/2)/(d^2/c)^(1/4)))
```

Maxima [A]

time = 0.48, size = 297, normalized size = 0.94

$$-\frac{b\left(\frac{c\left(\frac{6\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\,\sqrt{c}\right)}{2\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}\,d}+\frac{6\sqrt{2}\,c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\,\sqrt{c}\right)}{2\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}\,d}+\frac{3\sqrt{2}\,c^{\frac{3}{4}}\log\left(\sqrt{c}\,dx+\sqrt{2}\,\sqrt{dx}\,c^{\frac{1}{4}}\sqrt{d}+d\right)}{d^{\frac{3}{2}}}-\frac{3\sqrt{2}\,c^{\frac{3}{4}}\log\left(\sqrt{c}\,dx-\sqrt{2}\,\sqrt{dx}\,c^{\frac{1}{4}}\sqrt{d}+d\right)}{d^{\frac{3}{2}}}-\frac{12\,c\arctan\left(\frac{\sqrt{dx}\,\sqrt{c}}{\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}\,d}+\frac{6\,c\log\left(\frac{\sqrt{dx}\,\sqrt{c}-\sqrt{\sqrt{c}\,d}}{\sqrt{dx}\,\sqrt{c}+\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}\,d}+\frac{16}{(dx)^{\frac{3}{2}}}\right)}{d^{2}}+\frac{12\,\mathrm{artanh}\left(cx^{2}\right)}{(dx)^{\frac{7}{2}}}\right)+\frac{12\,a}{(dx)^{\frac{7}{2}}}}{42\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/42*(b*(c*(6*sqrt(2)*c*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) + 2*sq
rt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*d) + 6*sqrt(2)*c*arctan(
-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) - 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d
```

```
))/(sqrt(sqrt(c)*d)*d) + 3*sqrt(2)*c^(3/4)*log(sqrt(c)*d*x + sqrt(2)*sqrt(d
*x)*c^(1/4)*sqrt(d) + d)/d^(3/2) - 3*sqrt(2)*c^(3/4)*log(sqrt(c)*d*x - sqrt
(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/d^(3/2) - 12*c*arctan(sqrt(d*x)*sqrt(c)/
sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*d) + 6*c*log((sqrt(d*x)*sqrt(c) - sqrt(sq
rt(c)*d))/(sqrt(d*x)*sqrt(c) + sqrt(sqrt(c)*d)))/(sqrt(sqrt(c)*d)*d) + 16/(
d*x)^(3/2))/d^2 + 12*arctanh(c*x^2)/(d*x)^(7/2)) + 12*a/(d*x)^(7/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(212) = 424$.
time = 0.45, size = 450, normalized size = 1.42

$$-\frac{12\,d^5x^4\left(\frac{b^4c^7}{d^{18}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,bc^2d^{13}\left(\frac{b^4c^7}{d^{18}}\right)^{\frac{3}{4}}-\sqrt{d^{10}\sqrt{\frac{b^4c^7}{d^{18}}}+b^2c^4dx}\,d^{13}\left(\frac{b^4c^7}{d^{18}}\right)^{\frac{3}{4}}}{b^4c^7}\right)+12\,d^5x^4\left(-\frac{b^4c^7}{d^{18}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,bc^2d^{13}\left(-\frac{b^4c^7}{d^{18}}\right)^{\frac{3}{4}}-\sqrt{d^{10}\sqrt{-\frac{b^4c^7}{d^{18}}}+b^2c^4dx}\,d^{13}\left(-\frac{b^4c^7}{d^{18}}\right)^{\frac{3}{4}}}{b^4c^7}\right)-3\,d^5x^4\left(\frac{b^4c^7}{d^{18}}\right)^{\frac{1}{4}}\log\left(d^5\left(\frac{b^4c^7}{d^{18}}\right)^{\frac{1}{4}}+\sqrt{dx}\,bc^2\right)+3\,d^5x^4\left(\frac{b^4c^7}{d^{18}}\right)^{\frac{1}{4}}\log\left(-d^5\left(\frac{b^4c^7}{d^{18}}\right)^{\frac{1}{4}}+\sqrt{dx}\,bc^2\right)+3\,d^5x^4\left(-\frac{b^4c^7}{d^{18}}\right)^{\frac{1}{4}}\log\left(d^5\left(-\frac{b^4c^7}{d^{18}}\right)^{\frac{1}{4}}+\sqrt{dx}\,bc^2\right)-3\,d^5x^4\left(-\frac{b^4c^7}{d^{18}}\right)^{\frac{1}{4}}\log\left(-d^5\left(-\frac{b^4c^7}{d^{18}}\right)^{\frac{1}{4}}+\sqrt{dx}\,bc^2\right)+\left(8\,bcx^2+3\,b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+6\,a\right)\sqrt{dx}}{21\,d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/21*(12*d^5*x^4*(b^4*c^7/d^18)^(1/4)*arctan(-(sqrt(d*x)*b*c^2*d^13*(b^4*c
^7/d^18)^(3/4) - sqrt(d^10*sqrt(b^4*c^7/d^18) + b^2*c^4*d*x)*d^13*(b^4*c^7/
d^18)^(3/4))/(b^4*c^7)) + 12*d^5*x^4*(-b^4*c^7/d^18)^(1/4)*arctan(-(sqrt(d*
x)*b*c^2*d^13*(-b^4*c^7/d^18)^(3/4) - sqrt(d^10*sqrt(-b^4*c^7/d^18) + b^2*c
^4*d*x)*d^13*(-b^4*c^7/d^18)^(3/4))/(b^4*c^7)) - 3*d^5*x^4*(b^4*c^7/d^18)^(
1/4)*log(d^5*(b^4*c^7/d^18)^(1/4) + sqrt(d*x)*b*c^2) + 3*d^5*x^4*(b^4*c^7/d
^18)^(1/4)*log(-d^5*(b^4*c^7/d^18)^(1/4) + sqrt(d*x)*b*c^2) + 3*d^5*x^4*(-b
^4*c^7/d^18)^(1/4)*log(d^5*(-b^4*c^7/d^18)^(1/4) + sqrt(d*x)*b*c^2) - 3*d^5
*x^4*(-b^4*c^7/d^18)^(1/4)*log(-d^5*(-b^4*c^7/d^18)^(1/4) + sqrt(d*x)*b*c^2
) + (8*b*c*x^2 + 3*b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a)*sqrt(d*x))/(d^5*x
^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/(d*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(212) = 424$.
time = 5.28, size = 519, normalized size = 1.64

$$-\frac{\frac{6\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}bc\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{d^4}+\frac{6\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}bc\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{d^4}-\frac{6\sqrt{2}\left(-c^3d^2\right)^{\frac{1}{4}}bc\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{d^4}-\frac{6\sqrt{2}\left(-c^3d^2\right)^{\frac{1}{4}}bc\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{d^4}+\frac{3\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}bc\log\left(dx+\sqrt{2}\sqrt{dx}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{d^2}{c}}\right)}{d^4}-\frac{3\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}bc\log\left(dx-\sqrt{2}\sqrt{dx}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{d^2}{c}}\right)}{d^4}-\frac{3\sqrt{2}\left(-c^3d^2\right)^{\frac{1}{4}}bc\log\left(dx+\sqrt{2}\sqrt{dx}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}+\sqrt{-\frac{d^2}{c}}\right)}{d^4}+\frac{3\sqrt{2}\left(-c^3d^2\right)^{\frac{1}{4}}bc\log\left(dx-\sqrt{2}\sqrt{dx}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}+\sqrt{-\frac{d^2}{c}}\right)}{d^4}+\frac{6\,b\log\left(-\frac{cd^2x^2+d^2}{cd^2x^2-d^2}\right)}{\sqrt{dx}\,d^3x^3}+\frac{4\left(4\,bcd^2x^2+3\,ad^2\right)}{\sqrt{dx}\,d^5x^3}}{42\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/42*(6*sqrt(2)*(c^3*d^2)^(1/4)*b*c*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1
/4) + 2*sqrt(d*x))/(d^2/c)^(1/4))/d^4 + 6*sqrt(2)*(c^3*d^2)^(1/4)*b*c*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/d^4 - 6
*sqrt(2)*(-c^3*d^2)^(1/4)*b*c*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) +
2*sqrt(d*x))/(-d^2/c)^(1/4))/d^4 - 6*sqrt(2)*(-c^3*d^2)^(1/4)*b*c*arctan(-1
/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4))/d^4 + 3*s
qrt(2)*(c^3*d^2)^(1/4)*b*c*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(1/4) + sqrt
(d^2/c))/d^4 - 3*sqrt(2)*(c^3*d^2)^(1/4)*b*c*log(d*x - sqrt(2)*sqrt(d*x)*(d
^2/c)^(1/4) + sqrt(d^2/c))/d^4 - 3*sqrt(2)*(-c^3*d^2)^(1/4)*b*c*log(d*x + s
qrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/d^4 + 3*sqrt(2)*(-c^3*d^2)^
(1/4)*b*c*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/d^4 +
6*b*log(-(c*d^2*x^2 + d^2)/(c*d^2*x^2 - d^2))/(sqrt(d*x)*d^3*x^3) + 4*(4*b*
c*d^2*x^2 + 3*a*d^2)/(sqrt(d*x)*d^5*x^3))/d
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{atanh}(c\,x^2)}{(d\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/(d*x)^(9/2),x)
```

```
[Out] int((a + b*atanh(c*x^2))/(d*x)^(9/2), x)
```

3.90 $\int \sqrt{dx} \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$

Optimal. Leaf size=6327

result too large to display

```
[Out] 2/3*a*b*arctan(-1+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*(d*x)^(1/2)/c^(3/4)/x^(1
/2)-8/9*a*b*x*(d*x)^(1/2)+4/9*b^2*x*ln(-c*x^2+1)*(d*x)^(1/2)+4/9*b*x*(2*a-b
*ln(-c*x^2+1))*(d*x)^(1/2)+1/6*b^2*x*ln(c*x^2+1)^2*(d*x)^(1/2)+2/3*a*b*arct
an(1+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*(d*x)^(1/2)/c^(3/4)/x^(1/2)+1/3*a*b*l
n(1+x*c^(1/2)-c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*(d*x)^(1/2)/c^(3/4)/x^(1/2)-
1/3*a*b*ln(1+x*c^(1/2)+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*(d*x)^(1/2)/c^(3/4)
/x^(1/2)+1/3*b^2*polylog(2,1+2*(-c)^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(
1+(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(
3/4)/x^(1/2)-1/3*b^2*polylog(2,1+2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/
(1+c^(1/4)*x^(1/2))/(-c^(1/4)+(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(
1/2)+1/3*b^2*polylog(2,1-2*(-c)^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-
c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/
x^(1/2)-1/3*b^2*polylog(2,1-2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^
(1/4)*x^(1/2))/(c^(1/4)+(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)+1
/6*x*(2*a-b*ln(-c*x^2+1))^2*(d*x)^(1/2)-2/3*b^2*arctanh((-c)^(1/4)*x^(1/2))
^2*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)-2/3*b^2*arctanh(c^(1/4)*x^(1/2))^2*(d*x)^
(1/2)/c^(3/4)/x^(1/2)+2/3*b^2*polylog(2,1-2/(1-(-c)^(1/4)*x^(1/2)))*(d*x)^(
1/2)/(-c)^(3/4)/x^(1/2)+2/3*b^2*polylog(2,1-2/(1+(-c)^(1/4)*x^(1/2)))*(d*x)
^(1/2)/(-c)^(3/4)/x^(1/2)+2/3*b^2*polylog(2,1-2/(1-c^(1/4)*x^(1/2)))*(d*x)^
(1/2)/c^(3/4)/x^(1/2)-1/3*b^2*polylog(2,1-2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/
((-c)^(1/4)-c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)
+2/3*b^2*polylog(2,1-2/(1+c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-1/3
*b^2*polylog(2,1+2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))/((-c)^(1/4)-c^(1/4))/(1+c
^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-1/3*b^2*polylog(2,1-2*c^(1/4)*
(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+c^(1/4))/(1+c^(1/4)*x^(1/2)))*(d*x)^(1/2
)/c^(3/4)/x^(1/2)+2/3*a*b*x*ln(c*x^2+1)*(d*x)^(1/2)-1/3*b^2*x*ln(-c*x^2+1)*
ln(c*x^2+1)*(d*x)^(1/2)-1/3*b^2*polylog(2,1-2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2)
)/((-c)^(1/4)+c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/
2)+1/3*I*b^2*polylog(2,1-2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-I*c^(
1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+1/3*I*b^2*po
lylog(2,1+2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))/(I*(-c)^(1/4)-c^(1/4))/(1-I*c^(1
/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)+1/3*I*b^2*polylog(2,1-2*c^(1/4)*(
1+(-c)^(1/4)*x^(1/2))/(I*(-c)^(1/4)+c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*(d*x)^(
1/2)/c^(3/4)/x^(1/2)+1/3*I*b^2*polylog(2,1-2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))
/((-c)^(1/4)+I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^
(1/2)+1/3*I*b^2*polylog(2,1+2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*
c^(1/4)*x^(1/2))/(-c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1
/2)+1/3*I*b^2*polylog(2,1-2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^
(1/4)*x^(1/2))/(c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)
```

+1/3*I*b^2*polylog(2,1+2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^
(1/4)*x^(1/2))/(-(-c)^(1/4)+I*(-c^(1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(
1/2)+1/3*I*b^2*polylog(2,1-2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(
-c)^(1/4)*x^(1/2))/((-c)^(1/4)+I*(-c^(1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/
x^(1/2)+2/3*b^2*arctan((-c)^(1/4)*x^(1/2))*ln((1-I)*(1+(-c)^(1/4)*x^(1/2))/
(1-I*(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+4/3*b^2*arctanh(c^
(1/4)*x^(1/2))*ln(2/(1-c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-2/3*b^
2*arctan((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4
)-I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+2/3*b
^2*arctanh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1
/4)-c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+4/3*b^2
*arctan(c^(1/4)*x^(1/2))*ln(2/(1-I*c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^
(1/2)-2/3*b^2*arctan(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))/
(I*(-c)^(1/4)-c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-2
/3*b^2*arctan(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/(I*(-c)^
(1/4)+c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)+2/3*b^2*a
rctan(c^(1/4)*x^(1/2))*ln((1+I)*(1-c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*
(d*x)^(1/2)/c^(3/4)/x^(1/2)-4/3*b^2*arctan(c^(1/4)*x^(1/2))*ln(2/(1+I*c^(1/
4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-4/3*b^2*arctanh(c^(1/4)*x^(1/2))*l
n(2/(1+c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)+2/3*b^2*arctanh(c^(1/4
)*x^(1/2))*ln(-2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))/((-c)^(1/4)-c^(1/4))/(1+c^(
1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)+2/3*b^2*arctanh(c^(1/4)*x^(1/2))
*ln(2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+c^(1/4))/(1+c^(1/4)*x^(1/2
)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-2/3*b^2*arctan((-c)^(1/4)*x^(1/2))*ln(2*(-c
)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2))
)*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+2/3*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(2*(
-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*
(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+2/3*b^2*arctan(c^(1/4)*x^(1/2))*ln((1-I)*(1+
c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-2/3*b^2
*arctan(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I
*c^(1/4)*x^(1/2))/(-c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(
1/2)-2/3*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(-2*(-c)^(1/4)*(1-x^(1/2)*(-(-c)
^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*(d
*x)^(1/2)/(-c)^(3/4)/x^(1/2)+2/3*b^2*arctanh(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)
(1-x^(1/2)(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(-c^(1/4)+(-(-c)^(1/2)
)^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-2/3*b^2*arctan(c^(1/4)*x^(1/2))*ln(2*
c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(1/2))/(c^(1/4)+I*(-
(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-2/3*b^2*arctanh((-c)^(1/4)*
x^(1/2))*ln(2*(-c)^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1
/2))/((-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+2/3*b
^2*arctanh(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1
+c^(1/4)*x^(1/2))/(c^(1/4)+(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2
)-2/3*b^2*arctan((-c)^(1/4)*x^(1/2))*ln(-2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))
^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+I*(-c^(1/2))^(1/2)))*(d*x)^(1
/2)/(-c)^(3/4)/x^(1/2)+2/3*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(-2*(-c)^(1/4)

```
*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+(-c^(1/2)
)^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)-2/3*b^2*arctanh(c^(1/4)*x^(1/2))*l
n(-2*c^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(-c^(1/4)+(-c
^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-2/3*b^2*arctan((-c)^(1/4)*x^(1/
2))*ln(2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/(
(-c)^(1/4)+I*(-c^(1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+2/3*b^2*arct
anh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c
)^(1/4)*x^(1/2))/((-c)^(1/4)+(-c^(1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1
/2)-2/3*b^2*arctanh(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/
2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-c^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(
1/2)+2/3*b^2*arctan((-c)^(1/4)*x^(1/2))*ln(-c*x^2+1)*(d*x)^(1/2)/(-c)^(3/4)
/x^(1/2)-2/3*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(-c*x^2+1)*(d*x)^(1/2)/(-c)^
(3/4)/x^(1/2)+2/3*b*arctan(c^(1/4)*x^(1/2))*(2*a-b*ln(-c*x^2+1))*(d*x)^(1/2
)/c^(3/4)/x^(1/2)-2/3*b*arctanh(c^(1/4)*x^(1/2))*(2*a-b*ln(-c*x^2+1))*(d*x)
^(1/2)/c^(3/4)/x^(1/2)-2/3*b^2*arctan((-c)^(1/4)*x^(1/2))*ln(c*x^2+1)*(d*x)
^(1/2)/(-c)^(3/4)/x^(1/2)+2/3*b^2*arctan(c^(1/4)*x^(1/2))*ln(c*x^2+1)*(d*x)
^(1/2)/c^(3/4)/x^(1/2)+2/3*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(c*x^2+1)*(d*x
)^(1/2)/(-c)^(3/4)/x^(1/2)-2/3*b^2*arctanh(c^(1/4)*x^(1/2))*ln(c*x^2+1)*(d*
x)^(1/2)/c^(3/4)/x^(1/2)+4/3*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(2/(1-(-c)^(
1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+4/3*b^2*arctan((-c)^(1/4)*x^(
1/2))*ln(2/(1-I*(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+2/3*b^2
*arctan((-c)^(1/4)*x^(1/2))*ln((1+I)*(1-(-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)
*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)-4/3*b^2*arctan((-c)^(1/4)*x^(1/2)
)*ln(2/(1+I*(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)-4/3*b^2*arc
tanh((-c)^(1/4)*x^(1/2))*ln(2/(1+(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4
)/x^(1/2)-2/3*I*b^2*arctan((-c)^(1/4)*x^(1/2))^2*(d*x)^(1/2)/(-c)^(3/4)/x^(
1/2)-2/3*I*b^2*arctan(c^(1/4)*x^(1/2))^2*(d*x)^(1/2)/c^(3/4)/x^(1/2)-2/3*I*
b^2*polylog(2,1-2/(1-I*(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)-
1/3*I*b^2*polylog(2,1-(1+I)*(1-(-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))
)*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)-2/3*I*b^2*polylog(2,1-2/(1+I*(-c)^(1/4)*x^
(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)-1/3*I*b^2*polylog(2,1+(-1+I)*(1+(-c)
^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)-2/
3*I*b^2*polylog(2,1-2/(1-I*c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-1/
3*I*b^2*polylog(2,1-(1+I)*(1-c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*(d*x)^
(1/2)/c^(3/4)/x^(1/2)-2/3*I*b^2*polylog(2,1-2/(1+I*c^(1/4)*x^(1/2)))*(d*x)^
(1/2)/c^(3/4)/x^(1/2)-1/3*I*b^2*polylog(2,1+(-1+I)*(1+c^(1/4)*x^(1/2))/(1-I
*c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-1/3*b^2*polylog(2,1+2*(-c)^(
1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+(-c^(
1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+1/3*b^2*polylog(2,1+2*c^(1/4)*
(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(-c^(1/4)+(-c^(1/2))^(1/2)
))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-1/3*b^2*polylog(2,1-2*(-c)^(1/4)*(1+x^(1/2)*
(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-c^(1/2))^(1/2)))*(d*
x)^(1/2)/(-c)^(3/4)/x^(1/2)+1/3*b^2*polylog(2,1-2*c^(1/4)*(1+x^(1/2)*(-c^(1
/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-c^(1/2))^(1/2)))*(d*x)^(1/2)/c^(
3/4)/x^(1/2)
```

Rubi [A]

time = 10.66, antiderivative size = 6327, normalized size of antiderivative = 1.00, number of steps used = 238, number of rules used = 34, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.700, Rules used = {6051, 6043, 6041, 2507, 2526, 2505, 327, 304, 209, 212, 2520, 12, 266, 6857, 6131, 6055, 2449, 2352, 6139, 6057, 2497, 5048, 4966, 5040, 4964, 6874, 303, 1176, 631, 210, 1179, 642, 30, 2637}

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[d*x]*(a + b*ArcTanh[c*x^2])^2,x]

[Out] (-8*a*b*x*Sqrt[d*x])/9 - (2*Sqrt[2]*a*b*Sqrt[d*x]*ArcTan[1 - Sqrt[2]*c^(1/4
)*Sqrt[x]])/(3*c^(3/4)*Sqrt[x]) + (2*Sqrt[2]*a*b*Sqrt[d*x]*ArcTan[1 + Sqrt[
2]*c^(1/4)*Sqrt[x]])/(3*c^(3/4)*Sqrt[x]) - (((2*I)/3)*b^2*Sqrt[d*x]*ArcTan[
(-c)^(1/4)*Sqrt[x]]^2)/((-c)^(3/4)*Sqrt[x]) - (((2*I)/3)*b^2*Sqrt[d*x]*ArcT
an[c^(1/4)*Sqrt[x]]^2)/(c^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1
/4)*Sqrt[x]]^2)/(3*(-c)^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTanh[c^(1/4)*S
qrt[x]]^2)/(3*c^(3/4)*Sqrt[x]) + (4*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1/4)*Sqrt[x
]]*Log[2/(1 - (-c)^(1/4)*Sqrt[x])])/(3*(-c)^(3/4)*Sqrt[x]) + (4*b^2*Sqrt[d*
x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - I*(-c)^(1/4)*Sqrt[x])])/(3*(-c)^(3
/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/
4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 - I*(-
c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTan[(-c)^
(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sq
rt[c]] + (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) +
 (2*b^2*Sqrt[d*x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 + I)*(1 - (-c)^(1/4)*S
qrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/(3*(-c)^(3/4)*Sqrt[x]) - (4*b^2*Sqrt[
d*x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + I*(-c)^(1/4)*Sqrt[x])])/(3*(-c)^
(3/4)*Sqrt[x]) - (4*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + (-
c)^(1/4)*Sqrt[x])])/(3*(-c)^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTanh[(-c)^
(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sq
rt[-c]] - (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) -
(2*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-S
qrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x])
)])/(3*(-c)^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*L
og[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] - (-c)^(1/
4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*A
rcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/
((Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sq
rt[x]) + (2*b^2*Sqrt[d*x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 - I)*(1 + (-c)
^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/(3*(-c)^(3/4)*Sqrt[x]) + (4*b
^2*Sqrt[d*x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[2/(1 - c^(1/4)*Sqrt[x])])/(3*c^(3
/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4
)*(1 - c^(1/4)*Sqrt[x]))/(((-c)^(1/4) - I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x
```

```
]))])/(3*(-c)^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]
*Log[(2*(-c)^(1/4)*(1 - c^(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + (-c)
^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) + (4*b^2*Sqrt[d*x]*ArcTan[c^(1/4)
*Sqrt[x]]*Log[2/(1 - I*c^(1/4)*Sqrt[x])])/(3*c^(3/4)*Sqrt[x]) - (2*b^2*Sqrt
[d*x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x])
)/((I*Sqrt[-Sqrt[-c]] - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(3*c^(3/4)*Sqrt
[x]) - (2*b^2*Sqrt[d*x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-S
qrt[-c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[-c]] + c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))]
)/(3*c^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(-2*c^
(1/4)*(1 - (-c)^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) - c^(1/4))*(1 - I*c^(1/4)*Sq
rt[x]))])/(3*c^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTan[c^(1/4)*Sqrt[x]]*Lo
g[(2*c^(1/4)*(1 + (-c)^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) + c^(1/4))*(1 - I*c^(
1/4)*Sqrt[x]))])/(3*c^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*ArcTan[c^(1/4)*Sqrt
[x]]*Log[((1 + I)*(1 - c^(1/4)*Sqrt[x]))/(1 - I*c^(1/4)*Sqrt[x])])/(3*c^(3/
4)*Sqrt[x]) - (4*b^2*Sqrt[d*x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[2/(1 + I*c^(1/4)
*Sqrt[x])])/(3*c^(3/4)*Sqrt[x]) - (4*b^2*Sqrt[d*x]*ArcTanh[c^(1/4)*Sqrt[x]]
*Log[2/(1 + c^(1/4)*Sqrt[x])])/(3*c^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*ArcTa
nh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-
Sqrt[-c]] - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(3*c^(3/4)*Sqrt[x]) + (2*b^2*
Sqrt[d*x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt
[x]))/((Sqrt[-Sqrt[-c]] + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(3*c^(3/4)*Sqrt
[x]) - (2*b^2*Sqrt[d*x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[
-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(3
*c^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4
)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] + c^(1/4))*(1 + c^(1/4)*Sq
rt[x]))])/(3*c^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*ArcTanh[c^(1/4)*Sqrt[x]]*L
og[(-2*c^(1/4)*(1 - (-c)^(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + c^(1/
4)*Sqrt[x]))])/(3*c^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*ArcTanh[c^(1/4)*Sqrt[
x]]*Log[(2*c^(1/4)*(1 + (-c)^(1/4)*Sqrt[x]))/(((-c)^(1/4) + c^(1/4))*(1 + c
^(1/4)*Sqrt[x]))])/(3*c^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTan[(-c)^(1/4)
*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + c^(1/4)*Sqrt[x]))/(((-c)^(1/4) + I*c^(1/4)
)*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*A
rcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + c^(1/4)*Sqrt[x]))/(((-c)^
(1/4) + c^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3...
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
```

```
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
```

```
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6041

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p
, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6043

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
With[{k = Denominator[m]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*ArcTa
nh[c*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]
 && IGtQ[n, 0] && FractionQ[m]
```

Rule 6051

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_)*(x_))^(m_), x_Sym
bol] :> Dist[d^IntPart[m]*((d*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a + b
*ArcTanh[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] &&
 (EqQ[p, 1] || RationalQ[m, n])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx=\int \sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx$$

Mathematica [F]
time = 48.32, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[d*x]*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] Integrate[Sqrt[d*x]*(a + b*ArcTanh[c*x^2])^2, x]
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\,\left(a+b\,\mathrm{arctanh}\left(c\,x^{2}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*sqrt(d)*x^(3/2)*log(-c*x^2 + 1)^2 + 1/6*a^2*c*sqrt(d)*(4*x^(3/2)/c
- 3*(I*(log(I*c^(1/4)*sqrt(x) + 1) - log(-I*c^(1/4)*sqrt(x) + 1))/c^(3/4) -
 log((sqrt(c)*sqrt(x) - c^(1/4))/(sqrt(c)*sqrt(x) + c^(1/4)))/c^(3/4))/c) +
 3*b^2*c*sqrt(d)*integrate(1/12*x^(5/2)*log(c*x^2 + 1)^2/(c*x^2 - 1), x) -
6*b^2*c*sqrt(d)*integrate(1/12*x^(5/2)*log(c*x^2 + 1)*log(-c*x^2 + 1)/(c*x^
2 - 1), x) + 12*a*b*c*sqrt(d)*integrate(1/12*x^(5/2)*log(c*x^2 + 1)/(c*x^2
- 1), x) - 12*a*b*c*sqrt(d)*integrate(1/12*x^(5/2)*log(-c*x^2 + 1)/(c*x^2 -
 1), x) - 8*b^2*c*sqrt(d)*integrate(1/12*x^(5/2)*log(-c*x^2 + 1)/(c*x^2 - 1
), x) + 1/2*a^2*sqrt(d)*(I*(log(I*c^(1/4)*sqrt(x) + 1) - log(-I*c^(1/4)*sqr
t(x) + 1))/c^(3/4) - log((sqrt(c)*sqrt(x) - c^(1/4))/(sqrt(c)*sqrt(x) + c^(
1/4)))/c^(3/4)) - 3*b^2*sqrt(d)*integrate(1/12*sqrt(x)*log(c*x^2 + 1)^2/(c*
x^2 - 1), x) + 6*b^2*sqrt(d)*integrate(1/12*sqrt(x)*log(c*x^2 + 1)*log(-c*x
^2 + 1)/(c*x^2 - 1), x) - 12*a*b*sqrt(d)*integrate(1/12*sqrt(x)*log(c*x^2 +
 1)/(c*x^2 - 1), x) + 12*a*b*sqrt(d)*integrate(1/12*sqrt(x)*log(-c*x^2 + 1)
/(c*x^2 - 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*sqrt(d*x), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(a+b*atanh(c*x**2))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x)*(b*arctanh(c*x^2) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d\,x}\,\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a + b*atanh(c*x^2))^2,x)
[Out] int((d*x)^(1/2)*(a + b*atanh(c*x^2))^2, x)
```

3.91 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{\sqrt{dx}}\,dx$

Optimal. Leaf size=6177

result too large to display

```
[Out] 2*a*b*arctan(-1+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/c^(1/4)/(d*x)^(1/2
)+2*a^2*x/(d*x)^(1/2)+2*a*b*arctan(1+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1/
2)/c^(1/4)/(d*x)^(1/2)-a*b*ln(1+x*c^(1/2)-c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*
x^(1/2)/c^(1/4)/(d*x)^(1/2)+a*b*ln(1+x*c^(1/2)+c^(1/4)*2^(1/2)*x^(1/2))*2^(
1/2)*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(c*x^2
+1)*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-2*b^2*arctanh(c^(1/4)*x^(1/2))*ln(c*x^2+
1)*x^(1/2)/c^(1/4)/(d*x)^(1/2)+4*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(2/(1-(-
c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-4*b^2*arctan((-c)^(1/4)*x
^(1/2))*ln(2/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-2*b^2
*arctan((-c)^(1/4)*x^(1/2))*ln((1+I)*(1-(-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)
*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+4*b^2*arctan((-c)^(1/4)*x^(1/2))*
ln(2/(1+I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-4*b^2*arctanh
((-c)^(1/4)*x^(1/2))*ln(2/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^
(1/2)-2*b^2*arctan((-c)^(1/4)*x^(1/2))*ln((1-I)*(1+(-c)^(1/4)*x^(1/2))/(1-I
*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+4*b^2*arctanh(c^(1/4)*
x^(1/2))*ln(2/(1-c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*arctan
((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-I*c^(1
/4))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*b^2*arctanh
((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-c^(1/4
))/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-4*b^2*arctan(c^(1
/4)*x^(1/2))*ln(2/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*
arctan(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))/(I*(-c)^(1/4)-
c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*arctan(c^
(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/(I*(-c)^(1/4)+c^(1/4))/(
1-I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-2*b^2*arctan(c^(1/4)*x^(1
/2))*ln((1+I)*(1-c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d
*x)^(1/2)+1/2*b^2*x*ln(-c*x^2+1)^2/(d*x)^(1/2)+1/2*b^2*x*ln(c*x^2+1)^2/(d*x
)^(1/2)-I*b^2*polylog(2,1+2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^
(1/4)*x^(1/2))/(-c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2
)-I*b^2*polylog(2,1-2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*
x^(1/2))/(c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-I*b^2
*polylog(2,1+2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1
/2))/(-(-c)^(1/4)+I*(-c^(1/2))^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-I*b^2
*polylog(2,1-2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1
/2))/((-c)^(1/4)+I*(-c^(1/2))^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+4*b^2*
arctan(c^(1/4)*x^(1/2))*ln(2/(1+I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(
1/2)-4*b^2*arctanh(c^(1/4)*x^(1/2))*ln(2/(1+c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/
4)/(d*x)^(1/2)+2*b^2*arctanh(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-(-c)^(1/4)*x
```

^(1/2))/((-c)^(1/4)-c^(1/4))/(1+c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/
2)+2*b^2*arctanh(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/((-c)
^(1/4)+c^(1/4))/(1+c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*arct
an((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+I*c^
(1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*b^2*arcta
nh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+c^(1
/4))/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-2*b^2*arctan(c^
(1/4)*x^(1/2))*ln((1-I)*(1+c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/
c^(1/4)/(d*x)^(1/2)+2*b^2*arctan(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)*
(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(1/2))/(-c^(1/4)+I*(-(-c)^(1/2))^(1/2))
)*x^(1/2)/c^(1/4)/(d*x)^(1/2)-2*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(-2*(-c)^
(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+(
-(-c)^(1/2))^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*b^2*arctanh(c^(1/4)*x
^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(
-c^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*arctan(c^(
1/4)*x^(1/2))*ln(2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(
1/2))/(c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-2*b^2*ar
ctanh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(
1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/(-c)^(1/4)/
(d*x)^(1/2)+2*b^2*arctanh(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+x^(1/2)*(-(-c)^(
1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/c^(
1/4)/(d*x)^(1/2)+2*b^2*arctan((-c)^(1/4)*x^(1/2))*ln(-2*(-c)^(1/4)*(1-x^(1/
2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+I*(-c^(1/2))^(1/
2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(-2
(-c)^(1/4)(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4
)+(-c^(1/2))^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-2*b^2*arctanh(c^(1/4)*x
^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(-c^
(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*arctan((-c)^(1/4
)*x^(1/2))*ln(2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(
1/2))/((-c)^(1/4)+I*(-c^(1/2))^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*b^2
*arctanh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(
1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*
x)^(1/2)-2*b^2*arctanh(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+x^(1/2)*(-c^(1/2))^
(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/c^(1/4)/(d*x
)^(1/2)+I*b^2*polylog(2,1-(1+I)*(1-(-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1
/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+I*b^2*polylog(2,1+(-1+I)*(1+(-c)^(1/4)
x^(1/2))/(1-I(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+I*b^2*po
lylog(2,1-(1+I)*(1-c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/
(d*x)^(1/2)+I*b^2*polylog(2,1+(-1+I)*(1+c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/
2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-I*b^2*polylog(2,1-2*(-c)^(1/4)*(1-c^(1/4)*
x^(1/2))/((-c)^(1/4)-I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4
)/(d*x)^(1/2)-I*b^2*polylog(2,1+2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))/(I*(-c)^(1
/4)-c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-I*b^2*polyl
og(2,1-2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/(I*(-c)^(1/4)+c^(1/4))/(1-I*c^(1/4)
*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-I*b^2*polylog(2,1-2*(-c)^(1/4)*(1+c^

```
(1/4)*x^(1/2))/((-c)^(1/4)+I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c
)^(1/4)/(d*x)^(1/2)-4*a*b*arctan(c^(1/4)*x^(1/2))*x^(1/2)/c^(1/4)/(d*x)^(1/
2)-4*a*b*arctanh(c^(1/4)*x^(1/2))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-2*b^2*arctan(
(-c)^(1/4)*x^(1/2))*ln(-c*x^2+1)*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*b^2*arcta
n(c^(1/4)*x^(1/2))*ln(-c*x^2+1)*x^(1/2)/c^(1/4)/(d*x)^(1/2)-2*b^2*arctanh((
-c)^(1/4)*x^(1/2))*ln(-c*x^2+1)*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*b^2*arctan
h(c^(1/4)*x^(1/2))*ln(-c*x^2+1)*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*arctan((-
c)^(1/4)*x^(1/2))*ln(c*x^2+1)*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-2*b^2*arctan(c
^(1/4)*x^(1/2))*ln(c*x^2+1)*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*polylog(2,1-2
/(1-(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*b^2*polylog(2,1-2
/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*b^2*polylog(2,1-2
/(1-c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-b^2*polylog(2,1-2*(-c)^(1
/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2
)/(-c)^(1/4)/(d*x)^(1/2)+2*b^2*polylog(2,1-2/(1+c^(1/4)*x^(1/2)))*x^(1/2)/c
^(1/4)/(d*x)^(1/2)-b^2*polylog(2,1+2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))/((-c)^(
1/4)-c^(1/4))/(1+c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-b^2*polylog(
2,1-2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+c^(1/4))/(1+c^(1/4)*x^(1/2
)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-2*a*b*x*ln(-c*x^2+1)/(d*x)^(1/2)+2*a*b*x*ln
(c*x^2+1)/(d*x)^(1/2)-2*b^2*arctanh((-c)^(1/4)*x^(1/2))^2*x^(1/2)/(-c)^(1/4
)/(d*x)^(1/2)-2*b^2*arctanh(c^(1/4)*x^(1/2))^2*x^(1/2)/c^(1/4)/(d*x)^(1/2)+
2*I*b^2*polylog(2,1-2/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1
/2)+2*I*b^2*polylog(2,1-2/(1+I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x
)^(1/2)+2*I*b^2*polylog(2,1-2/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^
(1/2)+2*I*b^2*polylog(2,1-2/(1+I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1
/2)+2*I*b^2*arctan((-c)^(1/4)*x^(1/2))^2*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*I
*b^2*arctan(c^(1/4)*x^(1/2))^2*x^(1/2)/c^(1/4)/(d*x)^(1/2)-b^2*x*ln(-c*x^2+
1)*ln(c*x^2+1)/(d*x)^(1/2)-b^2*polylog(2,1-2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))
/((-c)^(1/4)+c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2
)+b^2*polylog(2,1+2*(-c)^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1/4
)*x^(1/2))/(-(-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2
)-b^2*polylog(2,1+2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1
/2))/(-c^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+b^2*polylo
g(2,1-2*(-c)^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(
(-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-b^2*polylog
(2,1-2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)
+(-(-c)^(1/2))^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-b^2*polylog(2,1+2*(-c)^(
1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+(-c^(
1/2))^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+b^2*polylog(2,1+2*c^(1/4)*(1-x
^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(-c^(1/4)+(-c^(1/2))^(1/2)))*x
^(1/2)/c^(1/4)/(d*x)^(1/2)-b^2*polylog(2,1-2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2
))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/(-c
)^(1/4)/(d*x)^(1/2)+b^2*polylog(2,1-2*c^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/
(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)
```

Rubi [A]

time = 9.23, antiderivative size = 6177, normalized size of antiderivative = 1.00, number of steps used = 241, number of rules used = 33, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.650, Rules used = {6051, 6043, 6023, 2498, 327, 218, 212, 209, 2500, 2526, 2521, 2520, 12, 266, 6857, 6131, 6055, 2449, 2352, 6139, 6057, 2497, 5048, 4966, 5040, 4964, 217, 1179, 642, 1176, 631, 210, 2636}

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/Sqrt[d*x],x]
```

```
[Out] (2*a^2*x)/Sqrt[d*x] - (2*Sqrt[2]*a*b*Sqrt[x]*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqr
t[x]])/(c^(1/4)*Sqrt[d*x]) + (2*Sqrt[2]*a*b*Sqrt[x]*ArcTan[1 + Sqrt[2]*c^(1
/4)*Sqrt[x]])/(c^(1/4)*Sqrt[d*x]) + ((2*I)*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sq
rt[x]]^2)/((-c)^(1/4)*Sqrt[d*x]) - (4*a*b*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]])/
(c^(1/4)*Sqrt[d*x]) + ((2*I)*b^2*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]^2)/(c^(1/4
)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]^2)/((-c)^(1/4)*Sq
rt[d*x]) - (4*a*b*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]])/(c^(1/4)*Sqrt[d*x]) - (
2*b^2*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]^2)/(c^(1/4)*Sqrt[d*x]) + (4*b^2*Sqrt
[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - (-c)^(1/4)*Sqrt[x])])/((-c)^(1/4
)*Sqrt[d*x]) - (4*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - I*(-c)^
(1/4)*Sqrt[x])])/((-c)^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*
Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]
] - (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/((-c)^(1/4)*Sqrt[d*x]) + (2*b
^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]
*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(
(-c)^(1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 +
 I)*(1 - (-c)^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/((-c)^(1/4)*Sqrt
[d*x]) + (4*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + I*(-c)^(1/4)*
Sqrt[x])])/((-c)^(1/4)*Sqrt[d*x]) - (4*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[
x]]*Log[2/(1 + (-c)^(1/4)*Sqrt[x])])/((-c)^(1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[x
]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[
x]))/((Sqrt[-Sqrt[-c]] - (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/((-c)^(1/4
)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)
*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] + (-c)^(1/4))*(1 + (-c)^(
1/4)*Sqrt[x]))])/((-c)^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)
*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]]
 - (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/((-c)^(1/4)*Sqrt[d*x]) + (2*b^2*
Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*S
qrt[x]))/((Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/((-c)^(
1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 - I)*(1
 + (-c)^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/((-c)^(1/4)*Sqrt[d*x])
 + (4*b^2*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[2/(1 - c^(1/4)*Sqrt[x])])/(c
^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(
1/4)*(1 - c^(1/4)*Sqrt[x]))/(((-c)^(1/4) - I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqr
```

```
t[x]))])/((-c)^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]
]*Log[(2*(-c)^(1/4)*(1 - c^(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + (-c
)^(1/4)*Sqrt[x]))])/((-c)^(1/4)*Sqrt[d*x]) - (4*b^2*Sqrt[x]*ArcTan[c^(1/4)*
Sqrt[x]]*Log[2/(1 - I*c^(1/4)*Sqrt[x])])/(c^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[
x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/(
(I*Sqrt[-Sqrt[-c]] - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(c^(1/4)*Sqrt[d*x]
) + (2*b^2*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sqrt[-
c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[-c]] + c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(c^
(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(
1 - (-c)^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))
])/(c^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(2*c^(1
/4)*(1 + (-c)^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) + c^(1/4))*(1 - I*c^(1/4)*Sqrt
[x]))])/(c^(1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[((
1 + I)*(1 - c^(1/4)*Sqrt[x]))/(1 - I*c^(1/4)*Sqrt[x])])/(c^(1/4)*Sqrt[d*x])
 + (4*b^2*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[2/(1 + I*c^(1/4)*Sqrt[x])])/(
c^(1/4)*Sqrt[d*x]) - (4*b^2*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[2/(1 + c^(
1/4)*Sqrt[x])])/(c^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x
]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] - c^(1/
4))*(1 + c^(1/4)*Sqrt[x]))])/(c^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTanh[c
^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[
-c]] + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(c^(1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[
x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/(
(Sqrt[-Sqrt[c]] - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(c^(1/4)*Sqrt[d*x]) - (
2*b^2*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sqrt[c]]*S
qrt[x]))/((Sqrt[-Sqrt[c]] + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(c^(1/4)*Sqrt
[d*x]) + (2*b^2*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - (-c)^
(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(c^(1/4)*S
qrt[d*x]) + (2*b^2*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + (-c
)^(1/4)*Sqrt[x]))/(((-c)^(1/4) + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(c^(1/4)
*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(
1 + c^(1/4)*Sqrt[x]))/(((-c)^(1/4) + I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))
])/((-c)^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[
(2*(-c)^(1/4)*(1 + c^(1/4)*Sqrt[x]))/(((-c)^(1/...
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
```

```
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2636

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[x*Log[w]*(D[v, x]/v), x], x] - Int[SimplifyIntegrand[x*Log[v]*(D[
w, x]/w), x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
```

```
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6023

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[ExpandI
ntegrand[(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p, x], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6043

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
With[{k = Denominator[m]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*ArcTa
nh[c*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]
 && IGtQ[n, 0] && FractionQ[m]
```

Rule 6051

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_)*(x_))^(m_), x_Sym
bol] :> Dist[d^IntPart[m]*((d*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a + b
*ArcTanh[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] &&
 (EqQ[p, 1] || RationalQ[m, n])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{\sqrt{dx}}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{\sqrt{dx}}\,dx$$

Mathematica [F]
time = 45.60, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{\sqrt{dx}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/Sqrt[d*x],x]
[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/Sqrt[d*x], x]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\,x^{2}\right)\right)^{2}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*c*((-I*(log(I*c^(1/4)*sqrt(x) + 1) - log(-I*c^(1/4)*sqrt(x) + 1))/
c^(1/4) - log((sqrt(c)*sqrt(x) - c^(1/4))/(sqrt(c)*sqrt(x) + c^(1/4)))/c^(1
/4))/(c*sqrt(d)) - 4*sqrt(x)/(c*sqrt(d))) + b^2*c*integrate(1/4*x^(3/2)*log
(c*x^2 + 1)^2/(c*sqrt(d)*x^2 - sqrt(d)), x) - 2*b^2*c*integrate(1/4*x^(3/2)
*log(c*x^2 + 1)*log(-c*x^2 + 1)/(c*sqrt(d)*x^2 - sqrt(d)), x) + 4*a*b*c*int
egrate(1/4*x^(3/2)*log(c*x^2 + 1)/(c*sqrt(d)*x^2 - sqrt(d)), x) - 4*a*b*c*i
ntegrate(1/4*x^(3/2)*log(-c*x^2 + 1)/(c*sqrt(d)*x^2 - sqrt(d)), x) - 8*b^2*
c*integrate(1/4*x^(3/2)*log(-c*x^2 + 1)/(c*sqrt(d)*x^2 - sqrt(d)), x) + 1/2
*b^2*sqrt(x)*log(-c*x^2 + 1)^2/sqrt(d) - b^2*integrate(1/4*log(c*x^2 + 1)^2
/((c*sqrt(d)*x^2 - sqrt(d))*sqrt(x)), x) + 2*b^2*integrate(1/4*log(c*x^2 +
1)*log(-c*x^2 + 1)/((c*sqrt(d)*x^2 - sqrt(d))*sqrt(x)), x) - 4*a*b*integrat
e(1/4*log(c*x^2 + 1)/((c*sqrt(d)*x^2 - sqrt(d))*sqrt(x)), x) + 4*a*b*integr
ate(1/4*log(-c*x^2 + 1)/((c*sqrt(d)*x^2 - sqrt(d))*sqrt(x)), x) + 1/2*a^2*(
-I*(log(I*c^(1/4)*sqrt(x) + 1) - log(-I*c^(1/4)*sqrt(x) + 1))/c^(1/4) - log
((sqrt(c)*sqrt(x) - c^(1/4))/(sqrt(c)*sqrt(x) + c^(1/4)))/c^(1/4))/sqrt(d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*sqrt(d*x)/(d*x
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(c x^{2}\right)\right)^{2}}{\sqrt{d x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/(d*x)**(1/2),x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/sqrt(d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/sqrt(d*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)}^2}{\sqrt{d\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/(d*x)^(1/2),x)
```

```
[Out] int((a + b*atanh(c*x^2))^2/(d*x)^(1/2), x)
```

3.92 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=6334

result too large to display

```
[Out] -2*a*b*ln(c*x^2+1)/d/(d*x)^(1/2)+b^2*ln(-c*x^2+1)*ln(c*x^2+1)/d/(d*x)^(1/2)
-1/2*(2*a-b*ln(-c*x^2+1))^2/d/(d*x)^(1/2)+2*a*b*c^(1/4)*arctan(1+c^(1/4)*2^
(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/d/(d*x)^(1/2)+a*b*c^(1/4)*ln(1+x*c^(1/2)-c^(
1/4)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/d/(d*x)^(1/2)-a*b*c^(1/4)*ln(1+x*c^(1
/2)+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/d/(d*x)^(1/2)+2*a*b*c^(1/4)*ar
ctan(-1+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/d/(d*x)^(1/2)+2*I*b^2*(-c)
^(1/4)*polylog(2,1-2/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*I*b^
2*(-c)^(1/4)*polylog(2,1-2/(1+I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+
2*I*b^2*c^(1/4)*polylog(2,1-2/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+
2*I*b^2*c^(1/4)*polylog(2,1-2/(1+I*c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+
2*I*b^2*(-c)^(1/4)*arctan((-c)^(1/4)*x^(1/2))^2*x^(1/2)/d/(d*x)^(1/2)+2*I*b
^2*c^(1/4)*arctan(c^(1/4)*x^(1/2))^2*x^(1/2)/d/(d*x)^(1/2)+2*b^2*(-c)^(1/4)
*arctan((-c)^(1/4)*x^(1/2))*ln(c*x^2+1)*x^(1/2)/d/(d*x)^(1/2)-2*b^2*c^(1/4)
*arctan(c^(1/4)*x^(1/2))*ln(c*x^2+1)*x^(1/2)/d/(d*x)^(1/2)-2*b^2*(-c)^(1/4)
*arctanh((-c)^(1/4)*x^(1/2))*ln(c*x^2+1)*x^(1/2)/d/(d*x)^(1/2)+2*b^2*(-c)^(
1/4)*arctanh((-c)^(1/4)*x^(1/2))^2*x^(1/2)/d/(d*x)^(1/2)+2*b^2*c^(1/4)*arct
anh(c^(1/4)*x^(1/2))^2*x^(1/2)/d/(d*x)^(1/2)-2*b^2*(-c)^(1/4)*polylog(2,1-2
/(1-(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*(-c)^(1/4)*polylog(2,1
-2/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*c^(1/4)*polylog(2,1-
2/(1-c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+b^2*(-c)^(1/4)*polylog(2,1-2*(
-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*
x^(1/2)/d/(d*x)^(1/2)-2*b^2*c^(1/4)*polylog(2,1-2/(1+c^(1/4)*x^(1/2)))*x^(1
/2)/d/(d*x)^(1/2)+b^2*c^(1/4)*polylog(2,1+2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))/
((-c)^(1/4)-c^(1/4))/(1+c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+b^2*c^(1/4)
*polylog(2,1-2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+c^(1/4))/(1+c^(1/
4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+b^2*(-c)^(1/4)*polylog(2,1-2*(-c)^(1/4)*
(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/
(d*x)^(1/2)-b^2*(-c)^(1/4)*polylog(2,1+2*(-c)^(1/4)*(1-x^(1/2)*(-(-c)^(1/2)
)^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/
d/(d*x)^(1/2)+b^2*c^(1/4)*polylog(2,1+2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1
/2))/(1+c^(1/4)*x^(1/2))/(-c^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1
/2)-b^2*(-c)^(1/4)*polylog(2,1-2*(-c)^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))
/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(
1/2)+b^2*c^(1/4)*polylog(2,1-2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c
^(1/4)*x^(1/2))/(c^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+b^2*(-
c)^(1/4)*polylog(2,1+2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4
)*x^(1/2))/(-(-c)^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-b^2*c^(1/4
)*polylog(2,1+2*c^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(-
c^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+b^2*(-c)^(1/4)*polylog(2,1
```

-2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/
4)+(-c^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-b^2*c^(1/4)*polylog(2,1-2*c^(1/
4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-c^(1/2))^(1/
2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*c^(1/4)*arctanh(c^(1/4)*x^(1/2))*ln(-2*c^(
1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(-c^(1/4)+(-c^(1/2))^
(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*(-c)^(1/4)*arctan((-c)^(1/4)*x^(1/2))*l
n(2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/((-c)^
(1/4)+I*(-c^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*(-c)^(1/4)*arctanh((
-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/
4)*x^(1/2))/((-c)^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*c^(1
/4)*arctanh(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+c
^(1/4)*x^(1/2))/(c^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+I*b^2*(-c
)^(1/4)*polylog(2,1-(1+I)*(1-(-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*
x^(1/2)/d/(d*x)^(1/2)+I*b^2*(-c)^(1/4)*polylog(2,1+(-1+I)*(1+(-c)^(1/4)*x^(
1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+I*b^2*c^(1/4)*polylog
(2,1-(1+I)*(1-c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)
+I*b^2*c^(1/4)*polylog(2,1+(-1+I)*(1+c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2))
)*x^(1/2)/d/(d*x)^(1/2)-I*b^2*(-c)^(1/4)*polylog(2,1-2*(-c)^(1/4)*(1-c^(1/4
)*x^(1/2))/((-c)^(1/4)-I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)
^(1/2)-I*b^2*c^(1/4)*polylog(2,1+2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))/(I*(-c)^(
1/4)-c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-I*b^2*c^(1/4)*po
lylog(2,1-2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/(I*(-c)^(1/4)+c^(1/4))/(1-I*c^(1
/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-I*b^2*(-c)^(1/4)*polylog(2,1-2*(-c)^(1/
4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(
1/2)/d/(d*x)^(1/2)-I*b^2*c^(1/4)*polylog(2,1+2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1
/2))^(1/2))/(1-I*c^(1/4)*x^(1/2))/(-c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*x^(1/2)
/d/(d*x)^(1/2)-I*b^2*c^(1/4)*polylog(2,1-2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))
^(1/2))/(1-I*c^(1/4)*x^(1/2))/(c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(d
*x)^(1/2)-I*b^2*(-c)^(1/4)*polylog(2,1+2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(
1/2))/(1-I*(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+I*(-c^(1/2))^(1/2)))*x^(1/2)/d/
(d*x)^(1/2)-I*b^2*(-c)^(1/4)*polylog(2,1-2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))
^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+I*(-c^(1/2))^(1/2)))*x^(1/2)/d
/(d*x)^(1/2)-1/2*b^2*ln(c*x^2+1)^2/d/(d*x)^(1/2)+2*b^2*c^(1/4)*arctanh(c^(1
/4)*x^(1/2))*ln(c*x^2+1)*x^(1/2)/d/(d*x)^(1/2)-4*b^2*(-c)^(1/4)*arctanh((-c
)^(1/4)*x^(1/2))*ln(2/(1-(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-4*b^2*(
-c)^(1/4)*arctan((-c)^(1/4)*x^(1/2))*ln(2/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)
/d/(d*x)^(1/2)-2*b^2*(-c)^(1/4)*arctan((-c)^(1/4)*x^(1/2))*ln((1+I)*(1-(-c)
^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+4*b^2*(-c)^
(1/4)*arctan((-c)^(1/4)*x^(1/2))*ln(2/(1+I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(
d*x)^(1/2)+4*b^2*(-c)^(1/4)*arctanh((-c)^(1/4)*x^(1/2))*ln(2/(1+(-c)^(1/4)*
x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*(-c)^(1/4)*arctan((-c)^(1/4)*x^(1/2))
ln((1-I)(1+(-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^
(1/2)-4*b^2*c^(1/4)*arctanh(c^(1/4)*x^(1/2))*ln(2/(1-c^(1/4)*x^(1/2)))*x^(1
/2)/d/(d*x)^(1/2)+2*b^2*(-c)^(1/4)*arctan((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/
4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(

```
1/2)/d/(d*x)^(1/2)-2*b^2*(-c)^(1/4)*arctanh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(
1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*x^(1/
2)/d/(d*x)^(1/2)-4*b^2*c^(1/4)*arctan(c^(1/4)*x^(1/2))*ln(2/(1-I*c^(1/4)*x^
(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*c^(1/4)*arctan(c^(1/4)*x^(1/2))*ln(-2*c
^(1/4)*(1-(-c)^(1/4)*x^(1/2))/(I*(-c)^(1/4)-c^(1/4))/(1-I*c^(1/4)*x^(1/2)))
*x^(1/2)/d/(d*x)^(1/2)+2*b^2*c^(1/4)*arctan(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(
1+(-c)^(1/4)*x^(1/2))/(I*(-c)^(1/4)+c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)
/d/(d*x)^(1/2)-2*b^2*c^(1/4)*arctan(c^(1/4)*x^(1/2))*ln((1+I)*(1-c^(1/4)*x^
(1/2))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+4*b^2*c^(1/4)*arctan(c^
(1/4)*x^(1/2))*ln(2/(1+I*c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+4*b^2*c^(1
/4)*arctanh(c^(1/4)*x^(1/2))*ln(2/(1+c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2
)-2*b^2*c^(1/4)*arctanh(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2
))/((-c)^(1/4)-c^(1/4))/(1+c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*c^
(1/4)*arctanh(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/((-c)^(1
/4)+c^(1/4))/(1+c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*(-c)^(1/4)*ar
ctan((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+I*
c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*(-c)^(1/4)*a
rctanh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+
c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*c^(1/4)*arctan
(c^(1/4)*x^(1/2))*ln((1-I)*(1+c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*x^(1/
2)/d/(d*x)^(1/2)+2*b^2*c^(1/4)*arctan(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-x^(
1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(1/2))/(-c^(1/4)+I*(-(-c)^(1/2))^(
1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*(-c)^(1/4)*arctanh((-c)^(1/4)*x^(1/2))*l
n(-2*(-c)^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-
c)^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*c^(1/4)*arctanh(
c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x
^(1/2))/(-c^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*c^(1/4)
*arctan(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*
c^(1/4)*x^(1/2))/(c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b
^2*(-c)^(1/4)*arctanh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+x^(1/2)*(-(-c)
^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(
1/2)/d/(d*x)^(1/2)-2*b^2*(-c)^(1/4)*arctan((-c)^(1/4)*x^(1/2))*ln(-c*x^2+1)
*x^(1/2)/d/(d*x)^(1/2)+2*b^2*(-c)^(1/4)*arctanh((-c)^(1/4)*x^(1/2))*ln(-c*x
^2+1)*x^(1/2)/d/(d*x)^(1/2)-2*b*c^(1/4)*arctan(c^(1/4)*x^(1/2))*(2*a-b*ln(-
c*x^2+1))*x^(1/2)/d/(d*x)^(1/2)+2*b*c^(1/4)*arctanh(c^(1/4)*x^(1/2))*(2*a-b
*ln(-c*x^2+1))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*c^(1/4)*arctanh(c^(1/4)*x^(1/2))
*ln(2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+
(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*(-c)^(1/4)*arctan((-c)^(1
/4)*x^(1/2))*ln(-2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*
x^(1/2))/(-(-c)^(1/4)+I*(-c^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*(-c)
^(1/4)*arctanh((-c)^(1/4)*x^(1/2))*ln(-2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(
1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/d/(d*x
)^(1/2)
```

Rubi [A]

time = 9.67, antiderivative size = 6334, normalized size of antiderivative = 1.00, number of steps used = 197, number of rules used = 33, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} =$ 1.650, Rules used = {6051, 6043, 6041, 2507, 2526, 212, 2520, 12, 266, 6857, 6131, 6055, 2449, 2352, 6139, 6057, 2497, 209, 5048, 4966, 5040, 4964, 2505, 304, 6874, 303, 1176, 631, 210, 1179, 642, 30, 2637}

Too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/(d*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[2]*a*b*c^(1/4)*Sqrt[x]*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt[x]])/(d*Sqr
t[d*x]) + (2*Sqrt[2]*a*b*c^(1/4)*Sqrt[x]*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]
])/(d*Sqrt[d*x]) + ((2*I)*b^2*(-c)^(1/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]
^2)/(d*Sqrt[d*x]) + ((2*I)*b^2*c^(1/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]^2)/(
d*Sqrt[d*x]) + (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]^2)/(d*
Sqrt[d*x]) + (2*b^2*c^(1/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]^2)/(d*Sqrt[d*x
]) - (4*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - (-c)^
(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) - (4*b^2*(-c)^(1/4)*Sqrt[x]*ArcTan[(-c)^(1/4
)*Sqrt[x]]*Log[2/(1 - I*(-c)^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) + (2*b^2*(-c)^(
1/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[
c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))]
)/(d*Sqrt[d*x]) + (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[
(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] + (-c)^(1/4)
)*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) - (2*b^2*(-c)^(1/4)*Sqrt[x]*A
rcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 + I)*(1 - (-c)^(1/4)*Sqrt[x]))/(1 - I*(-c
)^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) + (4*b^2*(-c)^(1/4)*Sqrt[x]*ArcTan[(-c)^(1
/4)*Sqrt[x]]*Log[2/(1 + I*(-c)^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) + (4*b^2*(-c)
^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + (-c)^(1/4)*Sqrt[x])])
/(d*Sqrt[d*x]) + (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[
(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] - (-c)^(1/4
))*(1 + (-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) + (2*b^2*(-c)^(1/4)*Sqrt[x]*Ar
cTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/
((Sqrt[-Sqrt[-c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) -
 (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(
1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 + (-c)^(1/4)
*Sqrt[x]))])/(d*Sqrt[d*x]) - (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*S
qrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] +
(-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) - (2*b^2*(-c)^(1/4)*S
qrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 - I)*(1 + (-c)^(1/4)*Sqrt[x]))/(1
 - I*(-c)^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) - (4*b^2*c^(1/4)*Sqrt[x]*ArcTanh[c
^(1/4)*Sqrt[x]]*Log[2/(1 - c^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) + (2*b^2*(-c)^(
1/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 - c^(1/4)*Sqrt
[x]))/(((-c)^(1/4) - I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x])
 - (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*
```

```
(1 - c^(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/
(d*Sqrt[d*x]) - (4*b^2*c^(1/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[2/(1 - I
*c^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) + (2*b^2*c^(1/4)*Sqrt[x]*ArcTan[c^(1/4)*S
qrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[-c]]
 - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) + (2*b^2*c^(1/4)*Sqrt[x
]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((I
*Sqrt[-Sqrt[-c]] + c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) + (2*b
^2*c^(1/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - (-c)^(1/4)*
Sqrt[x]))/((I*(-c)^(1/4) - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]
) + (2*b^2*c^(1/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + (-c)
^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) + c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(d*Sq
rt[d*x]) - (2*b^2*c^(1/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[((1 + I)*(1 -
 c^(1/4)*Sqrt[x]))/(1 - I*c^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) + (4*b^2*c^(1/4)
*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[2/(1 + I*c^(1/4)*Sqrt[x])])/(d*Sqrt[d*
x]) + (4*b^2*c^(1/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[2/(1 + c^(1/4)*Sq
rt[x])])/(d*Sqrt[d*x]) - (2*b^2*c^(1/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Lo
g[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] - c^(1/4))*(
1 + c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) - (2*b^2*c^(1/4)*Sqrt[x]*ArcTanh[c^(1
/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]
] + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) + (2*b^2*c^(1/4)*Sqrt[x
]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((
Sqrt[-Sqrt[c]] - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) + (2*b^2*c
^(1/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sqrt[c]]*
Sqrt[x]))/((Sqrt[-Sqrt[c]] + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]
) - (2*b^2*c^(1/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - (-
c)^(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(d*Sqrt
[d*x]) - (2*b^2*c^(1/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1
+ (-c)^(1/4)*Sqrt[x]))/(((-c)^(1/4) + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(d*
Sqrt[d*x]) + (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-
c)^(1/4)*(1 + c^(1/4)*Sqrt[x]))/(((-c)^(1/4) + I*c^(1/4))*(1 - I*(-c)^(1/4)
*Sqrt[x]))])/(d*Sqrt[d*x]) - (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*S
qrt[x]]*Log[(2*(-c)^(1/4)*(1 + c^(1/4)*Sqrt[x]))/(((-c)^(1/4) + c^(1/4))*(1
 + (-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) - (2*b^...
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 30</u>

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

<u>Rule 209</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
```

```
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
```

```
b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6041

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p
, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6043

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
With[{k = Denominator[m]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*ArcTa
nh[c*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]
 && IGtQ[n, 0] && FractionQ[m]
```

Rule 6051

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_)*(x_))^(m_), x_Sym
bol] :> Dist[d^IntPart[m]*((d*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a + b
*ArcTanh[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] &&
 (EqQ[p, 1] || RationalQ[m, n])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{3/2}}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{3/2}}\,dx$$

Mathematica [F]
time = 48.96, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/(d*x)^(3/2),x]
[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/(d*x)^(3/2), x]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\,x^2\right)\right)^2}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*c*integrate(1/4*x^(3/2)*log(c*x^2 + 1)^2/(c*d^(3/2)*x^3 - d^(3/2)*x), x
) - 2*b^2*c*integrate(1/4*x^(3/2)*log(c*x^2 + 1)*log(-c*x^2 + 1)/(c*d^(3/2)
*x^3 - d^(3/2)*x), x) + 4*a*b*c*integrate(1/4*x^(3/2)*log(c*x^2 + 1)/(c*d^(
3/2)*x^3 - d^(3/2)*x), x) - 4*a*b*c*integrate(1/4*x^(3/2)*log(-c*x^2 + 1)/(
c*d^(3/2)*x^3 - d^(3/2)*x), x) + 8*b^2*c*integrate(1/4*x^(3/2)*log(-c*x^2 +
 1)/(c*d^(3/2)*x^3 - d^(3/2)*x), x) + 1/2*a^2*(c*(I*(log(I*c^(1/4)*sqrt(x)
+ 1) - log(-I*c^(1/4)*sqrt(x) + 1))/c^(3/4) - log((sqrt(c)*sqrt(x) - c^(1/4
))/(sqrt(c)*sqrt(x) + c^(1/4)))/c^(3/4))/d^(3/2) - 4/(d^(3/2)*sqrt(x))) - b
^2*integrate(1/4*log(c*x^2 + 1)^2/((c*d^(3/2)*x^3 - d^(3/2)*x)*sqrt(x)), x)
 + 2*b^2*integrate(1/4*log(c*x^2 + 1)*log(-c*x^2 + 1)/((c*d^(3/2)*x^3 - d^(
3/2)*x)*sqrt(x)), x) - 4*a*b*integrate(1/4*log(c*x^2 + 1)/((c*d^(3/2)*x^3 -
 d^(3/2)*x)*sqrt(x)), x) + 4*a*b*integrate(1/4*log(-c*x^2 + 1)/((c*d^(3/2)*
x^3 - d^(3/2)*x)*sqrt(x)), x) - 1/2*a^2*c*(I*(log(I*c^(1/4)*sqrt(x) + 1) -
log(-I*c^(1/4)*sqrt(x) + 1))/c^(3/4) - log((sqrt(c)*sqrt(x) - c^(1/4))/(sqr
t(c)*sqrt(x) + c^(1/4)))/c^(3/4))/d^(3/2) - 1/2*b^2*log(-c*x^2 + 1)^2/(d^(3
/2)*sqrt(x))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*sqrt(d*x)/(d^2
*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\operatorname{atanh}\left(cx^{2}\right)\right)^{2}}{\left(dx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/(d*x)**(3/2),x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/(d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/(d*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)^2}{{\left(d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/(d*x)^(3/2),x)
```

```
[Out] int((a + b*atanh(c*x^2))^2/(d*x)^(3/2), x)
```

3.93 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=6520

result too large to display

```
[Out] 2/3*a*b*c^(3/4)*arctan(-1+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/d^2/(d*x
)^(1/2)-1/6*(2*a-b*ln(-c*x^2+1))^2/d^2/x/(d*x)^(1/2)+2/3*a*b*c^(3/4)*arctan
(1+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/d^2/(d*x)^(1/2)-1/3*a*b*c^(3/4)
*ln(1+x*c^(1/2)-c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/d^2/(d*x)^(1/2)+1/
3*a*b*c^(3/4)*ln(1+x*c^(1/2)+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/d^2/(
d*x)^(1/2)-2/3*a*b*ln(c*x^2+1)/d^2/x/(d*x)^(1/2)+1/3*b^2*ln(-c*x^2+1)*ln(c*
x^2+1)/d^2/x/(d*x)^(1/2)+2/3*b^2*(-c)^(3/4)*arctanh((-c)^(1/4)*x^(1/2))^2*x
^(1/2)/d^2/(d*x)^(1/2)+2/3*b^2*c^(3/4)*arctanh(c^(1/4)*x^(1/2))^2*x^(1/2)/d
^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*polylog(2,1-2/(1-(-c)^(1/4)*x^(1/2)))*x^(
1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*polylog(2,1-2/(1+(-c)^(1/4)*x^(1/2)
))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*arctanh((-c)^(1/4)*x^(1/2))*l
n(2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-c^(1/4))/(1+(-c)^(1/4)*x^(1/
2)))*x^(1/2)/d^2/(d*x)^(1/2)+4/3*b^2*c^(3/4)*arctan(c^(1/4)*x^(1/2))*ln(2/(
1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*c^(3/4)*arctan(c^(1/4
)*x^(1/2))*ln(-2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))/(I*(-c)^(1/4)-c^(1/4))/(1-I
*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*c^(3/4)*arctan(c^(1/4)*x
^(1/2))*ln(2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/(I*(-c)^(1/4)+c^(1/4))/(1-I*c^(
1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b^2*c^(3/4)*arctan(c^(1/4)*x^(1/
2))*ln((1+I)*(1-c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(
1/2)-4/3*b^2*c^(3/4)*arctan(c^(1/4)*x^(1/2))*ln(2/(1+I*c^(1/4)*x^(1/2)))*x^
(1/2)/d^2/(d*x)^(1/2)+4/3*b^2*c^(3/4)*arctanh(c^(1/4)*x^(1/2))*ln(2/(1+c^(1
/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*c^(3/4)*arctanh(c^(1/4)*x^(1/
2))*ln(-2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))/((-c)^(1/4)-c^(1/4))/(1+c^(1/4)*x^
(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*c^(3/4)*arctanh(c^(1/4)*x^(1/2))*ln
(2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+c^(1/4))/(1+c^(1/4)*x^(1/2)))
*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*arctan((-c)^(1/4)*x^(1/2))*ln(2
*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1
/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*arctanh((-c)^(1/4)*x^(1/2)
)*ln(2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+c^(1/4))/(1+(-c)^(1/4)*x^
(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b^2*c^(3/4)*arctan(c^(1/4)*x^(1/2))*ln(
(1-I)*(1+c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/
3*b^2*c^(3/4)*arctan(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2)
)^(1/2))/(1-I*c^(1/4)*x^(1/2))/(-c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*x^(1/2)/d^
2/(d*x)^(1/2)+2/3*b^2*(-c)^(3/4)*arctanh((-c)^(1/4)*x^(1/2))*ln(-2*(-c)^(1/
4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+(-(-
c)^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*c^(3/4)*arctanh(c^(1/4)*x
^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(
-c^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*I*b^2*c^(3/4)*po
lylog(2,1-2/(1+I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-1/3*I*b^2*c^(3/4
```

```
)*polylog(2,1+(-1+I)*(1+c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2
/(d*x)^(1/2)+1/3*I*b^2*(-c)^(3/4)*polylog(2,1-2*(-c)^(1/4)*(1-c^(1/4)*x^(1/
2))/((-c)^(1/4)-I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2
)+1/3*I*b^2*c^(3/4)*polylog(2,1+2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))/(I*(-c)^(1
/4)-c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*I*b^2*c^(3/
4)*polylog(2,1-2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/(I*(-c)^(1/4)+c^(1/4))/(1-I
*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*I*b^2*(-c)^(3/4)*polylog(2,1
-2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+I*c^(1/4))/(1-I*(-c)^(1/4)*x^
(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*I*b^2*c^(3/4)*polylog(2,1+2*c^(1/4)*(1-
x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(1/2))/(-c^(1/4)+I*(-(-c)^(1/2)
)^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*I*b^2*c^(3/4)*polylog(2,1-2*c^(1/4)*(
1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(1/2))/(c^(1/4)+I*(-(-c)^(1/2
))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*I*b^2*(-c)^(3/4)*polylog(2,1+2*(-c)^
(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+I*
(-c^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*I*b^2*(-c)^(3/4)*polylog(2,1
-2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/((-c)^(
1/4)+I*(-c^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*c^(3/4)*arctan(c^
(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^
(1/2))/(c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b^2*(-c
)^(3/4)*arctanh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+x^(1/2)*(-(-c)^(1/2)
)^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d
^2/(d*x)^(1/2)-2/3*b^2*c^(3/4)*arctanh(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+x^(
1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-(-c)^(1/2))^(1/2))
)*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*arctan((-c)^(1/4)*x^(1/2))*ln(
-2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/(-(-c)^
(1/4)+I*(-c^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*arcta
nh((-c)^(1/4)*x^(1/2))*ln(-2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c
)^(1/4)*x^(1/2))/(-(-c)^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+2/
3*b^2*c^(3/4)*arctanh(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)*(-c^(1/2))^
(1/2))/(1+c^(1/4)*x^(1/2))/(-c^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(
1/2)-2/3*b^2*(-c)^(3/4)*arctan((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+x^(1/
2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+I*(-c^(1/2))^(1/2
)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*arctanh((-c)^(1/4)*x^(1/2))*
ln(2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(
1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b^2*c^(3/4)*arctanh(c^(
1/4)*x^(1/2))*ln(2*c^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))
/(c^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*c^(3/4)*polylo
g(2,1-2/(1-c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*b^2*(-c)^(3/4)*pol
ylog(2,1-2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-c^(1/4))/(1+(-c)^(1/4
)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*c^(3/4)*polylog(2,1-2/(1+c^(1/4
)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*b^2*c^(3/4)*polylog(2,1+2*c^(1/4)*(
1-(-c)^(1/4)*x^(1/2))/((-c)^(1/4)-c^(1/4))/(1+c^(1/4)*x^(1/2)))*x^(1/2)/d^2
/(d*x)^(1/2)+1/3*b^2*c^(3/4)*polylog(2,1-2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/(
(-c)^(1/4)+c^(1/4))/(1+c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*b^2*(-
c)^(3/4)*polylog(2,1-2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+c^(1/4))/
```

(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-1/3*b^2*(-c)^(3/4)*polylog(
2,1+2*(-c)^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(
-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*b^2*c^(3/4)*pol
ylog(2,1+2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(-c^
(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-1/3*b^2*(-c)^(3/4)*poly
log(2,1-2*(-c)^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))
/((-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*b^2*c^(3/4)*
polylog(2,1-2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(
c^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*b^2*(-c)^(3/4)*po
lylog(2,1+2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/
(-(-c)^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-1/3*b^2*c^(3/4)*pol
ylog(2,1+2*c^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(-c^(1/
4)+(-c^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*b^2*(-c)^(3/4)*polylog(2,
1-2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1
/4)+(-c^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-1/3*b^2*c^(3/4)*polylog(2,1-
2*c^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-c^(1/
2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*I*b^2*(-c)^(3/4)*arctan((-c)^(1/4)*
x^(1/2))^2*x^(1/2)/d^2/(d*x)^(1/2)-2/3*I*b^2*c^(3/4)*arctan(c^(1/4)*x^(1/2)
)^2*x^(1/2)/d^2/(d*x)^(1/2)-2/3*I*b^2*(-c)^(3/4)*polylog(2,1-2/(1-I*(-c)^(1
/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-1/3*I*b^2*(-c)^(3/4)*polylog(2,1-(1+I
)*(1-(-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-
2/3*I*b^2*(-c)^(3/4)*polylog(2,1-2/(1+I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d
*x)^(1/2)-1/3*I*b^2*(-c)^(3/4)*polylog(2,1+(-1+I)*(1+(-c)^(1/4)*x^(1/2))/(1
-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*I*b^2*c^(3/4)*polylog(2
,1-2/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-1/3*I*b^2*c^(3/4)*polyl
og(2,1-(1+I)*(1-c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(
1/2)+2/3*b^2*(-c)^(3/4)*arctan((-c)^(1/4)*x^(1/2))*ln(-c*x^2+1)*x^(1/2)/d^2
/(d*x)^(1/2)+2/3*b^2*(-c)^(3/4)*arctanh((-c)^(1/4)*x^(1/2))*ln(-c*x^2+1)*x^
(1/2)/d^2/(d*x)^(1/2)+2/3*b*c^(3/4)*arctan(c^(1/4)*x^(1/2))*(2*a-b*ln(-c*x^
2+1))*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b*c^(3/4)*arctanh(c^(1/4)*x^(1/2))*(2*a-b
*ln(-c*x^2+1))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*arctan((-c)^(1/4)
*x^(1/2))*ln(c*x^2+1)*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b^2*c^(3/4)*arctan(c^(1/4
)*x^(1/2))*ln(c*x^2+1)*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*arctanh((
-c)^(1/4)*x^(1/2))*ln(c*x^2+1)*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b^2*c^(3/4)*arct
anh(c^(1/4)*x^(1/2))*ln(c*x^2+1)*x^(1/2)/d^2/(d*x)^(1/2)-4/3*b^2*(-c)^(3/4)
*arctanh((-c)^(1/4)*x^(1/2))*ln(2/(1-(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)
^(1/2)+4/3*b^2*(-c)^(3/4)*arctan((-c)^(1/4)*x^(1/2))*ln(2/(1-I*(-c)^(1/4)*x
^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b^2*(-c)^(3/4)*arctan((-c)^(1/4)*x^(1/
2))*ln((1+I)*(1-(-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(
d*x)^(1/2)-4/3*b^2*(-c)^(3/4)*arctan((-c)^(1/4)*x^(1/2))*ln(2/(1+I*(-c)^(1/
4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+4/3*b^2*(-c)^(3/4)*arctanh((-c)^(1/4)*
x^(1/2))*ln(2/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b^2*(-c)^
(3/4)*arctan((-c)^(1/4)*x^(1/2))*ln((1-I)*(1+(-c)^(1/4)*x^(1/2))/(1-I*(-c)^
(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-4/3*b^2*c^(3/4)*arctanh(c^(1/4)*x^(
1/2))*ln(2/(1-c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*

```
arctan((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-
I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-1/6*b^2*ln(c*x
^2+1)^2/d^2/x/(d*x)^(1/2)
```

Rubi [A]

time = 8.94, antiderivative size = 6520, normalized size of antiderivative = 1.00, number of steps used = 197, number of rules used = 33, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.650, Rules used = {6051, 6043, 6041, 2507, 2521, 212, 2520, 12, 266, 6857, 6131, 6055, 2449, 2352, 6139, 6057, 2497, 209, 5048, 4966, 5040, 4964, 2505, 218, 6874, 217, 1179, 642, 1176, 631, 210, 30, 2637}

Too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/(d*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[2]*a*b*c^(3/4)*Sqrt[x]*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt[x]])/(3*d^2
*Sqrt[d*x]) + (2*Sqrt[2]*a*b*c^(3/4)*Sqrt[x]*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqr
t[x]])/(3*d^2*Sqrt[d*x]) - (((2*I)/3)*b^2*(-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1
/4)*Sqrt[x]]^2)/(d^2*Sqrt[d*x]) - (((2*I)/3)*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(
1/4)*Sqrt[x]]^2)/(d^2*Sqrt[d*x]) + (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(
1/4)*Sqrt[x]]^2)/(3*d^2*Sqrt[d*x]) + (2*b^2*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)
*Sqrt[x]]^2)/(3*d^2*Sqrt[d*x]) - (4*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/
4)*Sqrt[x]]*Log[2/(1 - (-c)^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[d*x]) + (4*b^2*(-c
)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - I*(-c)^(1/4)*Sqrt[x])
])/(3*d^2*Sqrt[d*x]) - (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]
*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] - (-c)
^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(3*d^2*Sqrt[d*x]) - (2*b^2*(-c)^(3/4)
*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*S
qrt[x]))/((I*Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(3*
d^2*Sqrt[d*x]) + (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(
(1 + I)*(1 - (-c)^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[
d*x]) - (4*b^2*(-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + I*(
-c)^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[d*x]) + (4*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[
(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + (-c)^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[d*x]) + (2
*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 -
 Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] - (-c)^(1/4))*(1 + (-c)^(1/4)*
Sqrt[x]))])/(3*d^2*Sqrt[d*x]) + (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4
)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c
]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*d^2*Sqrt[d*x]) - (2*b^2*(-c
)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-S
qrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))]
)/(3*d^2*Sqrt[d*x]) - (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]
*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] + (-c)^(1
/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*d^2*Sqrt[d*x]) + (2*b^2*(-c)^(3/4)*Sqrt
```

```
[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 - I)*(1 + (-c)^(1/4)*Sqrt[x]))/(1 -
I*(-c)^(1/4)*Sqrt[x])]])/(3*d^2*Sqrt[d*x]) - (4*b^2*c^(3/4)*Sqrt[x]*ArcTanh[
c^(1/4)*Sqrt[x]]*Log[2/(1 - c^(1/4)*Sqrt[x])]])/(3*d^2*Sqrt[d*x]) - (2*b^2*(
-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 - c^(1/4)
*Sqrt[x]))/(((-c)^(1/4) - I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))]])/(3*d^2*S
qrt[d*x]) - (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-
c)^(1/4)*(1 - c^(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + (-c)^(1/4)*Sqr
t[x]))]])/(3*d^2*Sqrt[d*x]) + (4*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]
*Log[2/(1 - I*c^(1/4)*Sqrt[x])]])/(3*d^2*Sqrt[d*x]) - (2*b^2*c^(3/4)*Sqrt[x]
*ArcTan[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((I
*Sqrt[-Sqrt[-c]] - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))]])/(3*d^2*Sqrt[d*x]) -
(2*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sq
rt[-c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[-c]] + c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))]])
/(3*d^2*Sqrt[d*x]) - (2*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(-2
*c^(1/4)*(1 - (-c)^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) - c^(1/4))*(1 - I*c^(1/4)
*Sqrt[x]))]])/(3*d^2*Sqrt[d*x]) - (2*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt
[x]]*Log[(2*c^(1/4)*(1 + (-c)^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) + c^(1/4))*(1
- I*c^(1/4)*Sqrt[x]))]])/(3*d^2*Sqrt[d*x]) + (2*b^2*c^(3/4)*Sqrt[x]*ArcTan[c
^(1/4)*Sqrt[x]]*Log[((1 + I)*(1 - c^(1/4)*Sqrt[x]))/(1 - I*c^(1/4)*Sqrt[x])
]])/(3*d^2*Sqrt[d*x]) - (4*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[2
/(1 + I*c^(1/4)*Sqrt[x])]])/(3*d^2*Sqrt[d*x]) + (4*b^2*c^(3/4)*Sqrt[x]*ArcTa
nh[c^(1/4)*Sqrt[x]]*Log[2/(1 + c^(1/4)*Sqrt[x])]])/(3*d^2*Sqrt[d*x]) - (2*b^
2*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[
-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))]])/(3*d^2
*Sqrt[d*x]) - (2*b^2*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4
)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] + c^(1/4))*(1 + c^(1/4)*
Sqrt[x]))]])/(3*d^2*Sqrt[d*x]) + (2*b^2*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt
[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] - c^(1/
4))*(1 + c^(1/4)*Sqrt[x]))]])/(3*d^2*Sqrt[d*x]) + (2*b^2*c^(3/4)*Sqrt[x]*Arc
Tanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-
Sqrt[c]] + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))]])/(3*d^2*Sqrt[d*x]) - (2*b^2*c^(
3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - (-c)^(1/4)*Sqrt[
x]))/(((-c)^(1/4) - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))]])/(3*d^2*Sqrt[d*x]) - (
2*b^2*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + (-c)^(1/
4)*Sqrt[x]))/(((-c)^(1/4) + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))]])/(3*d^2*Sqrt[d
*x]) - (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/
4)*(1 + c^(1/4)*Sqrt[x]))/(((-c)^(1/4) + I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[
x]))]])/(3*d^2*Sqrt[d*x]) - (2*b^2*(-c)^(3/4)*Sq...
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6041

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p
, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6043

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
With[{k = Denominator[m]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*ArcTa
nh[c*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]
 && IGtQ[n, 0] && FractionQ[m]
```

Rule 6051

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_)*(x_))^(m_), x_Sym
bol] :> Dist[d^IntPart[m]*((d*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a + b
*ArcTanh[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] &&
 (EqQ[p, 1] || RationalQ[m, n])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
```

```
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{5/2}}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{5/2}}\,dx$$

Mathematica [F]
time = 39.65, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/(d*x)^(5/2),x]
[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/(d*x)^(5/2), x]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\,x^{2}\right)\right)^{2}}{\left(dx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x)
[Out] int((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x, algorithm="maxima")
[Out] 3*b^2*c*integrate(1/12*x^(3/2)*log(c*x^2 + 1)^2/(c*d^(5/2)*x^4 - d^(5/2)*x^
2), x) - 6*b^2*c*integrate(1/12*x^(3/2)*log(c*x^2 + 1)*log(-c*x^2 + 1)/(c*d
^(5/2)*x^4 - d^(5/2)*x^2), x) + 12*a*b*c*integrate(1/12*x^(3/2)*log(c*x^2 +
 1)/(c*d^(5/2)*x^4 - d^(5/2)*x^2), x) - 12*a*b*c*integrate(1/12*x^(3/2)*log
(-c*x^2 + 1)/(c*d^(5/2)*x^4 - d^(5/2)*x^2), x) + 8*b^2*c*integrate(1/12*x^(
3/2)*log(-c*x^2 + 1)/(c*d^(5/2)*x^4 - d^(5/2)*x^2), x) + 1/6*a^2*(3*(-I*c^(
3/4)*(log(I*c^(1/4)*sqrt(x) + 1) - log(-I*c^(1/4)*sqrt(x) + 1)) - c^(3/4)*l
og((sqrt(c)*sqrt(x) - c^(1/4))/(sqrt(c)*sqrt(x) + c^(1/4))))/d^(5/2) - 4/(d
^(5/2)*x^(3/2))) - 3*b^2*integrate(1/12*log(c*x^2 + 1)^2/((c*d^(5/2)*x^4 -
d^(5/2)*x^2)*sqrt(x)), x) + 6*b^2*integrate(1/12*log(c*x^2 + 1)*log(-c*x^2
+ 1)/((c*d^(5/2)*x^4 - d^(5/2)*x^2)*sqrt(x)), x) - 12*a*b*integrate(1/12*lo
g(c*x^2 + 1)/((c*d^(5/2)*x^4 - d^(5/2)*x^2)*sqrt(x)), x) + 12*a*b*integrate
(1/12*log(-c*x^2 + 1)/((c*d^(5/2)*x^4 - d^(5/2)*x^2)*sqrt(x)), x) - 1/2*a^2
*c*(-I*(log(I*c^(1/4)*sqrt(x) + 1) - log(-I*c^(1/4)*sqrt(x) + 1))/c^(1/4) -
 log((sqrt(c)*sqrt(x) - c^(1/4))/(sqrt(c)*sqrt(x) + c^(1/4)))/c^(1/4))/d^(5
/2) - 1/6*b^2*log(-c*x^2 + 1)^2/(d^(5/2)*x^(3/2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x, algorithm="fricas")
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*sqrt(d*x)/(d^3
*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(cx^{2}\right)\right)^{2}}{\left(dx\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/(d*x)**(5/2),x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/(d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/(d*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)}^2}{{\left(d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/(d*x)^(5/2),x)
```

```
[Out] int((a + b*atanh(c*x^2))^2/(d*x)^(5/2), x)
```

3.94 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 \,dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c*x^2))^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 \,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^2])^3,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x^2])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 \,dx=\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 \,dx$$

Mathematica [A]
time = 1.37, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 \,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2])^3, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(d x \right)^{m} \left(a +b \,\mathrm{arctanh}\left(c \,x^{2}\right)\right)^{3}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^2))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^2))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*d^m*x*x^m*log(-c*x^2 + 1)^3/(m + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1)
) + integrate(1/8*((b^3*c*d^m*(m + 1)*x^2 - b^3*d^m*(m + 1))*x^m*log(c*x^2
+ 1)^3 + 6*(a*b^2*c*d^m*(m + 1)*x^2 - a*b^2*d^m*(m + 1))*x^m*log(c*x^2 + 1)
^2 + 12*(a^2*b*c*d^m*(m + 1)*x^2 - a^2*b*d^m*(m + 1))*x^m*log(c*x^2 + 1) +
3*((b^3*c*d^m*(m + 1)*x^2 - b^3*d^m*(m + 1))*x^m*log(c*x^2 + 1) - 2*(a*b^2*
d^m*(m + 1) - (a*b^2*c*d^m*(m + 1) + b^3*c*d^m)*x^2)*x^m)*log(-c*x^2 + 1)^2
 - 3*((b^3*c*d^m*(m + 1)*x^2 - b^3*d^m*(m + 1))*x^m*log(c*x^2 + 1)^2 + 4*(a
*b^2*c*d^m*(m + 1)*x^2 - a*b^2*d^m*(m + 1))*x^m*log(c*x^2 + 1) + 4*(a^2*b*c
*d^m*(m + 1)*x^2 - a^2*b*d^m*(m + 1))*x^m)*log(-c*x^2 + 1))/(c*(m + 1)*x^2
- m - 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^2)^3 + 3*a*b^2*arctanh(c*x^2)^2 + 3*a^2*b*arctanh
(c*x^2) + a^3)*(d*x)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b \operatorname{atanh}\left(cx^2\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**2))**3,x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c*x**2))**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^3*(d*x)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c*x^2))^3,x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c*x^2))^3, x)
```

3.95 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c*x^2))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x^2])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx = \int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$$

Mathematica [A]
time = 0.85, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2])^2, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\,\mathrm{arctanh}\left(c\,x^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*d^m*x*x^m*log(-c*x^2 + 1)^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1))
 - integrate(-1/4*((b^2*c*d^m*(m + 1)*x^2 - b^2*d^m*(m + 1))*x^m*log(c*x^2
+ 1)^2 + 4*(a*b*c*d^m*(m + 1)*x^2 - a*b*d^m*(m + 1))*x^m*log(c*x^2 + 1) - 2
*((b^2*c*d^m*(m + 1)*x^2 - b^2*d^m*(m + 1))*x^m*log(c*x^2 + 1) - 2*(a*b*d^m
*(m + 1) - (a*b*c*d^m*(m + 1) + b^2*c*d^m)*x^2)*x^m)*log(-c*x^2 + 1))/(c*(m
 + 1)*x^2 - m - 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*(d*x)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b \operatorname{atanh}\left(cx^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c*x**2))**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2*(d*x)^m, x)
```

<u>**Mupad**</u> [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c*x^2))^2,x)
[Out] int((d*x)^m*(a + b*atanh(c*x^2))^2, x)
```

3.96 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=74

$$\frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{2bc(dx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{4};\frac{7+m}{4};c^2x^4\right)}{d^3(1+m)(3+m)}$$

```
[Out] (d*x)^(1+m)*(a+b*arctanh(c*x^2))/d/(1+m)-2*b*c*(d*x)^(3+m)*hypergeom([1, 3/
4+1/4*m],[7/4+1/4*m],c^2*x^4)/d^3/(1+m)/(3+m)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6049, 371}

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d(m+1)}-\frac{2bc(dx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{4};\frac{m+7}{4};c^2x^4\right)}{d^3(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTanh[c*x^2]))/(d*(1 + m)) - (2*b*c*(d*x)^(3 + m)*H
ypergeometric2F1[1, (3 + m)/4, (7 + m)/4, c^2*x^4])/(d^3*(1 + m)*(3 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx &= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{(2bc)\int\frac{x(dx)^{1+m}}{1-c^2x^4}\,dx}{d(1+m)}\\
&= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{(2bc)\int\frac{(dx)^{2+m}}{1-c^2x^4}\,dx}{d^2(1+m)}\\
&= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{2bc(dx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{4};\frac{7+m}{4};c^2x^4\right)}{d^3(1+m)(3+m)}
\end{aligned}$$

Mathematica [**A**]
time = 0.05, size = 64, normalized size = 0.86

$$-\frac{x(dx)^m \left(-\left((3+m)\left(a+b\tanh^{-1}\left(cx^2\right)\right)\right)+2bcx^2\,{}_2F_1\left(1,\frac{3+m}{4};\frac{7+m}{4};c^2x^4\right)\right)}{(1+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] -((x*(d*x)^m*(-((3 + m)*(a + b*ArcTanh[c*x^2])) + 2*b*c*x^2*Hypergeometric2
F1[1, (3 + m)/4, (7 + m)/4, c^2*x^4]))/((1 + m)*(3 + m)))
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\,\mathrm{arctanh}\left(c\,x^2\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^2)),x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^2)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/2*(4*c*d^m*integrate(x^2*x^m/(c^2*(m + 1)*x^4 - m - 1), x) + (d^m*x*x^m*l
og(c*x^2 + 1) - d^m*x*x^m*log(-c*x^2 + 1))/(m + 1))*b + (d*x)^(m + 1)*a/(d*
(m + 1))
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^2) + a)*(d*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b \operatorname{atanh}\left(cx^2\right)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**2)),x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \,\left(a + b\,\mathrm{atanh}\left(c\,x^2\right)\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c*x^2)),x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c*x^2)), x)
```

3.97 $\int \frac{(dx)^m}{a+b\tanh^{-1}(cx^2)}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{(dx)^m}{a+b\tanh^{-1}\left(cx^2\right)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctanh(c*x^2)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x^2]),x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x^2]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^2\right)}\,dx=\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^2\right)}\,dx$$

Mathematica [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^2]),x]
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^2]), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\,\mathrm{arctanh}\left(c\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x^2)),x)
[Out] int((d*x)^m/(a+b*arctanh(c*x^2)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^2)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arctanh(c*x^2) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^2)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arctanh(c*x^2) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b\operatorname{atanh}(cx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**2)),x)
[Out] Integral((d*x)**m/(a + b*atanh(c*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^2)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arctanh(c*x^2) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a + b\operatorname{atanh}\left(c\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c*x^2)),x)
[Out] int((d*x)^m/(a + b*atanh(c*x^2)), x)
```

3.98 $\int \frac{(dx)^m}{(a+b\tanh^{-1}(cx^2))^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctanh(c*x^2))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x^2])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^2])^2, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\,\mathrm{arctanh}\left(c\,x^2\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c*x^2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] (c^2*d^m*x^4 - d^m)*x^m/(b^2*c*x*log(c*x^2 + 1) - b^2*c*x*log(-c*x^2 + 1) +
 2*a*b*c*x) + integrate(-(c^2*d^m*(m + 3)*x^4 - d^m*(m - 1))*x^m/(b^2*c*x^2
*log(c*x^2 + 1) - b^2*c*x^2*log(-c*x^2 + 1) + 2*a*b*c*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x^2) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c*x^2))^2,x)
```

```
[Out] int((d*x)^m/(a + b*atanh(c*x^2))^2, x)
```

3.99 $\int x^{11} \left(a+b \tanh^{-1}\left(c x^3\right)\right) \, dx$

Optimal. Leaf size=54

$$\frac{bx^3}{12c^3}+\frac{bx^9}{36c}-\frac{b\tanh^{-1}\left(cx^3\right)}{12c^4}+\frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right)$$

```
[Out] 1/12*b*x^3/c^3+1/36*b*x^9/c-1/12*b*arctanh(c*x^3)/c^4+1/12*x^12*(a+b*arctan
h(c*x^3))
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6037, 281, 308, 212}

$$\frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{b\tanh^{-1}\left(cx^3\right)}{12c^4}+\frac{bx^3}{12c^3}+\frac{bx^9}{36c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (b*x^3)/(12*c^3) + (b*x^9)/(36*c) - (b*ArcTanh[c*x^3])/(12*c^4) + (x^12*(a
+ b*ArcTanh[c*x^3]))/12
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
```

```
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^{11}\left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx &= \frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{4}(bc)\int\frac{x^{14}}{1-c^2x^6}\,dx\\
&= \frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{12}(bc)\text{Subst}\left(\int\frac{x^4}{1-c^2x^2}\,dx,x,x^3\right)\\
&= \frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{12}(bc)\text{Subst}\left(\int\left(-\frac{1}{c^4}-\frac{x^2}{c^2}+\frac{1}{c^4\left(1-c^2x^2\right)}\right)\right.\\
&= \frac{bx^3}{12c^3}+\frac{bx^9}{36c}+\frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{b\text{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,x^3\right)}{12c^3}\\
&= \frac{bx^3}{12c^3}+\frac{bx^9}{36c}-\frac{b\tanh^{-1}\left(cx^3\right)}{12c^4}+\frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 78, normalized size = 1.44

$$\frac{bx^3}{12c^3}+\frac{bx^9}{36c}+\frac{ax^{12}}{12}+\frac{1}{12}bx^{12}\tanh^{-1}\left(cx^3\right)+\frac{b\log\left(1-cx^3\right)}{24c^4}-\frac{b\log\left(1+cx^3\right)}{24c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (b*x^3)/(12*c^3) + (b*x^9)/(36*c) + (a*x^12)/12 + (b*x^12*ArcTanh[c*x^3])/1
2 + (b*Log[1 - c*x^3])/(24*c^4) - (b*Log[1 + c*x^3])/(24*c^4)
```

Maple [A]
time = 0.03, size = 66, normalized size = 1.22

method	result	size
default	$\frac{x^{12}a}{12}+\frac{b\,x^{12}\,\mathrm{arctanh}(c\,x^3)}{12}+\frac{b\,x^9}{36c}+\frac{b\,x^3}{12c^3}+\frac{b\ln(c\,x^3-1)}{24c^4}-\frac{b\ln(c\,x^3+1)}{24c^4}$	66
risch	$\frac{x^{12}b\ln(c\,x^3+1)}{24}-\frac{x^{12}b\ln(-c\,x^3+1)}{24}+\frac{x^{12}a}{12}+\frac{b\,x^9}{36c}+\frac{b\,x^3}{12c^3}-\frac{b\ln(c\,x^3+1)}{24c^4}+\frac{b\ln(c\,x^3-1)}{24c^4}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a+b*arctanh(c*x^3)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*x^12*a+1/12*b*x^12*arctanh(c*x^3)+1/36*b*x^9/c+1/12*b*x^3/c^3+1/24*b/c
^4*ln(c*x^3-1)-1/24*b/c^4*ln(c*x^3+1)
```

Maxima [**A**]
time = 0.25, size = 69, normalized size = 1.28

$$\frac{1}{12}\,ax^{12} + \frac{1}{72}\left(6\,x^{12}\,\mathrm{artanh}\left(cx^{3}\right) + c\left(\frac{2\left(c^{2}x^{9} + 3\,x^{3}\right)}{c^{4}} - \frac{3\,\log\left(cx^{3} + 1\right)}{c^{5}} + \frac{3\,\log\left(cx^{3} - 1\right)}{c^{5}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/12*a*x^12 + 1/72*(6*x^12*arctanh(c*x^3) + c*(2*(c^2*x^9 + 3*x^3)/c^4 - 3*
log(c*x^3 + 1)/c^5 + 3*log(c*x^3 - 1)/c^5))*b
```

Fricas [**A**]
time = 0.37, size = 64, normalized size = 1.19

$$\frac{6\,ac^{4}x^{12} + 2\,bc^{3}x^{9} + 6\,bcx^{3} + 3\left(bc^{4}x^{12} - b\right)\log\left(-\frac{cx^{3}+1}{cx^{3}-1}\right)}{72\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/72*(6*a*c^4*x^12 + 2*b*c^3*x^9 + 6*b*c*x^3 + 3*(b*c^4*x^12 - b)*log(-(c*x
^3 + 1)/(c*x^3 - 1)))/c^4
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.43, size = 78, normalized size = 1.44

$$\frac{1}{24}\,bx^{12}\log\left(-\frac{cx^{3} + 1}{cx^{3} - 1}\right) + \frac{1}{12}\,ax^{12} + \frac{bx^{9}}{36\,c} + \frac{bx^{3}}{12\,c^{3}} - \frac{b\log\left(cx^{3} + 1\right)}{24\,c^{4}} + \frac{b\log\left(cx^{3} - 1\right)}{24\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/24*b*x^12*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/12*a*x^12 + 1/36*b*x^9/c + 1/
12*b*x^3/c^3 - 1/24*b*log(c*x^3 + 1)/c^4 + 1/24*b*log(c*x^3 - 1)/c^4
```

Mupad [B]
time = 1.11, size = 69, normalized size = 1.28

$$\frac{a\,x^{12}}{12}+\frac{b\,x^3}{12\,c^3}+\frac{b\,x^9}{36\,c}+\frac{b\,x^{12}\,\ln\left(c\,x^3+1\right)}{24}-\frac{b\,x^{12}\,\ln\left(1-c\,x^3\right)}{24}+\frac{b\,\mathrm{atan}(c\,x^3\,1\mathrm{i})\,1\mathrm{i}}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*atanh(c*x^3)),x)
```

```
[Out] (a*x^12)/12 + (b*x^3)/(12*c^3) + (b*x^9)/(36*c) + (b*atan(c*x^3*1i)*1i)/(12
*c^4) + (b*x^12*log(c*x^3 + 1))/24 - (b*x^12*log(1 - c*x^3))/24
```

3.100 $\int x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx$

Optimal. Leaf size=48

$$\frac{bx^6}{18c}+\frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{18c^3}$$

```
[Out] 1/18*b*x^6/c+1/9*x^9*(a+b*arctanh(c*x^3))+1/18*b*ln(-c^2*x^6+1)/c^3
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {6037, 272, 45}

$$\frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{18c^3}+\frac{bx^6}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (b*x^6)/(18*c) + (x^9*(a + b*ArcTanh[c*x^3]))/9 + (b*Log[1 - c^2*x^6])/(18*
c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx &= \frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{3}(bc)\int\frac{x^{11}}{1-c^2x^6}\,dx \\
&= \frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{18}(bc)\text{Subst}\left(\int\frac{x}{1-c^2x}\,dx,x,x^6\right) \\
&= \frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{18}(bc)\text{Subst}\left(\int\left(-\frac{1}{c^2}-\frac{1}{c^2\left(-1+c^2x\right)}\right)\,dx,x,
\end{aligned}
$$

$$
= \frac{bx^6}{18c}+\frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{18c^3}
$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 1.10

$$\frac{bx^6}{18c}+\frac{ax^9}{9}+\frac{1}{9}bx^9\tanh^{-1}\left(cx^3\right)+\frac{b\log\left(1-c^2x^6\right)}{18c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (b*x^6)/(18*c) + (a*x^9)/9 + (b*x^9*ArcTanh[c*x^3])/9 + (b*Log[1 - c^2*x^6]
)/(18*c^3)
```

Maple [A]
time = 0.05, size = 45, normalized size = 0.94

method	result	size
default	$\frac{x^9 a}{9}+\frac{b\,x^9\,\mathrm{arctanh}(c\,x^3)}{9}+\frac{b\,x^6}{18c}+\frac{b\ln(c^2x^6-1)}{18c^3}$	45
risch	$\frac{x^9 b\ln(c\,x^3+1)}{18}-\frac{x^9 b\ln(-c\,x^3+1)}{18}+\frac{x^9 a}{9}+\frac{b\,x^6}{18c}+\frac{b\ln(c^2x^6-1)}{18c^3}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*arctanh(c*x^3)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*x^9*a+1/9*b*x^9*arctanh(c*x^3)+1/18*b*x^6/c+1/18*b/c^3*ln(c^2*x^6-1)
```

Maxima [A]
time = 0.27, size = 46, normalized size = 0.96

$$\frac{1}{9}\,ax^9+\frac{1}{18}\left(2\,x^9\,\mathrm{artanh}\left(cx^3\right)+\left(\frac{x^6}{c^2}+\frac{\log\left(c^2x^6-1\right)}{c^4}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/9*a*x^9 + 1/18*(2*x^9*arctanh(c*x^3) + (x^6/c^2 + log(c^2*x^6 - 1)/c^4)*c
)*b
```

Fricas [A]
time = 0.34, size = 62, normalized size = 1.29

$$\frac{bc^3x^9\log\left(-\frac{cx^3+1}{cx^3-1}\right)+2\,ac^3x^9+bc^2x^6+b\log\left(c^2x^6-1\right)}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/18*(b*c^3*x^9*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*a*c^3*x^9 + b*c^2*x^6 + b
*log(c^2*x^6 - 1))/c^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 57, normalized size = 1.19

$$\frac{1}{18}\,bx^9\log\left(-\frac{cx^3+1}{cx^3-1}\right)+\frac{1}{9}\,ax^9+\frac{bx^6}{18\,c}+\frac{b\log\left(c^2x^6-1\right)}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/18*b*x^9*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/9*a*x^9 + 1/18*b*x^6/c + 1/18*
b*log(c^2*x^6 - 1)/c^3
```

Mupad [B]
time = 0.82, size = 61, normalized size = 1.27

$$\frac{a\,x^9}{9}+\frac{b\,\ln\left(c^2\,x^6-1\right)}{18\,c^3}+\frac{b\,x^6}{18\,c}+\frac{b\,x^9\,\ln\left(c\,x^3+1\right)}{18}-\frac{b\,x^9\,\ln\left(1-c\,x^3\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*atanh(c*x^3)),x)
```

```
[Out] (a*x^9)/9 + (b*log(c^2*x^6 - 1))/(18*c^3) + (b*x^6)/(18*c) + (b*x^9*log(c*x
^3 + 1))/18 - (b*x^9*log(1 - c*x^3))/18
```

3.101 $\int x^5 \left(a+b \tanh^{-1}\left(cx^3\right)\right) \, dx$

Optimal. Leaf size=43

$$\frac{bx^3}{6c} - \frac{b\tanh^{-1}\left(cx^3\right)}{6c^2} + \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)$$

```
[Out] 1/6*b*x^3/c-1/6*b*arctanh(c*x^3)/c^2+1/6*x^6*(a+b*arctanh(c*x^3))
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6037, 281, 327, 212}

$$\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right) - \frac{b\tanh^{-1}\left(cx^3\right)}{6c^2} + \frac{bx^3}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (b*x^3)/(6*c) - (b*ArcTanh[c*x^3])/(6*c^2) + (x^6*(a + b*ArcTanh[c*x^3]))/6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
```

```
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx &= \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{2}(bc)\int\frac{x^8}{1-c^2x^6}\,dx\\ &= \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{6}(bc)\text{Subst}\left(\int\frac{x^2}{1-c^2x^2}\,dx,x,x^3\right)\\ &= \frac{bx^3}{6c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{b\text{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,x^3\right)}{6c}\\ &= \frac{bx^3}{6c}-\frac{b\tanh^{-1}\left(cx^3\right)}{6c^2}+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 67, normalized size = 1.56

$$\frac{bx^3}{6c}+\frac{ax^6}{6}+\frac{1}{6}bx^6\tanh^{-1}\left(cx^3\right)+\frac{b\log\left(1-cx^3\right)}{12c^2}-\frac{b\log\left(1+cx^3\right)}{12c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x^3]),x]
[Out] (b*x^3)/(6*c) + (a*x^6)/6 + (b*x^6*ArcTanh[c*x^3])/6 + (b*Log[1 - c*x^3])/(
12*c^2) - (b*Log[1 + c*x^3])/(12*c^2)
```

Maple [A]
time = 0.05, size = 57, normalized size = 1.33

method	result	size
default	$\frac{x^6 a}{6}+\frac{b\,x^6\,\operatorname{arctanh}(c\,x^3)}{6}+\frac{b\,x^3}{6c}+\frac{b\ln(c\,x^3-1)}{12c^2}-\frac{b\ln(c\,x^3+1)}{12c^2}$	57
risch	$\frac{x^6 b\ln(c\,x^3+1)}{12}-\frac{x^6 b\ln(-c\,x^3+1)}{12}+\frac{x^6 a}{6}+\frac{b\,x^3}{6c}-\frac{b\ln(c\,x^3+1)}{12c^2}+\frac{b\ln(c\,x^3-1)}{12c^2}+\frac{b^2}{24a\,c^2}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x^3)),x,method=_RETURNVERBOSE)
[Out] 1/6*x^6*a+1/6*b*x^6*arctanh(c*x^3)+1/6*b*x^3/c+1/12*b/c^2*ln(c*x^3-1)-1/12*
b/c^2*ln(c*x^3+1)
```

Maxima [A]
time = 0.25, size = 58, normalized size = 1.35

$$\frac{1}{6}\,ax^6+\frac{1}{12}\left(2\,x^6\,\operatorname{artanh}\left(cx^3\right)+c\left(\frac{2\,x^3}{c^2}-\frac{\log\left(cx^3+1\right)}{c^3}+\frac{\log\left(cx^3-1\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/12*(2*x^6*arctanh(c*x^3) + c*(2*x^3/c^2 - log(c*x^3 + 1)/c^3
+ log(c*x^3 - 1)/c^3))*b
```

Fricas [A]
time = 0.33, size = 54, normalized size = 1.26

$$\frac{2\,ac^2x^6+2\,bcx^3+\left(bc^2x^6-b\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{12\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*a*c^2*x^6 + 2*b*c*x^3 + (b*c^2*x^6 - b)*log(-(c*x^3 + 1)/(c*x^3 - 1
)))/c^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(37) = 74.
time = 0.42, size = 181, normalized size = 4.21

$$\frac{1}{3}\,c\left(\frac{(cx^3+1)b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{(cx^3-1)\left(\frac{(cx^3+1)^2c^3}{(cx^3-1)^2}-\frac{2\,(cx^3+1)c^3}{cx^3-1}+c^3\right)}+\frac{\frac{2\,(cx^3+1)a}{cx^3-1}+\frac{(cx^3+1)b}{cx^3-1}-b}{\frac{(cx^3+1)^2c^3}{(cx^3-1)^2}-\frac{2\,(cx^3+1)c^3}{cx^3-1}+c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/3*c*((c*x^3 + 1)*b*log(-(c*x^3 + 1)/(c*x^3 - 1))/((c*x^3 - 1)*((c*x^3 + 1
)^2*c^3/(c*x^3 - 1)^2 - 2*(c*x^3 + 1)*c^3/(c*x^3 - 1) + c^3)) + (2*(c*x^3 +
 1)*a/(c*x^3 - 1) + (c*x^3 + 1)*b/(c*x^3 - 1) - b)/((c*x^3 + 1)^2*c^3/(c*x^
3 - 1)^2 - 2*(c*x^3 + 1)*c^3/(c*x^3 - 1) + c^3))
```

Mupad [B]
time = 0.96, size = 60, normalized size = 1.40

$$\frac{a\,x^6}{6}+\frac{b\,x^3}{6\,c}+\frac{b\,x^6\,\ln\left(c\,x^3+1\right)}{12}-\frac{b\,x^6\,\ln\left(1-c\,x^3\right)}{12}+\frac{b\,\mathrm{atan}(c\,x^3\,1\mathrm{i})\,1\mathrm{i}}{6\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atanh(c*x^3)),x)
```

```
[Out] (a*x^6)/6 + (b*x^3)/(6*c) + (b*atan(c*x^3*1i)*1i)/(6*c^2) + (b*x^6*log(c*x^
3 + 1))/12 - (b*x^6*log(1 - c*x^3))/12
```

3.102 $\int x^2 \left(a+b \tanh^{-1}\left(c x^3\right)\right) \, dx$

Optimal. Leaf size=37

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{6c}$$

```
[Out] 1/3*x^3*(a+b*arctanh(c*x^3))+1/6*b*ln(-c^2*x^6+1)/c
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6037, 266}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (x^3*(a + b*ArcTanh[c*x^3]))/3 + (b*Log[1 - c^2*x^6])/(6*c)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx&=\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)-(bc)\int\frac{x^5}{1-c^2x^6}\,dx\\&=\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{6c}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.14

$$\frac{ax^3}{3}+\frac{1}{3}bx^3\tanh^{-1}\left(cx^3\right)+\frac{b\log\left(1-c^2x^6\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^3]),x]
[Out] (a*x^3)/3 + (b*x^3*ArcTanh[c*x^3])/3 + (b*Log[1 - c^2*x^6])/(6*c)
```

Maple [A]
time = 0.02, size = 39, normalized size = 1.05

method	result	size
derivativedivides	$\frac{acx^3+bcx^3\operatorname{arctanh}(cx^3)+\frac{b\ln(-c^2x^6+1)}{2}}{3c}$	39
default	$\frac{acx^3+bcx^3\operatorname{arctanh}(cx^3)+\frac{b\ln(-c^2x^6+1)}{2}}{3c}$	39
risch	$\frac{x^3b\ln(cx^3+1)}{6}-\frac{bx^3\ln(-cx^3+1)}{6}+\frac{x^3a}{3}+\frac{b\ln(c^2x^6-1)}{6c}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^3)),x,method=_RETURNVERBOSE)
[Out] 1/3/c*(a*c*x^3+b*c*x^3*arctanh(c*x^3)+1/2*b*ln(-c^2*x^6+1))
```

Maxima [A]
time = 0.25, size = 37, normalized size = 1.00

$$\frac{1}{3}\,ax^3+\frac{(2\,cx^3\operatorname{artanh}(cx^3)+\log(-c^2x^6+1))b}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
[Out] 1/3*a*x^3 + 1/6*(2*c*x^3*arctanh(c*x^3) + log(-c^2*x^6 + 1))*b/c
```

Fricas [A]
time = 0.36, size = 50, normalized size = 1.35

$$\frac{bcx^3\log\left(-\frac{cx^3+1}{cx^3-1}\right)+2\,acx^3+b\log(c^2x^6-1)}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
[Out] 1/6*(b*c*x^3*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*a*c*x^3 + b*log(c^2*x^6 - 1)
)/c
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(33) = 66.

time = 0.44, size = 188, normalized size = 5.08

$$\frac{1}{3}ax^3 + \frac{1}{3}bc\left(\frac{\log\left(\frac{|-cx^3-1|}{|cx^3-1|}\right)}{c^2} - \frac{\log\left(\left|-\frac{cx^3+1}{cx^3-1}+1\right|\right)}{c^2} + \frac{\log\left(-\frac{\frac{c\left(\frac{cx^3+1}{cx^3-1}+1\right)}{\frac{(cx^3+1)c}{cx^3-1}-c}+1}{\frac{c\left(\frac{cx^3+1}{cx^3-1}+1\right)}{\frac{(cx^3+1)c}{cx^3-1}-c}-1}\right)}{c^2\left(\frac{cx^3+1}{cx^3-1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/3*a*x^3 + 1/3*b*c*(log(abs(-c*x^3 - 1)/abs(c*x^3 - 1))/c^2 - log(abs(-(c*
x^3 + 1)/(c*x^3 - 1) + 1))/c^2 + log(-(c*((c*x^3 + 1)/(c*x^3 - 1) + 1)/((c*
x^3 + 1)*c/(c*x^3 - 1) - c) + 1)/(c*((c*x^3 + 1)/(c*x^3 - 1) + 1)/((c*x^3 +
 1)*c/(c*x^3 - 1) - c) - 1))/(c^2*((c*x^3 + 1)/(c*x^3 - 1) - 1)))
```

Mupad [B]

time = 0.78, size = 52, normalized size = 1.41

$$\frac{a\,x^3}{3}+\frac{b\,\ln\left(c^2\,x^6-1\right)}{6\,c}+\frac{b\,x^3\,\ln\left(c\,x^3+1\right)}{6}-\frac{b\,x^3\,\ln\left(1-c\,x^3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^3)),x)
```

```
[Out] (a*x^3)/3 + (b*log(c^2*x^6 - 1))/(6*c) + (b*x^3*log(c*x^3 + 1))/6 - (b*x^3*
log(1 - c*x^3))/6
```

3.103 $\int \frac{a+b\tanh^{-1}(cx^3)}{x}\,dx$

Optimal. Leaf size=30

$$a\log(x) - \frac{1}{6}b\text{PolyLog}\left(2,-cx^3\right) + \frac{1}{6}b\text{PolyLog}\left(2,cx^3\right)$$

```
[Out] a*ln(x)-1/6*b*polylog(2,-c*x^3)+1/6*b*polylog(2,c*x^3)
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6035, 6031}

$$a\log(x) - \frac{1}{6}b\text{Li}_2\left(-cx^3\right) + \frac{1}{6}b\text{Li}_2\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x,x]
```

```
[Out] a*Log[x] - (b*PolyLog[2, -(c*x^3)])/6 + (b*PolyLog[2, c*x^3])/6
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx, x, x^3\right)\\
&= a\log(x) - \frac{1}{6}b\text{Li}_2\left(-cx^3\right) + \frac{1}{6}b\text{Li}_2\left(cx^3\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.93

$$a\log(x) + \frac{1}{6}b\left(-\text{PolyLog}\left(2,-cx^3\right) + \text{PolyLog}\left(2,cx^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x,x]
[Out] a*Log[x] + (b*(-PolyLog[2, -(c*x^3)] + PolyLog[2, c*x^3]))/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.04, size = 92, normalized size = 3.07

method	result
default	$a\ln(x)+b\ln(x)\operatorname{arctanh}(c\,x^3)-\frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(c\_Z^3+1\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{2}+\frac{b\left(\_R1=\right.}{}$
risch	$a\ln(x)-\frac{\ln(x)\ln(-c\,x^3+1)b}{2}+\frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(c\_Z^3-1\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{2}+\frac{\ln(x)\ln(c\,x^3+1)b}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x,x,method=_RETURNVERBOSE)
[Out] a*ln(x)+b*ln(x)*arctanh(c*x^3)-1/2*b*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x
)/_R1),_R1=RootOf(_Z^3*c+1))+1/2*b*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/
_R1),_R1=RootOf(_Z^3*c-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x,x, algorithm="maxima")
[Out] 1/2*b*integrate((log(c*x^3 + 1) - log(-c*x^3 + 1))/x, x) + a*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x,x, algorithm="fricas")
[Out] integral((b*arctanh(c*x^3) + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{atanh}\,(cx^3)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x,x)
```

```
[Out] Integral((a + b*atanh(c*x**3))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a+b\,\mathrm{atanh}\left(c\,x^3\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/x,x)
```

```
[Out] int((a + b*atanh(c*x^3))/x, x)
```

3.104 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^4}\,dx$

Optimal. Leaf size=40

$$-\frac{a+b\tanh^{-1}\left(cx^3\right)}{3x^3}+bc\log(x)-\frac{1}{6}bc\log\left(1-c^2x^6\right)$$

```
[Out] 1/3*(-a-b*arctanh(c*x^3))/x^3+b*c*ln(x)-1/6*b*c*ln(-c^2*x^6+1)
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6037, 272, 36, 29, 31}

$$-\frac{a+b\tanh^{-1}\left(cx^3\right)}{3x^3}-\frac{1}{6}bc\log\left(1-c^2x^6\right)+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^4,x]
```

```
[Out] -1/3*(a + b*ArcTanh[c*x^3])/x^3 + b*c*Log[x] - (b*c*Log[1 - c^2*x^6])/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx^3)}{x^4}\,dx &= -\frac{a+b\tanh^{-1}(cx^3)}{3x^3}+(bc)\int \frac{1}{x(1-c^2x^6)}\,dx\\ &= -\frac{a+b\tanh^{-1}(cx^3)}{3x^3}+\frac{1}{6}(bc)\text{Subst}\left(\int \frac{1}{x(1-c^2x)}\,dx,x,x^6\right)\\ &= -\frac{a+b\tanh^{-1}(cx^3)}{3x^3}+\frac{1}{6}(bc)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^6\right)+\frac{1}{6}(bc^3)\,\text{Subst}\left(\int \frac{1}{1-c^2x}\right.\\ &= -\frac{a+b\tanh^{-1}(cx^3)}{3x^3}+bc\log(x)-\frac{1}{6}bc\log\left(1-c^2x^6\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 45, normalized size = 1.12

$$-\frac{a}{3x^3}-\frac{b\tanh^{-1}(cx^3)}{3x^3}+bc\log(x)-\frac{1}{6}bc\log\left(1-c^2x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*ArcTanh[c*x^3])/(3*x^3) + b*c*Log[x] - (b*c*Log[1 - c^2*x^6
])/6
```

Maple [A]
time = 0.03, size = 49, normalized size = 1.22

method	result	size
default	$-\frac{a}{3x^3}-\frac{b\,\mathrm{arctanh}(c\,x^3)}{3x^3}-\frac{bc\ln(c\,x^3+1)}{6}-\frac{bc\ln(c\,x^3-1)}{6}+bc\ln(x)$	49
risch	$-\frac{b\ln(c\,x^3+1)}{6x^3}+\frac{6bc\ln(x)x^3-bc\ln(c^2x^6-1)x^3+b\ln(-c\,x^3+1)-2a}{6x^3}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctanh(c*x^3)-1/6*b*c*ln(c*x^3+1)-1/6*b*c*ln(c*x^3-1)
+b*c*ln(x)
```

Maxima [A]
time = 0.27, size = 41, normalized size = 1.02

$$-\frac{1}{6}\left(c\left(\log\left(c^2x^6-1\right)-\log\left(x^6\right)\right)+\frac{2\,\operatorname{artanh}\left(cx^3\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(c*(log(c^2*x^6 - 1) - log(x^6)) + 2*arctanh(c*x^3)/x^3)*b - 1/3*a/x^3
```

Fricas [A]
time = 0.34, size = 55, normalized size = 1.38

$$-\frac{bcx^3\log\left(c^2x^6-1\right)-6\,bcx^3\log\left(x\right)+b\log\left(-\frac{cx^3+1}{cx^3-1}\right)+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*c*x^3*log(c^2*x^6 - 1) - 6*b*c*x^3*log(x) + b*log(-(c*x^3 + 1)/(c*x
^3 - 1)) + 2*a)/x^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 51, normalized size = 1.28

$$-\frac{1}{6}\,bc\log\left(c^2x^6-1\right)+bc\log\left(x\right)-\frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{6\,x^3}-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*b*c*log(c^2*x^6 - 1) + b*c*log(x) - 1/6*b*log(-(c*x^3 + 1)/(c*x^3 - 1)
)/x^3 - 1/3*a/x^3
```

Mupad [B]
time = 0.85, size = 55, normalized size = 1.38

$$b\,c\,\ln\left(x\right)-\frac{a}{3\,x^3}-\frac{b\,c\,\ln\left(c^2\,x^6-1\right)}{6}-\frac{b\,\ln\left(c\,x^3+1\right)}{6\,x^3}+\frac{b\,\ln\left(1-c\,x^3\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/x^4,x)
```

```
[Out] b*c*log(x) - a/(3*x^3) - (b*c*log(c^2*x^6 - 1))/6 - (b*log(c*x^3 + 1))/(6*x
^3) + (b*log(1 - c*x^3))/(6*x^3)
```

3.105 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^7}\,dx$

Optimal. Leaf size=41

$$-\frac{bc}{6x^3}+\frac{1}{6}bc^2\tanh^{-1}\left(cx^3\right)-\frac{a+b\tanh^{-1}\left(cx^3\right)}{6x^6}$$

```
[Out] -1/6*b*c/x^3+1/6*b*c^2*arctanh(c*x^3)+1/6*(-a-b*arctanh(c*x^3))/x^6
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6037, 281, 331, 212}

$$-\frac{a+b\tanh^{-1}\left(cx^3\right)}{6x^6}+\frac{1}{6}bc^2\tanh^{-1}\left(cx^3\right)-\frac{bc}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^7,x]
```

```
[Out] -1/6*(b*c)/x^3 + (b*c^2*ArcTanh[c*x^3])/6 - (a + b*ArcTanh[c*x^3])/(6*x^6)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
```

```
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}(cx^3)}{x^7}\,dx &= -\frac{a+b\tanh^{-1}(cx^3)}{6x^6}+\frac{1}{2}(bc)\int \frac{1}{x^4(1-c^2x^6)}\,dx\\
&= -\frac{a+b\tanh^{-1}(cx^3)}{6x^6}+\frac{1}{6}(bc)\text{Subst}\left(\int \frac{1}{x^2(1-c^2x^2)}\,dx,x,x^3\right)\\
&= -\frac{bc}{6x^3}-\frac{a+b\tanh^{-1}(cx^3)}{6x^6}+\frac{1}{6}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx,x,x^3\right)\\
&= -\frac{bc}{6x^3}+\frac{1}{6}bc^2\tanh^{-1}\left(cx^3\right)-\frac{a+b\tanh^{-1}(cx^3)}{6x^6}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 65, normalized size = 1.59

$$-\frac{a}{6x^6}-\frac{bc}{6x^3}-\frac{b\tanh^{-1}(cx^3)}{6x^6}-\frac{1}{12}bc^2\log\left(1-cx^3\right)+\frac{1}{12}bc^2\log\left(1+cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^7,x]
```

```
[Out] -1/6*a/x^6 - (b*c)/(6*x^3) - (b*ArcTanh[c*x^3])/(6*x^6) - (b*c^2*Log[1 - c*
x^3])/12 + (b*c^2*Log[1 + c*x^3])/12
```

Maple [A]
time = 0.04, size = 55, normalized size = 1.34

method	result	size
default	$-\frac{a}{6x^6}-\frac{b\,\mathrm{arctanh}(c\,x^3)}{6x^6}+\frac{b\,c^2\ln(c\,x^3+1)}{12}-\frac{b\,c^2\ln(c\,x^3-1)}{12}-\frac{bc}{6x^3}$	55
risch	$-\frac{b\ln(c\,x^3+1)}{12x^6}-\frac{b\,c^2\ln(c\,x^3-1)x^6-b\,c^2\ln(c\,x^3+1)x^6+2bc\,x^3-b\ln(-c\,x^3+1)+2a}{12x^6}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a/x^6-1/6*b/x^6*arctanh(c*x^3)+1/12*b*c^2*ln(c*x^3+1)-1/12*b*c^2*ln(c*
x^3-1)-1/6*b*c/x^3
```

Maxima [A]
time = 0.25, size = 51, normalized size = 1.24

$$\frac{1}{12}\left(\left(\left(c\log\left(cx^3+1\right)-c\log\left(cx^3-1\right)-\frac{2}{x^3}\right)c-\frac{2\,\operatorname{artanh}\left(cx^3\right)}{x^6}\right)b-\frac{a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^7,x, algorithm="maxima")
```

```
[Out] 1/12*((c*log(c*x^3 + 1) - c*log(c*x^3 - 1) - 2/x^3)*c - 2*arctanh(c*x^3)/x^
6)*b - 1/6*a/x^6
```

Fricas [A]
time = 0.34, size = 49, normalized size = 1.20

$$-\frac{2\,bcx^3-\left(bc^2x^6-b\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(2*b*c*x^3 - (b*c^2*x^6 - b)*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*a)/x^6
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**7,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 67, normalized size = 1.63

$$\frac{1}{12}\,bc^2\log\left(cx^3+1\right)-\frac{1}{12}\,bc^2\log\left(cx^3-1\right)-\frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{12\,x^6}-\frac{bcx^3+a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^7,x, algorithm="giac")
```

```
[Out] 1/12*b*c^2*log(c*x^3 + 1) - 1/12*b*c^2*log(c*x^3 - 1) - 1/12*b*log(-(c*x^3
+ 1)/(c*x^3 - 1))/x^6 - 1/6*(b*c*x^3 + a)/x^6
```

Mupad [B]
time = 1.02, size = 52, normalized size = 1.27

$$\frac{b\,c^2\,\mathrm{atanh}\left(c\,x^3\right)}{6}-\frac{\frac{a}{6}+\frac{b\,\ln\left(c\,x^3+1\right)}{12}-\frac{b\,\ln\left(1-c\,x^3\right)}{12}+\frac{b\,c\,x^3}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/x^7,x)
```

```
[Out] (b*c^2*atanh(c*x^3))/6 - (a/6 + (b*log(c*x^3 + 1))/12 - (b*log(1 - c*x^3))/
12 + (b*c*x^3)/6)/x^6
```

3.106 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^{10}}\,dx$

Optimal. Leaf size=56

$$-\frac{bc}{18x^6}-\frac{a+b\tanh^{-1}\left(cx^3\right)}{9x^9}+\frac{1}{3}bc^3\log(x)-\frac{1}{18}bc^3\log\left(1-c^2x^6\right)$$

```
[Out] -1/18*b*c/x^6+1/9*(-a-b*arctanh(c*x^3))/x^9+1/3*b*c^3*ln(x)-1/18*b*c^3*ln(-
c^2*x^6+1)
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6037, 272, 46}

$$-\frac{a+b\tanh^{-1}\left(cx^3\right)}{9x^9}+\frac{1}{3}bc^3\log(x)-\frac{1}{18}bc^3\log\left(1-c^2x^6\right)-\frac{bc}{18x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^10,x]
```

```
[Out] -1/18*(b*c)/x^6 - (a + b*ArcTanh[c*x^3])/(9*x^9) + (b*c^3*Log[x])/3 - (b*c^
3*Log[1 - c^2*x^6])/18
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx^3)}{x^{10}}\,dx = -\frac{a+b\tanh^{-1}(cx^3)}{9x^9} + \frac{1}{3}(bc)\int \frac{1}{x^7(1-c^2x^6)}\,dx$$
$$= -\frac{a+b\tanh^{-1}(cx^3)}{9x^9} + \frac{1}{18}(bc)\text{Subst}\left(\int \frac{1}{x^2(1-c^2x)}\,dx, x, x^6\right)$$
$$= -\frac{a+b\tanh^{-1}(cx^3)}{9x^9} + \frac{1}{18}(bc)\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{c^2}{x}-\frac{c^4}{-1+c^2x}\right)dx, x, x^6\right)$$
$$= -\frac{bc}{18x^6} - \frac{a+b\tanh^{-1}(cx^3)}{9x^9} + \frac{1}{3}bc^3\log(x) - \frac{1}{18}bc^3\log\left(1-c^2x^6\right)$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 1.09

$$-\frac{a}{9x^9} - \frac{bc}{18x^6} - \frac{b\tanh^{-1}(cx^3)}{9x^9} + \frac{1}{3}bc^3\log(x) - \frac{1}{18}bc^3\log\left(1-c^2x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^10,x]
```

```
[Out] -1/9*a/x^9 - (b*c)/(18*x^6) - (b*ArcTanh[c*x^3])/(9*x^9) + (b*c^3*Log[x])/3
 - (b*c^3*Log[1 - c^2*x^6])/18
```

Maple [A]
time = 0.04, size = 63, normalized size = 1.12

method	result	size
default	$-\frac{a}{9x^9} - \frac{b\,\mathrm{arctanh}(c\,x^3)}{9x^9} - \frac{b\,c^3\ln(c\,x^3+1)}{18} - \frac{b\,c^3\ln(c\,x^3-1)}{18} - \frac{bc}{18x^6} + \frac{b\,c^3\ln(x)}{3}$	63
risch	$-\frac{b\ln(c\,x^3+1)}{18x^9} + \frac{6b\,c^3\ln(x)x^9 - b\,c^3\ln(c^2x^6-1)x^9 - bc\,x^3 + b\ln(-c\,x^3+1) - 2a}{18x^9}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a/x^9-1/9*b/x^9*arctanh(c*x^3)-1/18*b*c^3*ln(c*x^3+1)-1/18*b*c^3*ln(c*
x^3-1)-1/18*b*c/x^6+1/3*b*c^3*ln(x)
```

Maxima [A]
time = 0.26, size = 51, normalized size = 0.91

$$-\frac{1}{18}\left(\left(c^2\log\left(c^2x^6-1\right) - c^2\log\left(x^6\right) + \frac{1}{x^6}\right)c + \frac{2\,\mathrm{artanh}\left(cx^3\right)}{x^9}\right)b - \frac{a}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^10,x, algorithm="maxima")
```

```
[Out] -1/18*((c^2*log(c^2*x^6 - 1) - c^2*log(x^6) + 1/x^6)*c + 2*arctanh(c*x^3)/x
^9)*b - 1/9*a/x^9
```

Fricas [A]
time = 0.35, size = 65, normalized size = 1.16

$$-\frac{bc^3x^9\log\left(c^2x^6-1\right)-6\,bc^3x^9\log\left(x\right)+bcx^3+b\log\left(-\frac{cx^3+1}{cx^3-1}\right)+2\,a}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^10,x, algorithm="fricas")
```

```
[Out] -1/18*(b*c^3*x^9*log(c^2*x^6 - 1) - 6*b*c^3*x^9*log(x) + b*c*x^3 + b*log(-(
c*x^3 + 1)/(c*x^3 - 1)) + 2*a)/x^9
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**10,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 65, normalized size = 1.16

$$-\frac{1}{18}\,bc^3\log\left(c^2x^6-1\right)+\frac{1}{3}\,bc^3\log\left(x\right)-\frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{18\,x^9}-\frac{bcx^3+2\,a}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^10,x, algorithm="giac")
```

```
[Out] -1/18*b*c^3*log(c^2*x^6 - 1) + 1/3*b*c^3*log(x) - 1/18*b*log(-(c*x^3 + 1)/(
c*x^3 - 1))/x^9 - 1/18*(b*c*x^3 + 2*a)/x^9
```

Mupad [B]
time = 0.90, size = 67, normalized size = 1.20

$$\frac{b\,c^3\,\ln\left(x\right)}{3}-\frac{b\,c^3\,\ln\left(c^2\,x^6-1\right)}{18}-\frac{a}{9\,x^9}-\frac{b\,c}{18\,x^6}-\frac{b\,\ln\left(c\,x^3+1\right)}{18\,x^9}+\frac{b\,\ln\left(1-c\,x^3\right)}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/x^10,x)
```

```
[Out] (b*c^3*log(x))/3 - (b*c^3*log(c^2*x^6 - 1))/18 - a/(9*x^9) - (b*c)/(18*x^6)
 - (b*log(c*x^3 + 1))/(18*x^9) + (b*log(1 - c*x^3))/(18*x^9)
```

3.107 $\int x^3 \left(a + b \tanh^{-1}\left(cx^3\right)\right) \, dx$

Optimal. Leaf size=174

$$\frac{3bx}{4c} + \frac{\sqrt{3}\, b\text{ArcTan}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{c}\, x}{\sqrt{3}}\right)}{8c^{4/3}} - \frac{\sqrt{3}\, b\text{ArcTan}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{c}\, x}{\sqrt{3}}\right)}{8c^{4/3}} - \frac{b \tanh^{-1}\left(\sqrt[3]{c}\, x\right)}{4c^{4/3}} + \frac{1}{4}x^4 \left(a + b \tanh^{-1}\right.$$

```
[Out] 3/4*b*x/c-1/4*b*arctanh(c^(1/3)*x)/c^(4/3)+1/4*x^4*(a+b*arctanh(c*x^3))+1/1
6*b*ln(1-c^(1/3)*x+c^(2/3)*x^2)/c^(4/3)-1/16*b*ln(1+c^(1/3)*x+c^(2/3)*x^2)/
c^(4/3)-1/8*b*arctan(-1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(4/3)-1/
8*b*arctan(1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(4/3)
```

Rubi [A]

time = 0.16, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6037, 327, 216, 648, 632, 210, 642, 212}

$$\frac{1}{4}x^4 \left(a + b \tanh^{-1}\left(cx^3\right)\right) + \frac{\sqrt{3}\, b\text{ArcTan}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{c}\, x}{\sqrt{3}}\right)}{8c^{4/3}} - \frac{\sqrt{3}\, b\text{ArcTan}\left(\frac{2\sqrt[3]{c}\, x}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{8c^{4/3}} + \frac{b \log\left(c^{2/3}x^2 - \sqrt[3]{c}\, x + 1\right)}{16c^{4/3}} - \frac{b \log\left(c^{2/3}x^2 + \sqrt[3]{c}\, x + 1\right)}{16c^{4/3}} - \frac{b \tanh^{-1}\left(\sqrt[3]{c}\, x\right)}{4c^{4/3}} + \frac{3bx}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (3*b*x)/(4*c) + (Sqrt[3]*b*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3]])/(8*c^
(4/3)) - (Sqrt[3]*b*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/(8*c^(4/3))
- (b*ArcTanh[c^(1/3)*x])/(4*c^(4/3)) + (x^4*(a + b*ArcTanh[c*x^3]))/4 + (b*
Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(16*c^(4/3)) - (b*Log[1 + c^(1/3)*x + c^(
2/3)*x^2])/(16*c^(4/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
```

```
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int x^3 \left(a + b \tanh^{-1}\left(cx^3\right)\right) dx = \frac{1}{4} x^4 \left(a + b \tanh^{-1}\left(cx^3\right)\right) - \frac{1}{4}(3bc) \int \frac{x^6}{1 - c^2 x^6} dx$$

$$= \frac{3bx}{4c} + \frac{1}{4} x^4 \left(a + b \tanh^{-1}\left(cx^3\right)\right) - \frac{(3b) \int \frac{1}{1-c^2x^6} dx}{4c}$$

$$= \frac{3bx}{4c} + \frac{1}{4} x^4 \left(a + b \tanh^{-1}\left(cx^3\right)\right) - \frac{b \int \frac{1}{1-c^{2/3}x^2} dx}{4c} - \frac{b \int \frac{1 - \frac{\sqrt[3]{c}\, x}{2}}{1 - \sqrt[3]{c}\, x + c^{2/3} x^2} dx}{4c} -$$

$$= \frac{3bx}{4c} - \frac{b \tanh^{-1}\left(\sqrt[3]{c}\, x\right)}{4c^{4/3}} + \frac{1}{4} x^4 \left(a + b \tanh^{-1}\left(cx^3\right)\right) + \frac{b \int \frac{-\sqrt[3]{c} + 2c^{2/3} x}{1 - \sqrt[3]{c}\, x + c^{2/3} x^2} dx}{16c^{4/3}}$$

$$= \frac{3bx}{4c} - \frac{b \tanh^{-1}\left(\sqrt[3]{c}\, x\right)}{4c^{4/3}} + \frac{1}{4} x^4 \left(a + b \tanh^{-1}\left(cx^3\right)\right) + \frac{b \log\left(1 - \sqrt[3]{c}\, x + c^{2/}\right.}{16c^{4/3}}$$

$$= \frac{3bx}{4c} + \frac{\sqrt{3}\, b \tan^{-1}\left(\frac{1 - 2\sqrt[3]{c}\, x}{\sqrt{3}}\right)}{8c^{4/3}} - \frac{\sqrt{3}\, b \tan^{-1}\left(\frac{1 + 2\sqrt[3]{c}\, x}{\sqrt{3}}\right)}{8c^{4/3}} - \frac{b \tanh^{-1}\left(\sqrt[3]{c}\right.}{4c^{4/3}}$$

<u>Mathematica</u> [A]

time = 0.03, size = 196, normalized size = 1.13

$$\frac{3bx}{4c} + \frac{ax^4}{4} - \frac{\sqrt{3}\, b \text{ArcTan}\left(\frac{-1+2\sqrt[3]{c}\, x}{\sqrt{3}}\right)}{8c^{4/3}} - \frac{\sqrt{3}\, b \text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\, x}{\sqrt{3}}\right)}{8c^{4/3}} + \frac{1}{4} b x^4 \tanh^{-1}\left(cx^3\right) + \frac{b \log\left(1 - \sqrt[3]{c}\, x\right)}{8c^{4/3}} - \frac{b \log\left(1 + \sqrt[3]{c}\, x\right)}{8c^{4/3}} + \frac{b \log\left(1 - \sqrt[3]{c}\, x + c^{2/3} x^2\right)}{16c^{4/3}} - \frac{b \log\left(1 + \sqrt[3]{c}\, x + c^{2/3} x^2\right)}{16c^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (3*b*x)/(4*c) + (a*x^4)/4 - (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])/
(8*c^(4/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(8*c^(4/3)) + (
b*x^4*ArcTanh[c*x^3])/4 + (b*Log[1 - c^(1/3)*x])/(8*c^(4/3)) - (b*Log[1 + c
^(1/3)*x])/(8*c^(4/3)) + (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(16*c^(4/3))
- (b*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(16*c^(4/3))
```

<u>Maple</u> [A]

time = 0.04, size = 184, normalized size = 1.06

method	result
default	$\frac{x^4 a}{4} + \frac{b\, x^4 \operatorname{arctanh}\left(c\, x^3\right)}{4} + \frac{3bx}{4c} - \frac{b \ln\left(x + \left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{8c^2 \left(\frac{1}{c}\right)^{\frac{2}{3}}} + \frac{b \ln\left(x^2 - \left(\frac{1}{c}\right)^{\frac{1}{3}} x + \left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{16c^2 \left(\frac{1}{c}\right)^{\frac{2}{3}}} - \frac{b\sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{8c^2 \left(\frac{1}{c}\right)^{\frac{2}{3}}} +$

risch	$\frac{x^4 b \ln(c x^3+1)}{8}+\frac{x^4 a}{4}-\frac{b x^4 \ln(-c x^3+1)}{8}+\frac{3bx}{4c}+\frac{b \ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{8c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b \ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}} x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{16c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ \left(\ldots\right.}{\ldots}\right.}{8c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^3)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctanh(c*x^3)+3/4*b*x/c-1/8*b/c^2/(1/c)^(2/3)*ln(x+(1/
c)^(1/3))+1/16*b/c^2/(1/c)^(2/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))-1/8*b/c^
2/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))+1/8*b/c^2/(1/
c)^(2/3)*ln(x-(1/c)^(1/3))-1/16*b/c^2/(1/c)^(2/3)*ln(x^2+(1/c)^(1/3)*x+(1/c
)^(2/3))-1/8*b/c^2/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+
1))
```

Maxima [A]
time = 0.47, size = 162, normalized size = 0.93

$$\frac{1}{4}ax^4+\frac{1}{16}\left(4x^4\operatorname{artanh}\left(cx^3\right)-c\left(\frac{2\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{7}{3}}}+\frac{2\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{7}{3}}}-\frac{12x}{c^2}+\frac{\log\left(c^{\frac{2}{3}}x^2+c^{\frac{1}{3}}x+1\right)}{c^{\frac{7}{3}}}-\frac{\log\left(c^{\frac{2}{3}}x^2-c^{\frac{1}{3}}x+1\right)}{c^{\frac{7}{3}}}+\frac{2\log\left(\frac{c^{\frac{1}{3}}x+1}{c^{\frac{1}{3}}}\right)}{c^{\frac{7}{3}}}-\frac{2\log\left(\frac{c^{\frac{1}{3}}x-1}{c^{\frac{1}{3}}}\right)}{c^{\frac{7}{3}}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/16*(4*x^4*arctanh(c*x^3) - c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2
*c^(2/3)*x + c^(1/3))/c^(1/3))/c^(7/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^
(2/3)*x - c^(1/3))/c^(1/3))/c^(7/3) - 12*x/c^2 + log(c^(2/3)*x^2 + c^(1/3)*
x + 1)/c^(7/3) - log(c^(2/3)*x^2 - c^(1/3)*x + 1)/c^(7/3) + 2*log((c^(1/3)*
x + 1)/c^(1/3))/c^(7/3) - 2*log((c^(1/3)*x - 1)/c^(1/3))/c^(7/3)))*b
```

Fricas [A]
time = 0.40, size = 981, normalized size = 5.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] [1/16*(2*b*c^2*x^4*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^4 + sqrt(3)*b*
c*sqrt((-c)^(1/3)/c)*log((2*c*x^3 - sqrt(3)*(2*c*x^2 + (-c)^(2/3)*x + (-c)^
(1/3))*sqrt((-c)^(1/3)/c) + 3*(-c)^(1/3)*x - 1)/(c*x^3 + 1)) + sqrt(3)*b*c*
sqrt(-1/c^(2/3))*log((2*c*x^3 - sqrt(3)*(2*c*x^2 - c^(2/3)*x - c^(1/3))*sqr
t(-1/c^(2/3)) - 3*c^(1/3)*x + 1)/(c*x^3 - 1)) + 12*b*c*x + b*(-c)^(2/3)*log
```

```
(c*x^2 - (-c)^(2/3)*x - (-c)^(1/3)) - b*c^(2/3)*log(c*x^2 + c^(2/3)*x + c^(
1/3)) - 2*b*(-c)^(2/3)*log(c*x + (-c)^(2/3)) + 2*b*c^(2/3)*log(c*x - c^(2/3
)))/c^2, 1/16*(2*b*c^2*x^4*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^4 - 2*
sqrt(3)*b*c*sqrt(-(-c)^(1/3)/c)*arctan(1/3*sqrt(3)*(2*(-c)^(2/3)*x + (-c)^(
1/3))*sqrt(-(-c)^(1/3)/c)) + sqrt(3)*b*c*sqrt(-1/c^(2/3))*log((2*c*x^3 - sq
rt(3)*(2*c*x^2 - c^(2/3)*x - c^(1/3))*sqrt(-1/c^(2/3)) - 3*c^(1/3)*x + 1)/(
c*x^3 - 1)) + 12*b*c*x + b*(-c)^(2/3)*log(c*x^2 - (-c)^(2/3)*x - (-c)^(1/3)
) - b*c^(2/3)*log(c*x^2 + c^(2/3)*x + c^(1/3)) - 2*b*(-c)^(2/3)*log(c*x + (
-c)^(2/3)) + 2*b*c^(2/3)*log(c*x - c^(2/3)))/c^2, 1/16*(2*b*c^2*x^4*log(-(c
*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^4 + sqrt(3)*b*c*sqrt((-c)^(1/3)/c)*log((
2*c*x^3 - sqrt(3)*(2*c*x^2 + (-c)^(2/3)*x + (-c)^(1/3))*sqrt((-c)^(1/3)/c)
+ 3*(-c)^(1/3)*x - 1)/(c*x^3 + 1)) - 2*sqrt(3)*b*c^(2/3)*arctan(1/3*sqrt(3)
*(2*c^(2/3)*x + c^(1/3))/c^(1/3)) + 12*b*c*x + b*(-c)^(2/3)*log(c*x^2 - (-c
)^(2/3)*x - (-c)^(1/3)) - b*c^(2/3)*log(c*x^2 + c^(2/3)*x + c^(1/3)) - 2*b*
(-c)^(2/3)*log(c*x + (-c)^(2/3)) + 2*b*c^(2/3)*log(c*x - c^(2/3)))/c^2, 1/1
6*(2*b*c^2*x^4*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^4 - 2*sqrt(3)*b*c*
sqrt(-(-c)^(1/3)/c)*arctan(1/3*sqrt(3)*(2*(-c)^(2/3)*x + (-c)^(1/3))*sqrt(-
(-c)^(1/3)/c)) - 2*sqrt(3)*b*c^(2/3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x + c^(1
/3))/c^(1/3)) + 12*b*c*x + b*(-c)^(2/3)*log(c*x^2 - (-c)^(2/3)*x - (-c)^(1/
3)) - b*c^(2/3)*log(c*x^2 + c^(2/3)*x + c^(1/3)) - 2*b*(-c)^(2/3)*log(c*x +
 (-c)^(2/3)) + 2*b*c^(2/3)*log(c*x - c^(2/3)))/c^2]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 207, normalized size = 1.19

$$\frac{1}{16}\,bc^{7}\left(\frac{2\left(-\frac{1}{c}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{1}{c}\right)^{\frac{1}{3}}\right|\right)}{c^{8}}-\frac{2\sqrt{3}\,|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\,c^{\frac{1}{3}}\left(2x+\frac{1}{c^{\frac{1}{3}}}\right)\right)}{c^{9}}-\frac{2\sqrt{3}\left(-c^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{1}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{1}{c}\right)^{\frac{1}{3}}}\right)}{c^{9}}-\frac{|c|^{\frac{2}{3}}\log\left(x^{2}+\frac{x}{c^{\frac{1}{3}}}+\frac{1}{c^{\frac{2}{3}}}\right)}{c^{9}}+\frac{2\log\left(\left|x-\frac{1}{c^{\frac{1}{3}}}\right|\right)}{c^{\frac{25}{3}}}-\frac{\left(-c^{2}\right)^{\frac{1}{3}}\log\left(x^{2}+x\left(-\frac{1}{c}\right)^{\frac{1}{3}}+\left(-\frac{1}{c}\right)^{\frac{2}{3}}\right)}{c^{9}}\right)+\frac{1}{8}\,bx^{4}\log\left(-\frac{cx^{3}+1}{cx^{3}-1}\right)+\frac{1}{4}\,ax^{4}+\frac{3\,bx}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/16*b*c^7*(2*(-1/c)^(1/3)*log(abs(x - (-1/c)^(1/3)))/c^8 - 2*sqrt(3)*abs(c
)^(2/3)*arctan(1/3*sqrt(3)*c^(1/3)*(2*x + 1/c^(1/3)))/c^9 - 2*sqrt(3)*(-c^2
)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-1/c)^(1/3))/(-1/c)^(1/3))/c^9 - abs(c)^
(2/3)*log(x^2 + x/c^(1/3) + 1/c^(2/3))/c^9 + 2*log(abs(x - 1/c^(1/3)))/c^(2
```

```
5/3) - (-c^2)^(1/3)*log(x^2 + x*(-1/c)^(1/3) + (-1/c)^(2/3))/c^9) + 1/8*b*x
^4*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/4*a*x^4 + 3/4*b*x/c
```

Mupad [B]
time = 1.57, size = 125, normalized size = 0.72

$$\frac{a\,x^4}{4}+\frac{b\,\left(-\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)}{2}+\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)}{2}+\mathrm{atan}\left(c^{1/3}\,x\,1\mathrm{i}\right)\right)\,1\mathrm{i}}{4\,c^{4/3}}+\frac{3\,b\,x}{4\,c}+\frac{b\,x^4\,\ln\left(c\,x^3+1\right)}{8}-\frac{b\,x^4\,\ln\left(1-c\,x^3\right)}{8}-\frac{\sqrt{3}\,b\,\left(\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)+\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)\right)}{8\,c^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x^3)),x)
```

```
[Out] (a*x^4)/4 + (b*(atan((c^(1/3)*x*(3^(1/2) + 1i))/2)/2 - atan((c^(1/3)*x*(3^(
1/2) - 1i))/2)/2 + atan(c^(1/3)*x*1i))*1i)/(4*c^(4/3)) + (3*b*x)/(4*c) + (b
*x^4*log(c*x^3 + 1))/8 - (b*x^4*log(1 - c*x^3))/8 - (3^(1/2)*b*(atan((c^(1/
3)*x*(3^(1/2) - 1i))/2) + atan((c^(1/3)*x*(3^(1/2) + 1i))/2)))/(8*c^(4/3))
```

3.108 $\int \left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx$

Optimal. Leaf size=101

$$ax+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}+bx\tanh^{-1}\left(cx^3\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\log\left(1+c^{2/3}x^2+c^{4/3}x^4\right)}{4\sqrt[3]{c}}$$

```
[Out] a*x+b*x*arctanh(c*x^3)+1/2*b*ln(1-c^(2/3)*x^2)/c^(1/3)-1/4*b*ln(1+c^(2/3)*x
^2+c^(4/3)*x^4)/c^(1/3)+1/2*b*arctan(1/3*(1+2*c^(2/3)*x^2)*3^(1/2))*3^(1/2)
/c^(1/3)
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6021, 281, 298, 31, 648, 631, 210, 642}

$$ax+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{2\sqrt[3]{c}}+\frac{b\log\left(1-c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)}{4\sqrt[3]{c}}+bx\tanh^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c*x^3],x]
```

```
[Out] a*x + (Sqrt[3]*b*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])/(2*c^(1/3)) + b*x*Arc
Tanh[c*x^3] + (b*Log[1 - c^(2/3)*x^2])/(2*c^(1/3)) - (b*Log[1 + c^(2/3)*x^2
 + c^(4/3)*x^4])/(4*c^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx = ax + b\int \tanh^{-1}\left(cx^3\right)\,dx$$
$$= ax + bx\tanh^{-1}\left(cx^3\right) - (3bc)\int \frac{x^3}{1-c^2x^6}\,dx$$
$$= ax + bx\tanh^{-1}\left(cx^3\right) - \frac{1}{2}(3bc)\text{Subst}\left(\int \frac{x}{1-c^2x^3}\,dx, x, x^2\right)$$
$$= ax + bx\tanh^{-1}\left(cx^3\right) - \frac{1}{2}\left(b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{1}{1-c^{2/3}x}\,dx, x, x^2\right) + \frac{1}{2}\left(b\sqrt[3]{c}\right)\text{S}$$
$$= ax + bx\tanh^{-1}\left(cx^3\right) + \frac{b\log\left(1-c^{2/3}x^2\right)}{2\sqrt[3]{c}} - \frac{b\text{Subst}\left(\int \frac{c^{2/3}+2c^{4/3}x}{1+c^{2/3}x+c^{4/3}x^2}\,dx, x, x^2\right.}{4\sqrt[3]{c}}$$
$$= ax + bx\tanh^{-1}\left(cx^3\right) + \frac{b\log\left(1-c^{2/3}x^2\right)}{2\sqrt[3]{c}} - \frac{b\log\left(1+c^{2/3}x^2+c^{4/3}x^4\right)}{4\sqrt[3]{c}} - \frac{(3b}{}$$
$$= ax + \frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}} + bx\tanh^{-1}\left(cx^3\right) + \frac{b\log\left(1-c^{2/3}x^2\right)}{2\sqrt[3]{c}} - \frac{b\log(}{}$$

Mathematica [A]
time = 0.03, size = 136, normalized size = 1.35

$$ax + bx\tanh^{-1}\left(cx^3\right) - \frac{b\left(-2\sqrt{3}\,\text{ArcTan}\left(\frac{-1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right) + 2\sqrt{3}\,\text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right) - 2\log\left(1-\sqrt[3]{c}\,x\right) - 2\log\left(1+\sqrt[3]{c}\,x\right) + \log\left(1-\sqrt[3]{c}\,x+c^{2/3}x^2\right) + \log\left(1+\sqrt[3]{c}\,x+c^{2/3}x^2\right)\right)}{4\sqrt[3]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c*x^3],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x^3] - (b*(-2*Sqrt[3]*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]
] + 2*Sqrt[3]*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]] - 2*Log[1 - c^(1/3)*x] - 2*
Log[1 + c^(1/3)*x] + Log[1 - c^(1/3)*x + c^(2/3)*x^2] + Log[1 + c^(1/3)*x +
 c^(2/3)*x^2]))/(4*c^(1/3))
```

Maple [A]
time = 0.03, size = 99, normalized size = 0.98

method	result

default	$ax+bx\,\mathrm{arctanh}\left(c\,x^{3}\right)+\frac{b\ln\left(x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x^{4}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}$
risch	$ax+\frac{bx\ln\left(c\,x^{3}+1\right)}{2}+\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^{2}-\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{3\left(\frac{1}{c}\right)^{\frac{1}{3}}}\right)}{2c\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{bx\ln\left(-c\,x^{3}+\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x^3),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*x*arctanh(c*x^3)+1/2*b/c/(1/c^2)^(1/3)*ln(x^2-(1/c^2)^(1/3))-1/4*b/c/
(1/c^2)^(1/3)*ln(x^4+(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))+1/2*b*3^(1/2)/c/(1/c^
2)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/3)*x^2+1))
```

Maxima [A]
time = 0.46, size = 90, normalized size = 0.89

$$\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{4}{3}}x^{2}+c^{\frac{2}{3}}\right)}{3\,c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}-\frac{\log\left(c^{\frac{4}{3}}x^{4}+c^{\frac{2}{3}}x^{2}+1\right)}{c^{\frac{4}{3}}}+\frac{2\,\log\left(\frac{c^{\frac{2}{3}}x^{2}-1}{c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}\right)+4\,x\,\mathrm{artanh}\left(cx^{3}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^3),x, algorithm="maxima")
```

```
[Out] 1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 + c^(2/3))/c^(2/3))/c^(
4/3) - log(c^(4/3)*x^4 + c^(2/3)*x^2 + 1)/c^(4/3) + 2*log((c^(2/3)*x^2 - 1)
/c^(2/3))/c^(4/3)) + 4*x*arctanh(c*x^3))*b + a*x
```

Fricas [A]
time = 0.39, size = 260, normalized size = 2.57

$$\left[\frac{\sqrt{3}\,bc\sqrt{-\frac{1}{c^{\frac{2}{3}}}}\,\log\left(\frac{2\,c^{2}x^{6}-3\,c^{\frac{2}{3}}x^{2}+\sqrt{3}\left(2\,c^{\frac{5}{3}}x^{4}-cx^{2}-c^{\frac{1}{3}}\right)\sqrt{-\frac{1}{c^{\frac{2}{3}}}}+1}{c^{2}x^{6}-1}\right)+2\,bcx\log\left(-\frac{cx^{3}+1}{cx^{3}-1}\right)+4\,acx-bc^{\frac{2}{3}}\log\left(c^{2}x^{4}+c^{\frac{4}{3}}x^{2}+c^{\frac{2}{3}}\right)+2\,bc^{\frac{2}{3}}\log\left(cx^{2}-c^{\frac{1}{3}}\right)}{4\,c},\ \frac{2\,bcx\log\left(-\frac{cx^{3}+1}{cx^{3}-1}\right)+2\,\sqrt{3}\,bc^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,cx^{2}+c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)+4\,acx-bc^{\frac{2}{3}}\log\left(c^{2}x^{4}+c^{\frac{4}{3}}x^{2}+c^{\frac{2}{3}}\right)+2\,bc^{\frac{2}{3}}\log\left(cx^{2}-c^{\frac{1}{3}}\right)}{4\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*b*c*sqrt(-1/c^(2/3))*log((2*c^2*x^6 - 3*c^(2/3)*x^2 + sqrt(3)
*(2*c^(5/3)*x^4 - c*x^2 - c^(1/3))*sqrt(-1/c^(2/3)) + 1)/(c^2*x^6 - 1)) + 2
*b*c*x*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c*x - b*c^(2/3)*log(c^2*x^4 + c^
(4/3)*x^2 + c^(2/3)) + 2*b*c^(2/3)*log(c*x^2 - c^(1/3)))/c, 1/4*(2*b*c*x*lo
g(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*sqrt(3)*b*c^(2/3)*arctan(1/3*sqrt(3)*(2*c*x
^2 + c^(1/3))/c^(1/3)) + 4*a*c*x - b*c^(2/3)*log(c^2*x^4 + c^(4/3)*x^2 + c^
(2/3)) + 2*b*c^(2/3)*log(c*x^2 - c^(1/3)))/c]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x**3),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 109, normalized size = 1.08

$$\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\,|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^2}-\frac{|c|^{\frac{2}{3}}\log\left(x^4+\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^2}+\frac{2\log\left(\left|x^2-\frac{1}{|c|^{\frac{2}{3}}}\right|\right)}{|c|^{\frac{4}{3}}}\right)+2x\log\left(-\frac{cx^3+1}{cx^3-1}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^3),x, algorithm="giac")
```

```
[Out] 1/4*(c*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 + 1/abs(c)^(2/3))*
abs(c)^(2/3))/c^2 - abs(c)^(2/3)*log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)^(4/3
))/c^2 + 2*log(abs(x^2 - 1/abs(c)^(2/3)))/abs(c)^(4/3)) + 2*x*log(-(c*x^3 +
 1)/(c*x^3 - 1)))*b + a*x
```

Mupad [**B**]
time = 2.76, size = 107, normalized size = 1.06

$$a\,x+\frac{b\,\ln\left(c^{2/3}\,x^2-1\right)}{2\,c^{1/3}}-\frac{\ln\left(4\,c^{2/3}\,x^2+2-\sqrt{3}\,2\mathrm{i}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{4\,c^{1/3}}-\frac{\ln\left(4\,c^{2/3}\,x^2+2+\sqrt{3}\,2\mathrm{i}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{4\,c^{1/3}}+\frac{b\,x\,\ln\left(c\,x^3+1\right)}{2}-\frac{b\,x\,\ln\left(1-c\,x^3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atanh(c*x^3),x)
```

```
[Out] a*x + (b*log(c^(2/3)*x^2 - 1))/(2*c^(1/3)) - (log(4*c^(2/3)*x^2 - 3^(1/2)*2
i + 2)*(b + 3^(1/2)*b*1i))/(4*c^(1/3)) - (log(3^(1/2)*2i + 4*c^(2/3)*x^2 +
2)*(b - 3^(1/2)*b*1i))/(4*c^(1/3)) + (b*x*log(c*x^3 + 1))/2 - (b*x*log(1 -
c*x^3))/2
```

3.109 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^3}\,dx$

Optimal. Leaf size=165

$$-\frac{1}{4}\sqrt{3}\,bc^{2/3}\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}\right)+\frac{1}{4}\sqrt{3}\,bc^{2/3}\text{ArcTan}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}\right)+\frac{1}{2}bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{a+b\,\mathrm{t}}{}$$

```
[Out] 1/2*b*c^(2/3)*arctanh(c^(1/3)*x)+1/2*(-a-b*arctanh(c*x^3))/x^2-1/8*b*c^(2/3
)*ln(1-c^(1/3)*x+c^(2/3)*x^2)+1/8*b*c^(2/3)*ln(1+c^(1/3)*x+c^(2/3)*x^2)+1/4
*b*c^(2/3)*arctan(-1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)+1/4*b*c^(2/3)
*arctan(1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6037, 216, 648, 632, 210, 642, 212}

$$-\frac{a+b\tanh^{-1}(cx^3)}{2x^2}-\frac{1}{4}\sqrt{3}\,bc^{2/3}\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}\right)+\frac{1}{4}\sqrt{3}\,bc^{2/3}\text{ArcTan}\left(\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)-\frac{1}{8}bc^{2/3}\log\left(c^{2/3}x^2-\sqrt[3]{c}\,x+1\right)+\frac{1}{8}bc^{2/3}\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x+1\right)+\frac{1}{2}bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^3,x]
```

```
[Out] -1/4*(Sqrt[3]*b*c^(2/3)*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3]]) + (Sqrt[
3]*b*c^(2/3)*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/4 + (b*c^(2/3)*ArcT
anh[c^(1/3)*x])/2 - (a + b*ArcTanh[c*x^3])/(2*x^2) - (b*c^(2/3)*Log[1 - c^(
1/3)*x + c^(2/3)*x^2])/8 + (b*c^(2/3)*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
```

```
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{x^3}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{2x^2}+\frac{1}{2}(3bc)\int \frac{1}{1-c^2x^6}\,dx\\
&= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{2x^2}+\frac{1}{2}(bc)\int \frac{1}{1-c^{2/3}x^2}\,dx+\frac{1}{2}(bc)\int \frac{1-\frac{\sqrt[3]{c}\,x}{2}}{1-\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx\,+\\
&= \frac{1}{2}bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{a+b\tanh^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{8}\left(bc^{2/3}\right)\int \frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx\\
&= \frac{1}{2}bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{a+b\tanh^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{8}bc^{2/3}\log\left(1-\sqrt[3]{c}\,x+c^{2/3}x^2\right)+\\
&= -\frac{1}{4}\sqrt{3}\,bc^{2/3}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\,x}{\sqrt{3}}\right)+\frac{1}{4}\sqrt{3}\,bc^{2/3}\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)+\frac{1}{2}bc^{2/3}\,\mathrm{ta}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 187, normalized size = 1.13

$$-\frac{a}{2x^2}+\frac{1}{4}\sqrt{3}\,bc^{2/3}\text{ArcTan}\left(\frac{-1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)+\frac{1}{4}\sqrt{3}\,bc^{2/3}\text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)-\frac{b\tanh^{-1}(cx^3)}{2x^2}-\frac{1}{4}bc^{2/3}\log\left(1-\sqrt[3]{c}\,x\right)+\frac{1}{4}bc^{2/3}\log\left(1+\sqrt[3]{c}\,x\right)-\frac{1}{8}bc^{2/3}\log\left(1-\sqrt[3]{c}\,x+c^{2/3}x^2\right)+\frac{1}{8}bc^{2/3}\log\left(1+\sqrt[3]{c}\,x+c^{2/3}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^3,x]
```

```
[Out] -1/2*a/x^2 + (Sqrt[3]*b*c^(2/3)*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])/4 + (Sq
rt[3]*b*c^(2/3)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/4 - (b*ArcTanh[c*x^3])/(
2*x^2) - (b*c^(2/3)*Log[1 - c^(1/3)*x])/4 + (b*c^(2/3)*Log[1 + c^(1/3)*x])/
4 - (b*c^(2/3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/8 + (b*c^(2/3)*Log[1 + c^(
1/3)*x + c^(2/3)*x^2])/8
```

Maple [A]
time = 0.04, size = 159, normalized size = 0.96

method	result
default	$-\frac{a}{2x^2}-\frac{b\,\mathrm{arctanh}(c\,x^3)}{2x^2}+\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^2-\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x-\left(\frac{1}{c}\right.\right.}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}$
risch	$-\frac{b\ln(c\,x^3+1)}{4x^2}-\frac{a}{2x^2}+\frac{b\ln(-c\,x^3+1)}{4x^2}-\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctanh(c*x^3)+1/4*b/(1/c)^(2/3)*ln(x+(1/c)^(1/3))-1/8
*b/(1/c)^(2/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b/(1/c)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))-1/4*b/(1/c)^(2/3)*ln(x-(1/c)^(1/3))
+1/8*b/(1/c)^(2/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b/(1/c)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))
```

Maxima [A]
time = 0.46, size = 155, normalized size = 0.94

$$\frac{1}{8}\left(\left(\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}+\frac{\log\left(c^{\frac{2}{3}}x^2+c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}-\frac{\log\left(c^{\frac{2}{3}}x^2-c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}+\frac{2\log\left(\frac{c^{\frac{1}{3}}x+1}{c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}-\frac{2\log\left(\frac{c^{\frac{1}{3}}x-1}{c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}\right)c-\frac{4\,\mathrm{artanh}\left(cx^3\right)}{x^2}\right)b-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^3))/x^3,x, algorithm="maxima")

[Out] 1/8*((2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x + c^(1/3))/c^(1/3))/c^(1/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x - c^(1/3))/c^(1/3))/c^(1/3) + log(c^(2/3)*x^2 + c^(1/3)*x + 1)/c^(1/3) - log(c^(2/3)*x^2 - c^(1/3)*x + 1)/c^(1/3) + 2*log((c^(1/3)*x + 1)/c^(1/3))/c^(1/3) - 2*log((c^(1/3)*x - 1)/c^(1/3))/c^(1/3))*c - 4*arctanh(c*x^3)/x^2)*b - 1/2*a/x^2

Fricas [**A**]
time = 0.41, size = 228, normalized size = 1.38

$$-\frac{2\sqrt{3}\left(-c^{2}\right)^{\frac{1}{3}}bx^{2}\arctan\left(\frac{2\sqrt{3}\left(-c^{2}\right)^{\frac{2}{3}}x+\sqrt{3}\,c}{3\,c}\right)-2\sqrt{3}\,b\left(c^{2}\right)^{\frac{1}{3}}x^{2}\arctan\left(\frac{2\sqrt{3}\left(c^{2}\right)^{\frac{2}{3}}x-\sqrt{3}\,c}{3\,c}\right)+\left(-c^{2}\right)^{\frac{1}{3}}bx^{2}\log\left(c^{2}x^{2}-\left(-c^{2}\right)^{\frac{1}{3}}cx+\left(-c^{2}\right)^{\frac{2}{3}}\right)+b\left(c^{2}\right)^{\frac{1}{3}}x^{2}\log\left(c^{2}x^{2}-\left(c^{2}\right)^{\frac{1}{3}}cx+\left(c^{2}\right)^{\frac{2}{3}}\right)-2\left(-c^{2}\right)^{\frac{1}{3}}bx^{2}\log\left(cx+\left(-c^{2}\right)^{\frac{1}{3}}\right)-2\,b\left(c^{2}\right)^{\frac{1}{3}}x^{2}\log\left(cx+\left(c^{2}\right)^{\frac{1}{3}}\right)+2\,b\log\left(-\frac{cx^{3}+1}{cx^{3}-1}\right)+4\,a}{8\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^3))/x^3,x, algorithm="fricas")

[Out] -1/8*(2*sqrt(3)*(-c^2)^(1/3)*b*x^2*arctan(1/3*(2*sqrt(3)*(-c^2)^(2/3)*x + sqrt(3)*c)/c) - 2*sqrt(3)*b*(c^2)^(1/3)*x^2*arctan(1/3*(2*sqrt(3)*(c^2)^(2/3)*x - sqrt(3)*c)/c) + (-c^2)^(1/3)*b*x^2*log(c^2*x^2 - (-c^2)^(1/3)*c*x + (-c^2)^(2/3)) + b*(c^2)^(1/3)*x^2*log(c^2*x^2 - (c^2)^(1/3)*c*x + (c^2)^(2/3)) - 2*(-c^2)^(1/3)*b*x^2*log(c*x + (-c^2)^(1/3)) - 2*b*(c^2)^(1/3)*x^2*log(c*x + (c^2)^(1/3)) + 2*b*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a)/x^2

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x**3))/x**3,x)

[Out] Timed out

Giac [**A**]
time = 0.45, size = 165, normalized size = 1.00

$$\frac{1}{8}\left(\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}}+\frac{\log\left(x^{2}+\frac{x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{1}{3}}}-\frac{\log\left(x^{2}-\frac{x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{1}{3}}}+\frac{2\log\left(\left|x+\frac{1}{|c|^{\frac{1}{3}}}\right|\right)}{|c|^{\frac{1}{3}}}-\frac{2\log\left(\left|x-\frac{1}{|c|^{\frac{1}{3}}}\right|\right)}{|c|^{\frac{1}{3}}}\right)bc-\frac{b\log\left(-\frac{cx^{3}+1}{cx^{3}-1}\right)}{4\,x^{2}}-\frac{a}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^3))/x^3,x, algorithm="giac")

[Out] 1/8*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(1/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(1/3) + log(x^2 + x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3) -

```
log(x^2 - x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3) + 2*log(abs(x + 1/a
bs(c)^(1/3)))/abs(c)^(1/3) - 2*log(abs(x - 1/abs(c)^(1/3)))/abs(c)^(1/3))*b
*c - 1/4*b*log(-(c*x^3 + 1)/(c*x^3 - 1))/x^2 - 1/2*a/x^2
```

Mupad [B]
time = 1.47, size = 118, normalized size = 0.72

$$\frac{b\ln\left(1-c\,x^3\right)}{4\,x^2}-\frac{b\,c^{2/3}\left(-\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)}{2}+\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)}{2}+\mathrm{atan}\left(c^{1/3}\,x\,1\mathrm{i}\right)\right)\,1\mathrm{i}}{2}-\frac{b\ln\left(c\,x^3+1\right)}{4\,x^2}-\frac{a}{2\,x^2}+\frac{\sqrt{3}\,b\,c^{2/3}\left(\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)+\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/x^3,x)
```

```
[Out] (b*log(1 - c*x^3))/(4*x^2) - (b*c^(2/3)*(atan((c^(1/3)*x*(3^(1/2) + 1i))/2)
/2 - atan((c^(1/3)*x*(3^(1/2) - 1i))/2)/2 + atan(c^(1/3)*x*1i))*1i)/2 - (b*
log(c*x^3 + 1))/(4*x^2) - a/(2*x^2) + (3^(1/2)*b*c^(2/3)*(atan((c^(1/3)*x*(
3^(1/2) - 1i))/2) + atan((c^(1/3)*x*(3^(1/2) + 1i))/2)))/4
```

3.110 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^6}\,dx$

Optimal. Leaf size=115

$$-\frac{3bc}{10x^2}-\frac{1}{10}\sqrt{3}\,bc^{5/3}\text{ArcTan}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)-\frac{a+b\tanh^{-1}(cx^3)}{5x^5}-\frac{1}{10}bc^{5/3}\log\left(1-c^{2/3}x^2\right)+\frac{1}{20}bc^{5/3}\log\left(1+\right.$$

```
[Out] -3/10*b*c/x^2+1/5*(-a-b*arctanh(c*x^3))/x^5-1/10*b*c^(5/3)*ln(1-c^(2/3)*x^2
)+1/20*b*c^(5/3)*ln(1+c^(2/3)*x^2+c^(4/3)*x^4)-1/10*b*c^(5/3)*arctan(1/3*(1
+2*c^(2/3)*x^2)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6037, 281, 331, 298, 31, 648, 631, 210, 642}

$$-\frac{a+b\tanh^{-1}(cx^3)}{5x^5}-\frac{1}{10}\sqrt{3}\,bc^{5/3}\text{ArcTan}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)-\frac{1}{10}bc^{5/3}\log\left(1-c^{2/3}x^2\right)+\frac{1}{20}bc^{5/3}\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)-\frac{3bc}{10x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^6,x]
```

```
[Out] (-3*b*c)/(10*x^2) - (Sqrt[3]*b*c^(5/3)*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])
/10 - (a + b*ArcTanh[c*x^3])/(5*x^5) - (b*c^(5/3)*Log[1 - c^(2/3)*x^2])/10
+ (b*c^(5/3)*Log[1 + c^(2/3)*x^2 + c^(4/3)*x^4])/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}(cx^3)}{x^6}\,dx &= -\frac{a+b\tanh^{-1}(cx^3)}{5x^5}+\frac{1}{5}(3bc)\int \frac{1}{x^3(1-c^2x^6)}\,dx \\
&= -\frac{a+b\tanh^{-1}(cx^3)}{5x^5}+\frac{1}{10}(3bc)\text{Subst}\left(\int \frac{1}{x^2(1-c^2x^3)}\,dx,x,x^2\right) \\
&= -\frac{3bc}{10x^2}-\frac{a+b\tanh^{-1}(cx^3)}{5x^5}+\frac{1}{10}\left(3bc^3\right)\text{Subst}\left(\int \frac{x}{1-c^2x^3}\,dx,x,x^2\right) \\
&= -\frac{3bc}{10x^2}-\frac{a+b\tanh^{-1}(cx^3)}{5x^5}+\frac{1}{10}\left(bc^{7/3}\right)\text{Subst}\left(\int \frac{1}{1-c^{2/3}x}\,dx,x,x^2\right)-\frac{1}{10}(b \\
&= -\frac{3bc}{10x^2}-\frac{a+b\tanh^{-1}(cx^3)}{5x^5}-\frac{1}{10}bc^{5/3}\log\left(1-c^{2/3}x^2\right)+\frac{1}{20}\left(bc^{5/3}\right)\text{Subst}\left(\int \frac{}{1}\right. \\
&= -\frac{3bc}{10x^2}-\frac{a+b\tanh^{-1}(cx^3)}{5x^5}-\frac{1}{10}bc^{5/3}\log\left(1-c^{2/3}x^2\right)+\frac{1}{20}bc^{5/3}\log\left(1+c^{2/3}x\right. \\
&= -\frac{3bc}{10x^2}-\frac{1}{10}\sqrt{3}\,bc^{5/3}\tan^{-1}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)-\frac{a+b\tanh^{-1}(cx^3)}{5x^5}-\frac{1}{10}bc^{5/3}\log\left(\right.
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 196, normalized size = 1.70

$$-\frac{a}{5x^5}-\frac{3bc}{10x^2}-\frac{1}{10}\sqrt{3}\,bc^{5/3}\text{ArcTan}\left(\frac{-1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)+\frac{1}{10}\sqrt{3}\,bc^{5/3}\text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)-\frac{b\tanh^{-1}(cx^3)}{5x^5}-\frac{1}{10}bc^{5/3}\log\left(1-\sqrt[3]{c}\,x\right)-\frac{1}{10}bc^{5/3}\log\left(1+\sqrt[3]{c}\,x\right)+\frac{1}{20}bc^{5/3}\log\left(1-\sqrt[3]{c}\,x+c^{2/3}x^2\right)+\frac{1}{20}bc^{5/3}\log\left(1+\sqrt[3]{c}\,x+c^{2/3}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^6,x]
```

```
[Out] -1/5*a/x^5 - (3*b*c)/(10*x^2) - (Sqrt[3]*b*c^(5/3)*ArcTan[(-1 + 2*c^(1/3)*x
)/Sqrt[3]])/10 + (Sqrt[3]*b*c^(5/3)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/10 -
 (b*ArcTanh[c*x^3])/(5*x^5) - (b*c^(5/3)*Log[1 - c^(1/3)*x])/10 - (b*c^(5/3
)*Log[1 + c^(1/3)*x])/10 + (b*c^(5/3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/20
+ (b*c^(5/3)*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/20
```

Maple [A]
time = 0.05, size = 172, normalized size = 1.50

method	result
default	$-\frac{a}{5x^5}-\frac{b\,\text{arctanh}(c\,x^3)}{5x^5}-\frac{bc\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{10\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{bc\ln\left(x^2-\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{20\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{10\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{bc\ln(}{1}$

risch	$-\frac{b\ln(c\,x^3+1)}{10x^5}-\frac{a}{5x^5}+\frac{b\ln(-c\,x^3+1)}{10x^5}-\frac{bc\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{10\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{bc\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{20\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right.}{3}\right.}{10\left(\frac{1}{c}\right)^{\frac{2}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a/x^5-1/5*b/x^5*arctanh(c*x^3)-1/10*b*c/(1/c)^(2/3)*ln(x+(1/c)^(1/3))+
1/20*b*c/(1/c)^(2/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))-1/10*b*c/(1/c)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))-1/10*b*c/(1/c)^(2/3)*ln(x-
(1/c)^(1/3))+1/20*b*c/(1/c)^(2/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/10*b*
c/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))-3/10*b*c/x^2
```

Maxima [A]
time = 0.47, size = 100, normalized size = 0.87

$$-\frac{1}{20}\left(\left(2\sqrt{3}\,c^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{4}{3}}x^2+c^{\frac{2}{3}}\right)}{3\,c^{\frac{2}{3}}}\right)-c^{\frac{2}{3}}\log\left(c^{\frac{4}{3}}x^4+c^{\frac{2}{3}}x^2+1\right)+2\,c^{\frac{2}{3}}\log\left(\frac{c^{\frac{2}{3}}x^2-1}{c^{\frac{2}{3}}}\right)+\frac{6}{x^2}\right)c+\frac{4\,\mathrm{artanh}\left(cx^3\right)}{x^5}\right)b-\frac{a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*sqrt(3)*c^(2/3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 + c^(2/3))/c^(2
/3)) - c^(2/3)*log(c^(4/3)*x^4 + c^(2/3)*x^2 + 1) + 2*c^(2/3)*log((c^(2/3)*
x^2 - 1)/c^(2/3)) + 6/x^2)*c + 4*arctanh(c*x^3)/x^5)*b - 1/5*a/x^5
```

Fricas [A]
time = 0.37, size = 151, normalized size = 1.31

$$-\frac{2\sqrt{3}\left(-c^2\right)^{\frac{1}{3}}bcx^5\arctan\left(\frac{2}{3}\sqrt{3}\left(-c^2\right)^{\frac{1}{3}}x^2-\frac{1}{3}\sqrt{3}\right)+\left(-c^2\right)^{\frac{1}{3}}bcx^5\log\left(c^2x^4+\left(-c^2\right)^{\frac{2}{3}}x^2-\left(-c^2\right)^{\frac{1}{3}}\right)-2\left(-c^2\right)^{\frac{1}{3}}bcx^5\log\left(c^2x^2-\left(-c^2\right)^{\frac{2}{3}}\right)+6\,bcx^3+2\,b\log\left(-\frac{cx^3+1}{cx^3-1}\right)+4\,a}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(2*sqrt(3)*(-c^2)^(1/3)*b*c*x^5*arctan(2/3*sqrt(3)*(-c^2)^(1/3)*x^2 -
 1/3*sqrt(3)) + (-c^2)^(1/3)*b*c*x^5*log(c^2*x^4 + (-c^2)^(2/3)*x^2 - (-c^2
)^(1/3)) - 2*(-c^2)^(1/3)*b*c*x^5*log(c^2*x^2 - (-c^2)^(2/3)) + 6*b*c*x^3 +
 2*b*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a)/x^5
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**6,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 125, normalized size = 1.09

$$-\frac{1}{20}bc^3\left(\frac{2\sqrt{3}|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^2}-\frac{|c|^{\frac{2}{3}}\log\left(x^4+\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^2}+\frac{2\log\left(\left|x^2-\frac{1}{|c|^{\frac{2}{3}}}\right|\right)}{|c|^{\frac{4}{3}}}\right)-\frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{10x^5}-\frac{3bcx^3+2a}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^6,x, algorithm="giac")
```

```
[Out] -1/20*b*c^3*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 + 1/abs(c)^(2
/3))*abs(c)^(2/3))/c^2 - abs(c)^(2/3)*log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)
^(4/3))/c^2 + 2*log(abs(x^2 - 1/abs(c)^(2/3)))/abs(c)^(4/3)) - 1/10*b*log(-
(c*x^3 + 1)/(c*x^3 - 1))/x^5 - 1/10*(3*b*c*x^3 + 2*a)/x^5
```

Mupad [B]
time = 3.12, size = 135, normalized size = 1.17

$$\frac{b\ln\left(1-c\,x^3\right)}{10\,x^5}-\frac{b\,c^{5/3}\ln\left(c^{2/3}\,x^2-1\right)}{10}-\frac{b\ln\left(c\,x^3+1\right)}{10\,x^5}-\frac{\frac{3\,b\,c\,x^3}{2}+a}{5\,x^5}+\frac{b\,c^{5/3}\ln\left(\sqrt{3}\,c^{2/3}\,x^2-c^{2/3}\,x^2\,1\mathrm{i}-2\mathrm{i}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{20}-\frac{b\,c^{5/3}\ln\left(-c^{2/3}\,x^2\,1\mathrm{i}-\sqrt{3}\,c^{2/3}\,x^2-2\mathrm{i}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/x^6,x)
```

```
[Out] (b*log(1 - c*x^3))/(10*x^5) - (b*c^(5/3)*log(c^(2/3)*x^2 - 1))/10 - (b*log(
c*x^3 + 1))/(10*x^5) - (a + (3*b*c*x^3)/2)/(5*x^5) + (b*c^(5/3)*log(3^(1/2)
*c^(2/3)*x^2 - c^(2/3)*x^2*1i - 2i)*(3^(1/2)*1i + 1))/20 - (b*c^(5/3)*log(-
 c^(2/3)*x^2*1i - 3^(1/2)*c^(2/3)*x^2 - 2i)*(3^(1/2)*1i - 1))/20
```

3.111 $\int x^7\left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx$

Optimal. Leaf size=176

$$\frac{3bx^5}{40c}-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{16c^{8/3}}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{16c^{8/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\,x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\right.$$

```
[Out] 3/40*b*x^5/c-1/8*b*arctanh(c^(1/3)*x)/c^(8/3)+1/8*x^8*(a+b*arctanh(c*x^3))+
1/32*b*ln(1-c^(1/3)*x+c^(2/3)*x^2)/c^(8/3)-1/32*b*ln(1+c^(1/3)*x+c^(2/3)*x^
2)/c^(8/3)+1/16*b*arctan(-1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(8/3
)+1/16*b*arctan(1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(8/3)
```

Rubi [A]

time = 0.20, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.571$, Rules used = {6037, 327, 302, 648, 632, 210, 642, 212}

$$\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{16c^{8/3}}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{16c^{8/3}}+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}\,x+1\right)}{32c^{8/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x+1\right)}{32c^{8/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\,x\right)}{8c^{8/3}}+\frac{3bx^5}{40c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (3*b*x^5)/(40*c) - (Sqrt[3]*b*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3]])/(1
6*c^(8/3)) + (Sqrt[3]*b*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/(16*c^(8
/3)) - (b*ArcTanh[c^(1/3)*x])/(8*c^(8/3)) + (x^8*(a + b*ArcTanh[c*x^3]))/8
+ (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(32*c^(8/3)) - (b*Log[1 + c^(1/3)*x
+ c^(2/3)*x^2])/(32*c^(8/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
```

```
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int x^7\left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx=\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{8}(3bc)\int\frac{x^{10}}{1-c^2x^6}\,dx$$
$$=\frac{3bx^5}{40c}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{(3b)\int\frac{x^4}{1-c^2x^6}\,dx}{8c}$$
$$=\frac{3bx^5}{40c}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{b\int\frac{1}{1-c^{2/3}x^2}\,dx}{8c^{7/3}}-\frac{b\int\frac{-\frac{1}{2}-\frac{\sqrt[3]{c}\,x}{2}}{1-\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx}{8c^{7/3}}-$$
$$=\frac{3bx^5}{40c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\,x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\int\frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx}{32c^{8/3}}$$
$$=\frac{3bx^5}{40c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\,x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-\sqrt[3]{c}\,x+c^{2/}\right.}{32c^{8/3}}$$
$$=\frac{3bx^5}{40c}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{16c^{8/3}}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{16c^{8/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\right.}{8c^{8/3}}$$

Mathematica [A]
time = 0.02, size = 198, normalized size = 1.12

$$\frac{3bx^5}{40c}+\frac{ax^8}{8}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{-1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{16c^{8/3}}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{16c^{8/3}}+\frac{1}{8}bx^8\tanh^{-1}\left(cx^3\right)+\frac{b\log\left(1-\sqrt[3]{c}\,x\right)}{16c^{8/3}}-\frac{b\log\left(1+\sqrt[3]{c}\,x\right)}{16c^{8/3}}+\frac{b\log\left(1-\sqrt[3]{c}\,x+c^{2/3}x^2\right)}{32c^{8/3}}-\frac{b\log\left(1+\sqrt[3]{c}\,x+c^{2/3}x^2\right)}{32c^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (3*b*x^5)/(40*c) + (a*x^8)/8 + (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]
])/(16*c^(8/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(16*c^(8/3)
) + (b*x^8*ArcTanh[c*x^3])/8 + (b*Log[1 - c^(1/3)*x])/(16*c^(8/3)) - (b*Log
[1 + c^(1/3)*x])/(16*c^(8/3)) + (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(32*c^
(8/3)) - (b*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(32*c^(8/3))
```

Maple [A]
time = 0.06, size = 186, normalized size = 1.06

method	result
default	$\frac{x^8 a}{8}+\frac{x^8 b\,\text{arctanh}\left(c\,x^3\right)}{8}+\frac{3b\,x^5}{40c}-\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{16c^3\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\ln\left(x^2-\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{32c^3\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{16c^3\left(\frac{1}{c}\right)^{\frac{1}{3}}}+$

risch	$\frac{x^8 b \ln(c x^3+1)}{16}+\frac{x^8 a}{8}-\frac{b x^8 \ln(-c x^3+1)}{16}+\frac{3 b x^5}{40 c}+\frac{b \ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{16 c^3 \left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b \ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}} x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{32 c^3 \left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\, \arctan\left(\frac{\sqrt{3}}{\ldots}\right)}{16 c^3 \left(\frac{1}{c}\right)\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(a+b*arctanh(c*x^3)),x,method=_RETURNVERBOSE)

[Out] 1/8*x^8*a+1/8*x^8*b*arctanh(c*x^3)+3/40*b*x^5/c-1/16*b/c^3/(1/c)^(1/3)*ln(x+(1/c)^(1/3))+1/32*b/c^3/(1/c)^(1/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))+1/16*b/c^3*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))+1/16*b/c^3/(1/c)^(1/3)*ln(x-(1/c)^(1/3))-1/32*b/c^3/(1/c)^(1/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/16*b/c^3*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))

Maxima [A]
time = 0.47, size = 164, normalized size = 0.93

$$\frac{1}{8}\,ax^8+\frac{1}{160}\left(20\,x^8\,\mathrm{artanh}\left(cx^3\right)+\left(\frac{12\,x^5}{c^2}+\frac{10\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{11}{3}}}+\frac{10\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{11}{3}}}-\frac{5\,\log\left(c^{\frac{2}{3}}x^2+c^{\frac{1}{3}}x+1\right)}{c^{\frac{11}{3}}}+\frac{5\,\log\left(c^{\frac{2}{3}}x^2-c^{\frac{1}{3}}x+1\right)}{c^{\frac{11}{3}}}-\frac{10\,\log\left(\frac{c^{\frac{1}{3}}x+1}{c^{\frac{1}{3}}}\right)}{c^{\frac{11}{3}}}+\frac{10\,\log\left(\frac{c^{\frac{1}{3}}x-1}{c^{\frac{1}{3}}}\right)}{c^{\frac{11}{3}}}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(a+b*arctanh(c*x^3)),x, algorithm="maxima")

[Out] 1/8*a*x^8 + 1/160*(20*x^8*arctanh(c*x^3) + (12*x^5/c^2 + 10*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x + c^(1/3))/c^(1/3))/c^(11/3) + 10*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x - c^(1/3))/c^(1/3))/c^(11/3) - 5*log(c^(2/3)*x^2 + c^(1/3)*x + 1)/c^(11/3) + 5*log(c^(2/3)*x^2 - c^(1/3)*x + 1)/c^(11/3) - 10*log((c^(1/3)*x + 1)/c^(1/3))/c^(11/3) + 10*log((c^(1/3)*x - 1)/c^(1/3))/c^(11/3))*c)*b

Fricas [A]
time = 0.36, size = 248, normalized size = 1.41

$$\frac{10\,bc^4x^8\log\left(-\frac{cx^3+1}{cx^3-1}\right)+20\,ac^4x^8+12\,bc^3x^5+10\,\sqrt{3}\,bc\sqrt{-\left(-c^2\right)^{\frac{1}{3}}}\,\arctan\left(\frac{\sqrt{3}\left(2\,cx+\left(-c^2\right)^{\frac{1}{3}}\right)\sqrt{-\left(-c^2\right)^{\frac{1}{3}}}}{3\,c}\right)+10\,\sqrt{3}\,b\left(c^2\right)^{\frac{1}{6}}c\arctan\left(\frac{\sqrt{3}\left(c^2\right)^{\frac{1}{6}}\left(2\,cx+\left(c^2\right)^{\frac{1}{3}}\right)}{3\,c}\right)+5\left(-c^2\right)^{\frac{2}{3}}b\log\left(c^2x^2+\left(-c^2\right)^{\frac{1}{3}}cx+\left(-c^2\right)^{\frac{2}{3}}\right)-5\,b\left(c^2\right)^{\frac{2}{3}}\log\left(c^2x^2+\left(c^2\right)^{\frac{1}{3}}cx+\left(c^2\right)^{\frac{2}{3}}\right)-10\left(-c^2\right)^{\frac{2}{3}}b\log\left(cx-\left(-c^2\right)^{\frac{1}{3}}\right)+10\,b\left(c^2\right)^{\frac{2}{3}}\log\left(cx-\left(c^2\right)^{\frac{1}{3}}\right)}{160\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(a+b*arctanh(c*x^3)),x, algorithm="fricas")

[Out] 1/160*(10*b*c^4*x^8*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 20*a*c^4*x^8 + 12*b*c^3*x^5 + 10*sqrt(3)*b*c*sqrt(-(-c^2)^(1/3))*arctan(1/3*sqrt(3)*(2*c*x + (-c^2)^(1/3))*sqrt(-(-c^2)^(1/3))/c) + 10*sqrt(3)*b*(c^2)^(1/6)*c*arctan(1/3*sqrt(3)*(c^2)^(1/6)*(2*c*x + (c^2)^(1/3))/c) + 5*(-c^2)^(2/3)*b*log(c^2*x^2 +

```
(-c^2)^(1/3)*c*x + (-c^2)^(2/3)) - 5*b*(c^2)^(2/3)*log(c^2*x^2 + (c^2)^(1/3
)*c*x + (c^2)^(2/3)) - 10*(-c^2)^(2/3)*b*log(c*x - (-c^2)^(1/3)) + 10*b*(c^
2)^(2/3)*log(c*x - (c^2)^(1/3)))/c^4
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 208, normalized size = 1.18

$$\frac{1}{16}\,bx^{8}\log\left(-\frac{cx^{3}+1}{cx^{3}-1}\right)+\frac{1}{8}\,ax^{8}+\frac{3\,bx^{5}}{40\,c}-\frac{b\left(-\frac{1}{c}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{1}{c}\right)^{\frac{1}{3}}\right|\right)}{16\,c^{2}}+\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\,\left(2\,x+\left(-\frac{1}{c}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{1}{c}\right)^{\frac{1}{3}}}\right)}{16\,\left(-c^{2}\right)^{\frac{1}{3}}\,c^{2}}+\frac{\sqrt{3}\,b|c|^{\frac{4}{3}}\arctan\left(\frac{1}{3}\,\sqrt{3}\,c^{\frac{1}{3}}\left(2\,x+\frac{1}{c^{\frac{1}{3}}}\right)\right)}{16\,c^{4}}-\frac{b\log\left(x^{2}+x\left(-\frac{1}{c}\right)^{\frac{1}{3}}+\left(-\frac{1}{c}\right)^{\frac{2}{3}}\right)}{32\,\left(-c^{2}\right)^{\frac{1}{3}}\,c^{2}}-\frac{b\log\left(x^{2}+\frac{x}{c^{\frac{1}{3}}}+\frac{1}{c^{\frac{2}{3}}}\right)}{32\,c^{2}|c|^{\frac{2}{3}}}+\frac{b\log\left(\left|x-\frac{1}{c^{\frac{1}{3}}}\right|\right)}{16\,c^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/16*b*x^8*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/8*a*x^8 + 3/40*b*x^5/c - 1/16*
b*(-1/c)^(2/3)*log(abs(x - (-1/c)^(1/3)))/c^2 + 1/16*sqrt(3)*b*arctan(1/3*s
qrt(3)*(2*x + (-1/c)^(1/3))/(-1/c)^(1/3))/((-c^2)^(1/3)*c^2) + 1/16*sqrt(3)
*b*abs(c)^(4/3)*arctan(1/3*sqrt(3)*c^(1/3)*(2*x + 1/c^(1/3)))/c^4 - 1/32*b*
log(x^2 + x*(-1/c)^(1/3) + (-1/c)^(2/3))/((-c^2)^(1/3)*c^2) - 1/32*b*log(x^
2 + x/c^(1/3) + 1/c^(2/3))/(c^2*abs(c)^(2/3)) + 1/16*b*log(abs(x - 1/c^(1/3
)))/c^(8/3)
```

Mupad [B]
time = 1.26, size = 127, normalized size = 0.72

$$\frac{a\,x^{8}}{8}+\frac{b\left(-\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)}{2}+\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)}{2}+\mathrm{atan}\left(c^{1/3}\,x\,1\mathrm{i}\right)\right)\,1\mathrm{i}}{8\,c^{8/3}}+\frac{3\,b\,x^{5}}{40\,c}+\frac{b\,x^{8}\,\ln\left(c\,x^{3}+1\right)}{16}-\frac{b\,x^{8}\,\ln\left(1-c\,x^{3}\right)}{16}+\frac{\sqrt{3}\,b\,\left(\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)+\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)\right)}{16\,c^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*atanh(c*x^3)),x)
```

```
[Out] (a*x^8)/8 + (b*(atan((c^(1/3)*x*(3^(1/2) + 1i))/2)/2 - atan((c^(1/3)*x*(3^(
1/2) - 1i))/2)/2 + atan(c^(1/3)*x*1i))*1i)/(8*c^(8/3)) + (3*b*x^5)/(40*c) +
 (b*x^8*log(c*x^3 + 1))/16 - (b*x^8*log(1 - c*x^3))/16 + (3^(1/2)*b*(atan((
c^(1/3)*x*(3^(1/2) - 1i))/2) + atan((c^(1/3)*x*(3^(1/2) + 1i))/2)))/(16*c^(
8/3))
```

3.112 $\int x^4 \left(a+b \tanh^{-1}\left(cx^3\right)\right) \, dx$

Optimal. Leaf size=117

$$\frac{3bx^2}{10c}-\frac{\sqrt{3}\, b\text{ArcTan}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(1+c^{2/3}x^2+c^{4/3}x^4\right)}{20c^{5/3}}$$

```
[Out] 3/10*b*x^2/c+1/5*x^5*(a+b*arctanh(c*x^3))+1/10*b*ln(1-c^(2/3)*x^2)/c^(5/3)-
1/20*b*ln(1+c^(2/3)*x^2+c^(4/3)*x^4)/c^(5/3)-1/10*b*arctan(1/3*(1+2*c^(2/3)
*x^2)*3^(1/2))*3^(1/2)/c^(5/3)
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6037, 281, 327, 206, 31, 648, 631, 210, 642}

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}\, b\text{ArcTan}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{b\log\left(1-c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)}{20c^{5/3}}+\frac{3bx^2}{10c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (3*b*x^2)/(10*c) - (Sqrt[3]*b*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])/(10*c^(5
/3)) + (x^5*(a + b*ArcTanh[c*x^3]))/5 + (b*Log[1 - c^(2/3)*x^2])/(10*c^(5/3
)) - (b*Log[1 + c^(2/3)*x^2 + c^(4/3)*x^4])/(20*c^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int x^4\left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx=\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{5}(3bc)\int\frac{x^7}{1-c^2x^6}\,dx$$

$$=\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{10}(3bc)\text{Subst}\left(\int\frac{x^3}{1-c^2x^3}\,dx,x,x^2\right)$$

$$=\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{(3b)\text{Subst}\left(\int\frac{1}{1-c^2x^3}\,dx,x,x^2\right)}{10c}$$

$$=\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{b\text{Subst}\left(\int\frac{1}{1-c^{2/3}x}\,dx,x,x^2\right)}{10c}-\frac{b\text{Subst}\left(\right.}{}$$

$$=\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\text{Subst}\left(\int\frac{c^{2/3}+2c^4}{1+c^{2/3}x+c}\right.}{20c^{5/}}$$

$$=\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(1+c^{2/3}x^2+\right.}{20c^{5/3}}$$

$$=\frac{3bx^2}{10c}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^{2/}\right.}{10c^{5/3}}$$

Mathematica [A]
time = 0.02, size = 198, normalized size = 1.69

$$\frac{3bx^2}{10c}+\frac{ax^5}{5}-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{-1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{1}{5}bx^5\tanh^{-1}\left(cx^3\right)+\frac{b\log\left(1-\sqrt[3]{c}\,x\right)}{10c^{5/3}}+\frac{b\log\left(1+\sqrt[3]{c}\,x\right)}{10c^{5/3}}-\frac{b\log\left(1-\sqrt[3]{c}\,x+c^{2/3}x^2\right)}{20c^{5/3}}-\frac{b\log\left(1+\sqrt[3]{c}\,x+c^{2/3}x^2\right)}{20c^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (3*b*x^2)/(10*c) + (a*x^5)/5 - (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]
])/(10*c^(5/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(10*c^(5/3)
) + (b*x^5*ArcTanh[c*x^3])/5 + (b*Log[1 - c^(1/3)*x])/(10*c^(5/3)) + (b*Log
[1 + c^(1/3)*x])/(10*c^(5/3)) - (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(20*c^
(5/3)) - (b*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(20*c^(5/3))
```

Maple [A]
time = 0.03, size = 114, normalized size = 0.97

method	result

default	$\frac{a\,x^{5}}{5}+\frac{x^{5} b \,\mathrm{arctanh}\left(c\,x^{3}\right)}{5}+\frac{3 b\,x^{2}}{10 c}+\frac{b \ln\left(x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{10 c^{3} \left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}-\frac{b \ln\left(x^{4}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}} x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{20 c^{3} \left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2 x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{10 c^{3} \left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}$
risch	$\frac{x^{5} b \ln\left(c\,x^{3}+1\right)}{10}+\frac{a\,x^{5}}{5}-\frac{b\,x^{5} \ln\left(-c\,x^{3}+1\right)}{10}+\frac{3 b\,x^{2}}{10 c}+\frac{b \ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{10 c^{2} \left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b \ln\left(x^{2}+\left(\frac{1}{c}\right)^{\frac{1}{3}} x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{20 c^{2} \left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\ldots\right.}{\ldots}\right)}{10 c^{2} \left(\frac{1}{c}\right)^{\frac{1}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x^3)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*x^5+1/5*x^5*b*arctanh(c*x^3)+3/10*b*x^2/c+1/10*b/c^3/(1/c^2)^(2/3)*ln
(x^2-(1/c^2)^(1/3))-1/20*b/c^3/(1/c^2)^(2/3)*ln(x^4+(1/c^2)^(1/3)*x^2+(1/c^
2)^(2/3))-1/10*b/c^3/(1/c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1
/3)*x^2+1))
```

Maxima [A]
time = 0.46, size = 103, normalized size = 0.88

$$\frac{1}{5}\,ax^{5}+\frac{1}{20}\left(4\,x^{5}\,\mathrm{artanh}\left(cx^{3}\right)+c\left(\frac{6\,x^{2}}{c^{2}}-\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,c^{\frac{4}{3}}x^{2}+c^{\frac{2}{3}}\right)}{3\,c^{\frac{2}{3}}}\right)}{c^{\frac{8}{3}}}-\frac{\log\left(c^{\frac{4}{3}}x^{4}+c^{\frac{2}{3}}x^{2}+1\right)}{c^{\frac{8}{3}}}+\frac{2\,\log\left(\frac{c^{\frac{2}{3}}x^{2}-1}{c^{\frac{2}{3}}}\right)}{c^{\frac{8}{3}}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/20*(4*x^5*arctanh(c*x^3) + c*(6*x^2/c^2 - 2*sqrt(3)*arctan(1/
3*sqrt(3)*(2*c^(4/3)*x^2 + c^(2/3))/c^(2/3))/c^(8/3) - log(c^(4/3)*x^4 + c^
(2/3)*x^2 + 1)/c^(8/3) + 2*log((c^(2/3)*x^2 - 1)/c^(2/3))/c^(8/3)))*b
```

Fricas [A]
time = 0.37, size = 166, normalized size = 1.42

$$\frac{2\,bc^{3}x^{5}\log\left(-\frac{cx^{3}+1}{cx^{3}-1}\right)+4\,ac^{3}x^{5}+6\,bc^{2}x^{2}-2\,\sqrt{3}\,b(c^{2})^{\frac{1}{6}}c\arctan\left(-\frac{\sqrt{3}\,\left(4\,c^{2}x^{4}-2\,(c^{2})^{\frac{2}{3}}x^{2}+(c^{2})^{\frac{1}{3}}\right)(c^{2})^{\frac{1}{6}}}{8\,c^{3}x^{6}+c}\right)-b(c^{2})^{\frac{2}{3}}\log\left(c^{2}x^{4}+(c^{2})^{\frac{2}{3}}x^{2}+(c^{2})^{\frac{1}{3}}\right)+2\,b(c^{2})^{\frac{2}{3}}\log\left(c^{2}x^{2}-(c^{2})^{\frac{2}{3}}\right)}{20\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/20*(2*b*c^3*x^5*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^3*x^5 + 6*b*c^2*x^2
 - 2*sqrt(3)*b*(c^2)^(1/6)*c*arctan(-sqrt(3)*(4*c^2*x^4 - 2*(c^2)^(2/3)*x^2
 + (c^2)^(1/3))*(c^2)^(1/6)/(8*c^3*x^6 + c)) - b*(c^2)^(2/3)*log(c^2*x^4 +
(c^2)^(2/3)*x^2 + (c^2)^(1/3)) + 2*b*(c^2)^(2/3)*log(c^2*x^2 - (c^2)^(2/3))
)/c^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.42, size = 126, normalized size = 1.08

$$-\frac{1}{20}\,bc^9\left(\frac{2\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^{10}|c|^{\frac{2}{3}}}+\frac{\log\left(x^4+\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^{10}|c|^{\frac{2}{3}}}-\frac{2\,\log\left(\left|x^2-\frac{1}{|c|^{\frac{2}{3}}}\right|\right)}{c^{10}|c|^{\frac{2}{3}}}\right)+\frac{1}{10}\,bx^5\log\left(-\frac{cx^3+1}{cx^3-1}\right)+\frac{1}{5}\,ax^5+\frac{3\,bx^2}{10\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] -1/20*b*c^9*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1/abs(c)^(2/3))*abs(c)^(
2/3))/(c^10*abs(c)^(2/3)) + log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/(c
^10*abs(c)^(2/3)) - 2*log(abs(x^2 - 1/abs(c)^(2/3)))/(c^10*abs(c)^(2/3))) +
 1/10*b*x^5*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/5*a*x^5 + 3/10*b*x^2/c
```

Mupad [**B**]
time = 2.44, size = 124, normalized size = 1.06

$$\frac{a\,x^5}{5}+\frac{b\,\ln\left(1-c^{2/3}\,x^2\right)}{10\,c^{5/3}}+\frac{3\,b\,x^2}{10\,c}-\frac{\ln\left(2\,c^{2/3}\,x^2+1-\sqrt{3}\,1{}\mathrm{i}\right)\,\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}{20\,c^{5/3}}-\frac{\ln\left(2\,c^{2/3}\,x^2+1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}{20\,c^{5/3}}+\frac{b\,x^5\,\ln\left(c\,x^3+1\right)}{10}-\frac{b\,x^5\,\ln\left(1-c\,x^3\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atanh(c*x^3)),x)
```

```
[Out] (a*x^5)/5 + (b*log(1 - c^(2/3)*x^2))/(10*c^(5/3)) + (3*b*x^2)/(10*c) - (log
(2*c^(2/3)*x^2 - 3^(1/2)*1i + 1)*(b - 3^(1/2)*b*1i))/(20*c^(5/3)) - (log(3^
(1/2)*1i + 2*c^(2/3)*x^2 + 1)*(b + 3^(1/2)*b*1i))/(20*c^(5/3)) + (b*x^5*log
(c*x^3 + 1))/10 - (b*x^5*log(1 - c*x^3))/10
```

3.113 $\int x\left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx$

Optimal. Leaf size=165

$$-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{4c^{2/3}}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{4c^{2/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\,x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)$$

```
[Out] -1/2*b*arctanh(c^(1/3)*x)/c^(2/3)+1/2*x^2*(a+b*arctanh(c*x^3))+1/8*b*ln(1-c
^(1/3)*x+c^(2/3)*x^2)/c^(2/3)-1/8*b*ln(1+c^(1/3)*x+c^(2/3)*x^2)/c^(2/3)+1/4
*b*arctan(-1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/3)+1/4*b*arctan(
1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/3)
```

Rubi [A]
time = 0.17, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6037, 302, 648, 632, 210, 642, 212}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{4c^{2/3}}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{4c^{2/3}}+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}\,x+1\right)}{8c^{2/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x+1\right)}{8c^{2/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\,x\right)}{2c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] -1/4*(Sqrt[3]*b*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3]])/c^(2/3) + (Sqrt[
3]*b*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) - (b*ArcTanh[c^
(1/3)*x])/(2*c^(2/3)) + (x^2*(a + b*ArcTanh[c*x^3]))/2 + (b*Log[1 - c^(1/3)
*x + c^(2/3)*x^2])/(8*c^(2/3)) - (b*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(8*c^
(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
```

```
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{2}(3bc)\int\frac{x^4}{1-c^2x^6}\,dx$$

$$=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{b\int\frac{1}{1-c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}-\frac{b\int\frac{-\frac{1}{2}-\frac{\sqrt[3]{c}\,x}{2}}{1-\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}-\frac{b\int\frac{-\frac{1}{2}\cdots}{1+\sqrt[3]{c}\cdots}}{2\cdots}$$

$$=-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\,x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\int\frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx}{8c^{2/3}}-\frac{b\int\cdots}{\cdots}$$

$$=-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\,x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-\sqrt[3]{c}\,x+c^{2/3}x^2\right)}{8c^{2/3}}-\cdots$$

$$=-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{4c^{2/3}}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{4c^{2/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\,x\right)}{2c^{2/3}}+\frac{1}{2}\cdots$$

Mathematica [A]
time = 0.02, size = 187, normalized size = 1.13

$$\frac{ax^2}{2}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{-1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{4c^{2/3}}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{4c^{2/3}}+\frac{1}{2}bx^2\tanh^{-1}\left(cx^3\right)+\frac{b\log\left(1-\sqrt[3]{c}\,x\right)}{4c^{2/3}}-\frac{b\log\left(1+\sqrt[3]{c}\,x\right)}{4c^{2/3}}+\frac{b\log\left(1-\sqrt[3]{c}\,x+c^{2/3}x^2\right)}{8c^{2/3}}-\frac{b\log\left(1+\sqrt[3]{c}\,x+c^{2/3}x^2\right)}{8c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (a*x^2)/2 + (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) + (S
qrt[3]*b*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) + (b*x^2*ArcTanh[c*
x^3])/2 + (b*Log[1 - c^(1/3)*x])/(4*c^(2/3)) - (b*Log[1 + c^(1/3)*x])/(4*c^
(2/3)) + (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3)) - (b*Log[1 + c^(1
/3)*x + c^(2/3)*x^2])/(8*c^(2/3))
```

Maple [A]
time = 0.03, size = 177, normalized size = 1.07

method	result
default	$\frac{a\,x^2}{2}+\frac{b\,x^2\operatorname{arctanh}(c\,x^3)}{2}-\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\ln\left(x^2-\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8c\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\ln\left(x-\cdots\right.}{4c\left(\frac{1}{c}\cdots\right.}$

risch	$\frac{b\,x^{2}\ln(c\,x^{3}+1)}{4}+\frac{a\,x^{2}}{2}-\frac{b\,x^{2}\ln(-c\,x^{3}+1)}{4}+\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x^{2}+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8c\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}\right.}{3}\right.}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^3)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctanh(c*x^3)-1/4*b/c/(1/c)^(1/3)*ln(x+(1/c)^(1/3))+1/
8*b/c/(1/c)^(1/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b*3^(1/2)/c/(1/c)^(
1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))+1/4*b/c/(1/c)^(1/3)*ln(x-(1/c)
^(1/3))-1/8*b/c/(1/c)^(1/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b*3^(1/2)
/c/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))
```

Maxima [A]
time = 0.45, size = 155, normalized size = 0.94

$$\frac{1}{2}ax^{2}+\frac{1}{8}\left(4x^{2}\operatorname{artanh}\left(cx^{3}\right)+c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}-\frac{\log\left(c^{\frac{2}{3}}x^{2}+c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}+\frac{\log\left(c^{\frac{2}{3}}x^{2}-c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{2\log\left(\frac{c^{\frac{1}{3}}x+1}{c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}+\frac{2\log\left(\frac{c^{\frac{1}{3}}x-1}{c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/8*(4*x^2*arctanh(c*x^3) + c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*
c^(2/3)*x + c^(1/3))/c^(1/3))/c^(5/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(
2/3)*x - c^(1/3))/c^(1/3))/c^(5/3) - log(c^(2/3)*x^2 + c^(1/3)*x + 1)/c^(5/
3) + log(c^(2/3)*x^2 - c^(1/3)*x + 1)/c^(5/3) - 2*log((c^(1/3)*x + 1)/c^(1/
3))/c^(5/3) + 2*log((c^(1/3)*x - 1)/c^(1/3))/c^(5/3)))*b
```

Fricas [A]
time = 0.34, size = 238, normalized size = 1.44

$$\frac{2bc^{2}x^{2}\log\left(-\frac{cx^{3}+1}{cx^{3}-1}\right)+4ac^{2}x^{2}+2\sqrt{3}bc\sqrt{-\left(-c^{2}\right)^{\frac{1}{3}}}\arctan\left(\frac{\sqrt{3}\left(2cx+\left(-c^{2}\right)^{\frac{1}{3}}\right)\sqrt{-\left(-c^{2}\right)^{\frac{1}{3}}}}{3c}\right)+2\sqrt{3}b\left(c^{2}\right)^{\frac{1}{6}}c\arctan\left(\frac{\sqrt{3}\left(c^{2}\right)^{\frac{1}{6}}\left(2cx+\left(c^{2}\right)^{\frac{1}{3}}\right)}{3c}\right)+\left(-c^{2}\right)^{\frac{2}{3}}b\log\left(c^{2}x^{2}+\left(-c^{2}\right)^{\frac{1}{3}}cx+\left(-c^{2}\right)^{\frac{2}{3}}\right)-b\left(c^{2}\right)^{\frac{2}{3}}\log\left(c^{2}x^{2}+\left(c^{2}\right)^{\frac{1}{3}}cx+\left(c^{2}\right)^{\frac{2}{3}}\right)-2\left(-c^{2}\right)^{\frac{2}{3}}b\log\left(cx-\left(-c^{2}\right)^{\frac{1}{3}}\right)+2b\left(c^{2}\right)^{\frac{2}{3}}\log\left(cx-\left(c^{2}\right)^{\frac{1}{3}}\right)}{8c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b*c^2*x^2*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^2 + 2*sqrt(3)*b*
c*sqrt(-(-c^2)^(1/3))*arctan(1/3*sqrt(3)*(2*c*x + (-c^2)^(1/3))*sqrt(-(-c^2
)^(1/3))/c) + 2*sqrt(3)*b*(c^2)^(1/6)*c*arctan(1/3*sqrt(3)*(c^2)^(1/6)*(2*c
*x + (c^2)^(1/3))/c) + (-c^2)^(2/3)*b*log(c^2*x^2 + (-c^2)^(1/3)*c*x + (-c^
2)^(2/3)) - b*(c^2)^(2/3)*log(c^2*x^2 + (c^2)^(1/3)*c*x + (c^2)^(2/3)) - 2*
```

```
(-c^2)^(2/3)*b*log(c*x - (-c^2)^(1/3)) + 2*b*(c^2)^(2/3)*log(c*x - (c^2)^(1
/3)))/c^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 179, normalized size = 1.08

$$\frac{1}{4}\,bx^2\log\left(-\frac{cx^3+1}{cx^3-1}\right)+\frac{1}{2}\,ax^2+\frac{\sqrt{3}\,b|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{4\,c}+\frac{\sqrt{3}\,b|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{4\,c}-\frac{bc\log\left(x^2+\frac{x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{8\,|c|^{\frac{5}{3}}}+\frac{bc\log\left(x^2-\frac{x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{8\,|c|^{\frac{5}{3}}}-\frac{bc\log\left(\left|x+\frac{1}{|c|^{\frac{1}{3}}}\right|\right)}{4\,|c|^{\frac{5}{3}}}+\frac{b|c|^{\frac{1}{3}}\log\left(\left|x-\frac{1}{|c|^{\frac{1}{3}}}\right|\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^2*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/2*a*x^2 + 1/4*sqrt(3)*b*abs(c)^
(1/3)*arctan(1/3*sqrt(3)*(2*x + 1/abs(c)^(1/3))*abs(c)^(1/3))/c + 1/4*sqrt(
3)*b*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*x - 1/abs(c)^(1/3))*abs(c)^(1/3))/c
 - 1/8*b*c*log(x^2 + x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(5/3) + 1/8*b*
c*log(x^2 - x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(5/3) - 1/4*b*c*log(abs
(x + 1/abs(c)^(1/3)))/abs(c)^(5/3) + 1/4*b*abs(c)^(1/3)*log(abs(x - 1/abs(c
)^(1/3)))/c
```

Mupad [B]
time = 1.25, size = 118, normalized size = 0.72

$$\frac{a\,x^2}{2}+\frac{b\left(-\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)}{2}+\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)}{2}+\mathrm{atan}\left(c^{1/3}\,x\,1\mathrm{i}\right)\right)\,1\mathrm{i}}{2\,c^{2/3}}+\frac{b\,x^2\,\ln\left(c\,x^3+1\right)}{4}-\frac{b\,x^2\,\ln\left(1-c\,x^3\right)}{4}+\frac{\sqrt{3}\,b\,\left(\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)+\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)\right)}{4\,c^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^3)),x)
```

```
[Out] (a*x^2)/2 + (b*(atan((c^(1/3)*x*(3^(1/2) + 1i))/2)/2 - atan((c^(1/3)*x*(3^(
1/2) - 1i))/2)/2 + atan(c^(1/3)*x*1i))*1i)/(2*c^(2/3)) + (b*x^2*log(c*x^3 +
 1))/4 - (b*x^2*log(1 - c*x^3))/4 + (3^(1/2)*b*(atan((c^(1/3)*x*(3^(1/2) -
1i))/2) + atan((c^(1/3)*x*(3^(1/2) + 1i))/2)))/(4*c^(2/3))
```

3.114 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^2} dx$

Optimal. Leaf size=104

$$\frac{1}{2}\sqrt{3}\,b\sqrt[3]{c}\,\text{ArcTan}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)-\frac{a+b\tanh^{-1}(cx^3)}{x}-\frac{1}{2}b\sqrt[3]{c}\,\log\left(1-c^{2/3}x^2\right)+\frac{1}{4}b\sqrt[3]{c}\,\log\left(1+c^{2/3}x^2+c^{4/3}x^4\right)$$

```
[Out] (-a-b*arctanh(c*x^3))/x-1/2*b*c^(1/3)*ln(1-c^(2/3)*x^2)+1/4*b*c^(1/3)*ln(1+
c^(2/3)*x^2+c^(4/3)*x^4)+1/2*b*c^(1/3)*arctan(1/3*(1+2*c^(2/3)*x^2)*3^(1/2)
)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6037, 281, 206, 31, 648, 631, 210, 642}

$$-\frac{a+b\tanh^{-1}(cx^3)}{x}+\frac{1}{2}\sqrt{3}\,b\sqrt[3]{c}\,\text{ArcTan}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)-\frac{1}{2}b\sqrt[3]{c}\,\log\left(1-c^{2/3}x^2\right)+\frac{1}{4}b\sqrt[3]{c}\,\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^2,x]
```

```
[Out] (Sqrt[3]*b*c^(1/3)*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])/2 - (a + b*ArcTanh[
c*x^3])/x - (b*c^(1/3)*Log[1 - c^(2/3)*x^2])/2 + (b*c^(1/3)*Log[1 + c^(2/3)
*x^2 + c^(4/3)*x^4])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx^3)}{x^2}\,dx = -\frac{a+b\tanh^{-1}(cx^3)}{x} + (3bc)\int \frac{x}{1-c^2x^6}\,dx$$
$$= -\frac{a+b\tanh^{-1}(cx^3)}{x} + \frac{1}{2}(3bc)\text{Subst}\left(\int \frac{1}{1-c^2x^3}\,dx, x, x^2\right)$$
$$= -\frac{a+b\tanh^{-1}(cx^3)}{x} + \frac{1}{2}(bc)\text{Subst}\left(\int \frac{1}{1-c^{2/3}x}\,dx, x, x^2\right) + \frac{1}{2}(bc)\text{Subst}\Bigg(\int \frac{}{1}$$
$$= -\frac{a+b\tanh^{-1}(cx^3)}{x} - \frac{1}{2}b\sqrt[3]{c}\,\log\left(1-c^{2/3}x^2\right) + \frac{1}{4}\left(b\sqrt[3]{c}\right)\text{Subst}\Bigg(\int \frac{c^{2/3}+2c}{1+c^{2/3}x+}$$
$$= -\frac{a+b\tanh^{-1}(cx^3)}{x} - \frac{1}{2}b\sqrt[3]{c}\,\log\left(1-c^{2/3}x^2\right) + \frac{1}{4}b\sqrt[3]{c}\,\log\left(1+c^{2/3}x^2+c^{4/3}x^4\right)$$
$$= \frac{1}{2}\sqrt{3}\,b\sqrt[3]{c}\,\tan^{-1}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right) - \frac{a+b\tanh^{-1}(cx^3)}{x} - \frac{1}{2}b\sqrt[3]{c}\,\log\left(1-c^{2/3}x^2\right)$$

Mathematica [A]
time = 0.02, size = 183, normalized size = 1.76

$$-\frac{a}{x}+\frac{1}{2}\sqrt{3}\,b\sqrt[3]{c}\,\text{ArcTan}\left(\frac{-1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)-\frac{1}{2}\sqrt{3}\,b\sqrt[3]{c}\,\text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)-\frac{b\tanh^{-1}(cx^3)}{x}-\frac{1}{2}b\sqrt[3]{c}\,\log\left(1-\sqrt[3]{c}\,x\right)-\frac{1}{2}b\sqrt[3]{c}\,\log\left(1+\sqrt[3]{c}\,x\right)+\frac{1}{4}b\sqrt[3]{c}\,\log\left(1-\sqrt[3]{c}\,x+c^{2/3}x^2\right)+\frac{1}{4}b\sqrt[3]{c}\,\log\left(1+\sqrt[3]{c}\,x+c^{2/3}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^2,x]
```

```
[Out] -(a/x) + (Sqrt[3]*b*c^(1/3)*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])/2 - (Sqrt[3
]*b*c^(1/3)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/2 - (b*ArcTanh[c*x^3])/x - (
b*c^(1/3)*Log[1 - c^(1/3)*x])/2 - (b*c^(1/3)*Log[1 + c^(1/3)*x])/2 + (b*c^(
1/3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/4 + (b*c^(1/3)*Log[1 + c^(1/3)*x + c
^(2/3)*x^2])/4
```

Maple [A]
time = 0.04, size = 105, normalized size = 1.01

method	result
default	$-\frac{a}{x}-\frac{b\,\mathrm{arctanh}(c\,x^3)}{x}-\frac{b\ln\left(x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}+\frac{b\ln\left(x^4+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{2c\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}$

risch	$-\frac{b\ln(c\,x^3+1)}{2x}-\frac{a}{x}+\frac{b\ln(-c\,x^3+1)}{2x}-\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{2\left(\frac{1}{c}\right)^{\frac{1}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/x-b/x*arctanh(c*x^3)-1/2*b/c/(1/c^2)^(2/3)*ln(x^2-(1/c^2)^(1/3))+1/4*b/c
/(1/c^2)^(2/3)*ln(x^4+(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))+1/2*b/c/(1/c^2)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/3)*x^2+1))
```

Maxima [A]
time = 0.47, size = 94, normalized size = 0.90

$$\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2+c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{2}{3}}}+\frac{\log\left(c^{\frac{4}{3}}x^4+c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{2}{3}}}-\frac{2\log\left(\frac{c^{\frac{2}{3}}x^2-1}{c^{\frac{2}{3}}}\right)}{c^{\frac{2}{3}}}\right)-\frac{4\,\mathrm{artanh}\left(cx^3\right)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 + c^(2/3))/c^(2/3))/c^(
2/3) + log(c^(4/3)*x^4 + c^(2/3)*x^2 + 1)/c^(2/3) - 2*log((c^(2/3)*x^2 - 1)
/c^(2/3))/c^(2/3)) - 4*arctanh(c*x^3)/x)*b - a/x
```

Fricas [A]
time = 0.36, size = 117, normalized size = 1.12

$$-\frac{2\sqrt{3}\,b(-c)^{\frac{1}{3}}x\arctan\left(\frac{2}{3}\sqrt{3}\,(-c)^{\frac{2}{3}}x^2+\frac{1}{3}\sqrt{3}\right)+b(-c)^{\frac{1}{3}}x\log\left(c^2x^4-(-c)^{\frac{1}{3}}cx^2+(-c)^{\frac{2}{3}}\right)-2\,b(-c)^{\frac{1}{3}}x\log\left(cx^2+(-c)^{\frac{1}{3}}\right)+2\,b\log\left(-\frac{cx^3+1}{cx^3-1}\right)+4\,a}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*b*(-c)^(1/3)*x*arctan(2/3*sqrt(3)*(-c)^(2/3)*x^2 + 1/3*sqrt
(3)) + b*(-c)^(1/3)*x*log(c^2*x^4 - (-c)^(1/3)*c*x^2 + (-c)^(2/3)) - 2*b*(-
c)^(1/3)*x*log(c*x^2 + (-c)^(1/3)) + 2*b*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*
a)/x
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 106, normalized size = 1.02

$$\frac{1}{4}bc\left(\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{|c|^{\frac{2}{3}}}+\frac{\log\left(x^4+\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{|c|^{\frac{2}{3}}}-\frac{2\log\left(\left|x^2-\frac{1}{|c|^{\frac{2}{3}}}\right|\right)}{|c|^{\frac{2}{3}}}\right)-\frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{2x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1/abs(c)^(2/3))*abs(c)^(2/3)
)/abs(c)^(2/3) + log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/abs(c)^(2/3)
- 2*log(abs(x^2 - 1/abs(c)^(2/3)))/abs(c)^(2/3)) - 1/2*b*log(-(c*x^3 + 1)/(
c*x^3 - 1))/x - a/x
```

Mupad [B]
time = 2.39, size = 117, normalized size = 1.12

$$\frac{b\ln\left(1-cx^3\right)}{2x}-\frac{b\,c^{1/3}\ln\left(1-c^{2/3}x^2\right)}{2}-\frac{b\ln\left(cx^3+1\right)}{2x}-\frac{a}{x}-\frac{b\,c^{1/3}\ln\left(-\sqrt{3}-c^{2/3}x^2\,2\mathrm{i}-\mathrm{i}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4}+\frac{b\,c^{1/3}\ln\left(-\sqrt{3}+c^{2/3}x^2\,2\mathrm{i}+1\mathrm{i}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/x^2,x)
```

```
[Out] (b*log(1 - c*x^3))/(2*x) - (b*c^(1/3)*log(1 - c^(2/3)*x^2))/2 - (b*log(c*x^
3 + 1))/(2*x) - a/x - (b*c^(1/3)*log(- 3^(1/2) - c^(2/3)*x^2*2i - 1i)*(3^(1
/2)*1i - 1))/4 + (b*c^(1/3)*log(c^(2/3)*x^2*2i - 3^(1/2) + 1i)*(3^(1/2)*1i
+ 1))/4
```

3.115 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^5}\,dx$

Optimal. Leaf size=174

$$-\frac{3bc}{4x}+\frac{1}{8}\sqrt{3}\,bc^{4/3}\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}\right)-\frac{1}{8}\sqrt{3}\,bc^{4/3}\text{ArcTan}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}\right)+\frac{1}{4}bc^{4/3}\tanh^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{a}{}$$

```
[Out] -3/4*b*c/x+1/4*b*c^(4/3)*arctanh(c^(1/3)*x)+1/4*(-a-b*arctanh(c*x^3))/x^4-1
/16*b*c^(4/3)*ln(1-c^(1/3)*x+c^(2/3)*x^2)+1/16*b*c^(4/3)*ln(1+c^(1/3)*x+c^(
2/3)*x^2)-1/8*b*c^(4/3)*arctan(-1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)-
1/8*b*c^(4/3)*arctan(1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6037, 331, 302, 648, 632, 210, 642, 212}

$$-\frac{a+b\tanh^{-1}(cx^3)}{4x^4}+\frac{1}{8}\sqrt{3}\,bc^{4/3}\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}\right)-\frac{1}{8}\sqrt{3}\,bc^{4/3}\text{ArcTan}\left(\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)-\frac{1}{16}bc^{4/3}\log\left(c^{2/3}x^2-\sqrt[3]{c}\,x+1\right)+\frac{1}{16}bc^{4/3}\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x+1\right)+\frac{1}{4}bc^{4/3}\tanh^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{3bc}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^5,x]
```

```
[Out] (-3*b*c)/(4*x) + (Sqrt[3]*b*c^(4/3)*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3
]])/8 - (Sqrt[3]*b*c^(4/3)*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/8 + (
b*c^(4/3)*ArcTanh[c^(1/3)*x])/4 - (a + b*ArcTanh[c*x^3])/(4*x^4) - (b*c^(4/
3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/16 + (b*c^(4/3)*Log[1 + c^(1/3)*x + c^
(2/3)*x^2])/16
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
```

```
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx^3)}{x^5}\,dx = -\frac{a+b\tanh^{-1}(cx^3)}{4x^4}+\frac{1}{4}(3bc)\int \frac{1}{x^2(1-c^2x^6)}\,dx$$

$$= -\frac{3bc}{4x}-\frac{a+b\tanh^{-1}(cx^3)}{4x^4}+\frac{1}{4}(3bc^3)\int \frac{x^4}{1-c^2x^6}\,dx$$

$$= -\frac{3bc}{4x}-\frac{a+b\tanh^{-1}(cx^3)}{4x^4}+\frac{1}{4}(bc^{5/3})\int \frac{1}{1-c^{2/3}x^2}\,dx+\frac{1}{4}(bc^{5/3})\int \frac{-\frac{1}{2}-}{1-\sqrt[3]{c}\,x+}$$

$$= -\frac{3bc}{4x}+\frac{1}{4}bc^{4/3}\tanh^{-1}(\sqrt[3]{c}\,x)-\frac{a+b\tanh^{-1}(cx^3)}{4x^4}-\frac{1}{16}(bc^{4/3})\int \frac{-\sqrt[3]{c}+2c^2}{1-\sqrt[3]{c}\,x+c}$$

$$= -\frac{3bc}{4x}+\frac{1}{4}bc^{4/3}\tanh^{-1}(\sqrt[3]{c}\,x)-\frac{a+b\tanh^{-1}(cx^3)}{4x^4}-\frac{1}{16}bc^{4/3}\log(1-\sqrt[3]{c}\,x+c^2$$

$$= -\frac{3bc}{4x}+\frac{1}{8}\sqrt{3}\,bc^{4/3}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\,x}{\sqrt{3}}\right)-\frac{1}{8}\sqrt{3}\,bc^{4/3}\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)+\frac{1}{4}bc$$

Mathematica [A]
time = 0.04, size = 196, normalized size = 1.13

$$-\frac{a}{4x^4}-\frac{3bc}{4x}-\frac{1}{8}\sqrt{3}\,bc^{4/3}\text{ArcTan}\left(\frac{-1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)-\frac{1}{8}\sqrt{3}\,bc^{4/3}\text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)-\frac{b\tanh^{-1}(cx^3)}{4x^4}-\frac{1}{8}bc^{4/3}\log(1-\sqrt[3]{c}\,x)+\frac{1}{8}bc^{4/3}\log(1+\sqrt[3]{c}\,x)-\frac{1}{16}bc^{4/3}\log(1-\sqrt[3]{c}\,x+c^{2/3}x^2)+\frac{1}{16}bc^{4/3}\log(1+\sqrt[3]{c}\,x+c^{2/3}x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^5,x]
```

```
[Out] -1/4*a/x^4 - (3*b*c)/(4*x) - (Sqrt[3]*b*c^(4/3)*ArcTan[(-1 + 2*c^(1/3)*x)/S
qrt[3]])/8 - (Sqrt[3]*b*c^(4/3)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/8 - (b*A
rcTanh[c*x^3])/(4*x^4) - (b*c^(4/3)*Log[1 - c^(1/3)*x])/8 + (b*c^(4/3)*Log[
1 + c^(1/3)*x])/8 - (b*c^(4/3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/16 + (b*c^
(4/3)*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/16
```

Maple [A]
time = 0.05, size = 172, normalized size = 0.99

method	result
default	$-\frac{a}{4x^4}-\frac{b\,\mathrm{arctanh}(c\,x^3)}{4x^4}+\frac{bc\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{8\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{bc\ln\left(x^2-\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{16\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{8\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{bc\ln(x}{8(}$

risch	$-\frac{b\ln(c\,x^3+1)}{8x^4}-\frac{a}{4x^4}+\frac{b\ln(-c\,x^3+1)}{8x^4}-\frac{bc\ln\left(x-(\frac{1}{c})^{\frac{1}{3}}\right)}{8(\frac{1}{c})^{\frac{1}{3}}}+\frac{bc\ln\left(x^2+(\frac{1}{c})^{\frac{1}{3}}x+(\frac{1}{c})^{\frac{2}{3}}\right)}{16(\frac{1}{c})^{\frac{1}{3}}}-\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{(\frac{1}{c})^{\frac{1}{3}}}\cdots\right.}{3}\right.}{8(\frac{1}{c})^{\frac{1}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctanh(c*x^3)+1/8*b*c/(1/c)^(1/3)*ln(x+(1/c)^(1/3))-1
/16*b*c/(1/c)^(1/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))-1/8*b*c*3^(1/2)/(1/c)
^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))-1/8*b*c/(1/c)^(1/3)*ln(x-(1/
c)^(1/3))+1/16*b*c/(1/c)^(1/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))-1/8*b*c*3^
(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))-3/4*b*c/x
```

Maxima [A]
time = 0.45, size = 160, normalized size = 0.92

$$-\frac{1}{16}\left(\left(2\sqrt{3}\,c^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}x+c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)+2\sqrt{3}\,c^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}x-c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)-c^{\frac{1}{3}}\log\left(c^{\frac{2}{3}}x^2+c^{\frac{1}{3}}x+1\right)+c^{\frac{1}{3}}\log\left(c^{\frac{2}{3}}x^2-c^{\frac{1}{3}}x+1\right)-2\,c^{\frac{1}{3}}\log\left(\frac{c^{\frac{1}{3}}x+1}{c^{\frac{1}{3}}}\right)+2\,c^{\frac{1}{3}}\log\left(\frac{c^{\frac{1}{3}}x-1}{c^{\frac{1}{3}}}\right)+\frac{12}{x}\right)c+\frac{4\,\mathrm{artanh}\left(cx^3\right)}{x^4}\right)b-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^5,x, algorithm="maxima")
```

```
[Out] -1/16*((2*sqrt(3)*c^(1/3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x + c^(1/3))/c^(1/3
)) + 2*sqrt(3)*c^(1/3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x - c^(1/3))/c^(1/3))
- c^(1/3)*log(c^(2/3)*x^2 + c^(1/3)*x + 1) + c^(1/3)*log(c^(2/3)*x^2 - c^(1
/3)*x + 1) - 2*c^(1/3)*log((c^(1/3)*x + 1)/c^(1/3)) + 2*c^(1/3)*log((c^(1/3
)*x - 1)/c^(1/3)) + 12/x)*c + 4*arctanh(c*x^3)/x^4)*b - 1/4*a/x^4
```

Fricas [A]
time = 0.40, size = 196, normalized size = 1.13

$$-\frac{2\sqrt{3}\,b(-c)^{\frac{1}{3}}\,cx^4\arctan\left(\frac{2}{3}\sqrt{3}\,(-c)^{\frac{1}{3}}\,x-\frac{1}{3}\sqrt{3}\right)+2\sqrt{3}\,bc^{\frac{4}{3}}x^4\arctan\left(\frac{2}{3}\sqrt{3}\,c^{\frac{1}{3}}x-\frac{1}{3}\sqrt{3}\right)+b(-c)^{\frac{1}{3}}\,cx^4\log\left(cx^2+(-c)^{\frac{2}{3}}\,x-(-c)^{\frac{1}{3}}\right)+bc^{\frac{4}{3}}x^4\log\left(cx^2-c^{\frac{2}{3}}x+c^{\frac{1}{3}}\right)-2\,b(-c)^{\frac{1}{3}}\,cx^4\log\left(cx-(-c)^{\frac{2}{3}}\right)-2\,bc^{\frac{4}{3}}x^4\log\left(cx+c^{\frac{2}{3}}\right)+12\,bcx^3+2\,b\log\left(-\frac{cx^3+1}{cx^3-1}\right)+4\,a}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(2*sqrt(3)*b*(-c)^(1/3)*c*x^4*arctan(2/3*sqrt(3)*(-c)^(1/3)*x - 1/3*s
qrt(3)) + 2*sqrt(3)*b*c^(4/3)*x^4*arctan(2/3*sqrt(3)*c^(1/3)*x - 1/3*sqrt(3
)) + b*(-c)^(1/3)*c*x^4*log(c*x^2 + (-c)^(2/3)*x - (-c)^(1/3)) + b*c^(4/3)*
x^4*log(c*x^2 - c^(2/3)*x + c^(1/3)) - 2*b*(-c)^(1/3)*c*x^4*log(c*x - (-c)^
(2/3)) - 2*b*c^(4/3)*x^4*log(c*x + c^(2/3)) + 12*b*c*x^3 + 2*b*log(-(c*x^3
+ 1)/(c*x^3 - 1)) + 4*a)/x^4
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.62, size = 187, normalized size = 1.07

$$-\frac{1}{8}\sqrt{3}\,bc|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)-\frac{1}{8}\sqrt{3}\,bc|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)+\frac{bc^3\log\left(x^2+\frac{x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{16\,|c|^{\frac{5}{3}}}-\frac{bc^3\log\left(x^2-\frac{x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{16\,|c|^{\frac{5}{3}}}+\frac{1}{8}bc|c|^{\frac{1}{3}}\log\left(\left|x+\frac{1}{|c|^{\frac{1}{3}}}\right|\right)-\frac{bc^3\log\left(\left|x-\frac{1}{|c|^{\frac{1}{3}}}\right|\right)}{8\,|c|^{\frac{5}{3}}}-\frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{8\,x^4}-\frac{3\,bcx^3+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^5,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(3)*b*c*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*x + 1/abs(c)^(1/3))*abs
(c)^(1/3)) - 1/8*sqrt(3)*b*c*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*x - 1/abs(c
)^(1/3))*abs(c)^(1/3)) + 1/16*b*c^3*log(x^2 + x/abs(c)^(1/3) + 1/abs(c)^(2/
3))/abs(c)^(5/3) - 1/16*b*c^3*log(x^2 - x/abs(c)^(1/3) + 1/abs(c)^(2/3))/ab
s(c)^(5/3) + 1/8*b*c*abs(c)^(1/3)*log(abs(x + 1/abs(c)^(1/3))) - 1/8*b*c^3*
log(abs(x - 1/abs(c)^(1/3)))/abs(c)^(5/3) - 1/8*b*log(-(c*x^3 + 1)/(c*x^3 -
 1))/x^4 - 1/4*(3*b*c*x^3 + a)/x^4
```

Mupad [B]
time = 1.28, size = 125, normalized size = 0.72

$$\frac{b\ln\left(1-c\,x^3\right)}{8\,x^4}-\frac{b\,c^{4/3}\left(-\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)}{2}+\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)}{2}+\mathrm{atan}\left(c^{1/3}\,x\,1\mathrm{i}\right)\right)1\mathrm{i}}{4}-\frac{3\,b\,c}{4\,x}-\frac{b\ln\left(c\,x^3+1\right)}{8\,x^4}-\frac{a}{4\,x^4}-\frac{\sqrt{3}\,b\,c^{4/3}\left(\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)+\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/x^5,x)
```

```
[Out] (b*log(1 - c*x^3))/(8*x^4) - (b*c^(4/3)*(atan((c^(1/3)*x*(3^(1/2) + 1i))/2)
/2 - atan((c^(1/3)*x*(3^(1/2) - 1i))/2)/2 + atan(c^(1/3)*x*1i))*1i)/4 - (3*
b*c)/(4*x) - (b*log(c*x^3 + 1))/(8*x^4) - a/(4*x^4) - (3^(1/2)*b*c^(4/3)*(a
tan((c^(1/3)*x*(3^(1/2) - 1i))/2) + atan((c^(1/3)*x*(3^(1/2) + 1i))/2)))/8
```

3.116 $\int x^{11}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\,dx$

Optimal. Leaf size=125

$$\frac{abx^3}{6c^3}+\frac{b^2x^6}{36c^2}+\frac{b^2x^3\tanh^{-1}\left(cx^3\right)}{6c^3}+\frac{bx^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{18c}-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{12c^4}+\frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right.$$

```
[Out] 1/6*a*b*x^3/c^3+1/36*b^2*x^6/c^2+1/6*b^2*x^3*arctanh(c*x^3)/c^3+1/18*b*x^9*
(a+b*arctanh(c*x^3))/c-1/12*(a+b*arctanh(c*x^3))^2/c^4+1/12*x^12*(a+b*arcta
nh(c*x^3))^2+1/9*b^2*ln(-c^2*x^6+1)/c^4
```

Rubi [A]
time = 0.19, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6039, 6037, 6127, 272, 45, 6021, 266, 6095}

$$-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{12c^4}+\frac{abx^3}{6c^3}+\frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{bx^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{18c}+\frac{b^2x^3\tanh^{-1}\left(cx^3\right)}{6c^3}+\frac{b^2x^6}{36c^2}+\frac{b^2\log\left(1-c^2x^6\right)}{9c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (a*b*x^3)/(6*c^3) + (b^2*x^6)/(36*c^2) + (b^2*x^3*ArcTanh[c*x^3])/(6*c^3) +
 (b*x^9*(a + b*ArcTanh[c*x^3]))/(18*c) - (a + b*ArcTanh[c*x^3])^2/(12*c^4)
+ (x^12*(a + b*ArcTanh[c*x^3])^2)/12 + (b^2*Log[1 - c^2*x^6])/(9*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
```

```
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$
\begin{aligned}
\int x^{11}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\,dx &= \int \left(\frac{1}{4}x^{11}\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{1}{2}bx^{11}\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+\right.\right. \\
&= \frac{1}{4}\int x^{11}\left(2a-b\log\left(1-cx^3\right)\right)^2\,dx-\frac{1}{2}b\int x^{11}\left(-2a+b\log\left(1-cx^3\right)\right)\log \\
&= \frac{1}{12}\text{Subst}\left(\int x^3(2a-b\log(1-cx))^2\,dx,x,x^3\right)-\frac{1}{6}b\text{Subst}\left(\int x^3(-2a+\right. \\
&= \frac{1}{48}x^{12}\left(2a-b\log\left(1-cx^3\right)\right)^2+\frac{1}{24}bx^{12}\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right. \\
&= \frac{1}{48}x^{12}\left(2a-b\log\left(1-cx^3\right)\right)^2+\frac{1}{24}bx^{12}\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right. \\
&= \frac{1}{48}x^{12}\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{1}{288}b\left(2a-b\log\left(1-cx^3\right)\right)\left(\frac{48(1-cx^3)}{c^4}\right. \\
&= \frac{abx^3}{12c^3}-\frac{bx^6\left(2a-b\log\left(1-cx^3\right)\right)}{48c^2}+\frac{bx^9\left(2a-b\log\left(1-cx^3\right)\right)}{72c}-\frac{1}{96}bx^{12}(2 \\
&= \frac{abx^3}{12c^3}-\frac{bx^6\left(2a-b\log\left(1-cx^3\right)\right)}{48c^2}+\frac{bx^9\left(2a-b\log\left(1-cx^3\right)\right)}{72c}-\frac{1}{96}bx^{12}(2 \\
&= \frac{abx^3}{12c^3}+\frac{55b^2x^3}{288c^3}-\frac{5b^2x^6}{576c^2}+\frac{b^2x^9}{864c}-\frac{b^2x^{12}}{384}+\frac{b^2\left(1-cx^3\right)^2}{16c^4}-\frac{b^2\left(1-cx^3\right)^3}{54c^4}+ \\
&= \frac{abx^3}{12c^3}+\frac{55b^2x^3}{288c^3}-\frac{5b^2x^6}{576c^2}+\frac{b^2x^9}{864c}-\frac{b^2x^{12}}{384}+\frac{b^2\left(1-cx^3\right)^2}{16c^4}-\frac{b^2\left(1-cx^3\right)^3}{54c^4}+
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 146, normalized size = 1.17

$$
\frac{6abcx^3+b^2c^2x^6+2abc^3x^9+3a^2c^4x^{12}+2bcx^3(3ac^3x^9+b(3+c^2x^6))\tanh^{-1}\left(cx^3\right)+3b^2(-1+c^4x^{12})\tanh^{-1}\left(cx^3\right)^2+b(3a+4b)\log\left(1-cx^3\right)-3ab\log\left(1+cx^3\right)+4b^2\log\left(1+cx^3\right)}{36c^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (6*a*b*c*x^3 + b^2*c^2*x^6 + 2*a*b*c^3*x^9 + 3*a^2*c^4*x^12 + 2*b*c*x^3*(3*
a*c^3*x^9 + b*(3 + c^2*x^6))*ArcTanh[c*x^3] + 3*b^2*(-1 + c^4*x^12)*ArcTanh
[c*x^3]^2 + b*(3*a + 4*b)*Log[1 - c*x^3] - 3*a*b*Log[1 + c*x^3] + 4*b^2*Log
[1 + c*x^3])/(36*c^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(111) = 222$.
time = 0.17, size = 298, normalized size = 2.38

method	result
risch	$\frac{b^2(x^{12}c^4-1)\ln(cx^3+1)^2}{48c^4}+\frac{b(-3x^{12}b\ln(-cx^3+1)c^4+6ac^4x^{12}+2bc^3x^9+6bcx^3+3b\ln(-cx^3+1))\ln(cx^3+1)}{72c^4}+\frac{b^2x^{12}\ln(-cx^3+}{48}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a+b*arctanh(c*x^3))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*b^2*(c^4*x^12-1)/c^4*ln(c*x^3+1)^2+1/72*b*(-3*x^12*b*ln(-c*x^3+1)*c^4+
6*a*c^4*x^12+2*b*c^3*x^9+6*b*c*x^3+3*b*ln(-c*x^3+1))/c^4*ln(c*x^3+1)+1/48*b
^2*x^12*ln(-c*x^3+1)^2-1/12*a*b*x^12*ln(-c*x^3+1)+1/12*x^12*a^2-1/36/c*b^2*
x^9*ln(-c*x^3+1)+1/18/c*a*b*x^9+1/36*b^2*x^6/c^2-1/12/c^3*b^2*x^3*ln(-c*x^3
+1)+1/6*a*b*x^3/c^3-1/48/c^4*b^2*ln(-c*x^3+1)^2+1/12/c^4*b*ln(-c*x^3+1)*a+1
/9/c^4*b^2*ln(-c*x^3+1)-1/12/c^4*b*ln(-c*x^3-1)*a+1/9/c^4*b^2*ln(-c*x^3-1)
```

Maxima [A]
time = 0.28, size = 217, normalized size = 1.74

$$\frac{1}{12}\,b^2x^{12}\,\mathrm{artanh}\left(cx^3\right)^2+\frac{1}{12}\,a^2x^{12}+\frac{1}{36}\left(6\,x^{12}\,\mathrm{artanh}\left(cx^3\right)+c\left(\frac{2\left(c^2x^9+3\,x^3\right)}{c^4}-\frac{3\,\log\left(cx^3+1\right)}{c^5}+\frac{3\,\log\left(cx^3-1\right)}{c^5}\right)\right)ab+\frac{1}{144}\left(4\,c\left(\frac{2\left(c^2x^9+3\,x^3\right)}{c^4}-\frac{3\,\log\left(cx^3+1\right)}{c^5}+\frac{3\,\log\left(cx^3-1\right)}{c^5}\right)\mathrm{artanh}\left(cx^3\right)+\frac{4\,c^2x^6-2\left(3\,\log\left(cx^3-1\right)-8\right)\log\left(cx^3+1\right)+3\,\log\left(cx^3+1\right)^2+3\,\log\left(cx^3-1\right)^2+16\,\log\left(cx^3-1\right)}{c^4}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*x^12*arctanh(c*x^3)^2 + 1/12*a^2*x^12 + 1/36*(6*x^12*arctanh(c*x^3
) + c*(2*(c^2*x^9 + 3*x^3)/c^4 - 3*log(c*x^3 + 1)/c^5 + 3*log(c*x^3 - 1)/c^
5))*a*b + 1/144*(4*c*(2*(c^2*x^9 + 3*x^3)/c^4 - 3*log(c*x^3 + 1)/c^5 + 3*lo
g(c*x^3 - 1)/c^5)*arctanh(c*x^3) + (4*c^2*x^6 - 2*(3*log(c*x^3 - 1) - 8)*lo
g(c*x^3 + 1) + 3*log(c*x^3 + 1)^2 + 3*log(c*x^3 - 1)^2 + 16*log(c*x^3 - 1))
/c^4)*b^2
```

Fricas [A]
time = 0.36, size = 176, normalized size = 1.41

$$\frac{12\,a^2c^4x^{12}+8\,abc^3x^9+4\,b^2c^2x^6+24\,abcx^3+3\left(b^2c^4x^{12}-b^2\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)^2-4\left(3\,ab-4\,b^2\right)\log\left(cx^3+1\right)+4\left(3\,ab+4\,b^2\right)\log\left(cx^3-1\right)+4\left(3\,abc^4x^{12}+b^2c^3x^9+3\,b^2cx^3\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{144\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] 1/144*(12*a^2*c^4*x^12 + 8*a*b*c^3*x^9 + 4*b^2*c^2*x^6 + 24*a*b*c*x^3 + 3*(
b^2*c^4*x^12 - b^2)*log(-(c*x^3 + 1)/(c*x^3 - 1))^2 - 4*(3*a*b - 4*b^2)*log
(c*x^3 + 1) + 4*(3*a*b + 4*b^2)*log(c*x^3 - 1) + 4*(3*a*b*c^4*x^12 + b^2*c^
3*x^9 + 3*b^2*c*x^3)*log(-(c*x^3 + 1)/(c*x^3 - 1)))/c^4
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 175, normalized size = 1.40

$$\frac{1}{12}\,a^2x^{12}+\frac{abx^9}{18\,c}+\frac{b^2x^6}{36\,c^2}+\frac{1}{48}\left(b^2x^{12}-\frac{b^2}{c^4}\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)^2+\frac{abx^3}{6\,c^3}+\frac{1}{36}\left(3\,abx^{12}+\frac{b^2x^9}{c}+\frac{3\,b^2x^3}{c^3}\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)-\frac{(3\,ab-4\,b^2)\log\left(cx^3+1\right)}{36\,c^4}+\frac{(3\,ab+4\,b^2)\log\left(cx^3-1\right)}{36\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] 1/12*a^2*x^12 + 1/18*a*b*x^9/c + 1/36*b^2*x^6/c^2 + 1/48*(b^2*x^12 - b^2/c^
4)*log(-(c*x^3 + 1)/(c*x^3 - 1))^2 + 1/6*a*b*x^3/c^3 + 1/36*(3*a*b*x^12 + b
^2*x^9/c + 3*b^2*x^3/c^3)*log(-(c*x^3 + 1)/(c*x^3 - 1)) - 1/36*(3*a*b - 4*b
^2)*log(c*x^3 + 1)/c^4 + 1/36*(3*a*b + 4*b^2)*log(c*x^3 - 1)/c^4
```

Mupad [B]
time = 1.60, size = 335, normalized size = 2.68

$$\frac{a^2\,x^{12}}{12}+\frac{b^2\,\ln\left(c\,x^3-1\right)}{9\,c^4}+\frac{b^2\,\ln\left(c\,x^3+1\right)}{9\,c^4}-\frac{b^2\ln\left(c\,x^3+1\right)^2}{48\,c^4}-\frac{b^2\ln\left(1-c\,x^3\right)^2}{48\,c^4}+\frac{b^2\,x^6}{36\,c^2}+\frac{b^2\,x^{12}\ln\left(c\,x^3+1\right)^2}{48}+\frac{b^2\,x^{12}\ln\left(1-c\,x^3\right)^2}{48}+\frac{b^2\,x^3\,\ln\left(c\,x^3+1\right)}{12\,c^3}-\frac{b^2\,x^3\,\ln\left(1-c\,x^3\right)}{12\,c^3}+\frac{b^2\,x^9\,\ln\left(c\,x^3+1\right)}{36\,c}-\frac{b^2\,x^9\,\ln\left(1-c\,x^3\right)}{36\,c}+\frac{a\,b\,\ln\left(c\,x^3-1\right)}{12\,c^4}-\frac{a\,b\,\ln\left(c\,x^3+1\right)}{12\,c^4}+\frac{a\,b\,x^{12}\,\ln\left(c\,x^3+1\right)}{12}-\frac{a\,b\,x^{12}\,\ln\left(1-c\,x^3\right)}{12}+\frac{b^2\,\ln\left(c\,x^3+1\right)\,\ln\left(1-c\,x^3\right)}{24\,c^4}+\frac{a\,b\,x^3}{6\,c^3}+\frac{a\,b\,x^9}{18\,c}-\frac{b^2\,x^{12}\,\ln\left(c\,x^3+1\right)\,\ln\left(1-c\,x^3\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*atanh(c*x^3))^2,x)
```

```
[Out] (a^2*x^12)/12 + (b^2*log(c*x^3 - 1))/(9*c^4) + (b^2*log(c*x^3 + 1))/(9*c^4)
 - (b^2*log(c*x^3 + 1)^2)/(48*c^4) - (b^2*log(1 - c*x^3)^2)/(48*c^4) + (b^2
*x^6)/(36*c^2) + (b^2*x^12*log(c*x^3 + 1)^2)/48 + (b^2*x^12*log(1 - c*x^3)^
2)/48 + (b^2*x^3*log(c*x^3 + 1))/(12*c^3) - (b^2*x^3*log(1 - c*x^3))/(12*c^
3) + (b^2*x^9*log(c*x^3 + 1))/(36*c) - (b^2*x^9*log(1 - c*x^3))/(36*c) + (a
*b*log(c*x^3 - 1))/(12*c^4) - (a*b*log(c*x^3 + 1))/(12*c^4) + (a*b*x^12*log
(c*x^3 + 1))/12 - (a*b*x^12*log(1 - c*x^3))/12 + (b^2*log(c*x^3 + 1)*log(1
- c*x^3))/(24*c^4) + (a*b*x^3)/(6*c^3) + (a*b*x^9)/(18*c) - (b^2*x^12*log(c
*x^3 + 1)*log(1 - c*x^3))/24
```

3.117 $\int x^8 \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\, dx$

Optimal. Leaf size=146

$$\frac{b^2x^3}{9c^2}-\frac{b^2\tanh^{-1}\left(cx^3\right)}{9c^3}+\frac{bx^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{9c}+\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{9c^3}+\frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2-\frac{2b(a+$$

```
[Out] 1/9*b^2*x^3/c^2-1/9*b^2*arctanh(c*x^3)/c^3+1/9*b*x^6*(a+b*arctanh(c*x^3))/c
+1/9*(a+b*arctanh(c*x^3))^2/c^3+1/9*x^9*(a+b*arctanh(c*x^3))^2-2/9*b*(a+b*a
rctanh(c*x^3))*ln(2/(-c*x^3+1))/c^3-1/9*b^2*polylog(2,1-2/(-c*x^3+1))/c^3
```

Rubi [A]
time = 0.18, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {6039, 6037, 6127, 327, 212, 6131, 6055, 2449, 2352}

$$\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{9c^3}-\frac{2b\log\left(\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{9c^3}+\frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{bx^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{9c}-\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx^3}\right)}{9c^3}-\frac{b^2\tanh^{-1}\left(cx^3\right)}{9c^3}+\frac{b^2x^3}{9c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (b^2*x^3)/(9*c^2) - (b^2*ArcTanh[c*x^3])/(9*c^3) + (b*x^6*(a + b*ArcTanh[c*
x^3]))/(9*c) + (a + b*ArcTanh[c*x^3])^2/(9*c^3) + (x^9*(a + b*ArcTanh[c*x^3
])^2)/9 - (2*b*(a + b*ArcTanh[c*x^3])*Log[2/(1 - c*x^3)])/(9*c^3) - (b^2*Po
lyLog[2, 1 - 2/(1 - c*x^3)])/(9*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\,dx &= \int \left(\frac{1}{4}x^8\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{1}{2}bx^8\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right.\right.\\
&= \frac{1}{4}\int x^8\left(2a-b\log\left(1-cx^3\right)\right)^2\,dx-\frac{1}{2}b\int x^8\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1\right.\\
&= \frac{1}{12}\text{Subst}\left(\int x^2(2a-b\log(1-cx))^2\,dx,x,x^3\right)-\frac{1}{6}b\text{Subst}\left(\int x^2(-2a+b\,\text{l}\right.\\
&= \frac{1}{36}x^9\left(2a-b\log\left(1-cx^3\right)\right)^2+\frac{1}{18}bx^9\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\\
&= \frac{1}{36}x^9\left(2a-b\log\left(1-cx^3\right)\right)^2+\frac{1}{18}bx^9\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\\
&= \frac{1}{36}x^9\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{1}{108}b\left(2a-b\log\left(1-cx^3\right)\right)\left(\frac{18(1-cx^3)}{c^3}-\right.\\
&= -\frac{abx^3}{9c^2}+\frac{bx^6(2a-b\log\left(1-cx^3\right))}{36c}-\frac{1}{54}bx^9\left(2a-b\log\left(1-cx^3\right)\right)+\frac{1}{36}x^9(\\
&= -\frac{abx^3}{9c^2}+\frac{bx^6(2a-b\log\left(1-cx^3\right))}{36c}-\frac{1}{54}bx^9\left(2a-b\log\left(1-cx^3\right)\right)+\frac{1}{36}x^9(\\
&= -\frac{abx^3}{9c^2}+\frac{13b^2x^3}{108c^2}+\frac{b^2x^6}{216c}-\frac{b^2x^9}{162}+\frac{b^2(1-cx^3)^2}{24c^3}-\frac{b^2(1-cx^3)^3}{162c^3}+\frac{b^2\log\left(1\right.}{108c}\\
&= -\frac{abx^3}{9c^2}+\frac{13b^2x^3}{108c^2}+\frac{b^2x^6}{216c}-\frac{b^2x^9}{162}+\frac{b^2(1-cx^3)^2}{24c^3}-\frac{b^2(1-cx^3)^3}{162c^3}+\frac{b^2\log\left(1\right.}{108c}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 132, normalized size = 0.90

$$\frac{b^2cx^3+abc^2x^6+a^2c^3x^9+b^2(-1+c^3x^9)\tanh^{-1}\left(cx^3\right)^2+b\tanh^{-1}\left(cx^3\right)\left(-b+bc^2x^6+2ac^3x^9-2b\log\left(1+e^{-2\tanh^{-1}(cx^3)}\right)\right)+ab\log\left(-1+c^2x^6\right)+b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx^3)}\right)}{9c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (b^2*c*x^3 + a*b*c^2*x^6 + a^2*c^3*x^9 + b^2*(-1 + c^3*x^9)*ArcTanh[c*x^3]^
2 + b*ArcTanh[c*x^3]*(-b + b*c^2*x^6 + 2*a*c^3*x^9 - 2*b*Log[1 + E^(-2*ArcT
anh[c*x^3])]) + a*b*Log[-1 + c^2*x^6] + b^2*PolyLog[2, -E^(-2*ArcTanh[c*x^3
])])/(9*c^3)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^8 \left(a + b \operatorname{arctanh}\left(c x^3\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*arctanh(c*x^3))^2,x)
```

```
[Out] int(x^8*(a+b*arctanh(c*x^3))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/9*a^2*x^9 + 1/9*(2*x^9*arctanh(c*x^3) + (x^6/c^2 + log(c^2*x^6 - 1)/c^4)*
c)*a*b + 1/648*(18*x^9*log(-c*x^3 + 1)^2 - 2*c^4*(2*(c^2*x^9 + 3*x^3)/c^6 -
 3*log(c*x^3 + 1)/c^7 + 3*log(c*x^3 - 1)/c^7) + 3*(x^6/c^4 + log(c^2*x^6 -
1)/c^6)*c^3 + 1944*c^3*integrate(1/9*x^11*log(c*x^3 + 1)/(c^4*x^6 - c^2), x
) - 9*c^2*(2*x^3/c^4 - log(c*x^3 + 1)/c^5 + log(c*x^3 - 1)/c^5) - 6*c*((2*c
^2*x^9 + 3*c*x^6 + 6*x^3)/c^3 + 6*log(c*x^3 - 1)/c^4)*log(-c*x^3 + 1) + 972
*c*integrate(1/9*x^5*log(c*x^3 + 1)/(c^4*x^6 - c^2), x) + 6*(3*c^3*x^9*log(
c*x^3 + 1)^2 + (2*c^3*x^9 - 3*c^2*x^6 + 6*c*x^3 - 6*(c^3*x^9 + 1)*log(c*x^3
 + 1))*log(-c*x^3 + 1))/c^3 + (4*c^3*x^9 + 15*c^2*x^6 + 66*c*x^3 + 18*log(c
*x^3 - 1)^2 + 66*log(c*x^3 - 1))/c^3 - 18*log(9*c^4*x^6 - 9*c^2)/c^3 + 972*
integrate(1/9*x^2*log(c*x^3 + 1)/(c^4*x^6 - c^2), x))*b^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^8*arctanh(c*x^3)^2 + 2*a*b*x^8*arctanh(c*x^3) + a^2*x^8, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2*x^8, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^8 \left(a + b\,\mathrm{atanh}\left(c\,x^3\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*atanh(c*x^3))^2,x)
```

```
[Out] int(x^8*(a + b*atanh(c*x^3))^2, x)
```

3.118 $\int x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\,dx$

Optimal. Leaf size=91

$$\frac{abx^3}{3c}+\frac{b^2x^3\tanh^{-1}\left(cx^3\right)}{3c}-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{6c^2}+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{b^2\log\left(1-c^2x^6\right)}{6c^2}$$

```
[Out] 1/3*a*b*x^3/c+1/3*b^2*x^3*arctanh(c*x^3)/c-1/6*(a+b*arctanh(c*x^3))^2/c^2+1
/6*x^6*(a+b*arctanh(c*x^3))^2+1/6*b^2*ln(-c^2*x^6+1)/c^2
```

Rubi [A]
time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6039, 6037, 6127, 6021, 266, 6095}

$$-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{6c^2}+\frac{abx^3}{3c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{b^2\log\left(1-c^2x^6\right)}{6c^2}+\frac{b^2x^3\tanh^{-1}\left(cx^3\right)}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (a*b*x^3)/(3*c) + (b^2*x^3*ArcTanh[c*x^3])/(3*c) - (a + b*ArcTanh[c*x^3])^2
/(6*c^2) + (x^6*(a + b*ArcTanh[c*x^3])^2)/6 + (b^2*Log[1 - c^2*x^6])/(6*c^2
)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$\int x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\,dx=\int\left(\frac{1}{4}x^5\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{1}{2}bx^5\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right.\right.$$

$$=\frac{1}{4}\int x^5\left(2a-b\log\left(1-cx^3\right)\right)^2\,dx-\frac{1}{2}b\int x^5\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1\right.$$

$$=\frac{1}{12}\text{Subst}\left(\int x(2a-b\log(1-cx))^2\,dx,x,x^3\right)-\frac{1}{6}b\text{Subst}\left(\int x(-2a+b\log\right.$$

$$=\frac{1}{12}bx^6\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{1}{12}\text{Subst}\left(\int\left(\frac{(2a-b\log(1-}{c}\right.\right.$$

$$=\frac{1}{12}bx^6\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{\text{Subst}\left(\int(2a-b\log(1-cx))^2\,d\right.}{12c}$$

$$=\frac{1}{12}bx^6\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{1}{12}b\text{Subst}\left(\int x(-2a+b\log(1\right.$$

$$=\frac{abx^3}{6c}-\frac{1}{24}bx^6\left(2a-b\log\left(1-cx^3\right)\right)-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c^2}+$$

$$=\frac{abx^3}{2c}-\frac{b^2x^3}{4c}+\frac{b^2\left(1-cx^3\right)^2}{48c^2}+\frac{b^2\left(1+cx^3\right)^2}{48c^2}-\frac{1}{24}bx^6\left(2a-b\log\left(1-cx^3\right)\right)$$

$$=\frac{abx^3}{2c}-\frac{b^2x^6}{24}+\frac{b^2\left(1-cx^3\right)^2}{48c^2}+\frac{b^2\left(1+cx^3\right)^2}{48c^2}-\frac{b^2\log\left(1-cx^3\right)}{24c^2}+\frac{b^2(1-cx^3}{}$$

Mathematica [**A**]
time = 0.04, size = 106, normalized size = 1.16

$$\frac{2abcx^3+a^2c^2x^6+2bcx^3(b+acx^3)\tanh^{-1}(cx^3)+b^2(-1+c^2x^6)\tanh^{-1}(cx^3)^2+b(a+b)\log(1-cx^3)-ab\log(1+cx^3)+b^2\log(1+cx^3)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (2*a*b*c*x^3 + a^2*c^2*x^6 + 2*b*c*x^3*(b + a*c*x^3)*ArcTanh[c*x^3] + b^2*(
-1 + c^2*x^6)*ArcTanh[c*x^3]^2 + b*(a + b)*Log[1 - c*x^3] - a*b*Log[1 + c*x
^3] + b^2*Log[1 + c*x^3])/(6*c^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(81) = 162.
time = 0.22, size = 287, normalized size = 3.15

method	result
risch	$\frac{b^2(c^2x^6-1)\ln(cx^3+1)^2}{24c^2}+\frac{b(-2x^6b\ln(-cx^3+1)ac^2+4a^2c^2x^6+4abcx^3+2b\ln(-cx^3+1)a+b^2)\ln(cx^3+1)}{24ac^2}+\frac{b^2x^6\ln(-cx^3+1)^2}{24}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x^3))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*b^2*(c^2*x^6-1)/c^2*ln(c*x^3+1)^2+1/24*b*(-2*x^6*b*ln(-c*x^3+1)*a*c^2+
4*a^2*c^2*x^6+4*a*b*c*x^3+2*b*ln(-c*x^3+1)*a+b^2)/a/c^2*ln(c*x^3+1)+1/24*b^
2*x^6*ln(-c*x^3+1)^2-1/6*a*b*x^6*ln(-c*x^3+1)+1/6*x^6*a^2-1/6/c*b^2*x^3*ln(
-c*x^3+1)+1/3*a*b*x^3/c-1/24/c^2*b^2*ln(-c*x^3+1)^2-1/6*a/c^2*ln(-c*x^3-1)*
b+1/6/c^2*ln(-c*x^3-1)*b^2-1/24/a/c^2*ln(-c*x^3-1)*b^3+1/6*a/c^2*ln(-c*x^3+
1)*b+1/6/c^2*b^2*ln(-c*x^3+1)+1/6*b^2/c^2
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(81) = 162.
time = 0.26, size = 186, normalized size = 2.04

$$\frac{1}{6}b^2x^6\operatorname{artanh}(cx^3)^2+\frac{1}{6}a^2x^6+\frac{1}{6}\left(2x^6\operatorname{artanh}(cx^3)+c\left(\frac{2x^3}{c^2}-\frac{\log(cx^3+1)}{c^3}+\frac{\log(cx^3-1)}{c^3}\right)\right)ab+\frac{1}{24}\left(4c\left(\frac{2x^3}{c^2}-\frac{\log(cx^3+1)}{c^3}+\frac{\log(cx^3-1)}{c^3}\right)\operatorname{artanh}(cx^3)-\frac{2(\log(cx^3-1)-2)\log(cx^3+1)-\log(cx^3+1)^2-\log(cx^3-1)^2-4\log(cx^3-1)}{c^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6*arctanh(c*x^3)^2 + 1/6*a^2*x^6 + 1/6*(2*x^6*arctanh(c*x^3) + c*
(2*x^3/c^2 - log(c*x^3 + 1)/c^3 + log(c*x^3 - 1)/c^3))*a*b + 1/24*(4*c*(2*x
^3/c^2 - log(c*x^3 + 1)/c^3 + log(c*x^3 - 1)/c^3)*arctanh(c*x^3) - (2*(log(
c*x^3 - 1) - 2)*log(c*x^3 + 1) - log(c*x^3 + 1)^2 - log(c*x^3 - 1)^2 - 4*lo
g(c*x^3 - 1))/c^2)*b^2
```

Fricas [A]
time = 0.37, size = 138, normalized size = 1.52

$$\frac{4\,a^{2}c^{2}x^{6}+8\,abcx^{3}+\left(b^{2}c^{2}x^{6}-b^{2}\right)\log\left(-\frac{cx^{3}+1}{cx^{3}-1}\right)^{2}-4\,\left(ab-b^{2}\right)\log\left(cx^{3}+1\right)+4\,\left(ab+b^{2}\right)\log\left(cx^{3}-1\right)+4\,\left(abc^{2}x^{6}+b^{2}cx^{3}\right)\log\left(-\frac{cx^{3}+1}{cx^{3}-1}\right)}{24\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(4*a^2*c^2*x^6 + 8*a*b*c*x^3 + (b^2*c^2*x^6 - b^2)*log(-(c*x^3 + 1)/(c
*x^3 - 1))^2 - 4*(a*b - b^2)*log(c*x^3 + 1) + 4*(a*b + b^2)*log(c*x^3 - 1)
+ 4*(a*b*c^2*x^6 + b^2*c*x^3)*log(-(c*x^3 + 1)/(c*x^3 - 1)))/c^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(81) = 162.
time = 0.48, size = 361, normalized size = 3.97

$$\frac{1}{6}\left(\frac{\left(cx^{3}+1\right)b^{2}\log\left(-\frac{cx^{3}+1}{cx^{3}-1}\right)^{2}}{\left(cx^{3}-1\right)\left(\frac{\left(cx^{3}+1\right)^{2}c^{3}}{\left(cx^{3}-1\right)^{2}}-\frac{2\,\left(cx^{3}+1\right)c^{3}}{cx^{3}-1}+c^{3}\right)}+\frac{2\left(\frac{2\,\left(cx^{3}+1\right)ab}{cx^{3}-1}+\frac{\left(cx^{3}+1\right)b^{2}}{cx^{3}-1}-b^{2}\right)\log\left(-\frac{cx^{3}+1}{cx^{3}-1}\right)}{\frac{\left(cx^{3}+1\right)^{2}c^{3}}{\left(cx^{3}-1\right)^{2}}-\frac{2\,\left(cx^{3}+1\right)c^{3}}{cx^{3}-1}+c^{3}}+\frac{4\left(\frac{\left(cx^{3}+1\right)a^{2}}{cx^{3}-1}+\frac{\left(cx^{3}+1\right)ab}{cx^{3}-1}-ab\right)}{\frac{\left(cx^{3}+1\right)^{2}c^{3}}{\left(cx^{3}-1\right)^{2}}-\frac{2\,\left(cx^{3}+1\right)c^{3}}{cx^{3}-1}+c^{3}}-\frac{2\,b^{2}\log\left(-\frac{cx^{3}+1}{cx^{3}-1}+1\right)}{c^{3}}+\frac{2\,b^{2}\log\left(-\frac{cx^{3}+1}{cx^{3}-1}\right)}{c^{3}}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] 1/6*((c*x^3 + 1)*b^2*log(-(c*x^3 + 1)/(c*x^3 - 1))^2/((c*x^3 - 1)*((c*x^3 +
 1)^2*c^3/(c*x^3 - 1)^2 - 2*(c*x^3 + 1)*c^3/(c*x^3 - 1) + c^3)) + 2*(2*(c*x
^3 + 1)*a*b/(c*x^3 - 1) + (c*x^3 + 1)*b^2/(c*x^3 - 1) - b^2)*log(-(c*x^3 +
1)/(c*x^3 - 1))/((c*x^3 + 1)^2*c^3/(c*x^3 - 1)^2 - 2*(c*x^3 + 1)*c^3/(c*x^3
 - 1) + c^3) + 4*((c*x^3 + 1)*a^2/(c*x^3 - 1) + (c*x^3 + 1)*a*b/(c*x^3 - 1)
 - a*b)/((c*x^3 + 1)^2*c^3/(c*x^3 - 1)^2 - 2*(c*x^3 + 1)*c^3/(c*x^3 - 1) +
c^3) - 2*b^2*log(-(c*x^3 + 1)/(c*x^3 - 1) + 1)/c^3 + 2*b^2*log(-(c*x^3 + 1)
/(c*x^3 - 1))/c^3)*c
```

Mupad [B]
time = 1.25, size = 275, normalized size = 3.02

$$\frac{a^{2}\,x^{6}}{6}+\frac{b^{2}\,\ln\left(c\,x^{3}-1\right)}{6\,c^{2}}+\frac{b^{2}\,\ln\left(c\,x^{3}+1\right)}{6\,c^{2}}-\frac{b^{2}\,{\ln\left(c\,x^{3}+1\right)}^{2}}{24\,c^{2}}-\frac{b^{2}\,{\ln\left(1-c\,x^{3}\right)}^{2}}{24\,c^{2}}+\frac{b^{2}\,x^{6}\,{\ln\left(c\,x^{3}+1\right)}^{2}}{24}+\frac{b^{2}\,x^{6}\,{\ln\left(1-c\,x^{3}\right)}^{2}}{24}+\frac{b^{2}\,x^{3}\,\ln\left(c\,x^{3}+1\right)}{6\,c}-\frac{b^{2}\,x^{3}\,\ln\left(1-c\,x^{3}\right)}{6\,c}+\frac{a\,b\,\ln\left(c\,x^{3}-1\right)}{6\,c^{2}}-\frac{a\,b\,\ln\left(c\,x^{3}+1\right)}{6\,c^{2}}+\frac{a\,b\,x^{6}\,\ln\left(c\,x^{3}+1\right)}{6}-\frac{a\,b\,x^{6}\,\ln\left(1-c\,x^{3}\right)}{6}+\frac{b^{2}\,\ln\left(c\,x^{3}+1\right)\,\ln\left(1-c\,x^{3}\right)}{12\,c^{2}}+\frac{a\,b\,x^{3}}{3\,c}-\frac{b^{2}\,x^{6}\,\ln\left(c\,x^{3}+1\right)\,\ln\left(1-c\,x^{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atanh(c*x^3))^2,x)
```

```
[Out] (a^2*x^6)/6 + (b^2*log(c*x^3 - 1))/(6*c^2) + (b^2*log(c*x^3 + 1))/(6*c^2) -
 (b^2*log(c*x^3 + 1)^2)/(24*c^2) - (b^2*log(1 - c*x^3)^2)/(24*c^2) + (b^2*x
^6*log(c*x^3 + 1)^2)/24 + (b^2*x^6*log(1 - c*x^3)^2)/24 + (b^2*x^3*log(c*x^
3 + 1))/(6*c) - (b^2*x^3*log(1 - c*x^3))/(6*c) + (a*b*log(c*x^3 - 1))/(6*c^
2) - (a*b*log(c*x^3 + 1))/(6*c^2) + (a*b*x^6*log(c*x^3 + 1))/6 - (a*b*x^6*l
og(1 - c*x^3))/6 + (b^2*log(c*x^3 + 1)*log(1 - c*x^3))/(12*c^2) + (a*b*x^3)
/(3*c) - (b^2*x^6*log(c*x^3 + 1)*log(1 - c*x^3))/12
```

3.119 $\int x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\,dx$

Optimal. Leaf size=96

$$\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2-\frac{2b\left(a+b\tanh^{-1}\left(cx^3\right)\right)\log\left(\frac{2}{1-cx^3}\right)}{3c}-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)}{3c}$$

```
[Out] 1/3*(a+b*arctanh(c*x^3))^2/c+1/3*x^3*(a+b*arctanh(c*x^3))^2-2/3*b*(a+b*arct
anh(c*x^3))*ln(2/(-c*x^3+1))/c-1/3*b^2*polylog(2,1-2/(-c*x^3+1))/c
```

Rubi [A]

time = 0.10, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6039, 6021, 6131, 6055, 2449, 2352}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{3c}-\frac{2b\log\left(\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{3c}-\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx^3}\right)}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (a + b*ArcTanh[c*x^3])^2/(3*c) + (x^3*(a + b*ArcTanh[c*x^3])^2)/3 - (2*b*(a
 + b*ArcTanh[c*x^3])*Log[2/(1 - c*x^3)])/(3*c) - (b^2*PolyLog[2, 1 - 2/(1 -
 c*x^3)])/(3*c)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
```

```
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\,dx=\int\left(\frac{1}{4}x^2\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{1}{2}bx^2\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx\right.\right.$$

$$=\frac{1}{4}\int x^2\left(2a-b\log\left(1-cx^3\right)\right)^2\,dx-\frac{1}{2}b\int x^2\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1\right.$$

$$=\frac{1}{12}\text{Subst}\left(\int(2a-b\log(1-cx))^2\,dx,x,x^3\right)-\frac{1}{6}b\text{Subst}\left(\int(-2a+b\log(1\right.\right.$$

$$=\frac{1}{6}bx^3\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)-\frac{\text{Subst}\left(\int(2a-b\log(x))^2\,dx,x,\right.}{12c}$$

$$=-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c}+\frac{1}{6}bx^3\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+c\right.$$

$$=\frac{1}{3}abx^3+\frac{b^2x^3}{6}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c}-\frac{b^2\left(1+cx^3\right)\log\left(1+cx\right.}{6c}$$

$$=\frac{b^2x^3}{3}+\frac{b^2\left(1-cx^3\right)\log\left(1-cx^3\right)}{6c}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c}+\frac{b(2}{}$$

$$=\frac{b^2x^3}{6}+\frac{b^2\left(1-cx^3\right)\log\left(1-cx^3\right)}{6c}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c}+\frac{b(2}{}$$

$$=-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c}+\frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(\frac{1}{2}\left(1+cx^3\right.\right.}{6c}$$

Mathematica [A]

time = 0.11, size = 99, normalized size = 1.03

$$\frac{b^2(-1+cx^3)\tanh^{-1}(cx^3)^2+2b\tanh^{-1}(cx^3)\left(acx^3-b\log\left(1+e^{-2\tanh^{-1}(cx^3)}\right)\right)+a(acx^3+b\log(1-c^2x^6))+b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx^3)}\right)}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (b^2*(-1 + c*x^3)*ArcTanh[c*x^3]^2 + 2*b*ArcTanh[c*x^3]*(a*c*x^3 - b*Log[1
+ E^(-2*ArcTanh[c*x^3])]) + a*(a*c*x^3 + b*Log[1 - c^2*x^6]) + b^2*PolyLog[
2, -E^(-2*ArcTanh[c*x^3])])/(3*c)
```

Maple [A]
time = 0.19, size = 137, normalized size = 1.43

method	result
derivativedivides	$\frac{c\,x^3a^2+\operatorname{arctanh}(c\,x^3)^2b^2c\,x^3+b^2\operatorname{arctanh}(c\,x^3)^2-2\operatorname{arctanh}(c\,x^3)\ln\left(1+\frac{(c\,x^3+1)^2}{-c^2x^6+1}\right)b^2-\operatorname{polylog}\left(2,-\frac{(c\,x^3+1)^2}{-c^2x^6+1}\right)b^2+2a}{3c}$
default	$\frac{c\,x^3a^2+\operatorname{arctanh}(c\,x^3)^2b^2c\,x^3+b^2\operatorname{arctanh}(c\,x^3)^2-2\operatorname{arctanh}(c\,x^3)\ln\left(1+\frac{(c\,x^3+1)^2}{-c^2x^6+1}\right)b^2-\operatorname{polylog}\left(2,-\frac{(c\,x^3+1)^2}{-c^2x^6+1}\right)b^2+2a}{3c}$
risch	$\frac{a^2x^3}{3}-\frac{a^2}{3c}-\frac{b^2}{3c}-\frac{2ab}{3c}+\frac{\ln(-c\,x^3+1)^2x^3b^2}{12}-\frac{\ln(-c\,x^3+1)^2b^2}{12c}+\frac{\ln(-c\,x^3+1)b^2}{3c}+\frac{b^2\ln(c\,x^3+1)^2x^3}{12}+\frac{b^2\,\mathrm{l}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^3))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/c*(c*x^3*a^2+arctanh(c*x^3)^2*b^2*c*x^3+b^2*arctanh(c*x^3)^2-2*arctanh(
c*x^3)*ln(1+(c*x^3+1)^2/(-c^2*x^6+1))*b^2-polylog(2,-(c*x^3+1)^2/(-c^2*x^6+
1))*b^2+2*a*b*c*x^3*arctanh(c*x^3)+a*b*ln(-c^2*x^6+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/12*(x^3*log(-c*x^3 + 1)^2 - c^2*(2*x^3/c^2 - log(c*x^3 + 1)
/c^3 + log(c*x^3 - 1)/c^3) - 2*(x^3/c + log(c*x^3 - 1)/c^2)*c*log(-c*x^3 +
1) + 18*c*integrate(x^5*log(c*x^3 + 1)/(c^2*x^6 - 1), x) + (c*x^3*log(c*x^3
 + 1)^2 + 2*(c*x^3 - (c*x^3 + 1)*log(c*x^3 + 1))*log(-c*x^3 + 1))/c + (2*c*
x^3 + log(c*x^3 - 1)^2 + 2*log(c*x^3 - 1))/c - log(c^2*x^6 - 1)/c + 6*integ
rate(x^2*log(c*x^3 + 1)/(c^2*x^6 - 1), x))*b^2 + 1/3*(2*c*x^3*arctanh(c*x^3
) + log(-c^2*x^6 + 1))*a*b/c
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
[Out] integral(b^2*x^2*arctanh(c*x^3)^2 + 2*a*b*x^2*arctanh(c*x^3) + a^2*x^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**3))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^3) + a)^2*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{atanh}\left(c\,x^3\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^3))^2,x)
[Out] int(x^2*(a + b*atanh(c*x^3))^2, x)
```

3.120 $\int \frac{(a+b\tanh^{-1}(cx^3))^2}{x}\,dx$

Optimal. Leaf size=140

$$\frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)-\frac{1}{3}b\left(a+b\tanh^{-1}\left(cx^3\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)+\frac{1}{3}b(a+b$$

```
[Out] -2/3*(a+b*arctanh(c*x^3))^2*arctanh(-1+2/(-c*x^3+1))-1/3*b*(a+b*arctanh(c*x
^3))*polylog(2,1-2/(-c*x^3+1))+1/3*b*(a+b*arctanh(c*x^3))*polylog(2,-1+2/(-
c*x^3+1))+1/6*b^2*polylog(3,1-2/(-c*x^3+1))-1/6*b^2*polylog(3,-1+2/(-c*x^3+
1))
```

Rubi [A]
time = 0.23, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6035, 6033, 6199, 6095, 6205, 6745}

$$-\frac{1}{3}b\text{Li}_2\left(1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{1}{3}b\text{Li}_2\left(\frac{2}{1-cx^3}-1\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{2}{3}\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{1}{6}b^2\text{Li}_3\left(1-\frac{2}{1-cx^3}\right)-\frac{1}{6}b^2\text{Li}_3\left(\frac{2}{1-cx^3}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])^2/x,x]
```

```
[Out] (2*(a + b*ArcTanh[c*x^3])^2*ArcTanh[1 - 2/(1 - c*x^3)])/3 - (b*(a + b*ArcTa
nh[c*x^3])*PolyLog[2, 1 - 2/(1 - c*x^3)])/3 + (b*(a + b*ArcTanh[c*x^3])*Pol
yLog[2, -1 + 2/(1 - c*x^3)])/3 + (b^2*PolyLog[3, 1 - 2/(1 - c*x^3)])/6 - (b
^2*PolyLog[3, -1 + 2/(1 - c*x^3)])/6
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx,x,x^3\right)\\ &= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)-\frac{1}{3}(4bc)\text{Subst}\left(\int \frac{(a+b\tan\right.\\ &= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)+\frac{1}{3}(2bc)\text{Subst}\left(\int \frac{(a+b\tan\right.\\ &= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)-\frac{1}{3}b\left(a+b\tanh^{-1}\left(cx^3\right)\right)\text{Li}_2\\ &= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)-\frac{1}{3}b\left(a+b\tanh^{-1}\left(cx^3\right)\right)\text{Li}_2\end{aligned}$$

Mathematica [A]
time = 0.05, size = 183, normalized size = 1.31

$$\frac{1}{3}\left(2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)-4bc\left(\frac{1}{2}\left(\frac{\left(-a-b\tanh^{-1}\left(cx^3\right)\right)\text{PolyLog}\left(2,\frac{-1-cx^3}{-1+cx^3}\right)}{2c}+\frac{b\text{PolyLog}\left(3,\frac{-1-cx^3}{-1+cx^3}\right)}{4c}\right)+\frac{1}{2}\left(-\frac{\left(-a-b\tanh^{-1}\left(cx^3\right)\right)\text{PolyLog}\left(2,\frac{1+cx^3}{-1+cx^3}\right)}{2c}-\frac{b\text{PolyLog}\left(3,\frac{1+cx^3}{-1+cx^3}\right)}{4c}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^2/x,x]
```

```
[Out] (2*(a + b*ArcTanh[c*x^3])^2*ArcTanh[1 - 2/(1 - c*x^3)] - 4*b*c*((((-a - b*A
rcTanh[c*x^3])*PolyLog[2, (-1 - c*x^3)/(-1 + c*x^3)])/(2*c) + (b*PolyLog[3,
 (-1 - c*x^3)/(-1 + c*x^3)])/(4*c))/2 + (-1/2*((-a - b*ArcTanh[c*x^3])*Poly
Log[2, (1 + c*x^3)/(-1 + c*x^3)])/c - (b*PolyLog[3, (1 + c*x^3)/(-1 + c*x^3
)])/(4*c))/2))/3
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\,x^{3}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^2/x,x)
```

```
[Out] int((a+b*arctanh(c*x^3))^2/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + integrate(1/4*b^2*(log(c*x^3 + 1) - log(-c*x^3 + 1))^2/x + a*b
*(log(c*x^3 + 1) - log(-c*x^3 + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(cx^{3}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c*x**3))**2/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,x^3\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))^2/x,x)
```

```
[Out] int((a + b*atanh(c*x^3))^2/x, x)
```

3.121 $\int \frac{\left(a+b\tanh^{-1}(cx^3)\right)^2}{x^4}\,dx$

Optimal. Leaf size=90

$$\frac{1}{3}c\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{3x^3}+\frac{2}{3}bc\left(a+b\tanh^{-1}\left(cx^3\right)\right)\log\left(2-\frac{2}{1+cx^3}\right)-\frac{1}{3}b^2c\text{PolyLog}$$

```
[Out] 1/3*c*(a+b*arctanh(c*x^3))^2-1/3*(a+b*arctanh(c*x^3))^2/x^3+2/3*b*c*(a+b*ar
ctanh(c*x^3))*ln(2-2/(c*x^3+1))-1/3*b^2*c*polylog(2,-1+2/(c*x^3+1))
```

Rubi [A]

time = 0.13, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6039, 6037, 6135, 6079, 2497}

$$\frac{1}{3}c\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{3x^3}+\frac{2}{3}bc\log\left(2-\frac{2}{cx^3+1}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{3}b^2c\text{Li}_2\left(\frac{2}{cx^3+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])^2/x^4,x]
```

```
[Out] (c*(a + b*ArcTanh[c*x^3])^2)/3 - (a + b*ArcTanh[c*x^3])^2/(3*x^3) + (2*b*c*
(a + b*ArcTanh[c*x^3])*Log[2 - 2/(1 + c*x^3)])/3 - (b^2*c*PolyLog[2, -1 + 2
/(1 + c*x^3)])/3
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{x^4}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{4x^4} - \frac{b(-2a+b\log\left(1-cx^3\right))\log\left(1+cx^3\right)}{2x^4} + \frac{b^2\log$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{x^4}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{x^4}$$

$$= \frac{1}{12}\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^2}{x^2}\,dx, x, x^3\right) - \frac{1}{6}b\text{Subst}\left(\int \frac{(-2a+b\log(1$$

$$= -\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12x^3} - \frac{b(2a-b\log\left(1-cx^3\right))\log\left(1+cx^3\right)}{6x^3} -$$

$$= abc\log(x) - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12x^3} - \frac{b(2a-b\log\left(1-cx^3\right))\log(1$$

$$= abc\log(x) - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12x^3} - \frac{b(2a-b\log\left(1-cx^3\right))\log(1$$

$$= 2abc\log(x) - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12x^3} - \frac{1}{6}bc\left(2a-b\log\left(1-cx^3\right)\right)\text{lo}$$

$$= 2abc\log(x) - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12x^3} - \frac{1}{6}bc\left(2a-b\log\left(1-cx^3\right)\right)\text{lo}$$

$$= 2abc\log(x) - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12x^3} - \frac{1}{6}bc\left(2a-b\log\left(1-cx^3\right)\right)\text{lo}$$

Mathematica [A]
time = 0.11, size = 117, normalized size = 1.30

$$\frac{b^2\left(-1+cx^3\right)\tanh^{-1}\left(cx^3\right)^2 + 2b\tanh^{-1}\left(cx^3\right)\left(-a+bcx^3\log\left(1-e^{-2\tanh^{-1}\left(cx^3\right)}\right)\right) - a\left(a-2bcx^3\log\left(cx^3\right)+bcx^3\log\left(1-c^2x^6\right)\right) - b^2cx^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(cx^3\right)}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^2/x^4,x]
```

```
[Out] (b^2*(-1 + c*x^3)*ArcTanh[c*x^3]^2 + 2*b*ArcTanh[c*x^3]*(-a + b*c*x^3*Log[1
 - E^(-2*ArcTanh[c*x^3])]) - a*(a - 2*b*c*x^3*Log[c*x^3] + b*c*x^3*Log[1 -
c^2*x^6]) - b^2*c*x^3*PolyLog[2, E^(-2*ArcTanh[c*x^3])])/(3*x^3)
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\,x^{3}\right)\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^2/x^4,x)
```

```
[Out] int((a+b*arctanh(c*x^3))^2/x^4,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(c*(log(c^2*x^6 - 1) - log(x^6)) + 2*arctanh(c*x^3)/x^3)*a*b - 1/12*b^
2*(log(-c*x^3 + 1)^2/x^3 + 3*integrate(-((c*x^3 - 1)*log(c*x^3 + 1)^2 + 2*(
c*x^3 - (c*x^3 - 1)*log(c*x^3 + 1))*log(-c*x^3 + 1))/(c*x^7 - x^4), x)) - 1
/3*a^2/x^3
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2)/x^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**2/x**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,x^3\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))^2/x^4,x)
```

```
[Out] int((a + b*atanh(c*x^3))^2/x^4, x)
```

3.122 $\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{x^7}\,dx$

Optimal. Leaf size=88

$$-\frac{bc\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{3x^3}+\frac{1}{6}c^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{6x^6}+b^2c^2\log(x)-\frac{1}{6}b^2c^2\log\left(1-c^2x^6\right)$$

```
[Out] -1/3*b*c*(a+b*arctanh(c*x^3))/x^3+1/6*c^2*(a+b*arctanh(c*x^3))^2-1/6*(a+b*a
rctanh(c*x^3))^2/x^6+b^2*c^2*ln(x)-1/6*b^2*c^2*ln(-c^2*x^6+1)
```

Rubi [A]
time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6039, 6037, 6129, 272, 36, 29, 31, 6095}

$$\frac{1}{6}c^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2-\frac{bc\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{3x^3}-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{6x^6}-\frac{1}{6}b^2c^2\log\left(1-c^2x^6\right)+b^2c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])^2/x^7,x]
```

```
[Out] -1/3*(b*c*(a + b*ArcTanh[c*x^3]))/x^3 + (c^2*(a + b*ArcTanh[c*x^3])^2)/6 -
(a + b*ArcTanh[c*x^3])^2/(6*x^6) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 - c^2*x^
6])/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{x^7}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{4x^7} - \frac{b(-2a+b\log\left(1-cx^3\right))\log\left(1+cx^3\right)}{2x^7} + \frac{b^2\log^2}{4}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{x^7}\,dx - \frac{1}{2}b\int \frac{(-2a+b\log\left(1-cx^3\right))\log\left(1+cx^3\right)}{x^7}\,dx$$

$$= \frac{1}{12}\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^2}{x^3}\,dx, x, x^3\right) - \frac{1}{6}b\text{Subst}\left(\int \frac{(-2a+b\log(1-}{x}\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{24x^6} - \frac{b(2a-b\log\left(1-cx^3\right))\log\left(1+cx^3\right)}{12x^6} - \frac{b^2\log^2\left(1+\right.}{24x^6}$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{24x^6} - \frac{b(2a-b\log\left(1-cx^3\right))\log\left(1+cx^3\right)}{12x^6} - \frac{b^2\log^2\left(1+\right.}{24x^6}$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{24x^6} - \frac{b(2a-b\log\left(1-cx^3\right))\log\left(1+cx^3\right)}{12x^6} - \frac{b^2\log^2\left(1+\right.}{24x^6}$$

$$= -\frac{1}{2}abc^2\log(x) - \frac{bc(2a-b\log\left(1-cx^3\right))}{12x^3} - \frac{bc(1-cx^3)\left(2a-b\log\left(1-cx^3\right)\right)}{12x^3}$$

$$= \frac{1}{4}b^2c^2\log(x) - \frac{bc(2a-b\log\left(1-cx^3\right))}{12x^3} - \frac{bc(1-cx^3)\left(2a-b\log\left(1-cx^3\right)\right)}{12x^3} +$$

$$= \frac{1}{2}b^2c^2\log(x) - \frac{1}{12}b^2c^2\log\left(1-cx^3\right) - \frac{bc(2a-b\log\left(1-cx^3\right))}{12x^3} - \frac{bc(1-cx^3)\left(\right.}{}$$

Mathematica [A]
time = 0.05, size = 111, normalized size = 1.26

$$\frac{1}{6}\left(-\frac{a^2}{x^6} - \frac{2abc}{x^3} - \frac{2b(a+bcx^3)\tanh^{-1}\left(cx^3\right)}{x^6} + \frac{b^2(-1+c^2x^6)\tanh^{-1}\left(cx^3\right)^2}{x^6} + 6b^2c^2\log(x) - b(a+b)c^2\log\left(1-cx^3\right) + (a-b)bc^2\log\left(1+cx^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^2/x^7,x]
```

```
[Out] (-(a^2/x^6) - (2*a*b*c)/x^3 - (2*b*(a + b*c*x^3)*ArcTanh[c*x^3])/x^6 + (b^2
*(-1 + c^2*x^6)*ArcTanh[c*x^3]^2)/x^6 + 6*b^2*c^2*Log[x] - b*(a + b)*c^2*Lo
g[1 - c*x^3] + (a - b)*b*c^2*Log[1 + c*x^3])/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(80) = 160$.
time = 0.17, size = 257, normalized size = 2.92

method	result

risch	$\frac{b^2(c^2x^6-1)\ln(cx^3+1)^2}{24x^6} - \frac{b(bc^2\ln(-cx^3+1)x^6+2bcx^3-b\ln(-cx^3+1)+2a)\ln(cx^3+1)}{12x^6} + \frac{b^2c^2x^6\ln(-cx^3+1)^2+4bc^2\ln(cx^3-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^2/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*b^2*(c^2*x^6-1)/x^6*ln(c*x^3+1)^2-1/12*b*(b*c^2*ln(-c*x^3+1)*x^6+2*b*c
*x^3-b*ln(-c*x^3+1)+2*a)/x^6*ln(c*x^3+1)+1/24*(b^2*c^2*x^6*ln(-c*x^3+1)^2+4
*b*c^2*ln(c*x^3+1)*x^6*a-4*b^2*c^2*ln(c*x^3+1)*x^6-4*b*c^2*ln(c*x^3-1)*x^6*
a-4*b^2*c^2*ln(c*x^3-1)*x^6+24*b^2*c^2*ln(x)*x^6+4*b^2*c*x^3*ln(-c*x^3+1)-8
*a*b*c*x^3-b^2*ln(-c*x^3+1)^2+4*b*ln(-c*x^3+1)*a-4*a^2)/x^6
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(80) = 160$.
time = 0.28, size = 175, normalized size = 1.99

$$\frac{1}{6}\left(\left(c\log\left(cx^3+1\right)-c\log\left(cx^3-1\right)-\frac{2}{x^3}\right)c-\frac{2\,\mathrm{artanh}\left(cx^3\right)}{x^6}\right)ab+\frac{1}{24}\left(\left(2\left(\log\left(cx^3-1\right)-2\right)\log\left(cx^3+1\right)-\log\left(cx^3+1\right)^2-\log\left(cx^3-1\right)^2-4\log\left(cx^3-1\right)+24\log\left(x\right)\right)c^2+4\left(c\log\left(cx^3+1\right)-c\log\left(cx^3-1\right)-\frac{2}{x^3}\right)c\,\mathrm{artanh}\left(cx^3\right)\right)b^2-\frac{b^2\,\mathrm{artanh}\left(cx^3\right)^2}{6\,x^6}-\frac{a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*((c*log(c*x^3 + 1) - c*log(c*x^3 - 1) - 2/x^3)*c - 2*arctanh(c*x^3)/x^6
)*a*b + 1/24*((2*(log(c*x^3 - 1) - 2)*log(c*x^3 + 1) - log(c*x^3 + 1)^2 - l
og(c*x^3 - 1)^2 - 4*log(c*x^3 - 1) + 24*log(x))*c^2 + 4*(c*log(c*x^3 + 1) -
 c*log(c*x^3 - 1) - 2/x^3)*c*arctanh(c*x^3))*b^2 - 1/6*b^2*arctanh(c*x^3)^2
/x^6 - 1/6*a^2/x^6
```

Fricas [**A**]
time = 0.40, size = 151, normalized size = 1.72

$$\frac{24\,b^2c^2x^6\log\left(x\right)+4\,(ab-b^2)c^2x^6\log\left(cx^3+1\right)-4\,(ab+b^2)c^2x^6\log\left(cx^3-1\right)-8\,abcx^3+(b^2c^2x^6-b^2)\log\left(-\frac{cx^3+1}{cx^3-1}\right)^2-4\,a^2-4\,(b^2cx^3+ab)\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{24\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/24*(24*b^2*c^2*x^6*log(x) + 4*(a*b - b^2)*c^2*x^6*log(c*x^3 + 1) - 4*(a*b
 + b^2)*c^2*x^6*log(c*x^3 - 1) - 8*a*b*c*x^3 + (b^2*c^2*x^6 - b^2)*log(-(c*
x^3 + 1)/(c*x^3 - 1))^2 - 4*a^2 - 4*(b^2*c*x^3 + a*b)*log(-(c*x^3 + 1)/(c*x
^3 - 1)))/x^6
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**2/x**7,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2/x^7, x)
```

Mupad [**B**]
time = 1.54, size = 278, normalized size = 3.16

$$\frac{b^2\,c^2\,\ln\left(c\,x^3+1\right)^2}{24}-\frac{b^2\,c^2\,\ln\left(c\,x^3-1\right)}{6}-\frac{b^2\,c^2\,\ln\left(c\,x^3+1\right)}{6}-\frac{a^2}{6\,x^6}+\frac{b^2\,c^2\,\ln\left(1-c\,x^3\right)^2}{24}-\frac{b^2\,\ln\left(c\,x^3+1\right)^2}{24\,x^6}-\frac{b^2\,\ln\left(1-c\,x^3\right)^2}{24\,x^6}+b^2\,c^2\,\ln\left(x\right)-\frac{a\,b\,c^2\,\ln\left(c\,x^3-1\right)}{6}+\frac{a\,b\,c^2\,\ln\left(c\,x^3+1\right)}{6}-\frac{a\,b\,c}{3\,x^3}-\frac{a\,b\,\ln\left(c\,x^3+1\right)}{6\,x^6}+\frac{a\,b\,\ln\left(1-c\,x^3\right)}{6\,x^6}-\frac{b^2\,c^2\,\ln\left(c\,x^3+1\right)\,\ln\left(1-c\,x^3\right)}{12}-\frac{b^2\,c\,\ln\left(c\,x^3+1\right)}{6\,x^3}+\frac{b^2\,c\,\ln\left(1-c\,x^3\right)}{6\,x^3}+\frac{b^2\,\ln\left(c\,x^3+1\right)\,\ln\left(1-c\,x^3\right)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))^2/x^7,x)
```

```
[Out] (b^2*c^2*log(c*x^3 + 1)^2)/24 - (b^2*c^2*log(c*x^3 - 1))/6 - (b^2*c^2*log(c
*x^3 + 1))/6 - a^2/(6*x^6) + (b^2*c^2*log(1 - c*x^3)^2)/24 - (b^2*log(c*x^3
 + 1)^2)/(24*x^6) - (b^2*log(1 - c*x^3)^2)/(24*x^6) + b^2*c^2*log(x) - (a*b
*c^2*log(c*x^3 - 1))/6 + (a*b*c^2*log(c*x^3 + 1))/6 - (a*b*c)/(3*x^3) - (a*
b*log(c*x^3 + 1))/(6*x^6) + (a*b*log(1 - c*x^3))/(6*x^6) - (b^2*c^2*log(c*x
^3 + 1)*log(1 - c*x^3))/12 - (b^2*c*log(c*x^3 + 1))/(6*x^3) + (b^2*c*log(1
- c*x^3))/(6*x^3) + (b^2*log(c*x^3 + 1)*log(1 - c*x^3))/(12*x^6)
```

3.123 $\int \frac{\left(a+b\tanh^{-1}(cx^3)\right)^2}{x^{10}}\,dx$

Optimal. Leaf size=144

$$-\frac{b^2c^2}{9x^3}+\frac{1}{9}b^2c^3\tanh^{-1}\left(cx^3\right)-\frac{bc\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{9x^6}+\frac{1}{9}c^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{9x^9}+\frac{2}{9}bc$$

```
[Out] -1/9*b^2*c^2/x^3+1/9*b^2*c^3*arctanh(c*x^3)-1/9*b*c*(a+b*arctanh(c*x^3))/x^
6+1/9*c^3*(a+b*arctanh(c*x^3))^2-1/9*(a+b*arctanh(c*x^3))^2/x^9+2/9*b*c^3*(
a+b*arctanh(c*x^3))*ln(2-2/(c*x^3+1))-1/9*b^2*c^3*polylog(2,-1+2/(c*x^3+1))
```

Rubi [A]
time = 0.19, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6039, 6037, 6129, 331, 212, 6135, 6079, 2497}

$$\frac{1}{9}c^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{2}{9}bc^3\log\left(2-\frac{2}{cx^3+1}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{9x^9}-\frac{bc\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{9x^6}-\frac{1}{9}b^2c^3\text{Li}_2\left(\frac{2}{cx^3+1}-1\right)+\frac{1}{9}b^2c^3\tanh^{-1}\left(cx^3\right)-\frac{b^2c^2}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])^2/x^10,x]
```

```
[Out] -1/9*(b^2*c^2)/x^3 + (b^2*c^3*ArcTanh[c*x^3])/9 - (b*c*(a + b*ArcTanh[c*x^3
]))/(9*x^6) + (c^3*(a + b*ArcTanh[c*x^3])^2)/9 - (a + b*ArcTanh[c*x^3])^2/(
9*x^9) + (2*b*c^3*(a + b*ArcTanh[c*x^3])*Log[2 - 2/(1 + c*x^3)])/9 - (b^2*c
^3*PolyLog[2, -1 + 2/(1 + c*x^3)])/9
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{x^{10}}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{4x^{10}} - \frac{b\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{2x^{10}} + \frac{b^2\log\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{x^{10}}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{x^{10}}$$

$$= \frac{1}{12}\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^2}{x^4}\,dx, x, x^3\right) - \frac{1}{6}b\text{Subst}\left(\int \frac{(-2a+b\log(1\right.\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{36x^9} - \frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{18x^9} - \frac{b^2\log^2\left(1-\right.}{36x^9}$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{36x^9} - \frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{18x^9} - \frac{b^2\log^2\left(1-\right.}{36x^9}$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{36x^9} - \frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{18x^9} - \frac{b^2\log^2\left(1-\right.}{36x^9}$$

$$= \frac{1}{3}abc^3\log(x) - \frac{bc\left(2a-b\log\left(1-cx^3\right)\right)}{18x^6} + \frac{bc^2\left(2a-b\log\left(1-cx^3\right)\right)}{18x^3} - \frac{(2a-}{}$$

$$= \frac{1}{3}abc^3\log(x) - \frac{bc\left(2a-b\log\left(1-cx^3\right)\right)}{18x^6} + \frac{bc^2\left(2a-b\log\left(1-cx^3\right)\right)}{18x^3} - \frac{bc^2(1-}{}$$

$$= -\frac{b^2c^2}{18x^3} + \frac{2}{3}abc^3\log(x) + \frac{1}{6}b^2c^3\log(x) - \frac{bc\left(2a-b\log\left(1-cx^3\right)\right)}{18x^6} + \frac{bc^2(2a-}{}$$

$$= -\frac{b^2c^2}{18x^3} + \frac{2}{3}abc^3\log(x) + \frac{1}{6}b^2c^3\log(x) - \frac{bc\left(2a-b\log\left(1-cx^3\right)\right)}{18x^6} + \frac{bc^2(2a-}{}$$

Mathematica [A]
time = 0.27, size = 159, normalized size = 1.10

$$-\frac{a^2+abcx^3+b^2c^2x^6+b^2(1-c^3x^9)\tanh^{-1}(cx^3)^2+b\tanh^{-1}(cx^3)\left(2a+bcx^3-bc^3x^9-2bc^3x^9\log\left(1-e^{-2\tanh^{-1}(cx^3)}\right)\right)-2abc^3x^9\log(cx^3)+abc^3x^9\log(1-c^2x^6)+b^2c^3x^9\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx^3)}\right)}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^2/x^10,x]
```

```
[Out] -1/9*(a^2 + a*b*c*x^3 + b^2*c^2*x^6 + b^2*(1 - c^3*x^9)*ArcTanh[c*x^3]^2 +
b*ArcTanh[c*x^3]*(2*a + b*c*x^3 - b*c^3*x^9 - 2*b*c^3*x^9*Log[1 - E^(-2*Arc
Tanh[c*x^3])]) - 2*a*b*c^3*x^9*Log[c*x^3] + a*b*c^3*x^9*Log[1 - c^2*x^6] +
b^2*c^3*x^9*PolyLog[2, E^(-2*ArcTanh[c*x^3])])/x^9
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\,x^3\right)\right)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^2/x^10,x)
```

```
[Out] int((a+b*arctanh(c*x^3))^2/x^10,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/9*((c^2*log(c^2*x^6 - 1) - c^2*log(x^6) + 1/x^6)*c + 2*arctanh(c*x^3)/x^
9)*a*b - 1/36*b^2*(log(-c*x^3 + 1)^2/x^9 + 9*integrate(-1/3*(3*(c*x^3 - 1)*
log(c*x^3 + 1)^2 + 2*(c*x^3 - 3*(c*x^3 - 1)*log(c*x^3 + 1))*log(-c*x^3 + 1)
)/(c*x^13 - x^10), x)) - 1/9*a^2/x^9
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^10,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2)/x^10, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**2/x**10,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^10,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2/x^10, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atanh}(c\,x^3))^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))^2/x^10,x)
[Out] int((a + b*atanh(c*x^3))^2/x^10, x)
```

3.124 $\int x^8 \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3 \, dx$

Optimal. Leaf size=231

$$\frac{ab^2x^3}{3c^2}+\frac{b^3x^3\tanh^{-1}\left(cx^3\right)}{3c^2}-\frac{b\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{6c^3}+\frac{bx^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{6c}+\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{9c^3}+\frac{1}{9}x^9\left(a\right.$$

```
[Out] 1/3*a*b^2*x^3/c^2+1/3*b^3*x^3*arctanh(c*x^3)/c^2-1/6*b*(a+b*arctanh(c*x^3))
^2/c^3+1/6*b*x^6*(a+b*arctanh(c*x^3))^2/c+1/9*(a+b*arctanh(c*x^3))^3/c^3+1/
9*x^9*(a+b*arctanh(c*x^3))^3-1/3*b*(a+b*arctanh(c*x^3))^2*ln(2/(-c*x^3+1))/
c^3+1/6*b^3*ln(-c^2*x^6+1)/c^3-1/3*b^2*(a+b*arctanh(c*x^3))*polylog(2,1-2/(
-c*x^3+1))/c^3+1/6*b^3*polylog(3,1-2/(-c*x^3+1))/c^3
```

Rubi [A]
time = 0.37, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6039, 6037, 6127, 6021, 266, 6095, 6131, 6055, 6205, 6745}

$$-\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{3c^3}+\frac{ab^2x^3}{3c^2}+\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{9c^3}-\frac{b\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{6c^3}-\frac{b\log\left(\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{3c^3}+\frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3+\frac{bx^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{6c}+\frac{b^3\text{Li}_3\left(1-\frac{2}{1-cx^3}\right)}{6c^3}+\frac{b^3x^3\tanh^{-1}\left(cx^3\right)}{3c^2}+\frac{b^3\log\left(1-c^2x^6\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*ArcTanh[c*x^3])^3,x]
```

```
[Out] (a*b^2*x^3)/(3*c^2) + (b^3*x^3*ArcTanh[c*x^3])/(3*c^2) - (b*(a + b*ArcTanh[
c*x^3])^2)/(6*c^3) + (b*x^6*(a + b*ArcTanh[c*x^3])^2)/(6*c) + (a + b*ArcTan
h[c*x^3])^3/(9*c^3) + (x^9*(a + b*ArcTanh[c*x^3])^3)/9 - (b*(a + b*ArcTanh[
c*x^3])^2*Log[2/(1 - c*x^3)])/(3*c^3) + (b^3*Log[1 - c^2*x^6])/(6*c^3) - (b
^2*(a + b*ArcTanh[c*x^3])*PolyLog[2, 1 - 2/(1 - c*x^3)])/(3*c^3) + (b^3*Pol
yLog[3, 1 - 2/(1 - c*x^3)])/(6*c^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
```

```
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x^8\big(a+b\tanh^{-1}\left(cx^3\right)\big)^3\,dx = \int \bigg(\frac{1}{8}x^8\big(2a-b\log\left(1-cx^3\right)\big)^3+\frac{3}{8}bx^8\big(-2a+b\log\left(1-cx^3\right)\big)^2\log\left(1+c\right.$$

$$= \frac{1}{8}\int x^8\big(2a-b\log\left(1-cx^3\right)\big)^3\,dx+\frac{1}{8}(3b)\int x^8\big(-2a+b\log\left(1-cx^3\right)\big)^2\,\mathrm{lo}$$

$$= \frac{1}{24}\mathrm{Subst}\bigg(\int x^2(2a-b\log(1-cx))^3\,dx,x,x^3\bigg)+\frac{1}{8}b\mathrm{Subst}\bigg(\int x^2(-2a+b$$

$$= \frac{1}{24}bx^9\big(2a-b\log\left(1-cx^3\right)\big)^2\log\left(1+cx^3\right)+\frac{1}{24}b^2x^9\big(2a-b\log\left(1-cx^3\right)\big)$$

$$= \frac{1}{24}bx^9\big(2a-b\log\left(1-cx^3\right)\big)^2\log\left(1+cx^3\right)+\frac{1}{24}b^2x^9\big(2a-b\log\left(1-cx^3\right)\big)$$

$$= \frac{1}{24}bx^9\big(2a-b\log\left(1-cx^3\right)\big)^2\log\left(1+cx^3\right)+\frac{1}{24}b^2x^9\big(2a-b\log\left(1-cx^3\right)\big)$$

$$= -\frac{1}{72}bx^9\big(2a-b\log\left(1-cx^3\right)\big)^2-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c^3}+\frac{(1-}{}$$

$$= -\frac{1}{72}bx^9\big(2a-b\log\left(1-cx^3\right)\big)^2-\frac{b\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c^3}+\frac{b(1}{}$$

$$= \frac{ab^2x^3}{3c^2}-\frac{b^3x^3}{6c^2}+\frac{b^3\left(1-cx^3\right)^2}{32c^3}-\frac{b^3\left(1-cx^3\right)^3}{324c^3}+\frac{b^3\left(1+cx^3\right)^2}{32c^3}-\frac{b^3\left(1+cx^3\right.}{324c^3}$$

$$= \frac{ab^2x^3}{3c^2}+\frac{b^3\left(1-cx^3\right)^2}{32c^3}-\frac{b^3\left(1-cx^3\right)^3}{324c^3}+\frac{b^3\left(1+cx^3\right)^2}{32c^3}-\frac{b^3\left(1+cx^3\right)^3}{324c^3}+\frac{b^3(}{}$$

$$= \frac{ab^2x^3}{2c^2}-\frac{b^3x^3}{18c^2}+\frac{b^3\left(1-cx^3\right)^2}{24c^3}-\frac{b^3\left(1-cx^3\right)^3}{324c^3}+\frac{b^3\left(1+cx^3\right)^2}{24c^3}-\frac{b^3\left(1+cx^3\right.}{324c^3}$$

$$= \frac{ab^2x^3}{2c^2}-\frac{7b^3x^3}{216c^2}-\frac{5b^3x^6}{432c}+\frac{b^3x^9}{324}+\frac{b^3\left(1-cx^3\right)^2}{48c^3}+\frac{b^3\left(1+cx^3\right)^2}{24c^3}-\frac{b^3\left(1+c\right.}{324c^3}$$

$$= \frac{ab^2x^3}{2c^2}-\frac{7b^3x^3}{216c^2}-\frac{5b^3x^6}{432c}+\frac{b^3x^9}{324}+\frac{b^3\left(1-cx^3\right)^2}{48c^3}+\frac{b^3\left(1+cx^3\right)^2}{24c^3}-\frac{b^3\left(1+c\right.}{324c^3}$$

Mathematica [A]

time = 0.34, size = 334, normalized size = 1.45

$$\frac{6ab^2cx^3+3a^2bc^2x^6+2a^3c^3x^9-6ab^2\tanh^{-1}\left(cx^3\right)+6b^3cx^3\tanh^{-1}\left(cx^3\right)+6ab^2c^2x^6\tanh^{-1}\left(cx^3\right)+6a^2bc^3x^9\tanh^{-1}\left(cx^3\right)-6ab^2\tanh^{-1}\left(cx^3\right)^2-3b^3\tanh^{-1}\left(cx^3\right)^2+3b^3c^2x^6\tanh^{-1}\left(cx^3\right)^2+6ab^2c^3x^9\tanh^{-1}\left(cx^3\right)^2-2b^3\tanh^{-1}\left(cx^3\right)^3+2b^3c^3x^9\tanh^{-1}\left(cx^3\right)^3-12ab^2\tanh^{-1}\left(cx^3\right)\log\left(1+e^{-2\tanh^{-1}\left(cx^3\right)}\right)-6b^3\tanh^{-1}\left(cx^3\right)^2\log\left(1+e^{-2\tanh^{-1}\left(cx^3\right)}\right)+3a^2b\log\left(1-c^2x^6\right)+3b^3\log\left(1-c^2x^6\right)+6b^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(cx^3\right)}\right)+3b^3\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}\left(cx^3\right)}\right)}{18c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*ArcTanh[c*x^3])^3,x]
```

```
[Out] (6*a*b^2*c*x^3 + 3*a^2*b*c^2*x^6 + 2*a^3*c^3*x^9 - 6*a*b^2*ArcTanh[c*x^3] +
 6*b^3*c*x^3*ArcTanh[c*x^3] + 6*a*b^2*c^2*x^6*ArcTanh[c*x^3] + 6*a^2*b*c^3*
x^9*ArcTanh[c*x^3] - 6*a*b^2*ArcTanh[c*x^3]^2 - 3*b^3*ArcTanh[c*x^3]^2 + 3*
b^3*c^2*x^6*ArcTanh[c*x^3]^2 + 6*a*b^2*c^3*x^9*ArcTanh[c*x^3]^2 - 2*b^3*Arc
Tanh[c*x^3]^3 + 2*b^3*c^3*x^9*ArcTanh[c*x^3]^3 - 12*a*b^2*ArcTanh[c*x^3]*Lo
g[1 + E^(-2*ArcTanh[c*x^3])] - 6*b^3*ArcTanh[c*x^3]^2*Log[1 + E^(-2*ArcTanh
[c*x^3])] + 3*a^2*b*Log[1 - c^2*x^6] + 3*b^3*Log[1 - c^2*x^6] + 6*b^2*(a +
b*ArcTanh[c*x^3])*PolyLog[2, -E^(-2*ArcTanh[c*x^3])] + 3*b^3*PolyLog[3, -E^
(-2*ArcTanh[c*x^3])])/(18*c^3)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^{8}\left(a+b\,\mathrm{arctanh}\left(c\,x^{3}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*arctanh(c*x^3))^3,x)
```

```
[Out] int(x^8*(a+b*arctanh(c*x^3))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] 1/9*a^3*x^9 + 1/6*(2*x^9*arctanh(c*x^3) + (x^6/c^2 + log(c^2*x^6 - 1)/c^4)*
c)*a^2*b - 1/72*((b^3*c^3*x^9 - b^3)*log(-c*x^3 + 1)^3 - 3*(2*a*b^2*c^3*x^9
 + b^3*c^2*x^6 + (b^3*c^3*x^9 + b^3)*log(c*x^3 + 1))*log(-c*x^3 + 1)^2)/c^3
 - integrate(-1/8*((b^3*c^3*x^11 - b^3*c^2*x^8)*log(c*x^3 + 1)^3 + 6*(a*b^2
*c^3*x^11 - a*b^2*c^2*x^8)*log(c*x^3 + 1)^2 - (4*a*b^2*c^3*x^11 + 2*b^3*c^2
*x^8 + 3*(b^3*c^3*x^11 - b^3*c^2*x^8)*log(c*x^3 + 1)^2 - 2*(6*a*b^2*c^2*x^8
 - (6*a*b^2*c^3 + b^3*c^3)*x^11 - b^3*x^2)*log(c*x^3 + 1))*log(-c*x^3 + 1))
/(c^3*x^3 - c^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^8*arctanh(c*x^3)^3 + 3*a*b^2*x^8*arctanh(c*x^3)^2 + 3*a^2*b*
x^8*arctanh(c*x^3) + a^3*x^8, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atanh(c*x**3))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3*x^8, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^8\,\left(a+b\,\mathrm{atanh}\left(c\,x^3\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*atanh(c*x^3))^3,x)
```

```
[Out] int(x^8*(a + b*atanh(c*x^3))^3, x)
```

3.125 $\int x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\,dx$

Optimal. Leaf size=139

$$\frac{b\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{2c^2}+\frac{bx^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{2c}-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{6c^2}+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3-\frac{b^2(a+$$

```
[Out] 1/2*b*(a+b*arctanh(c*x^3))^2/c^2+1/2*b*x^3*(a+b*arctanh(c*x^3))^2/c-1/6*(a+
b*arctanh(c*x^3))^3/c^2+1/6*x^6*(a+b*arctanh(c*x^3))^3-b^2*(a+b*arctanh(c*x
^3))*ln(2/(-c*x^3+1))/c^2-1/2*b^3*polylog(2,1-2/(-c*x^3+1))/c^2
```

Rubi [A]
time = 0.21, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {6039, 6037, 6127, 6021, 6131, 6055, 2449, 2352, 6095}

$$-\frac{b^2\log\left(\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{c^2}-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{6c^2}+\frac{b\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{2c^2}+\frac{bx^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{2c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3-\frac{b^3\text{Li}_2\left(1-\frac{2}{1-cx^3}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c*x^3])^3,x]
```

```
[Out] (b*(a + b*ArcTanh[c*x^3])^2)/(2*c^2) + (b*x^3*(a + b*ArcTanh[c*x^3])^2)/(2*
c) - (a + b*ArcTanh[c*x^3])^3/(6*c^2) + (x^6*(a + b*ArcTanh[c*x^3])^3)/6 -
(b^2*(a + b*ArcTanh[c*x^3])*Log[2/(1 - c*x^3)])/c^2 - (b^3*PolyLog[2, 1 - 2
/(1 - c*x^3)])/(2*c^2)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\,dx &= \int \left(\frac{1}{8}x^5\left(2a-b\log\left(1-cx^3\right)\right)^3+\frac{3}{8}bx^5\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx\cdots\right.\right.\\
&= \frac{1}{8}\int x^5\left(2a-b\log\left(1-cx^3\right)\right)^3\,dx+\frac{1}{8}(3b)\int x^5\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\cdots\\
&= \frac{1}{24}\text{Subst}\left(\int x(2a-b\log(1-cx))^3\,dx,x,x^3\right)+\frac{1}{8}b\text{Subst}\left(\int x(-2a+b\log\cdots\right.\\
&= \frac{1}{16}bx^6\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{16}b^2x^6\left(2a-b\log\left(1-cx^3\right)\right)\text{l}\cdots\\
&= \frac{1}{16}bx^6\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{16}b^2x^6\left(2a-b\log\left(1-cx^3\right)\right)\text{l}\cdots\\
&= \frac{1}{16}bx^6\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{16}b^2x^6\left(2a-b\log\left(1-cx^3\right)\right)\text{l}\cdots\\
&= -\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c^2}+\frac{\left(1-cx^3\right)^2\left(2a-b\log\left(1-cx^3\right)\right)^3}{48c^2}-\frac{b(\cdots}{\cdots}\\
&= -\frac{3b\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{16c^2}+\frac{b\left(1-cx^3\right)^2\left(2a-b\log\left(1-cx^3\right)\right)^2}{32c^2}-\cdots\\
&= \frac{3ab^2x^3}{4c}+\frac{3b^3x^3}{8c}+\frac{b^3\left(1-cx^3\right)^2}{64c^2}-\frac{b^3\left(1+cx^3\right)^2}{64c^2}+\frac{b^2\left(1-cx^3\right)^2\left(2a-b\log\left(1\cdots\right.\right.}{32c^2}\\
&= \frac{3ab^2x^3}{4c}+\frac{3b^3x^3}{4c}+\frac{b^3\left(1-cx^3\right)^2}{64c^2}-\frac{b^3\left(1+cx^3\right)^2}{64c^2}+\frac{3b^3\left(1-cx^3\right)\log\left(1-cx^3\cdots\right.}{8c^2}\\
&= \frac{ab^2x^3}{2c}+\frac{5b^3x^3}{8c}+\frac{3b^3\left(1-cx^3\right)\log\left(1-cx^3\right)}{8c^2}-\frac{b\left(1-cx^3\right)\left(2a-b\log\left(1-c\cdots\right.\right.}{8c^2}\\
&= \frac{ab^2x^3}{2c}+\frac{b^3x^3}{2c}+\frac{b^3\left(1-cx^3\right)\log\left(1-cx^3\right)}{4c^2}-\frac{b\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\cdots\right.}{8c^2}
\end{aligned}
$$

<u>Mathematica</u> [A]
time = 0.20, size = 185, normalized size = 1.33

$$
\frac{6b^2(-1+cx^3)\left(a+b+acx^3\right)\tanh^{-1}\left(cx^3\right)^2+2b^3(-1+c^2x^6)\tanh^{-1}\left(cx^3\right)^3+6b\tanh^{-1}\left(cx^3\right)\left(acx^3(2b+acx^3)-2b^2\log\left(1+e^{-2\tanh^{-1}\left(cx^3\right)}\right)\right)+a(6abcx^3+2a^2c^2x^6+3ab\log\left(1-cx^3\right)-3ab\log\left(1+cx^3\right)+6b^2\log\left(1-c^2x^6\right))+6b^3\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(cx^3\right)}\right)}{12c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x^3])^3,x]
```

```
[Out] (6*b^2*(-1 + c*x^3)*(a + b + a*c*x^3)*ArcTanh[c*x^3]^2 + 2*b^3*(-1 + c^2*x^
6)*ArcTanh[c*x^3]^3 + 6*b*ArcTanh[c*x^3]*(a*c*x^3*(2*b + a*c*x^3) - 2*b^2*L
og[1 + E^(-2*ArcTanh[c*x^3])]) + a*(6*a*b*c*x^3 + 2*a^2*c^2*x^6 + 3*a*b*Log
[1 - c*x^3] - 3*a*b*Log[1 + c*x^3] + 6*b^2*Log[1 - c^2*x^6]) + 6*b^3*PolyLo
g[2, -E^(-2*ArcTanh[c*x^3])])/(12*c^2)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.37, size = 799, normalized size = 5.75

method	result	size
risch	Expression too large to display	799

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x^3))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*b^3*(c^2*x^6-1)/c^2*ln(c*x^3+1)^3+1/16*b^2*(-b*c^2*ln(-c*x^3+1)*x^6+2*
a*c^2*x^6+2*b*c*x^3+b*ln(-c*x^3+1)-2*a+2*b)/c^2*ln(c*x^3+1)^2+(1/16*b^3*(c^
2*x^6-1)/c^2*ln(-c*x^3+1)^2-1/16*b^2*(2*a*c*x^3+b)^2/a/c^2*ln(-c*x^3+1)+1/1
6*b*(4*a^3*c^2*x^6+8*a^2*b*c*x^3+4*ln(-c*x^3+1)*a^2*b+4*ln(-c*x^3+1)*a*b^2+
ln(-c*x^3+1)*b^3+4*a*b^2)/a/c^2)*ln(c*x^3+1)-1/4*b/c^2*ln(c*x^3+1)*a^2+1/2*
b^2/c^2*ln(c*x^3+1)*a+1/8*b^2/c^2*a*ln(c*x^3-1)+3/4*b^2/c*Sum(-2/3*(ln(x-_a
lpha)*ln(-c*x^3+1)+3*c*(-1/3*ln(x-_alpha)*(ln((RootOf(_Z^2+_Z*_alpha+_alpha
^2,index=1)-x+_alpha)/RootOf(_Z^2+_Z*_alpha+_alpha^2,index=1))+ln((RootOf(_
Z^2+_Z*_alpha+_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+_Z*_alpha+_alpha^2,in
dex=2))+ln(1/2*(x+_alpha)/_alpha))/c-1/3*(dilog((RootOf(_Z^2+_Z*_alpha+_alp
ha^2,index=1)-x+_alpha)/RootOf(_Z^2+_Z*_alpha+_alpha^2,index=1))+dilog((Roo
tOf(_Z^2+_Z*_alpha+_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+_Z*_alpha+_alpha
^2,index=2))+dilog(1/2*(x+_alpha)/_alpha))/c))*b/c,_alpha=RootOf(_Z^3*c+1))
+1/8*b^3/c*x^3*ln(-c*x^3+1)^2-1/4*a^2*b*x^6*ln(-c*x^3+1)+1/4*a^2*b/c^2*ln(c
*x^3-1)+1/8*a*b^2*x^6*ln(-c*x^3+1)^2+3/8*a*b^2/c^2*ln(-c*x^3+1)-1/8*a*b^2/c
^2*ln(-c*x^3+1)^2-1/2*a*b^2/c*x^3*ln(-c*x^3+1)-1/8*b^3/c^2*ln(-c*x^3+1)^2+1
/48*b^3/c^2*ln(-c*x^3+1)^3-1/8*b^3/c^2+1/6*a^3*x^6-1/4/c^2*b^3*ln(c*x^3-1)-
1/4/c^2*b^3*ln(c*x^3+1)+1/4/c^2*b^3*ln(-c*x^3+1)-1/48*b^3*x^6*ln(-c*x^3+1)^
3+1/2/c*a^2*b*x^3
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*b^2*x^6*arctanh(c*x^3)^2 + 1/6*a^3*x^6 + 1/4*(2*x^6*arctanh(c*x^3) +
c*(2*x^3/c^2 - log(c*x^3 + 1)/c^3 + log(c*x^3 - 1)/c^3))*a^2*b + 1/8*(4*c*(
```

```
2*x^3/c^2 - log(c*x^3 + 1)/c^3 + log(c*x^3 - 1)/c^3)*arctanh(c*x^3) - (2*(l
og(c*x^3 - 1) - 2)*log(c*x^3 + 1) - log(c*x^3 + 1)^2 - log(c*x^3 - 1)^2 - 4
*log(c*x^3 - 1))/c^2)*a*b^2 - 1/192*(4*x^6*log(-c*x^3 + 1)^3 + 3*(x^6/c^3 +
 log(c^2*x^6 - 1)/c^5)*c^3 - 6*c*((c*x^6 + 2*x^3)/c^2 + 2*log(c*x^3 - 1)/c^
3)*log(-c*x^3 + 1)^2 + 21*c^2*(2*x^3/c^3 - log(c*x^3 + 1)/c^4 + log(c*x^3 -
 1)/c^4) + c*(6*(c^2*x^6 + 6*c*x^3 + 2*log(c*x^3 - 1)^2 + 6*log(c*x^3 - 1))
*log(-c*x^3 + 1)/c^3 - (3*c^2*x^6 + 42*c*x^3 + 4*log(c*x^3 - 1)^3 + 18*log(
c*x^3 - 1)^2 + 42*log(c*x^3 - 1))/c^3) - 1728*c*integrate(1/4*x^5*log(c*x^3
 + 1)/(c^3*x^6 - c), x) - 2*(12*c*x^3*log(c*x^3 + 1)^2 + 2*(c^2*x^6 - 1)*lo
g(c*x^3 + 1)^3 - 3*(c^2*x^6 - 2*c*x^3 - 2*(c^2*x^6 - 1)*log(c*x^3 + 1) + 1)
*log(-c*x^3 + 1)^2 + 3*(c^2*x^6 + 6*c*x^3 - 2*(c^2*x^6 - 1)*log(c*x^3 + 1)^
2 - 8*(c*x^3 + 1)*log(c*x^3 + 1))*log(-c*x^3 + 1))/c^2 + 18*log(4*c^3*x^6 -
 4*c)/c^2 - 576*integrate(1/4*x^2*log(c*x^3 + 1)/(c^3*x^6 - c), x))*b^3
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^5*arctanh(c*x^3)^3 + 3*a*b^2*x^5*arctanh(c*x^3)^2 + 3*a^2*b*
x^5*arctanh(c*x^3) + a^3*x^5, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x**3))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3*x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left(a+b\,\mathrm{atanh}\left(c\,x^3\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atanh(c*x^3))^3,x)
[Out] int(x^5*(a + b*atanh(c*x^3))^3, x)
```

3.126 $\int x^2 \left(a + b \tanh^{-1}\left(cx^3\right)\right)^3 \, dx$

Optimal. Leaf size=130

$$\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3-\frac{b\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\log\left(\frac{2}{1-cx^3}\right)}{c}-\frac{b^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)$$

```
[Out] 1/3*(a+b*arctanh(c*x^3))^3/c+1/3*x^3*(a+b*arctanh(c*x^3))^3-b*(a+b*arctanh(
c*x^3))^2*ln(2/(-c*x^3+1))/c-b^2*(a+b*arctanh(c*x^3))*polylog(2,1-2/(-c*x^3
+1))/c+1/2*b^3*polylog(3,1-2/(-c*x^3+1))/c
```

Rubi [A]
time = 0.30, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6039, 6021, 6131, 6055, 6095, 6205, 6745}

$$-\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3+\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{3c}-\frac{b\log\left(\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{c}+\frac{b^3\text{Li}_3\left(1-\frac{2}{1-cx^3}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^3])^3,x]
```

```
[Out] (a + b*ArcTanh[c*x^3])^3/(3*c) + (x^3*(a + b*ArcTanh[c*x^3])^3)/3 - (b*(a +
 b*ArcTanh[c*x^3])^2*Log[2/(1 - c*x^3)])/c - (b^2*(a + b*ArcTanh[c*x^3])*Po
lyLog[2, 1 - 2/(1 - c*x^3)])/c + (b^3*PolyLog[3, 1 - 2/(1 - c*x^3)])/(2*c)
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\,dx &= \int\left(\frac{1}{8}x^2\left(2a-b\log\left(1-cx^3\right)\right)^3+\frac{3}{8}bx^2\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx\right.\right. \\
&= \frac{1}{8}\int x^2\left(2a-b\log\left(1-cx^3\right)\right)^3\,dx+\frac{1}{8}(3b)\int x^2\left(-2a+b\log\left(1-cx^3\right)\right)^2\log \\
&= \frac{1}{24}\text{Subst}\left(\int(2a-b\log(1-cx))^3\,dx,x,x^3\right)+\frac{1}{8}b\text{Subst}\left(\int(-2a+b\log(1\right. \\
&= \frac{1}{8}bx^3\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{8}b^2x^3\left(2a-b\log\left(1-cx^3\right)\right)\log \\
&= -\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c}+\frac{1}{8}bx^3\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+c\right. \\
&= -\frac{b(1-cx^3)\left(2a-b\log\left(1-cx^3\right)\right)^2}{8c}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c}+\frac{1}{8} \\
&= \frac{1}{2}ab^2x^3-\frac{b^3x^3}{4}-\frac{b(1-cx^3)\left(2a-b\log\left(1-cx^3\right)\right)^2}{8c}-\frac{\left(1-cx^3\right)\left(2a-b\log\right.}{24c} \\
&= \frac{1}{2}ab^2x^3+\frac{b^3(1-cx^3)\log\left(1-cx^3\right)}{4c}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c}+\frac{b(}{} \\
&= \frac{b^3x^3}{4}+\frac{b^3(1-cx^3)\log\left(1-cx^3\right)}{4c}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c}+\frac{b(2a}{} \\
&= -\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c}+\frac{b\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(\frac{1}{2}(1+cx^3\right.}{4c} \\
&= -\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c}+\frac{b\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(\frac{1}{2}(1+cx^3\right.}{4c}
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 191, normalized size = 1.47

$$
\frac{2a^3cx^3+6a^2bcx^3\tanh^{-1}\left(cx^3\right)+3a^2b\log\left(1-c^2x^6\right)+6ab^2\left(\tanh^{-1}\left(cx^3\right)\left(\left(-1+cx^3\right)\tanh^{-1}\left(cx^3\right)-2\log\left(1+e^{-2\tanh^{-1}\left(cx^3\right)}\right)\right)+\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(cx^3\right)}\right)\right)+b^3\left(2\tanh^{-1}\left(cx^3\right)^2\left(\left(-1+cx^3\right)\tanh^{-1}\left(cx^3\right)-3\log\left(1+e^{-2\tanh^{-1}\left(cx^3\right)}\right)\right)+6\tanh^{-1}\left(cx^3\right)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(cx^3\right)}\right)+3\text{PolyLog}\left(3,-e^{-2\tanh^{-1}\left(cx^3\right)}\right)\right)}{6c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^3])^3,x]
```

```
[Out] (2*a^3*c*x^3 + 6*a^2*b*c*x^3*ArcTanh[c*x^3] + 3*a^2*b*Log[1 - c^2*x^6] + 6*
a*b^2*(ArcTanh[c*x^3]*((-1 + c*x^3)*ArcTanh[c*x^3] - 2*Log[1 + E^(-2*ArcTan
h[c*x^3])]) + PolyLog[2, -E^(-2*ArcTanh[c*x^3])]) + b^3*(2*ArcTanh[c*x^3]^2
*((-1 + c*x^3)*ArcTanh[c*x^3] - 3*Log[1 + E^(-2*ArcTanh[c*x^3])]) + 6*ArcTa
```

```
nh[c*x^3]*PolyLog[2, -E^(-2*ArcTanh[c*x^3])] + 3*PolyLog[3, -E^(-2*ArcTanh[
c*x^3])]))/(6*c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(124) = 248$.
time = 0.26, size = 280, normalized size = 2.15

method	result
derivativedivides	$\frac{a^3 c x^3 + b^3 \operatorname{arctanh}(c x^3)^3 c x^3 + b^3 \operatorname{arctanh}(c x^3)^3 - 3b^3 \operatorname{arctanh}(c x^3)^2 \ln\left(1 + \frac{(c x^3+1)^2}{-c^2 x^6+1}\right) - 3b^3 \operatorname{arctanh}(c x^3) \operatorname{polylog}\left(2\right.}{}$
default	$\frac{a^3 c x^3 + b^3 \operatorname{arctanh}(c x^3)^3 c x^3 + b^3 \operatorname{arctanh}(c x^3)^3 - 3b^3 \operatorname{arctanh}(c x^3)^2 \ln\left(1 + \frac{(c x^3+1)^2}{-c^2 x^6+1}\right) - 3b^3 \operatorname{arctanh}(c x^3) \operatorname{polylog}\left(2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^3))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/c*(a^3*c*x^3+b^3*arctanh(c*x^3)^3*c*x^3+b^3*arctanh(c*x^3)^3-3*b^3*arct
anh(c*x^3)^2*ln(1+(c*x^3+1)^2/(-c^2*x^6+1))-3*b^3*arctanh(c*x^3)*polylog(2,
-(c*x^3+1)^2/(-c^2*x^6+1))+3/2*b^3*polylog(3,-(c*x^3+1)^2/(-c^2*x^6+1))+3*a
rctanh(c*x^3)^2*a*b^2*c*x^3+3*a*b^2*arctanh(c*x^3)^2-6*arctanh(c*x^3)*ln(1+
(c*x^3+1)^2/(-c^2*x^6+1))*a*b^2-3*polylog(2,-(c*x^3+1)^2/(-c^2*x^6+1))*a*b^
2+3*a^2*b*c*x^3*arctanh(c*x^3)+3/2*a^2*b*ln(-c^2*x^6+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*x^3 + 1/2*(2*c*x^3*arctanh(c*x^3) + log(-c^2*x^6 + 1))*a^2*b/c - 1/
24*((b^3*c*x^3 - b^3)*log(-c*x^3 + 1)^3 - 3*(2*a*b^2*c*x^3 + (b^3*c*x^3 + b
^3)*log(c*x^3 + 1))*log(-c*x^3 + 1)^2)/c - integrate(-1/8*((b^3*c*x^5 - b^3
*x^2)*log(c*x^3 + 1)^3 + 6*(a*b^2*c*x^5 - a*b^2*x^2)*log(c*x^3 + 1)^2 - 3*(
4*a*b^2*c*x^5 + (b^3*c*x^5 - b^3*x^2)*log(c*x^3 + 1)^2 + 2*((2*a*b^2*c + b^
3*c)*x^5 - (2*a*b^2 - b^3)*x^2)*log(c*x^3 + 1))*log(-c*x^3 + 1))/(c*x^3 - 1
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctanh(c*x^3)^3 + 3*a*b^2*x^2*arctanh(c*x^3)^2 + 3*a^2*b*
x^2*arctanh(c*x^3) + a^3*x^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**3))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{atanh}\left(c\,x^3\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^3))^3,x)
```

```
[Out] int(x^2*(a + b*atanh(c*x^3))^3, x)
```

3.127 $\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x}\,dx$

Optimal. Leaf size=210

$$\frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)-\frac{1}{2}b\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)+\frac{1}{2}b(a$$

```
[Out] -2/3*(a+b*arctanh(c*x^3))^3*arctanh(-1+2/(-c*x^3+1))-1/2*b*(a+b*arctanh(c*x
^3))^2*polylog(2,1-2/(-c*x^3+1))+1/2*b*(a+b*arctanh(c*x^3))^2*polylog(2,-1+
2/(-c*x^3+1))+1/2*b^2*(a+b*arctanh(c*x^3))*polylog(3,1-2/(-c*x^3+1))-1/2*b^
2*(a+b*arctanh(c*x^3))*polylog(3,-1+2/(-c*x^3+1))-1/4*b^3*polylog(4,1-2/(-c
*x^3+1))+1/4*b^3*polylog(4,-1+2/(-c*x^3+1))
```

Rubi [A]

time = 0.41, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6035, 6033, 6199, 6095, 6205, 6209, 6745}

$$\frac{1}{2}b^2\text{Li}_3\left(1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{2}b^2\text{Li}_3\left(\frac{2}{1-cx^3}-1\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{2}b\text{Li}_2\left(1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{1}{2}b\text{Li}_2\left(\frac{2}{1-cx^3}-1\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{2}{3}\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3-\frac{1}{4}b^3\text{Li}_4\left(1-\frac{2}{1-cx^3}\right)+\frac{1}{4}b^3\text{Li}_4\left(\frac{2}{1-cx^3}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])^3/x,x]
```

```
[Out] (2*(a + b*ArcTanh[c*x^3])^3*ArcTanh[1 - 2/(1 - c*x^3)])/3 - (b*(a + b*ArcTa
nh[c*x^3])^2*PolyLog[2, 1 - 2/(1 - c*x^3)])/2 + (b*(a + b*ArcTanh[c*x^3])^2
*PolyLog[2, -1 + 2/(1 - c*x^3)])/2 + (b^2*(a + b*ArcTanh[c*x^3])*PolyLog[3,
 1 - 2/(1 - c*x^3)])/2 - (b^2*(a + b*ArcTanh[c*x^3])*PolyLog[3, -1 + 2/(1 -
 c*x^3)])/2 - (b^3*PolyLog[4, 1 - 2/(1 - c*x^3)])/4 + (b^3*PolyLog[4, -1 +
2/(1 - c*x^3)])/4
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6209

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}\,dx, x, x^3\right)$$

$$= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right) - (2bc)\text{Subst}\left(\int \frac{\left(a+b\tanh\right.}{}\right.$$

$$= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right) + (bc)\text{Subst}\left(\int \frac{\left(a+b\tanh^{-}\right.}{1}\right.$$

$$= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right) - \frac{1}{2}b\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\text{Li}$$

$$= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right) - \frac{1}{2}b\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\text{Li}$$

$$= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right) - \frac{1}{2}b\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\text{Li}$$

Mathematica [A]
time = 0.12, size = 214, normalized size = 1.02

$$\frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1+\frac{2}{-1+cx^3}\right)+\frac{1}{4}b\left(2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\text{PolyLog}\left(2,\frac{1+cx^3}{1-cx^3}\right)-2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\text{PolyLog}\left(2,\frac{1+cx^3}{-1+cx^3}\right)+b\left(-2\left(a+b\tanh^{-1}\left(cx^3\right)\right)\text{PolyLog}\left(3,\frac{1+cx^3}{1-cx^3}\right)+2\left(a+b\tanh^{-1}\left(cx^3\right)\right)\text{PolyLog}\left(3,\frac{1+cx^3}{-1+cx^3}\right)+b\left(\text{PolyLog}\left(4,\frac{1+cx^3}{1-cx^3}\right)-\text{PolyLog}\left(4,\frac{1+cx^3}{-1+cx^3}\right)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^3/x,x]
```

```
[Out] (2*(a + b*ArcTanh[c*x^3])^3*ArcTanh[1 + 2/(-1 + c*x^3)])/3 + (b*(2*(a + b*A
rcTanh[c*x^3])^2*PolyLog[2, (1 + c*x^3)/(1 - c*x^3)] - 2*(a + b*ArcTanh[c*x
^3])^2*PolyLog[2, (1 + c*x^3)/(-1 + c*x^3)] + b*(-2*(a + b*ArcTanh[c*x^3])*
PolyLog[3, (1 + c*x^3)/(1 - c*x^3)] + 2*(a + b*ArcTanh[c*x^3])*PolyLog[3, (
1 + c*x^3)/(-1 + c*x^3)] + b*(PolyLog[4, (1 + c*x^3)/(1 - c*x^3)] - PolyLog
[4, (1 + c*x^3)/(-1 + c*x^3)]))))/4
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\,x^3\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^3/x,x)
```

```
[Out] int((a+b*arctanh(c*x^3))^3/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + integrate(1/8*b^3*(log(c*x^3 + 1) - log(-c*x^3 + 1))^3/x + 3/4
*a*b^2*(log(c*x^3 + 1) - log(-c*x^3 + 1))^2/x + 3/2*a^2*b*(log(c*x^3 + 1) -
 log(-c*x^3 + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^3)^3 + 3*a*b^2*arctanh(c*x^3)^2 + 3*a^2*b*arctanh
(c*x^3) + a^3)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{atanh}(cx^3))^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**3/x,x)
```

```
[Out] Integral((a + b*atanh(c*x**3))**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\operatorname{atanh}(c\,x^3))^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))^3/x,x)
```

```
[Out] int((a + b*atanh(c*x^3))^3/x, x)
```

3.128 $\int \frac{(a+b\tanh^{-1}(cx^3))^3}{x^4}\,dx$

Optimal. Leaf size=120

$$\frac{1}{3}c\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{3x^3}+bc\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\log\left(2-\frac{2}{1+cx^3}\right)-b^2c\left(a+b\tanh\right.$$

```
[Out] 1/3*c*(a+b*arctanh(c*x^3))^3-1/3*(a+b*arctanh(c*x^3))^3/x^3+b*c*(a+b*arctan
h(c*x^3))^2*ln(2-2/(c*x^3+1))-b^2*c*(a+b*arctanh(c*x^3))*polylog(2,-1+2/(c*
x^3+1))-1/2*b^3*c*polylog(3,-1+2/(c*x^3+1))
```

Rubi [A]
time = 0.22, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6039, 6037, 6135, 6079, 6095, 6203, 6745}

$$-b^2c\text{Li}_2\left(\frac{2}{cx^3+1}-1\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{1}{3}c\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{3x^3}+bc\log\left(2-\frac{2}{cx^3+1}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2-\frac{1}{2}b^3c\text{Li}_3\left(\frac{2}{cx^3+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])^3/x^4,x]
```

```
[Out] (c*(a + b*ArcTanh[c*x^3])^3)/3 - (a + b*ArcTanh[c*x^3])^3/(3*x^3) + b*c*(a
+ b*ArcTanh[c*x^3])^2*Log[2 - 2/(1 + c*x^3)] - b^2*c*(a + b*ArcTanh[c*x^3])
*PolyLog[2, -1 + 2/(1 + c*x^3)] - (b^3*c*PolyLog[3, -1 + 2/(1 + c*x^3)])/2
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
```

```
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x^4}\,dx &= \int \left(\frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{8x^4}+\frac{3b\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)}{8x^4}-\frac{3b^2(-\right.\\
&= \frac{1}{8}\int \frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{x^4}\,dx+\frac{1}{8}(3b)\int \frac{\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx\right.}{x^4}\\
&= \frac{1}{24}\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^3}{x^2}\,dx,x,x^3\right)+\frac{1}{8}b\text{Subst}\left(\int \frac{(-2a+b\log(1-}{x}\right.\\
&= -\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24x^3}-\frac{b^3\left(1+cx^3\right)\log^3\left(1+cx^3\right)}{24x^3}+\frac{1}{8}b\text{Subst}\left(\int\right.\\
&= \frac{1}{8}bc\log\left(cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24x^3}+\frac{1}{8}b^3\\
&= \frac{1}{8}bc\log\left(cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24x^3}+\frac{1}{8}b^3\\
&= \frac{1}{8}bc\log\left(cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24x^3}+\frac{1}{8}b^3\\
&= \frac{1}{8}bc\log\left(cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24x^3}+\frac{1}{8}b^3
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.27, size = 223, normalized size = 1.86

$$-\frac{a^3}{3x^3}-\frac{a^2b\tanh^{-1}\left(cx^3\right)}{x^3}+3a^2bc\log(x)-\frac{1}{2}a^2bc\log\left(1-c^2x^6\right)+ab^2c\left(\tanh^{-1}\left(cx^3\right)\left(\left(1-\frac{1}{cx^3}\right)\tanh^{-1}\left(cx^3\right)+2\log\left(1-e^{-2\tanh^{-1}\left(cx^3\right)}\right)\right)-\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(cx^3\right)}\right)\right)+\frac{1}{3}b^3c\left(\frac{i\pi^3}{8}-\tanh^{-1}\left(cx^3\right)^3-\frac{\tanh^{-1}\left(cx^3\right)^3}{cx^3}+3\tanh^{-1}\left(cx^3\right)^2\log\left(1-e^{2\tanh^{-1}\left(cx^3\right)}\right)+3\tanh^{-1}\left(cx^3\right)\text{PolyLog}\left(2,e^{2\tanh^{-1}\left(cx^3\right)}\right)-\frac{3}{2}\text{PolyLog}\left(3,e^{2\tanh^{-1}\left(cx^3\right)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (a^2*b*ArcTanh[c*x^3])/x^3 + 3*a^2*b*c*Log[x] - (a^2*b*c*Log
[1 - c^2*x^6])/2 + a*b^2*c*(ArcTanh[c*x^3]*((1 - 1/(c*x^3))*ArcTanh[c*x^3]
+ 2*Log[1 - E^(-2*ArcTanh[c*x^3])]) - PolyLog[2, E^(-2*ArcTanh[c*x^3])]) +
(b^3*c*((I/8)*Pi^3 - ArcTanh[c*x^3]^3 - ArcTanh[c*x^3]^3/(c*x^3) + 3*ArcTan
h[c*x^3]^2*Log[1 - E^(2*ArcTanh[c*x^3])] + 3*ArcTanh[c*x^3]*PolyLog[2, E^(2
*ArcTanh[c*x^3])] - (3*PolyLog[3, E^(2*ArcTanh[c*x^3])])/2))/3
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\,x^{3}\right)\right)^{3}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^3/x^4,x)
```

```
[Out] int((a+b*arctanh(c*x^3))^3/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^6 - 1) - log(x^6)) + 2*arctanh(c*x^3)/x^3)*a^2*b - 1/3*a
^3/x^3 - 1/24*((b^3*c*x^3 - b^3)*log(-c*x^3 + 1)^3 + 3*(2*a*b^2 + (b^3*c*x^
3 + b^3)*log(c*x^3 + 1))*log(-c*x^3 + 1)^2)/x^3 - integrate(-1/8*((b^3*c*x^
3 - b^3)*log(c*x^3 + 1)^3 + 6*(a*b^2*c*x^3 - a*b^2)*log(c*x^3 + 1)^2 + 3*(4
*a*b^2*c*x^3 - (b^3*c*x^3 - b^3)*log(c*x^3 + 1)^2 + 2*(b^3*c^2*x^6 - (2*a*b
^2*c - b^3*c)*x^3 + 2*a*b^2)*log(c*x^3 + 1))*log(-c*x^3 + 1))/(c*x^7 - x^4)
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^3)^3 + 3*a*b^2*arctanh(c*x^3)^2 + 3*a^2*b*arctanh
(c*x^3) + a^3)/x^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**3/x**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^4,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^3) + a)^3/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,x^3\right)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))^3/x^4,x)
[Out] int((a + b*atanh(c*x^3))^3/x^4, x)
```

3.129 $\int \frac{\left(a+b\tanh^{-1}(cx^3)\right)^3}{x^7} dx$

Optimal. Leaf size=136

$$\frac{1}{2}bc^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2-\frac{bc\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{2x^3}+\frac{1}{6}c^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{6x^6}+b^2c$$

```
[Out] 1/2*b*c^2*(a+b*arctanh(c*x^3))^2-1/2*b*c*(a+b*arctanh(c*x^3))^2/x^3+1/6*c^2
*(a+b*arctanh(c*x^3))^3-1/6*(a+b*arctanh(c*x^3))^3/x^6+b^2*c^2*(a+b*arctanh
(c*x^3))*ln(2-2/(c*x^3+1))-1/2*b^3*c^2*polylog(2,-1+2/(c*x^3+1))
```

Rubi [A]
time = 0.24, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6039, 6037, 6129, 6135, 6079, 2497, 6095}

$$b^2c^2\log\left(2-\frac{2}{cx^3+1}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{1}{2}bc^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{1}{6}c^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3-\frac{bc\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{2x^3}-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{6x^6}-\frac{1}{2}b^3c^2\text{Li}_2\left(\frac{2}{cx^3+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])^3/x^7,x]
```

```
[Out] (b*c^2*(a + b*ArcTanh[c*x^3])^2)/2 - (b*c*(a + b*ArcTanh[c*x^3])^2)/(2*x^3)
 + (c^2*(a + b*ArcTanh[c*x^3])^3)/6 - (a + b*ArcTanh[c*x^3])^3/(6*x^6) + b^
2*c^2*(a + b*ArcTanh[c*x^3])*Log[2 - 2/(1 + c*x^3)] - (b^3*c^2*PolyLog[2, -
1 + 2/(1 + c*x^3)])/2
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
```

fy[(m + 1)/n]]

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x^7}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{8x^7}+\frac{3b\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)}{8x^7}-\frac{3b^2}{\cdots}\right.$$

$$=\frac{1}{8}\int \frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{x^7}\,dx+\frac{1}{8}(3b)\int \frac{\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+c\cdots\right.}{x^7}$$

$$=\frac{1}{24}\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^3}{x^3}\,dx,x,x^3\right)+\frac{1}{8}b\text{Subst}\left(\int \frac{(-2a+b\log(1\cdots}{\cdots}\right.$$

$$=-\frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{48x^6}-\frac{b^3\log^3\left(1+cx^3\right)}{48x^6}+\frac{1}{8}b\text{Subst}\left(\int \frac{(-2a+b\log(1-\cdots}{\cdots}\right.$$

$$=-\frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{48x^6}-\frac{b^3\log^3\left(1+cx^3\right)}{48x^6}-\frac{1}{16}b\text{Subst}\left(\int \frac{(2a-b\log(x))^{2}\cdots}{x\left(\frac{1}{c}-\frac{x}{c}\right)^2}\right.$$

$$=-\frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{48x^6}-\frac{b^3\log^3\left(1+cx^3\right)}{48x^6}-\frac{1}{16}b\text{Subst}\left(\int \frac{(2a-b\log(x))^{2}\cdots}{\left(\frac{1}{c}-\frac{x}{c}\right)^2}\right.$$

$$=-\frac{bc\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{16x^3}-\frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{48x^6}-\frac{b^3c(1+cx^3\cdots}{1\cdots}$$

$$=\frac{3}{4}ab^2c^2\log(x)-\frac{bc\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{16x^3}+\frac{1}{16}bc^2\log\left(cx^3\right)\left(2a-b\cdots\right.$$

$$=\frac{3}{4}ab^2c^2\log(x)-\frac{bc\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{16x^3}+\frac{1}{16}bc^2\log\left(cx^3\right)\left(2a-b\cdots\right.$$

$$=\frac{3}{4}ab^2c^2\log(x)-\frac{bc\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{16x^3}+\frac{1}{16}bc^2\log\left(cx^3\right)\left(2a-b\cdots\right.$$

<u>Mathematica</u> [A]
time = 0.19, size = 218, normalized size = 1.60

$$\frac{6b^2(-1+cx^3)\left(a+acx^3+bcx^3\right)\tanh^{-1}\left(cx^3\right)^2+2b^3(-1+c^2x^6)\tanh^{-1}\left(cx^3\right)^3-6b\tanh^{-1}\left(cx^3\right)\left(a^2+2abcx^3-2b^2c^2x^6\log\left(1-e^{-2\tanh^{-1}(cx^3)}\right)\right)+a\left(-2a^2-6abcx^3-3abc^2x^6\log\left(1-cx^3\right)+3abc^2x^6\log\left(1+cx^3\right)+12b^2c^2x^6\log\left(\frac{cx^3}{\sqrt{1-c^2x^6}}\right)\right)-6b^3c^2x^6\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx^3)}\right)}{12x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^3/x^7,x]
```

```
[Out] (6*b^2*(-1 + c*x^3)*(a + a*c*x^3 + b*c*x^3)*ArcTanh[c*x^3]^2 + 2*b^3*(-1 +
c^2*x^6)*ArcTanh[c*x^3]^3 - 6*b*ArcTanh[c*x^3]*(a^2 + 2*a*b*c*x^3 - 2*b^2*c
^2*x^6*Log[1 - E^(-2*ArcTanh[c*x^3])]) + a*(-2*a^2 - 6*a*b*c*x^3 - 3*a*b*c^
2*x^6*Log[1 - c*x^3] + 3*a*b*c^2*x^6*Log[1 + c*x^3] + 12*b^2*c^2*x^6*Log[(c
*x^3)/Sqrt[1 - c^2*x^6]]) - 6*b^3*c^2*x^6*PolyLog[2, E^(-2*ArcTanh[c*x^3])]
)/(12*x^6)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,\mathrm{arctanh}\left(c\,x^{3}\right)\right)^{3}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^3/x^7,x)
[Out] int((a+b*arctanh(c*x^3))^3/x^7,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^7,x, algorithm="maxima")
[Out] 1/4*((c*log(c*x^3 + 1) - c*log(c*x^3 - 1) - 2/x^3)*c - 2*arctanh(c*x^3)/x^6
)*a^2*b + 1/8*((2*(log(c*x^3 - 1) - 2)*log(c*x^3 + 1) - log(c*x^3 + 1)^2 -
log(c*x^3 - 1)^2 - 4*log(c*x^3 - 1) + 24*log(x))*c^2 + 4*(c*log(c*x^3 + 1)
- c*log(c*x^3 - 1) - 2/x^3)*c*arctanh(c*x^3))*a*b^2 - 1/48*b^3*(((c^2*x^6 -
 1)*log(-c*x^3 + 1)^3 + 3*(2*c*x^3 - (c^2*x^6 - 1)*log(c*x^3 + 1))*log(-c*x
^3 + 1)^2)/x^6 + 6*integrate(-((c*x^3 - 1)*log(c*x^3 + 1)^3 + 3*(2*c^2*x^6
- (c*x^3 - 1)*log(c*x^3 + 1)^2 - (c^3*x^9 - c*x^3)*log(c*x^3 + 1))*log(-c*x
^3 + 1))/(c*x^10 - x^7), x)) - 1/2*a*b^2*arctanh(c*x^3)^2/x^6 - 1/6*a^3/x^6
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^7,x, algorithm="fricas")
[Out] integral((b^3*arctanh(c*x^3)^3 + 3*a*b^2*arctanh(c*x^3)^2 + 3*a^2*b*arctanh
(c*x^3) + a^3)/x^7, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**3/x**7,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^7,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^3) + a)^3/x^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,x^3\right)\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))^3/x^7,x)
[Out] int((a + b*atanh(c*x^3))^3/x^7, x)
```

3.130 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3 dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left((dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c*x^3))^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3 dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^3])^3,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x^3])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3 dx = \int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3 dx$$

Mathematica [A]
time = 1.39, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3 dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3])^3, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\,\mathrm{arctanh}\left(c\,x^3\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^3))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^3))^3,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*d^m*x*x^m*log(-c*x^3 + 1)^3/(m + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1)
) + integrate(1/8*((b^3*c*d^m*(m + 1)*x^3 - b^3*d^m*(m + 1))*x^m*log(c*x^3
+ 1)^3 + 6*(a*b^2*c*d^m*(m + 1)*x^3 - a*b^2*d^m*(m + 1))*x^m*log(c*x^3 + 1)
^2 + 12*(a^2*b*c*d^m*(m + 1)*x^3 - a^2*b*d^m*(m + 1))*x^m*log(c*x^3 + 1) +
3*((b^3*c*d^m*(m + 1)*x^3 - b^3*d^m*(m + 1))*x^m*log(c*x^3 + 1) - (2*a*b^2*
d^m*(m + 1) - (2*a*b^2*c*d^m*(m + 1) + 3*b^3*c*d^m)*x^3)*x^m)*log(-c*x^3 +
1)^2 - 3*((b^3*c*d^m*(m + 1)*x^3 - b^3*d^m*(m + 1))*x^m*log(c*x^3 + 1)^2 +
4*(a*b^2*c*d^m*(m + 1)*x^3 - a*b^2*d^m*(m + 1))*x^m*log(c*x^3 + 1) + 4*(a^2
*b*c*d^m*(m + 1)*x^3 - a^2*b*d^m*(m + 1))*x^m)*log(-c*x^3 + 1))/(c*(m + 1)*
x^3 - m - 1), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^3)^3 + 3*a*b^2*arctanh(c*x^3)^2 + 3*a^2*b*arctanh
(c*x^3) + a^3)*(d*x)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**3))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3*(d*x)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atanh}\left(c\,x^3\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c*x^3))^3,x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c*x^3))^3, x)
```

3.131 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c*x^3))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^3])^2,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x^3])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx = \int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx$$

Mathematica [A]
time = 0.84, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3])^2, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\,\mathrm{arctanh}\left(c\,x^3\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^3))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^3))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*d^m*x*x^m*log(-c*x^3 + 1)^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1))
 - integrate(-1/4*((b^2*c*d^m*(m + 1)*x^3 - b^2*d^m*(m + 1))*x^m*log(c*x^3
+ 1)^2 + 4*(a*b*c*d^m*(m + 1)*x^3 - a*b*d^m*(m + 1))*x^m*log(c*x^3 + 1) - 2
*((b^2*c*d^m*(m + 1)*x^3 - b^2*d^m*(m + 1))*x^m*log(c*x^3 + 1) - (2*a*b*d^m
*(m + 1) - (2*a*b*c*d^m*(m + 1) + 3*b^2*c*d^m)*x^3)*x^m)*log(-c*x^3 + 1))/(
c*(m + 1)*x^3 - m - 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2)*(d*x)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2*(d*x)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atanh}\left(c\,x^3\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c*x^3))^2,x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c*x^3))^2, x)
```

3.132 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right) \, dx$

Optimal. Leaf size=74

$$\frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{d(1+m)}-\frac{3bc(dx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{6};\frac{10+m}{6};c^2x^6\right)}{d^4(1+m)(4+m)}$$

```
[Out] (d*x)^(1+m)*(a+b*arctanh(c*x^3))/d/(1+m)-3*b*c*(d*x)^(4+m)*hypergeom([1, 2/
3+1/6*m],[5/3+1/6*m],c^2*x^6)/d^4/(1+m)/(4+m)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6049, 371}

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{d(m+1)}-\frac{3bc(dx)^{m+4}\,{}_2F_1\left(1,\frac{m+4}{6};\frac{m+10}{6};c^2x^6\right)}{d^4(m+1)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTanh[c*x^3]))/(d*(1 + m)) - (3*b*c*(d*x)^(4 + m)*H
ypergeometric2F1[1, (4 + m)/6, (10 + m)/6, c^2*x^6])/(d^4*(1 + m)*(4 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m\left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx &= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{d(1+m)}-\frac{(3bc)\int\frac{x^2(dx)^{1+m}}{1-c^2x^6}\,dx}{d(1+m)}\\
&= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{d(1+m)}-\frac{(3bc)\int\frac{(dx)^{3+m}}{1-c^2x^6}\,dx}{d^3(1+m)}\\
&= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{d(1+m)}-\frac{3bc(dx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{6};\frac{10+m}{6};c^2x^6\right)}{d^4(1+m)(4+m)}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 64, normalized size = 0.86

$$-\frac{x(dx)^m\left(-\left((4+m)\left(a+b\tanh^{-1}\left(cx^3\right)\right)\right)+3bcx^3\,{}_2F_1\left(1,\frac{4+m}{6};\frac{10+m}{6};c^2x^6\right)\right)}{(1+m)(4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] -((x*(d*x)^m*(-((4 + m)*(a + b*ArcTanh[c*x^3])) + 3*b*c*x^3*Hypergeometric2
F1[1, (4 + m)/6, (10 + m)/6, c^2*x^6]))/((1 + m)*(4 + m)))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx)^m\left(a+b\,\mathrm{arctanh}\left(c\,x^3\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^3)),x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^3)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/2*(6*c*d^m*integrate(x^3*x^m/(c^2*(m + 1)*x^6 - m - 1), x) + (d^m*x*x^m*l
og(c*x^3 + 1) - d^m*x*x^m*log(-c*x^3 + 1))/(m + 1))*b + (d*x)^(m + 1)*a/(d*
(m + 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^3) + a)*(d*x)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**3)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3)),x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^3) + a)*(d*x)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atanh}\left(c\,x^3\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c*x^3)),x)
[Out] int((d*x)^m*(a + b*atanh(c*x^3)), x)
```

3.133 $\int \frac{(dx)^m}{a+b\tanh^{-1}(cx^3)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(dx)^m}{a+b\tanh^{-1}\left(cx^3\right)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctanh(c*x^3)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x^3]),x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x^3]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^3\right)}\,dx=\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^3\right)}\,dx$$

Mathematica [A]
time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^3]),x]
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^3]), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\,\mathrm{arctanh}\left(c\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x^3)),x)
[Out] int((d*x)^m/(a+b*arctanh(c*x^3)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^3)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arctanh(c*x^3) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^3)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arctanh(c*x^3) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**3)),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^3)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arctanh(c*x^3) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a+b\,\mathrm{atanh}\left(c\,x^3\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c*x^3)),x)
[Out] int((d*x)^m/(a + b*atanh(c*x^3)), x)
```

3.134 $\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctanh(c*x^3))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x^3])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^3])^2, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\,\mathrm{arctanh}\left(c\,x^3\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x^3))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c*x^3))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^m*x^6 - d^m)*x^m/(b^2*c*x^2*log(c*x^3 + 1) - b^2*c*x^2*log(-c*x^
3 + 1) + 2*a*b*c*x^2) + integrate(-2/3*(c^2*d^m*(m + 4)*x^6 - d^m*(m - 2))*
x^m/(b^2*c*x^3*log(c*x^3 + 1) - b^2*c*x^3*log(-c*x^3 + 1) + 2*a*b*c*x^3), x
)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x^3) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{\left(a+b\,\mathrm{atanh}\left(c\,x^3\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c*x^3))^2,x)
[Out] int((d*x)^m/(a + b*atanh(c*x^3))^2, x)
```

3.135 $\int x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)\,dx$

Optimal. Leaf size=50

$$\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{4}bc^4\tanh^{-1}\left(\frac{x}{c}\right)$$

```
[Out] 1/4*b*c^3*x+1/12*b*c*x^3+1/4*x^4*(a+b*arctanh(c/x))-1/4*b*c^4*arctanh(x/c)
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6037, 269, 308, 213}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{4}bc^4\tanh^{-1}\left(\frac{x}{c}\right)+\frac{1}{4}bc^3x+\frac{1}{12}bcx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c/x]),x]
```

```
[Out] (b*c^3*x)/4 + (b*c*x^3)/12 + (x^4*(a + b*ArcTanh[c/x]))/4 - (b*c^4*ArcTanh[
x/c])/4
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)\,dx &= \frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\frac{x^2}{1-\frac{c^2}{x^2}}\,dx\\
&= \frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\frac{x^4}{-c^2+x^2}\,dx\\
&= \frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\left(c^2+x^2+\frac{c^4}{-c^2+x^2}\right)dx\\
&= \frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}\left(bc^5\right)\int\frac{1}{-c^2+x^2}\,dx\\
&= \frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{4}bc^4\tanh^{-1}\left(\frac{x}{c}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 67, normalized size = 1.34

$$\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{ax^4}{4}+\frac{1}{4}bx^4\tanh^{-1}\left(\frac{c}{x}\right)+\frac{1}{8}bc^4\log(-c+x)-\frac{1}{8}bc^4\log(c+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c/x]),x]
```

```
[Out] (b*c^3*x)/4 + (b*c*x^3)/12 + (a*x^4)/4 + (b*x^4*ArcTanh[c/x])/4 + (b*c^4*Lo
g[-c + x])/8 - (b*c^4*Log[c + x])/8
```

Maple [A]
time = 0.20, size = 69, normalized size = 1.38

method	result
derivativedivides	$-c^4\left(-\frac{a\,x^4}{4c^4}-\frac{b\,x^4\,\mathrm{arctanh}\left(\frac{c}{x}\right)}{4c^4}+\frac{b\ln\left(1+\frac{c}{x}\right)}{8}-\frac{b\,x^3}{12c^3}-\frac{bx}{4c}-\frac{b\ln\left(\frac{c}{x}-1\right)}{8}\right)$
default	$-c^4\left(-\frac{a\,x^4}{4c^4}-\frac{b\,x^4\,\mathrm{arctanh}\left(\frac{c}{x}\right)}{4c^4}+\frac{b\ln\left(1+\frac{c}{x}\right)}{8}-\frac{b\,x^3}{12c^3}-\frac{bx}{4c}-\frac{b\ln\left(\frac{c}{x}-1\right)}{8}\right)$
risch	$\frac{x^4b\ln(x+c)}{8}-\frac{x^4b\ln(c-x)}{8}-\frac{i\pi b\,x^4}{8}-\frac{i\pi b\,x^4\mathrm{csgn}\left(\frac{i(x+c)}{x}\right)^3}{16}+\frac{i\pi b\,x^4\mathrm{csgn}\left(\frac{i(c-x)}{x}\right)^2}{8}-\frac{i\pi b\,x^4\mathrm{csgn}\left(\frac{i(c-x)}{x}\right)^3}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c/x)),x,method=_RETURNVERBOSE)
```

```
[Out] -c^4*(-1/4*a/c^4*x^4-1/4*b/c^4*x^4*arctanh(c/x)+1/8*b*ln(1+c/x)-1/12*b*x^3/
c^3-1/4*b*x/c-1/8*b*ln(c/x-1))
```

Maxima [A]
time = 0.26, size = 57, normalized size = 1.14

$$\frac{1}{4}\,ax^4 + \frac{1}{24}\left(6\,x^4\,\operatorname{artanh}\left(\frac{c}{x}\right) - \left(3\,c^3\log\left(c+x\right) - 3\,c^3\log\left(-c+x\right) - 6\,c^2x - 2\,x^3\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/24*(6*x^4*arctanh(c/x) - (3*c^3*log(c + x) - 3*c^3*log(-c + x
) - 6*c^2*x - 2*x^3)*c)*b
```

Fricas [A]
time = 0.35, size = 48, normalized size = 0.96

$$\frac{1}{4}\,bc^3x + \frac{1}{12}\,bcx^3 + \frac{1}{4}\,ax^4 - \frac{1}{8}\left(bc^4 - bx^4\right)\log\left(-\frac{c+x}{c-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x)),x, algorithm="fricas")
```

```
[Out] 1/4*b*c^3*x + 1/12*b*c*x^3 + 1/4*a*x^4 - 1/8*(b*c^4 - b*x^4)*log(-(c + x)/(
c - x))
```

Sympy [A]
time = 0.16, size = 46, normalized size = 0.92

$$\frac{ax^4}{4} - \frac{bc^4\operatorname{atanh}\left(\frac{c}{x}\right)}{4} + \frac{bc^3x}{4} + \frac{bcx^3}{12} + \frac{bx^4\operatorname{atanh}\left(\frac{c}{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c/x)),x)
```

```
[Out] a*x**4/4 - b*c**4*atanh(c/x)/4 + b*c**3*x/4 + b*c*x**3/12 + b*x**4*atanh(c/
x)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(42) = 84.
time = 0.43, size = 262, normalized size = 5.24

$$-\frac{\frac{3\left(\frac{b(c+x)^3c^5}{(c-x)^3}+\frac{b(c+x)c^5}{c-x}\right)\log\left(-\frac{c+x}{c-x}\right)}{\frac{(c+x)^4}{(c-x)^4}+\frac{4\,(c+x)^3}{(c-x)^3}+\frac{6\,(c+x)^2}{(c-x)^2}+\frac{4\,(c+x)}{c-x}+1}+\frac{2\,bc^5+\frac{6\,a(c+x)^3c^5}{(c-x)^3}+\frac{3\,b(c+x)^3c^5}{(c-x)^3}+\frac{6\,b(c+x)^2c^5}{(c-x)^2}+\frac{6\,a(c+x)c^5}{c-x}+\frac{5\,b(c+x)c^5}{c-x}}{\frac{(c+x)^4}{(c-x)^4}+\frac{4\,(c+x)^3}{(c-x)^3}+\frac{6\,(c+x)^2}{(c-x)^2}+\frac{4\,(c+x)}{c-x}+1}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x)),x, algorithm="giac")
```

```
[Out] -1/3*(3*(b*(c + x)^3*c^5/(c - x)^3 + b*(c + x)*c^5/(c - x))*log(-(c + x)/(c
 - x))/((c + x)^4/(c - x)^4 + 4*(c + x)^3/(c - x)^3 + 6*(c + x)^2/(c - x)^2
 + 4*(c + x)/(c - x) + 1) + (2*b*c^5 + 6*a*(c + x)^3*c^5/(c - x)^3 + 3*b*(c
 + x)^3*c^5/(c - x)^3 + 6*b*(c + x)^2*c^5/(c - x)^2 + 6*a*(c + x)*c^5/(c -
x) + 5*b*(c + x)*c^5/(c - x))/((c + x)^4/(c - x)^4 + 4*(c + x)^3/(c - x)^3
+ 6*(c + x)^2/(c - x)^2 + 4*(c + x)/(c - x) + 1))/c
```

Mupad [B]
time = 0.78, size = 45, normalized size = 0.90

$$\frac{a\,x^4}{4}-\frac{b\,c^4\,\mathrm{atanh}\left(\frac{c}{x}\right)}{4}+\frac{b\,x^4\,\mathrm{atanh}\left(\frac{c}{x}\right)}{4}+\frac{b\,c\,x^3}{12}+\frac{b\,c^3\,x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c/x)),x)
```

```
[Out] (a*x^4)/4 - (b*c^4*atanh(c/x))/4 + (b*x^4*atanh(c/x))/4 + (b*c*x^3)/12 + (b
*c^3*x)/4
```

3.136 $\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)\,dx$

Optimal. Leaf size=45

$$\frac{1}{6}bcx^2+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{6}bc^3\log\left(c^2-x^2\right)$$

```
[Out] 1/6*b*c*x^2+1/3*x^3*(a+b*arctanh(c/x))+1/6*b*c^3*ln(c^2-x^2)
```

Rubi [A]

time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {6037, 269, 272, 45}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{6}bc^3\log\left(c^2-x^2\right)+\frac{1}{6}bcx^2$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c/x]),x]
```

```
[Out] (b*c*x^2)/6 + (x^3*(a + b*ArcTanh[c/x]))/3 + (b*c^3*Log[c^2 - x^2])/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)\,dx &= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{3}(bc)\int\frac{x}{1-\frac{c^2}{x^2}}\,dx\\
&= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{3}(bc)\int\frac{x^3}{-c^2+x^2}\,dx\\
&= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{6}(bc)\text{Subst}\left(\int\frac{x}{-c^2+x}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{6}(bc)\text{Subst}\left(\int\left(1-\frac{c^2}{c^2-x}\right)\,dx,x,x^2\right)\\
&= \frac{1}{6}bcx^2+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{6}bc^3\log\left(c^2-x^2\right)
\end{aligned}
$$

Mathematica [**A**]
time = 0.01, size = 50, normalized size = 1.11

$$
\frac{1}{6}bcx^2+\frac{ax^3}{3}+\frac{1}{3}bx^3\tanh^{-1}\left(\frac{c}{x}\right)+\frac{1}{6}bc^3\log\left(-c^2+x^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c/x]),x]
```

```
[Out] (b*c*x^2)/6 + (a*x^3)/3 + (b*x^3*ArcTanh[c/x])/3 + (b*c^3*Log[-c^2 + x^2])/
6
```

Maple [**A**]
time = 0.11, size = 71, normalized size = 1.58

method	result
derivativedivides	$-c^3\left(-\frac{a\,x^3}{3c^3}-\frac{b\,x^3\,\operatorname{arctanh}\left(\frac{c}{x}\right)}{3c^3}-\frac{b\ln\left(1+\frac{c}{x}\right)}{6}-\frac{b\,x^2}{6c^2}+\frac{b\ln\left(\frac{c}{x}\right)}{3}-\frac{b\ln\left(\frac{c}{x}-1\right)}{6}\right)$
default	$-c^3\left(-\frac{a\,x^3}{3c^3}-\frac{b\,x^3\,\operatorname{arctanh}\left(\frac{c}{x}\right)}{3c^3}-\frac{b\ln\left(1+\frac{c}{x}\right)}{6}-\frac{b\,x^2}{6c^2}+\frac{b\ln\left(\frac{c}{x}\right)}{3}-\frac{b\ln\left(\frac{c}{x}-1\right)}{6}\right)$
risch	$\frac{x^3b\ln(x+c)}{6}-\frac{x^3b\ln(c-x)}{6}+\frac{i\pi b\,x^3\operatorname{csgn}\left(\frac{i(c-x)}{x}\right)^2}{6}-\frac{i\pi b\,x^3\operatorname{csgn}(i(c-x))\operatorname{csgn}\left(\frac{i(c-x)}{x}\right)^2}{12}-\frac{i\pi b\,x^3}{6}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c/x)),x,method=_RETURNVERBOSE)
```

```
[Out] -c^3*(-1/3*a/c^3*x^3-1/3*b/c^3*x^3*arctanh(c/x)-1/6*b*ln(1+c/x)-1/6*b/c^2*x
^2+1/3*b*ln(c/x)-1/6*b*ln(c/x-1))
```

Maxima [A]
time = 0.25, size = 42, normalized size = 0.93

$$\frac{1}{3}\,ax^3+\frac{1}{6}\left(2\,x^3\,\operatorname{artanh}\left(\frac{c}{x}\right)+\left(c^2\log\left(-c^2+x^2\right)+x^2\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctanh(c/x) + (c^2*log(-c^2 + x^2) + x^2)*c)*b
```

Fricas [A]
time = 0.37, size = 49, normalized size = 1.09

$$\frac{1}{6}\,bc^3\log\left(-c^2+x^2\right)+\frac{1}{6}\,bx^3\log\left(-\frac{c+x}{c-x}\right)+\frac{1}{6}\,bcx^2+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x)),x, algorithm="fricas")
```

```
[Out] 1/6*b*c^3*log(-c^2 + x^2) + 1/6*b*x^3*log(-(c + x)/(c - x)) + 1/6*b*c*x^2 +
 1/3*a*x^3
```

Sympy [A]
time = 0.15, size = 49, normalized size = 1.09

$$\frac{ax^3}{3}+\frac{bc^3\log\left(-c+x\right)}{3}+\frac{bc^3\operatorname{atanh}\left(\frac{c}{x}\right)}{3}+\frac{bcx^2}{6}+\frac{bx^3\operatorname{atanh}\left(\frac{c}{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c/x)),x)
```

```
[Out] a*x**3/3 + b*c**3*log(-c + x)/3 + b*c**3*atanh(c/x)/3 + b*c*x**2/6 + b*x**3
*atanh(c/x)/3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(39) = 78.
time = 0.44, size = 227, normalized size = 5.04

$$-\frac{bc^4\log\left(-\frac{c+x}{c-x}-1\right)-bc^4\log\left(-\frac{c+x}{c-x}\right)+\frac{\left(bc^4+\frac{3\,b(c+x)^2c^4}{(c-x)^2}\right)\log\left(-\frac{c+x}{c-x}\right)}{\frac{(c+x)^3}{(c-x)^3}+\frac{3\,(c+x)^2}{(c-x)^2}+\frac{3\,(c+x)}{c-x}+1}+\frac{2\left(ac^4+\frac{3\,a(c+x)^2c^4}{(c-x)^2}+\frac{b(c+x)^2c^4}{(c-x)^2}+\frac{b(c+x)c^4}{c-x}\right)}{\frac{(c+x)^3}{(c-x)^3}+\frac{3\,(c+x)^2}{(c-x)^2}+\frac{3\,(c+x)}{c-x}+1}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x)),x, algorithm="giac")
```

```
[Out] -1/3*(b*c^4*log(-(c + x)/(c - x) - 1) - b*c^4*log(-(c + x)/(c - x)) + (b*c^
4 + 3*b*(c + x)^2*c^4/(c - x)^2)*log(-(c + x)/(c - x))/((c + x)^3/(c - x)^3
 + 3*(c + x)^2/(c - x)^2 + 3*(c + x)/(c - x) + 1) + 2*(a*c^4 + 3*a*(c + x)^
2*c^4/(c - x)^2 + b*(c + x)^2*c^4/(c - x)^2 + b*(c + x)*c^4/(c - x))/((c +
x)^3/(c - x)^3 + 3*(c + x)^2/(c - x)^2 + 3*(c + x)/(c - x) + 1))/c
```

Mupad [B]
time = 0.72, size = 42, normalized size = 0.93

$$\frac{a\,x^3}{3}+\frac{b\,c^3\,\ln\left(x^2-c^2\right)}{6}+\frac{b\,x^3\,\mathrm{atanh}\left(\frac{c}{x}\right)}{3}+\frac{b\,c\,x^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c/x)),x)
[Out] (a*x^3)/3 + (b*c^3*log(x^2 - c^2))/6 + (b*x^3*atanh(c/x))/3 + (b*c*x^2)/6
```

3.137 $\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)\,dx$

Optimal. Leaf size=39

$$\frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\tanh^{-1}\left(\frac{x}{c}\right)$$

[Out] 1/2*b*c*x+1/2*x^2*(a+b*arctanh(c/x))-1/2*b*c^2*arctanh(x/c)

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {6037, 199, 327, 213}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\tanh^{-1}\left(\frac{x}{c}\right)+\frac{bcx}{2}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*ArcTanh[c/x]),x]

[Out] (b*c*x)/2 + (x^2*(a + b*ArcTanh[c/x]))/2 - (b*c^2*ArcTanh[x/c])/2

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
```

```
&& IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)dx &= \frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{2}(bc)\int\frac{1}{1-\frac{c^2}{x^2}}\,dx\\
&= \frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{2}(bc)\int\frac{x^2}{-c^2+x^2}\,dx\\
&= \frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{2}\left(bc^3\right)\int\frac{1}{-c^2+x^2}\,dx\\
&= \frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\tanh^{-1}\left(\frac{x}{c}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 1.44

$$\frac{bcx}{2}+\frac{ax^2}{2}+\frac{1}{2}bx^2\tanh^{-1}\left(\frac{c}{x}\right)+\frac{1}{4}bc^2\log(-c+x)-\frac{1}{4}bc^2\log(c+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c/x]),x]
[Out] (b*c*x)/2 + (a*x^2)/2 + (b*x^2*ArcTanh[c/x])/2 + (b*c^2*Log[-c + x])/4 - (b
*c^2*Log[c + x])/4
```

Maple [A]
time = 0.12, size = 60, normalized size = 1.54

method	result
derivativedivides	$-c^2\left(-\frac{a\,x^2}{2c^2}-\frac{b\,x^2\,\operatorname{arctanh}\left(\frac{c}{x}\right)}{2c^2}+\frac{b\ln\left(1+\frac{c}{x}\right)}{4}-\frac{b\ln\left(\frac{c}{x}-1\right)}{4}-\frac{bx}{2c}\right)$
default	$-c^2\left(-\frac{a\,x^2}{2c^2}-\frac{b\,x^2\,\operatorname{arctanh}\left(\frac{c}{x}\right)}{2c^2}+\frac{b\ln\left(1+\frac{c}{x}\right)}{4}-\frac{b\ln\left(\frac{c}{x}-1\right)}{4}-\frac{bx}{2c}\right)$
risch	$\frac{b\,x^2\ln(x+c)}{4}-\frac{b\,x^2\ln(c-x)}{4}-\frac{i\pi b\,x^2\operatorname{csgn}\left(\frac{i}{x}\right)\operatorname{csgn}(i(x+c))\operatorname{csgn}\left(\frac{i(x+c)}{x}\right)}{8}-\frac{i\pi b\,x^2\operatorname{csgn}\left(\frac{i(c-x)}{x}\right)^3}{8}+\frac{i\pi b\,x^2\operatorname{csgn}(i($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c/x)),x,method=_RETURNVERBOSE)
[Out] -c^2*(-1/2*a/c^2*x^2-1/2*b/c^2*x^2*arctanh(c/x)+1/4*b*ln(1+c/x)-1/4*b*ln(c/
x-1)-1/2*b*x/c)
```

Maxima [A]
time = 0.25, size = 44, normalized size = 1.13

$$\frac{1}{2}\,ax^2+\frac{1}{4}\left(2\,x^2\,\mathrm{artanh}\left(\frac{c}{x}\right)-\left(c\log\left(c+x\right)-c\log\left(-c+x\right)-2\,x\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(2*x^2*arctanh(c/x) - (c*log(c + x) - c*log(-c + x) - 2*x)*
c)*b
```

Fricas [A]
time = 0.34, size = 39, normalized size = 1.00

$$\frac{1}{2}\,bcx+\frac{1}{2}\,ax^2-\frac{1}{4}\left(bc^2-bx^2\right)\log\left(-\frac{c+x}{c-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x)),x, algorithm="fricas")
```

```
[Out] 1/2*b*c*x + 1/2*a*x^2 - 1/4*(b*c^2 - b*x^2)*log(-(c + x)/(c - x))
```

Sympy [A]
time = 0.12, size = 36, normalized size = 0.92

$$\frac{ax^2}{2}-\frac{bc^2\,\mathrm{atanh}\left(\frac{c}{x}\right)}{2}+\frac{bcx}{2}+\frac{bx^2\,\mathrm{atanh}\left(\frac{c}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c/x)),x)
```

```
[Out] a*x**2/2 - b*c**2*atanh(c/x)/2 + b*c*x/2 + b*x**2*atanh(c/x)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(33) = 66.
time = 0.40, size = 130, normalized size = 3.33

$$-\frac{\frac{b(c+x)c^3\log\left(-\frac{c+x}{c-x}\right)}{(c-x)\left(\frac{(c+x)^2}{(c-x)^2}+\frac{2\,(c+x)}{c-x}+1\right)}+\frac{bc^3+\frac{2\,a(c+x)c^3}{c-x}+\frac{b(c+x)c^3}{c-x}}{\frac{(c+x)^2}{(c-x)^2}+\frac{2\,(c+x)}{c-x}+1}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x)),x, algorithm="giac")
```

```
[Out] -(b*(c + x)*c^3*log(-(c + x)/(c - x))/((c - x)*((c + x)^2/(c - x)^2 + 2*(c
+ x)/(c - x) + 1)) + (b*c^3 + 2*a*(c + x)*c^3/(c - x) + b*(c + x)*c^3/(c -
x))/((c + x)^2/(c - x)^2 + 2*(c + x)/(c - x) + 1))/c
```

Mupad [B]
time = 0.74, size = 36, normalized size = 0.92

$$\frac{a\,x^2}{2}-\frac{b\,c^2\,\mathrm{atanh}\left(\frac{c}{x}\right)}{2}+\frac{b\,x^2\,\mathrm{atanh}\left(\frac{c}{x}\right)}{2}+\frac{b\,c\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c/x)),x)
[Out] (a*x^2)/2 - (b*c^2*atanh(c/x))/2 + (b*x^2*atanh(c/x))/2 + (b*c*x)/2
```

3.138 $\int \left(a+b \tanh^{-1}\left(\frac{c}{x}\right)\right) dx$

Optimal. Leaf size=29

$$ax+bx\tanh^{-1}\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2-x^2\right)$$

```
[Out] a*x+b*x*arctanh(c/x)+1/2*b*c*ln(c^2-x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6021, 269, 266}

$$ax+\frac{1}{2}bc\log\left(c^2-x^2\right)+bx\tanh^{-1}\left(\frac{c}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c/x],x]
```

```
[Out] a*x + b*x*ArcTanh[c/x] + (b*c*Log[c^2 - x^2])/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)\,dx &= ax+b\int \tanh^{-1}\left(\frac{c}{x}\right)\,dx\\
&= ax+bx\tanh^{-1}\left(\frac{c}{x}\right)+(bc)\int \frac{1}{\left(1-\frac{c^2}{x^2}\right)x}\,dx\\
&= ax+bx\tanh^{-1}\left(\frac{c}{x}\right)+(bc)\int \frac{x}{-c^2+x^2}\,dx\\
&= ax+bx\tanh^{-1}\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2-x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 29, normalized size = 1.00

$$ax+bx\tanh^{-1}\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c/x],x]
```

```
[Out] a*x + b*x*ArcTanh[c/x] + (b*c*Log[c^2 - x^2])/2
```

Maple [A]
time = 0.07, size = 48, normalized size = 1.66

method	result
default	$ax+bx\,\mathrm{arctanh}\left(\frac{c}{x}\right)+\frac{bc\ln\left(1+\frac{c}{x}\right)}{2}-bc\ln\left(\frac{c}{x}\right)+\frac{bc\ln\left(\frac{c}{x}-1\right)}{2}$
derivativedivides	$-c\left(-\frac{ax}{c}-\frac{bx\,\mathrm{arctanh}\left(\frac{c}{x}\right)}{c}-\frac{b\ln\left(1+\frac{c}{x}\right)}{2}+b\ln\left(\frac{c}{x}\right)-\frac{b\ln\left(\frac{c}{x}-1\right)}{2}\right)$
risch	$ax+\frac{bx\ln(x+c)}{2}-\frac{b\ln(c-x)x}{2}-\frac{ib\pi\,\mathrm{csgn}\left(\frac{i}{x}\right)\mathrm{csgn}\left(\frac{i(c-x)}{x}\right)^2x}{4}-\frac{ib\pi\mathrm{csgn}\left(\frac{i(c-x)}{x}\right)^3x}{4}+\frac{ib\pi\mathrm{csgn}\left(\frac{i(c-x)}{x}\right)^2x}{2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c/x),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*x*arctanh(c/x)+1/2*b*c*ln(1+c/x)-b*c*ln(c/x)+1/2*b*c*ln(c/x-1)
```

Maxima [A]
time = 0.25, size = 29, normalized size = 1.00

$$\frac{1}{2}\left(2\,x\,\mathrm{artanh}\left(\frac{c}{x}\right)+c\log\left(-c^2+x^2\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c/x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*x*arctanh(c/x) + c*log(-c^2 + x^2))*b + a*x
```

Fricas [A]
time = 0.34, size = 35, normalized size = 1.21

$$\frac{1}{2}\,bc\log\left(-c^2+x^2\right)+\frac{1}{2}\,bx\log\left(-\frac{c+x}{c-x}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c/x),x, algorithm="fricas")
```

```
[Out] 1/2*b*c*log(-c^2 + x^2) + 1/2*b*x*log(-(c + x)/(c - x)) + a*x
```

Sympy [A]
time = 0.10, size = 24, normalized size = 0.83

$$ax+b\Big(c\log\left(-c+x\right)+c\,\text{atanh}\left(\frac{c}{x}\right)+x\,\text{atanh}\left(\frac{c}{x}\right)\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c/x),x)
```

```
[Out] a*x + b*(c*log(-c + x) + c*atanh(c/x) + x*atanh(c/x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(27) = 54.
time = 0.41, size = 150, normalized size = 5.17

$$ax+\frac{\left(c^2\left(\log\left(\frac{|-c-x|}{|c-x|}\right)-\log\left(\left|-\frac{c+x}{c-x}-1\right|\right)\right)-\frac{c^2\log\left(-\frac{\frac{c\left(\frac{c+x}{(c-x)c}+\frac{1}{c}\right)}{\frac{c+x}{c-x}-1}+1}{\frac{c\left(\frac{c+x}{(c-x)c}+\frac{1}{c}\right)}{\frac{c+x}{c-x}-1}-1}\right)}{\frac{c+x}{c-x}+1}\right)b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c/x),x, algorithm="giac")
```

```
[Out] a*x + (c^2*(log(abs(-c - x)/abs(c - x)) - log(abs(-(c + x)/(c - x) - 1))) -
 c^2*log(-(c*((c + x)/((c - x)*c) + 1/c)/((c + x)/(c - x) - 1) + 1)/(c*((c
+ x)/((c - x)*c) + 1/c)/((c + x)/(c - x) - 1) - 1))/((c + x)/(c - x) + 1))*
b/c
```

Mupad [B]
time = 0.68, size = 27, normalized size = 0.93

$$a\,x+b\,x\,\mathrm{atanh}\left(\frac{c}{x}\right)+\frac{b\,c\,\ln\left(x^2-c^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atanh(c/x),x)
```

```
[Out] a*x + b*x*atanh(c/x) + (b*c*log(x^2 - c^2))/2
```

3.139 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x}\,dx$

Optimal. Leaf size=30

$$a\log(x)+\frac{1}{2}b\text{PolyLog}\left(2,-\frac{c}{x}\right)-\frac{1}{2}b\text{PolyLog}\left(2,\frac{c}{x}\right)$$

```
[Out] a*ln(x)+1/2*b*polylog(2,-c/x)-1/2*b*polylog(2,c/x)
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6035, 6031}

$$a\log(x)+\frac{1}{2}b\text{Li}_2\left(-\frac{c}{x}\right)-\frac{1}{2}b\text{Li}_2\left(\frac{c}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])/x,x]
```

```
[Out] a*Log[x] + (b*PolyLog[2, -(c/x)])/2 - (b*PolyLog[2, c/x])/2
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x}\,dx &= -\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx,x,\frac{1}{x}\right)\\ &= a\log(x)+\frac{1}{2}b\text{Li}_2\left(-\frac{c}{x}\right)-\frac{1}{2}b\text{Li}_2\left(\frac{c}{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.93

$$a\log(x)+\frac{1}{2}b\left(\text{PolyLog}\left(2,-\frac{c}{x}\right)-\text{PolyLog}\left(2,\frac{c}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])/x,x]
[Out] a*Log[x] + (b*(PolyLog[2, -(c/x)] - PolyLog[2, c/x]))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(26) = 52$.
time = 0.11, size = 63, normalized size = 2.10

method	result
derivativedivides	$-a\ln\left(\frac{c}{x}\right)-b\ln\left(\frac{c}{x}\right)\operatorname{arctanh}\left(\frac{c}{x}\right)+\frac{b\operatorname{dilog}\left(1+\frac{c}{x}\right)}{2}+\frac{b\ln\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{2}+\frac{b\operatorname{dilog}\left(\frac{c}{x}\right)}{2}$
default	$-a\ln\left(\frac{c}{x}\right)-b\ln\left(\frac{c}{x}\right)\operatorname{arctanh}\left(\frac{c}{x}\right)+\frac{b\operatorname{dilog}\left(1+\frac{c}{x}\right)}{2}+\frac{b\ln\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{2}+\frac{b\operatorname{dilog}\left(\frac{c}{x}\right)}{2}$
risch	$\frac{b\ln(x)\ln(x+c)}{2}-\frac{i\pi\ln(-x)b\operatorname{csgn}\left(\frac{i}{x}\right)\operatorname{csgn}\left(\frac{i(c-x)}{x}\right)^{2}}{4}+\frac{i\pi\ln(-x)b\operatorname{csgn}\left(\frac{i}{x}\right)\operatorname{csgn}(i(c-x))\operatorname{csgn}\left(\frac{i(c-x)}{x}\right)}{4}-\frac{i\pi\ln(-x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))/x,x,method=_RETURNVERBOSE)
[Out] -a*ln(c/x)-b*ln(c/x)*arctanh(c/x)+1/2*b*dilog(1+c/x)+1/2*b*ln(c/x)*ln(1+c/x
)+1/2*b*dilog(c/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x,x, algorithm="maxima")
[Out] 1/2*b*integrate((log(c/x + 1) - log(-c/x + 1))/x, x) + a*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x,x, algorithm="fricas")
[Out] integral((b*arctanh(c/x) + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\operatorname{atanh}\left(\frac{c}{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))/x,x)
```

```
[Out] Integral((a + b*atanh(c/x))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))/x,x)
```

```
[Out] int((a + b*atanh(c/x))/x, x)
```

3.140 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=35

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x}-\frac{b\log\left(1-\frac{c^2}{x^2}\right)}{2c}$$

```
[Out] (-a-b*arctanh(c/x))/x-1/2*b*ln(1-c^2/x^2)/c
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6037, 266}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x}-\frac{b\log\left(1-\frac{c^2}{x^2}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])/x^2,x]
```

```
[Out] -((a + b*ArcTanh[c/x])/x) - (b*Log[1 - c^2/x^2])/(2*c)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x^2}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x}-(bc)\int \frac{1}{\left(1-\frac{c^2}{x^2}\right)x^3}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x}-\frac{b\log\left(1-\frac{c^2}{x^2}\right)}{2c}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.09

$$-\frac{a}{x}-\frac{b\tanh^{-1}\left(\frac{c}{x}\right)}{x}-\frac{b\log\left(1-\frac{c^2}{x^2}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])/x^2,x]
[Out] -(a/x) - (b*ArcTanh[c/x])/x - (b*Log[1 - c^2/x^2])/(2*c)
```

Maple [A]
time = 0.10, size = 39, normalized size = 1.11

method	result
derivativedivides	$-\frac{\frac{ca}{x}+\frac{bc\,\mathrm{arctanh}\left(\frac{c}{x}\right)}{x}+\frac{b\ln\left(1-\frac{c^2}{x^2}\right)}{2}}{c}$
default	$-\frac{\frac{ca}{x}+\frac{bc\,\mathrm{arctanh}\left(\frac{c}{x}\right)}{x}+\frac{b\ln\left(1-\frac{c^2}{x^2}\right)}{2}}{c}$
risch	$-\frac{b\ln(x+c)}{2x}+\frac{i\pi bc\,\mathrm{csgn}(i(c-x))\mathrm{csgn}\left(\frac{i(c-x)}{x}\right)^2-i\pi bc\,\mathrm{csgn}\left(\frac{i}{x}\right)\mathrm{csgn}\left(\frac{i(x+c)}{x}\right)^2+i\pi bc\,\mathrm{csgn}\left(\frac{i}{x}\right)\mathrm{csgn}(i(x+c))\mathrm{csgn}\left(\frac{i(x+c)}{x}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))/x^2,x,method=_RETURNVERBOSE)
[Out] -1/c*(c/x*a+b*c/x*arctanh(c/x)+1/2*b*ln(1-c^2/x^2))
```

Maxima [A]
time = 0.26, size = 37, normalized size = 1.06

$$-\frac{b\left(\frac{2\,c\,\mathrm{artanh}\left(\frac{c}{x}\right)}{x}+\log\left(-\frac{c^2}{x^2}+1\right)\right)}{2\,c}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^2,x, algorithm="maxima")
[Out] -1/2*b*(2*c*arctanh(c/x)/x + log(-c^2/x^2 + 1))/c - a/x
```

Fricas [A]
time = 0.36, size = 48, normalized size = 1.37

$$-\frac{bx\log\left(-c^2+x^2\right)-2\,bx\log\left(x\right)+bc\log\left(-\frac{c+x}{c-x}\right)+2\,ac}{2\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*log(-c^2 + x^2) - 2*b*x*log(x) + b*c*log(-(c + x)/(c - x)) + 2*a*
c)/(c*x)
```

Sympy [A]
time = 0.30, size = 39, normalized size = 1.11

$$\begin{cases} -\frac{a}{x} - \frac{b \operatorname{atanh}\left(\frac{c}{x}\right)}{x} + \frac{b \log(x)}{c} - \frac{b \log(-c+x)}{c} - \frac{b \operatorname{atanh}\left(\frac{c}{x}\right)}{c} & \text{for}\: c \neq 0 \\ -\frac{a}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))/x**2,x)
```

```
[Out] Piecewise((-a/x - b*atanh(c/x)/x + b*log(x)/c - b*log(-c + x)/c - b*atanh(c
/x)/c, Ne(c, 0)), (-a/x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(33) = 66.
time = 0.42, size = 87, normalized size = 2.49

$$\frac{b \log\left(-\frac{c+x}{c-x} + 1\right) - b \log\left(-\frac{c+x}{c-x}\right) - \frac{b \log\left(-\frac{c+x}{c-x}\right)}{\frac{c+x}{c-x} - 1} - \frac{2a}{\frac{c+x}{c-x} - 1}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^2,x, algorithm="giac")
```

```
[Out] (b*log(-(c + x)/(c - x) + 1) - b*log(-(c + x)/(c - x)) - b*log(-(c + x)/(c
- x))/((c + x)/(c - x) - 1) - 2*a/((c + x)/(c - x) - 1))/c
```

Mupad [B]
time = 0.74, size = 43, normalized size = 1.23

$$\frac{b\,x\,\ln(x) - \frac{b\,x\,\ln(x^2-c^2)}{2}}{c\,x} - \frac{a + b\,\operatorname{atanh}\left(\frac{c}{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))/x^2,x)
```

```
[Out] (b*x*log(x) - (b*x*log(x^2 - c^2))/2)/(c*x) - (a + b*atanh(c/x))/x
```

3.141 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x^3}\,dx$

Optimal. Leaf size=43

$$-\frac{b}{2cx}-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2}+\frac{b\tanh^{-1}\left(\frac{x}{c}\right)}{2c^2}$$

```
[Out] -1/2*b/c/x+1/2*(-a-b*arctanh(c/x))/x^2+1/2*b*arctanh(x/c)/c^2
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6037, 269, 331, 213}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2}+\frac{b\tanh^{-1}\left(\frac{x}{c}\right)}{2c^2}-\frac{b}{2cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])/x^3,x]
```

```
[Out] -1/2*b/(c*x) - (a + b*ArcTanh[c/x])/(2*x^2) + (b*ArcTanh[x/c])/(2*c^2)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
```

```
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x^3}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2}-\frac{1}{2}(bc)\int \frac{1}{\left(1-\frac{c^2}{x^2}\right)x^4}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2}-\frac{1}{2}(bc)\int \frac{1}{x^2\left(-c^2+x^2\right)}\,dx\\ &= -\frac{b}{2cx}-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2}-\frac{b\int \frac{1}{-c^2+x^2}\,dx}{2c}\\ &= -\frac{b}{2cx}-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2}+\frac{b\tanh^{-1}\left(\frac{x}{c}\right)}{2c^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 60, normalized size = 1.40

$$-\frac{a}{2x^2}-\frac{b}{2cx}-\frac{b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2}-\frac{b\log(-c+x)}{4c^2}+\frac{b\log(c+x)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])/x^3,x]
[Out] -1/2*a/x^2 - b/(2*c*x) - (b*ArcTanh[c/x])/(2*x^2) - (b*Log[-c + x])/(4*c^2)
 + (b*Log[c + x])/(4*c^2)
```

Maple [A]
time = 0.10, size = 60, normalized size = 1.40

method	result
derivativedivides	$-\frac{\frac{a\,c^2}{2x^2}+\frac{b\,c^2\,\mathrm{arctanh}\left(\frac{c}{x}\right)}{2x^2}+\frac{bc}{2x}+\frac{b\ln\left(\frac{c}{x}-1\right)}{4}-\frac{b\ln\left(1+\frac{c}{x}\right)}{4}}{c^2}$
default	$-\frac{\frac{a\,c^2}{2x^2}+\frac{b\,c^2\,\mathrm{arctanh}\left(\frac{c}{x}\right)}{2x^2}+\frac{bc}{2x}+\frac{b\ln\left(\frac{c}{x}-1\right)}{4}-\frac{b\ln\left(1+\frac{c}{x}\right)}{4}}{c^2}$
risch	$-\frac{b\ln(x+c)}{4x^2}-\frac{i\pi b\,c^2\mathrm{csgn}\left(\frac{i}{x}\right)\mathrm{csgn}(i(c-x))\mathrm{csgn}\left(\frac{i(c-x)}{x}\right)+2i\pi b\,c^2\mathrm{csgn}\left(\frac{i(c-x)}{x}\right)^2-i\pi b\,c^2\mathrm{csgn}\left(\frac{i(x+c)}{x}\right)^3-2i\pi b\,c^2-i\pi b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))/x^3,x,method=_RETURNVERBOSE)
[Out] -1/c^2*(1/2*a*c^2/x^2+1/2*b*c^2/x^2*arctanh(c/x)+1/2*b*c/x+1/4*b*ln(c/x-1)-
1/4*b*ln(1+c/x))
```

Maxima [A]
time = 0.26, size = 52, normalized size = 1.21

$$\frac{1}{4}\left(c\left(\frac{\log\left(c+x\right)}{c^3}-\frac{\log\left(-c+x\right)}{c^3}-\frac{2}{c^2x}\right)-\frac{2\,\mathrm{artanh}\left(\frac{c}{x}\right)}{x^2}\right)b-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(c*(log(c + x)/c^3 - log(-c + x)/c^3 - 2/(c^2*x)) - 2*arctanh(c/x)/x^2)
*b - 1/2*a/x^2
```

Fricas [A]
time = 0.34, size = 46, normalized size = 1.07

$$-\frac{2\,ac^2+2\,bcx+\left(bc^2-bx^2\right)\log\left(-\frac{c+x}{c-x}\right)}{4\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*c^2 + 2*b*c*x + (b*c^2 - b*x^2)*log(-(c + x)/(c - x)))/(c^2*x^2)
```

Sympy [A]
time = 0.33, size = 44, normalized size = 1.02

$$\begin{cases}-\frac{a}{2x^2}-\frac{b\,\mathrm{atanh}\left(\frac{c}{x}\right)}{2x^2}-\frac{b}{2cx}+\frac{b\,\mathrm{atanh}\left(\frac{c}{x}\right)}{2c^2} & \text{for}\ c\neq 0\\ -\frac{a}{2x^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) - b*atanh(c/x)/(2*x**2) - b/(2*c*x) + b*atanh(c/x)/(
2*c**2), Ne(c, 0)), (-a/(2*x**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(37) = 74.
time = 0.41, size = 123, normalized size = 2.86

$$-\frac{\frac{b(c+x)\log\left(-\frac{c+x}{c-x}\right)}{\left(\frac{(c+x)^2c}{(c-x)^2}-\frac{2\,(c+x)c}{c-x}+c\right)(c-x)}-\frac{b-\frac{2\,a(c+x)}{c-x}-\frac{b(c+x)}{c-x}}{\frac{(c+x)^2c}{(c-x)^2}-\frac{2\,(c+x)c}{c-x}+c}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^3,x, algorithm="giac")
```

```
[Out] -(b*(c + x)*log(-(c + x)/(c - x))/((((c + x)^2*c/(c - x)^2 - 2*(c + x)*c/(c
- x) + c)*(c - x)) - (b - 2*a*(c + x)/(c - x) - b*(c + x)/(c - x))/(((c + x)
^2*c/(c - x)^2 - 2*(c + x)*c/(c - x) + c))/c
```

Mupad [B]
time = 0.71, size = 49, normalized size = 1.14

$$\frac{b\,c\,\mathrm{atan}\left(\frac{x}{\sqrt{-c^2}}\right)}{2\,{\left(-c^2\right)}^{3/2}}-\frac{b}{2\,c\,x}-\frac{b\,\mathrm{atanh}\left(\frac{c}{x}\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))/x^3,x)
```

```
[Out] (b*c*atan(x/(-c^2)^(1/2)))/(2*(-c^2)^(3/2)) - b/(2*c*x) - (b*atanh(c/x))/(2
*x^2) - a/(2*x^2)
```

3.142 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x^4}\,dx$

Optimal. Leaf size=57

$$-\frac{b}{6cx^2}-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2-x^2\right)}{6c^3}$$

```
[Out] -1/6*b/c/x^2+1/3*(-a-b*arctanh(c/x))/x^3+1/3*b*ln(x)/c^3-1/6*b*ln(c^2-x^2)/
c^3
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {6037, 269, 272, 46}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2-x^2\right)}{6c^3}-\frac{b}{6cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])/x^4,x]
```

```
[Out] -1/6*b/(c*x^2) - (a + b*ArcTanh[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) - (b*Log
[c^2 - x^2])/(6*c^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x^4}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{3}(bc)\int \frac{1}{\left(1-\frac{c^2}{x^2}\right)x^5}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{3}(bc)\int \frac{1}{x^3\left(-c^2+x^2\right)}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{6}(bc)\text{Subst}\left(\int \frac{1}{x^2\left(-c^2+x\right)}\,dx,x,x^2\right)\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{6}(bc)\text{Subst}\left(\int \left(-\frac{1}{c^4\left(c^2-x\right)}-\frac{1}{c^2x^2}-\frac{1}{c^4x}\right)dx,x,x^2\right)\\ &= -\frac{b}{6cx^2}-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2-x^2\right)}{6c^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 62, normalized size = 1.09

$$-\frac{a}{3x^3}-\frac{b}{6cx^2}-\frac{b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(-c^2+x^2\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])/x^4,x]
[Out] -1/3*a/x^3 - b/(6*c*x^2) - (b*ArcTanh[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) -
(b*Log[-c^2 + x^2])/(6*c^3)
```

Maple [A]
time = 0.10, size = 62, normalized size = 1.09

method	result
derivativedivides	$-\frac{\frac{a c^3}{3x^3}+\frac{b c^3 \operatorname{arctanh}\left(\frac{c}{x}\right)}{3x^3}+\frac{b c^2}{6x^2}+\frac{b\ln\left(\frac{c}{x}-1\right)}{6}+\frac{b\ln\left(1+\frac{c}{x}\right)}{6}}{c^3}$
default	$-\frac{\frac{a c^3}{3x^3}+\frac{b c^3 \operatorname{arctanh}\left(\frac{c}{x}\right)}{3x^3}+\frac{b c^2}{6x^2}+\frac{b\ln\left(\frac{c}{x}-1\right)}{6}+\frac{b\ln\left(1+\frac{c}{x}\right)}{6}}{c^3}$
risch	$-\frac{b\ln(x+c)}{6x^3}-\frac{-i\pi b c^3\operatorname{csgn}\left(\frac{i}{x}\right)\operatorname{csgn}\left(\frac{i(c-x)}{x}\right)^2-i\pi b c^3\operatorname{csgn}\left(\frac{i}{x}\right)\operatorname{csgn}(i(x+c))\operatorname{csgn}\left(\frac{i(x+c)}{x}\right)+i\pi b c^3\operatorname{csgn}(i(x+c))\operatorname{csgn}\left(\frac{i}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^3*(1/3*a*c^3/x^3+1/3*b*c^3/x^3*arctanh(c/x)+1/6*b*c^2/x^2+1/6*b*ln(c/x
-1)+1/6*b*ln(1+c/x))
```

Maxima [A]
time = 0.26, size = 55, normalized size = 0.96

$$-\frac{1}{6}\left(c\left(\frac{\log\left(-c^2+x^2\right)}{c^4}-\frac{\log\left(x^2\right)}{c^4}+\frac{1}{c^2x^2}\right)+\frac{2\,\mathrm{artanh}\left(\frac{c}{x}\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(c*(log(-c^2 + x^2)/c^4 - log(x^2)/c^4 + 1/(c^2*x^2)) + 2*arctanh(c/x)
/x^3)*b - 1/3*a/x^3
```

Fricas [A]
time = 0.36, size = 62, normalized size = 1.09

$$-\frac{bx^3\log\left(-c^2+x^2\right)-2\,bx^3\log\left(x\right)+bc^3\log\left(-\frac{c+x}{c-x}\right)+2\,ac^3+bc^2x}{6\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*x^3*log(-c^2 + x^2) - 2*b*x^3*log(x) + b*c^3*log(-(c + x)/(c - x))
+ 2*a*c^3 + b*c^2*x)/(c^3*x^3)
```

Sympy [A]
time = 0.43, size = 68, normalized size = 1.19

$$\begin{cases} -\frac{a}{3x^3}-\frac{b\,\mathrm{atanh}\left(\frac{c}{x}\right)}{3x^3}-\frac{b}{6cx^2}+\frac{b\log\left(x\right)}{3c^3}-\frac{b\log\left(-c+x\right)}{3c^3}-\frac{b\,\mathrm{atanh}\left(\frac{c}{x}\right)}{3c^3} & \text{for}\ c\neq 0 \\ -\frac{a}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3) - b*atanh(c/x)/(3*x**3) - b/(6*c*x**2) + b*log(x)/(3
*c**3) - b*log(-c + x)/(3*c**3) - b*atanh(c/x)/(3*c**3), Ne(c, 0)), (-a/(3*
x**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(49) = 98.

time = 0.42, size = 234, normalized size = 4.11

$$-\frac{\frac{\left(b+\frac{3\,b(c+x)^2}{(c-x)^2}\right)\log\left(-\frac{c+x}{c-x}\right)}{\frac{(c+x)^3c^2}{(c-x)^3}-\frac{3\,(c+x)^2c^2}{(c-x)^2}+\frac{3\,(c+x)c^2}{c-x}-c^2}+\frac{2\left(a+\frac{3\,a(c+x)^2}{(c-x)^2}+\frac{b(c+x)^2}{(c-x)^2}-\frac{b(c+x)}{c-x}\right)}{\frac{(c+x)^3c^2}{(c-x)^3}-\frac{3\,(c+x)^2c^2}{(c-x)^2}+\frac{3\,(c+x)c^2}{c-x}-c^2}-\frac{b\log\left(-\frac{c+x}{c-x}+1\right)}{c^2}+\frac{b\log\left(-\frac{c+x}{c-x}\right)}{c^2}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^4,x, algorithm="giac")
```

```
[Out] -1/3*((b + 3*b*(c + x)^2/(c - x)^2)*log(-(c + x)/(c - x))/((c + x)^3*c^2/(c
 - x)^3 - 3*(c + x)^2*c^2/(c - x)^2 + 3*(c + x)*c^2/(c - x) - c^2) + 2*(a +
 3*a*(c + x)^2/(c - x)^2 + b*(c + x)^2/(c - x)^2 - b*(c + x)/(c - x))/((c +
 x)^3*c^2/(c - x)^3 - 3*(c + x)^2*c^2/(c - x)^2 + 3*(c + x)*c^2/(c - x) - c
^2) - b*log(-(c + x)/(c - x) + 1)/c^2 + b*log(-(c + x)/(c - x))/c^2)/c
```

Mupad [B]

time = 0.75, size = 59, normalized size = 1.04

$$-\frac{\frac{a}{3}+\frac{b\,\mathrm{atanh}\left(\frac{c}{x}\right)}{3}}{x^3}-\frac{\frac{b\,x^3\,\ln\left(x^2-c^2\right)}{6}-\frac{b\,x^3\,\ln\left(x\right)}{3}+\frac{b\,c^2\,x}{6}}{c^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))/x^4,x)
```

```
[Out] - (a/3 + (b*atanh(c/x))/3)/x^3 - ((b*x^3*log(x^2 - c^2))/6 - (b*x^3*log(x))
/3 + (b*c^2*x)/6)/(c^3*x^3)
```

3.143 $\int x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal. Leaf size=123

$$\frac{1}{12}b^2c^2x^2+\frac{1}{2}bc^3x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{6}bcx^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{4}c^4\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{4}x^4\left(a+b\coth\right.$$

```
[Out] 1/12*b^2*c^2*x^2+1/2*b*c^3*x*(a+b*arccoth(x/c))+1/6*b*c*x^3*(a+b*arccoth(x/
c))-1/4*c^4*(a+b*arccoth(x/c))^2+1/4*x^4*(a+b*arccoth(x/c))^2+1/3*b^2*c^4*l
n(1-c^2/x^2)+2/3*b^2*c^4*ln(x)
```

Rubi [A]
time = 0.20, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {6039, 6037, 6129, 272, 46, 36, 29, 31, 6095}

$$-\frac{1}{4}c^4\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}bc^3x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{4}x^4\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{6}bcx^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{2}{3}b^2c^4\log(x)+\frac{1}{12}b^2c^2x^2+\frac{1}{3}b^2c^4\log\left(1-\frac{c^2}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] (b^2*c^2*x^2)/12 + (b*c^3*x*(a + b*ArcCoth[x/c]))/2 + (b*c*x^3*(a + b*ArcCo
th[x/c]))/6 - (c^4*(a + b*ArcCoth[x/c])^2)/4 + (x^4*(a + b*ArcCoth[x/c])^2)
/4 + (b^2*c^4*Log[1 - c^2/x^2])/3 + (2*b^2*c^4*Log[x])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2 dx &= \int \left(\frac{1}{4}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{2}bx^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)+\right.\\
&= \frac{1}{4}\int x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2 dx+\frac{1}{2}b\int x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\right.\\
&= -\left(\frac{1}{4}\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^2}{x^5}dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int\left(2ax^3\log\left(1+\frac{c}{x}\right.\right.\\
&= \frac{1}{16}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{16}b^2x^4\log^2\left(\frac{c+x}{x}\right)+(ab)\int x^3\log\left(1+\frac{c}{x}\right.\\
&= \frac{1}{16}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{4}abx^4\log\left(1+\frac{c}{x}\right)-\frac{1}{8}b^2x^4\log\left(1-\frac{c}{x}\right)\log\left(\right.\\
&= \frac{1}{16}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{4}abx^4\log\left(1+\frac{c}{x}\right)-\frac{1}{8}b^2x^4\log\left(1-\frac{c}{x}\right)\log\left(\right.\\
&= \frac{1}{24}bcx^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)+\frac{1}{16}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{4}abx^4\log\left(1\right.\\
&= \frac{1}{4}abc^3x-\frac{1}{8}abc^2x^2+\frac{1}{12}abcx^3+\frac{1}{16}bc^2x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)+\frac{1}{24}bcx^3\left(2a\right.\\
&= \frac{1}{4}abc^3x-\frac{1}{16}b^2c^3x-\frac{1}{8}abc^2x^2+\frac{1}{24}b^2c^2x^2+\frac{1}{12}abcx^3+\frac{1}{24}b^2c^4\log\left(1-\frac{c}{x}\right)\\
&= \frac{1}{4}abc^3x-\frac{1}{8}abc^2x^2+\frac{1}{24}b^2c^2x^2+\frac{1}{12}abcx^3+\frac{5}{48}b^2c^4\log\left(1-\frac{c}{x}\right)-\frac{1}{8}b^2c^3x\,\text{lo}\\
&= \frac{1}{4}abc^3x-\frac{1}{8}abc^2x^2+\frac{1}{24}b^2c^2x^2+\frac{1}{12}abcx^3+\frac{5}{48}b^2c^4\log\left(1-\frac{c}{x}\right)-\frac{1}{8}b^2c^3x\,\text{lo}\\
&= \frac{1}{4}abc^3x-\frac{1}{8}abc^2x^2+\frac{1}{12}b^2c^2x^2+\frac{1}{12}abcx^3+\frac{5}{48}b^2c^4\log\left(1-\frac{c}{x}\right)-\frac{1}{8}b^2c^3x\,\text{lo}\\
&= \frac{1}{4}abc^3x-\frac{1}{8}abc^2x^2+\frac{1}{12}b^2c^2x^2+\frac{1}{12}abcx^3+\frac{5}{48}b^2c^4\log\left(1-\frac{c}{x}\right)-\frac{1}{8}b^2c^3x\,\text{lo}\\
&= \frac{1}{4}abc^3x-\frac{1}{8}abc^2x^2+\frac{1}{12}b^2c^2x^2+\frac{1}{12}abcx^3+\frac{5}{48}b^2c^4\log\left(1-\frac{c}{x}\right)-\frac{1}{8}b^2c^3x\,\text{lo}
\end{aligned}$$

<u>Mathematica</u> [A]
time = 0.06, size = 131, normalized size = 1.07

$$\frac{1}{12}\left(6abc^3x+b^2c^2x^2+2abcx^3+3a^2x^4+2bx\left(3ax^3+bc\left(3c^2+x^2\right)\right)\tanh^{-1}\left(\frac{c}{x}\right)+3b^2\left(-c^4+x^4\right)\tanh^{-1}\left(\frac{c}{x}\right)^2+b(3a+4b)c^4\log(-c+x)-3abc^4\log(c+x)+4b^2c^4\log(c+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] (6*a*b*c^3*x + b^2*c^2*x^2 + 2*a*b*c*x^3 + 3*a^2*x^4 + 2*b*x*(3*a*x^3 + b*c
*(3*c^2 + x^2))*ArcTanh[c/x] + 3*b^2*(-c^4 + x^4)*ArcTanh[c/x]^2 + b*(3*a +
 4*b)*c^4*Log[-c + x] - 3*a*b*c^4*Log[c + x] + 4*b^2*c^4*Log[c + x])/12
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(109) = 218$.
time = 1.03, size = 310, normalized size = 2.52

method	result
derivativedivides	$-c^4\left(-\frac{a^2x^4}{4c^4} - \frac{b^2x^4 \operatorname{arctanh}\left(\frac{c}{x}\right)^2}{4c^4} + \frac{b^2 \operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{4} - \frac{b^2 \operatorname{arctanh}\left(\frac{c}{x}\right)x^3}{6c^3} - \frac{b^2 \operatorname{arctanh}\left(\frac{c}{x}\right)x}{2c} - b^2 \text{arc}\right.$
default	$-c^4\left(-\frac{a^2x^4}{4c^4} - \frac{b^2x^4 \operatorname{arctanh}\left(\frac{c}{x}\right)^2}{4c^4} + \frac{b^2 \operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{4} - \frac{b^2 \operatorname{arctanh}\left(\frac{c}{x}\right)x^3}{6c^3} - \frac{b^2 \operatorname{arctanh}\left(\frac{c}{x}\right)x}{2c} - b^2 \text{arc}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c/x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -c^4*(-1/4*a^2/c^4*x^4-1/4*b^2/c^4*x^4*arctanh(c/x)^2+1/4*b^2*arctanh(c/x)*
ln(1+c/x)-1/6*b^2*arctanh(c/x)/c^3*x^3-1/2*b^2*arctanh(c/x)/c*x-1/4*b^2*arc
tanh(c/x)*ln(c/x-1)-1/16*b^2*ln(c/x-1)^2+1/8*b^2*ln(c/x-1)*ln(1/2*c/x+1/2)-
1/16*b^2*ln(1+c/x)^2+1/8*b^2*ln(-1/2*c/x+1/2)*ln(1+c/x)-1/8*b^2*ln(-1/2*c/x
+1/2)*ln(1/2*c/x+1/2)-1/3*b^2*ln(1+c/x)-1/12*b^2*x^2/c^2+2/3*b^2*ln(c/x)-1/
3*b^2*ln(c/x-1)-1/2*a*b/c^4*x^4*arctanh(c/x)+1/4*a*b*ln(1+c/x)-1/6*a*b*x^3/
c^3-1/2*a*b*x/c-1/4*a*b*ln(c/x-1))
```

Maxima [A]
time = 0.25, size = 189, normalized size = 1.54

$$\frac{1}{4}b^2x^4\operatorname{artanh}\left(\frac{c}{x}\right)^2 + \frac{1}{4}a^2x^4 + \frac{1}{12}\left(6x^4\operatorname{artanh}\left(\frac{c}{x}\right) - \left(3c^3\log\left(c+x\right) - 3c^3\log\left(-c+x\right) - 6c^2x - 2x^3\right)c\right)ab + \frac{1}{48}\left(\left(3c^2\log\left(c+x\right)^2 + 3c^2\log\left(-c+x\right)^2 + 16c^2\log\left(c+x\right) + 4x^2 - 2\left(3c^2\log\left(c+x\right) - 8c^2\right)\log\left(-c+x\right)\right)c^2 - 4\left(3c^3\log\left(c+x\right) - 3c^3\log\left(-c+x\right) - 6c^2x - 2x^3\right)c\operatorname{artanh}\left(\frac{c}{x}\right)\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctanh(c/x)^2 + 1/4*a^2*x^4 + 1/12*(6*x^4*arctanh(c/x) - (3*c^
3*log(c + x) - 3*c^3*log(-c + x) - 6*c^2*x - 2*x^3)*c)*a*b + 1/48*((3*c^2*l
og(c + x)^2 + 3*c^2*log(-c + x)^2 + 16*c^2*log(c + x) + 4*x^2 - 2*(3*c^2*lo
g(c + x) - 8*c^2)*log(-c + x))*c^2 - 4*(3*c^3*log(c + x) - 3*c^3*log(-c + x
) - 6*c^2*x - 2*x^3)*c*arctanh(c/x))*b^2
```

Fricas [A]
time = 0.34, size = 149, normalized size = 1.21

$$\frac{1}{2}abc^3x + \frac{1}{12}b^2c^2x^2 + \frac{1}{6}abcx^3 + \frac{1}{4}a^2x^4 - \frac{1}{12}\left(3ab - 4b^2\right)c^4\log\left(c+x\right) + \frac{1}{12}\left(3ab + 4b^2\right)c^4\log\left(-c+x\right) - \frac{1}{16}\left(b^2c^4 - b^2x^4\right)\log\left(-\frac{c+x}{c-x}\right)^2 + \frac{1}{12}\left(3b^2c^3x + b^2cx^3 + 3abx^4\right)\log\left(-\frac{c+x}{c-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*a*b*c^3*x + 1/12*b^2*c^2*x^2 + 1/6*a*b*c*x^3 + 1/4*a^2*x^4 - 1/12*(3*a*
b - 4*b^2)*c^4*log(c + x) + 1/12*(3*a*b + 4*b^2)*c^4*log(-c + x) - 1/16*(b^
2*c^4 - b^2*x^4)*log(-(c + x)/(c - x))^2 + 1/12*(3*b^2*c^3*x + b^2*c*x^3 +
3*a*b*x^4)*log(-(c + x)/(c - x))
```

Sympy [A]
time = 0.23, size = 158, normalized size = 1.28

$$\frac{a^2x^4}{4}-\frac{abc^4\operatorname{atanh}\left(\frac{c}{x}\right)}{2}+\frac{abc^3x}{2}+\frac{abcx^3}{6}+\frac{abx^4\operatorname{atanh}\left(\frac{c}{x}\right)}{2}+\frac{2b^2c^4\log\left(-c+x\right)}{3}-\frac{b^2c^4\operatorname{atanh}^2\left(\frac{c}{x}\right)}{4}+\frac{2b^2c^4\operatorname{atanh}\left(\frac{c}{x}\right)}{3}+\frac{b^2c^3x\operatorname{atanh}\left(\frac{c}{x}\right)}{2}+\frac{b^2c^2x^2}{12}+\frac{b^2cx^3\operatorname{atanh}\left(\frac{c}{x}\right)}{6}+\frac{b^2x^4\operatorname{atanh}^2\left(\frac{c}{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c/x))**2,x)
```

```
[Out] a**2*x**4/4 - a*b*c**4*atanh(c/x)/2 + a*b*c**3*x/2 + a*b*c*x**3/6 + a*b*x**
4*atanh(c/x)/2 + 2*b**2*c**4*log(-c + x)/3 - b**2*c**4*atanh(c/x)**2/4 + 2*
b**2*c**4*atanh(c/x)/3 + b**2*c**3*x*atanh(c/x)/2 + b**2*c**2*x**2/12 + b**
2*c*x**3*atanh(c/x)/6 + b**2*x**4*atanh(c/x)**2/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(109) = 218.
time = 0.43, size = 552, normalized size = 4.49

$$-\frac{4\,b^2c^5\log\left(-\frac{c+x}{c-x}-1\right)-4\,b^2c^5\log\left(-\frac{c+x}{c-x}\right)+\frac{3\left(\frac{b^2(c+x)^3c^5}{(c-x)^3}+\frac{b^2(c+x)c^5}{c-x}\right)\log\left(-\frac{c+x}{c-x}\right)^2}{\frac{(c+x)^4}{(c-x)^4}+\frac{4\,(c+x)^3}{(c-x)^3}+\frac{6\,(c+x)^2}{(c-x)^2}+\frac{4\,(c+x)}{c-x}+1}+\frac{2\left(2\,b^2c^5+\frac{6\,ab(c+x)^3c^5}{(c-x)^3}+\frac{3\,b^2(c+x)^3c^5}{(c-x)^3}+\frac{6\,b^2(c+x)^2c^5}{(c-x)^2}+\frac{6\,ab(c+x)c^5}{c-x}+\frac{5\,b^2(c+x)c^5}{c-x}\right)\log\left(-\frac{c+x}{c-x}\right)}{\frac{(c+x)^4}{(c-x)^4}+\frac{4\,(c+x)^3}{(c-x)^3}+\frac{6\,(c+x)^2}{(c-x)^2}+\frac{4\,(c+x)}{c-x}+1}+\frac{2\left(4\,abc^5+\frac{6\,a^2(c+x)^3c^5}{(c-x)^3}+\frac{6\,ab(c+x)^3c^5}{(c-x)^3}+\frac{b^2(c+x)^3c^5}{(c-x)^3}+\frac{12\,ab(c+x)^2c^5}{(c-x)^2}+\frac{2\,b^2(c+x)^2c^5}{(c-x)^2}+\frac{6\,a^2(c+x)c^5}{c-x}+\frac{10\,ab(c+x)c^5}{c-x}+\frac{b^2(c+x)c^5}{c-x}\right)}{\frac{(c+x)^4}{(c-x)^4}+\frac{4\,(c+x)^3}{(c-x)^3}+\frac{6\,(c+x)^2}{(c-x)^2}+\frac{4\,(c+x)}{c-x}+1}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^2,x, algorithm="giac")
```

```
[Out] -1/6*(4*b^2*c^5*log(-(c + x)/(c - x) - 1) - 4*b^2*c^5*log(-(c + x)/(c - x))
 + 3*(b^2*(c + x)^3*c^5/(c - x)^3 + b^2*(c + x)*c^5/(c - x))*log(-(c + x)/(
c - x))^2/((c + x)^4/(c - x)^4 + 4*(c + x)^3/(c - x)^3 + 6*(c + x)^2/(c - x
)^2 + 4*(c + x)/(c - x) + 1) + 2*(2*b^2*c^5 + 6*a*b*(c + x)^3*c^5/(c - x)^3
 + 3*b^2*(c + x)^3*c^5/(c - x)^3 + 6*b^2*(c + x)^2*c^5/(c - x)^2 + 6*a*b*(c
 + x)*c^5/(c - x) + 5*b^2*(c + x)*c^5/(c - x))*log(-(c + x)/(c - x))/((c +
x)^4/(c - x)^4 + 4*(c + x)^3/(c - x)^3 + 6*(c + x)^2/(c - x)^2 + 4*(c + x)/
(c - x) + 1) + 2*(4*a*b*c^5 + 6*a^2*(c + x)^3*c^5/(c - x)^3 + 6*a*b*(c + x)
^3*c^5/(c - x)^3 + b^2*(c + x)^3*c^5/(c - x)^3 + 12*a*b*(c + x)^2*c^5/(c -
x)^2 + 2*b^2*(c + x)^2*c^5/(c - x)^2 + 6*a^2*(c + x)*c^5/(c - x) + 10*a*b*(
c + x)*c^5/(c - x) + b^2*(c + x)*c^5/(c - x))/((c + x)^4/(c - x)^4 + 4*(c +
 x)^3/(c - x)^3 + 6*(c + x)^2/(c - x)^2 + 4*(c + x)/(c - x) + 1))/c
```

Mupad [B]
time = 0.89, size = 142, normalized size = 1.15

$$\frac{a^2\,x^4}{4}-\frac{b^2\,c^4\,{\mathrm{atanh}\left(\frac{c}{x}\right)}^2}{4}+\frac{b^2\,x^4\,{\mathrm{atanh}\left(\frac{c}{x}\right)}^2}{4}+\frac{b^2\,c^4\,\ln\left(x^2-c^2\right)}{3}+\frac{b^2\,c^2\,x^2}{12}+\frac{b^2\,c\,x^3\,\mathrm{atanh}\left(\frac{c}{x}\right)}{6}+\frac{b^2\,c^3\,x\,\mathrm{atanh}\left(\frac{c}{x}\right)}{2}+\frac{a\,b\,c\,x^3}{6}+\frac{a\,b\,c^3\,x}{2}-\frac{a\,b\,c^4\,\mathrm{atanh}\left(\frac{c}{x}\right)}{2}+\frac{a\,b\,x^4\,\mathrm{atanh}\left(\frac{c}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c/x))^2,x)
```

```
[Out] (a^2*x^4)/4 - (b^2*c^4*atanh(c/x)^2)/4 + (b^2*x^4*atanh(c/x)^2)/4 + (b^2*c^
4*log(x^2 - c^2))/3 + (b^2*c^2*x^2)/12 + (b^2*c*x^3*atanh(c/x))/6 + (b^2*c^
3*x*atanh(c/x))/2 + (a*b*c*x^3)/6 + (a*b*c^3*x)/2 - (a*b*c^4*atanh(c/x))/2
+ (a*b*x^4*atanh(c/x))/2
```

3.144 $\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal. Leaf size=142

$$\frac{1}{3}b^2c^2x-\frac{1}{3}b^2c^3\coth^{-1}\left(\frac{x}{c}\right)+\frac{1}{3}bcx^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{3}c^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{3}x^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2$$

```
[Out] 1/3*b^2*c^2*x-1/3*b^2*c^3*arccoth(x/c)+1/3*b*c*x^2*(a+b*arccoth(x/c))-1/3*c
^3*(a+b*arccoth(x/c))^2+1/3*x^3*(a+b*arccoth(x/c))^2-2/3*b*c^3*(a+b*arccoth
(x/c))*ln(2-2/(1+c/x))+1/3*b^2*c^3*polylog(2,-1+2/(1+c/x))
```

Rubi [A]
time = 0.19, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {6039, 6037, 6129, 331, 212, 6135, 6079, 2497}

$$-\frac{1}{3}c^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{2}{3}bc^3\log\left(2-\frac{2}{\frac{c}{x}+1}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{3}x^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{3}bcx^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{3}b^2c^3\text{Li}_2\left(\frac{2}{\frac{c}{x}+1}-1\right)-\frac{1}{3}b^2c^3\coth^{-1}\left(\frac{x}{c}\right)+\frac{1}{3}b^2c^2x$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] (b^2*c^2*x)/3 - (b^2*c^3*ArcCoth[x/c])/3 + (b*c*x^2*(a + b*ArcCoth[x/c]))/3
 - (c^3*(a + b*ArcCoth[x/c])^2)/3 + (x^3*(a + b*ArcCoth[x/c])^2)/3 - (2*b*c
^3*(a + b*ArcCoth[x/c])*Log[2 - 2/(1 + c/x)])/3 + (b^2*c^3*PolyLog[2, -1 +
2/(1 + c/x)])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2 dx = \int \left(\frac{1}{4}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{2}bx^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)+\right.$$
$$=\frac{1}{4}\int x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2 dx+\frac{1}{2}b\int x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\right.$$
$$=-\left(\frac{1}{4}\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^2}{x^4}dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int\left(2ax^2\log\left(1+\frac{c}{x}\right.\right.$$
$$=\frac{1}{12}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{12}b^2x^3\log^2\left(\frac{c+x}{x}\right)+(ab)\int x^2\log\left(1+\frac{c}{x}\right.$$
$$=\frac{1}{12}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{3}abx^3\log\left(1+\frac{c}{x}\right)-\frac{1}{6}b^2x^3\log\left(1-\frac{c}{x}\right)\log\left(\right.$$
$$=\frac{1}{12}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{3}abx^3\log\left(1+\frac{c}{x}\right)-\frac{1}{6}b^2x^3\log\left(1-\frac{c}{x}\right)\log\left(\right.$$
$$=\frac{1}{12}bcx^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)+\frac{1}{12}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{3}abx^3\log\left(1\right.$$
$$=-\frac{1}{3}abc^2x+\frac{1}{6}abcx^2+\frac{1}{6}bc^2\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)+\frac{1}{12}bcx^2\left(2a-\right.$$
$$=-\frac{1}{3}abc^2x+\frac{1}{6}b^2c^2x+\frac{1}{6}abcx^2+\frac{1}{12}b^2c^3\log\left(1-\frac{c}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1-\frac{c}{x}\right)-$$
$$=-\frac{1}{3}abc^2x+\frac{1}{6}b^2c^2x+\frac{1}{6}abcx^2+\frac{1}{12}b^2c^3\log\left(1-\frac{c}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1-\frac{c}{x}\right)-$$
$$=-\frac{1}{3}abc^2x+\frac{1}{6}b^2c^2x+\frac{1}{6}abcx^2+\frac{1}{12}b^2c^3\log\left(1-\frac{c}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1-\frac{c}{x}\right)-$$
$$=-\frac{1}{3}abc^2x+\frac{1}{3}b^2c^2x+\frac{1}{6}abcx^2+\frac{1}{12}b^2c^3\log\left(1-\frac{c}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1-\frac{c}{x}\right)-$$
$$=-\frac{1}{3}abc^2x+\frac{1}{3}b^2c^2x+\frac{1}{6}abcx^2+\frac{1}{12}b^2c^3\log\left(1-\frac{c}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1-\frac{c}{x}\right)-$$
$$=-\frac{1}{3}abc^2x+\frac{1}{3}b^2c^2x+\frac{1}{6}abcx^2+\frac{1}{12}b^2c^3\log\left(1-\frac{c}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1-\frac{c}{x}\right)-$$

<u>Mathematica</u> [A]
time = 0.22, size = 145, normalized size = 1.02

$$\frac{1}{3}\left(b^2c^2x+abcx^2+a^2x^3+b^2\left(-c^3+x^3\right)\tanh^{-1}\left(\frac{c}{x}\right)^2+b\tanh^{-1}\left(\frac{c}{x}\right)\left(-bc^3+bcx^2+2ax^3-2bc^3\log\left(1-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)+abc^3\log\left(1-\frac{c^2}{x^2}\right)-2abc^3\log\left(\frac{c}{x}\right)+b^2c^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c/x])^2,x]
```

[Out] (b^2*c^2*x + a*b*c*x^2 + a^2*x^3 + b^2*(-c^3 + x^3)*ArcTanh[c/x]^2 + b*ArcTanh[c/x]*(-(b*c^3) + b*c*x^2 + 2*a*x^3 - 2*b*c^3*Log[1 - E^(-2*ArcTanh[c/x])]) + a*b*c^3*Log[1 - c^2/x^2] - 2*a*b*c^3*Log[c/x] + b^2*c^3*PolyLog[2, E^(-2*ArcTanh[c/x])])/3

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(128) = 256$.
time = 0.37, size = 358, normalized size = 2.52

method	result
derivativedivides	$-c^3\left(-\frac{a^2x^3}{3c^3}-\frac{b^2x^3\operatorname{arctanh}\left(\frac{c}{x}\right)^2}{3c^3}-\frac{b^2\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{3}-\frac{b^2\operatorname{arctanh}\left(\frac{c}{x}\right)x^2}{3c^2}+\frac{2b^2\ln\left(\frac{c}{x}\right)\operatorname{arctanh}\left(\frac{c}{x}\right)}{3}-\right.$
default	$-c^3\left(-\frac{a^2x^3}{3c^3}-\frac{b^2x^3\operatorname{arctanh}\left(\frac{c}{x}\right)^2}{3c^3}-\frac{b^2\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{3}-\frac{b^2\operatorname{arctanh}\left(\frac{c}{x}\right)x^2}{3c^2}+\frac{2b^2\ln\left(\frac{c}{x}\right)\operatorname{arctanh}\left(\frac{c}{x}\right)}{3}-\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*arctanh(c/x))^2,x,method=_RETURNVERBOSE)

[Out] -c^3*(-1/3*a^2/c^3*x^3-1/3*b^2/c^3*x^3*arctanh(c/x)^2-1/3*b^2*arctanh(c/x)*ln(1+c/x)-1/3*b^2*arctanh(c/x)/c^2*x^2+2/3*b^2*ln(c/x)*arctanh(c/x)-1/3*b^2*arctanh(c/x)*ln(c/x-1)+1/6*b^2*ln(1+c/x)-1/6*b^2*ln(c/x-1)-1/3*b^2/c*x-1/12*b^2*ln(c/x-1)^2+1/3*b^2*dilog(1/2*c/x+1/2)+1/6*b^2*ln(c/x-1)*ln(1/2*c/x+1/2)+1/12*b^2*ln(1+c/x)^2+1/6*b^2*ln(-1/2*c/x+1/2)*ln(1/2*c/x+1/2)-1/6*b^2*ln(-1/2*c/x+1/2)*ln(1+c/x)-1/3*b^2*dilog(1+c/x)-1/3*b^2*ln(c/x)*ln(1+c/x)-1/3*b^2*dilog(c/x)-2/3*a*b/c^3*x^3*arctanh(c/x)-1/3*a*b*ln(1+c/x)-1/3*a*b/c^2*x^2+2/3*a*b*ln(c/x)-1/3*a*b*ln(c/x-1))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arctanh(c/x))^2,x, algorithm="maxima")

[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arctanh(c/x) + (c^2*log(-c^2 + x^2) + x^2)*c)*a*b + 1/12*(6*c^4*integrate(-1/3*log(c + x)/(c^2 - x^2), x) + x^3*log(c + x)^2 + 6*c^3*integrate(-1/3*x*log(c + x)/(c^2 - x^2), x) - (c*log(c + x) - c*log(-c + x) - 2*x)*c^2 - (c^3 - x^3)*log(-c + x)^2 + (c^2*log(-c^2 + x^2) + x^2)*c + 12*c*integrate(-1/3*x^3*log(c + x)/(c^2 - x^2), x) - 2*(c*x^2 + (c^3 + x^3)*log(c + x))*log(-c + x))*b^2

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctanh(c/x)^2 + 2*a*b*x^2*arctanh(c/x) + a^2*x^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b\operatorname{atanh}\left(\frac{c}{x}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c/x))**2,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c/x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^2*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c/x))^2,x)
```

```
[Out] int(x^2*(a + b*atanh(c/x))^2, x)
```

3.145 $\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal. Leaf size=83

$$bcx\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{2}c^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}x^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}b^2c^2\log\left(1-\frac{c^2}{x^2}\right)+b^2c^2\log$$

[Out] b*c*x*(a+b*arccoth(x/c))-1/2*c^2*(a+b*arccoth(x/c))^2+1/2*x^2*(a+b*arccoth(x/c))^2+1/2*b^2*c^2*ln(1-c^2/x^2)+b^2*c^2*ln(x)

Rubi [A]
time = 0.11, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.571$, Rules used = {6039, 6037, 6129, 272, 36, 29, 31, 6095}

$$-\frac{1}{2}c^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}x^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+bcx\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}b^2c^2\log\left(1-\frac{c^2}{x^2}\right)+b^2c^2\log(x)$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*ArcTanh[c/x])^2,x]

[Out] b*c*x*(a + b*ArcCoth[x/c]) - (c^2*(a + b*ArcCoth[x/c])^2)/2 + (x^2*(a + b*ArcCoth[x/c])^2)/2 + (b^2*c^2*Log[1 - c^2/x^2])/2 + b^2*c^2*Log[x]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2 dx = \int \left(\frac{1}{4}x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{2}bx\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)+\right.$$

$$=\frac{1}{4}\int x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2 dx+\frac{1}{2}b\int x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right.$$

$$=-\left(\frac{1}{4}\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^2}{x^3}dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int\left(2ax\log\left(1+\frac{c}{x}\right)\right.$$

$$=\frac{1}{8}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{8}b^2x^2\log^2\left(\frac{c+x}{x}\right)+(ab)\int x\log\left(1+\frac{c}{x}\right)$$

$$=\frac{1}{8}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x}\right)\log\left(\right.$$

$$=\frac{1}{8}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x}\right)\log\left(\right.$$

$$=\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)+\frac{1}{8}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{2}abx^2$$

$$=\frac{1}{2}abcx+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(\right.$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(\right.$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(\right.$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(\right.$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(\right.$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(\right.$$

Mathematica [A]
time = 0.03, size = 92, normalized size = 1.11

$$\frac{1}{2}\left(2abcx+a^2x^2+2bx(bc+ax)\tanh^{-1}\left(\frac{c}{x}\right)+b^2\left(-c^2+x^2\right)\tanh^{-1}\left(\frac{c}{x}\right)^2+b(a+b)c^2\log(-c+x)-abc^2\log(c+x)+b^2c^2\log(c+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] (2*a*b*c*x + a^2*x^2 + 2*b*x*(b*c + a*x)*ArcTanh[c/x] + b^2*(-c^2 + x^2)*Ar
cTanh[c/x]^2 + b*(a + b)*c^2*Log[-c + x] - a*b*c^2*Log[c + x] + b^2*c^2*Log
[c + x])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(77) = 154$.
time = 0.76, size = 271, normalized size = 3.27

method	result
derivativedivides	$-c^2\left(-\frac{a^2x^2}{2c^2}-\frac{b^2x^2\operatorname{arctanh}\left(\frac{c}{x}\right)^2}{2c^2}+\frac{b^2\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{2}-\frac{b^2\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)}{2}-\frac{b^2\operatorname{arctanh}\left(\frac{c}{x}\right)x}{c}-\right.$ b
default	$-c^2\left(-\frac{a^2x^2}{2c^2}-\frac{b^2x^2\operatorname{arctanh}\left(\frac{c}{x}\right)^2}{2c^2}+\frac{b^2\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{2}-\frac{b^2\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)}{2}-\frac{b^2\operatorname{arctanh}\left(\frac{c}{x}\right)x}{c}-\right.$ b
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c/x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -c^2*(-1/2*a^2/c^2*x^2-1/2*b^2/c^2*x^2*arctanh(c/x)^2+1/2*b^2*arctanh(c/x)*
ln(1+c/x)-1/2*b^2*arctanh(c/x)*ln(c/x-1)-b^2*arctanh(c/x)/c*x-1/8*b^2*ln(c/
x-1)^2+1/4*b^2*ln(c/x-1)*ln(1/2*c/x+1/2)-1/2*b^2*ln(1+c/x)+b^2*ln(c/x)-1/2*
b^2*ln(c/x-1)-1/8*b^2*ln(1+c/x)^2+1/4*b^2*ln(-1/2*c/x+1/2)*ln(1+c/x)-1/4*b^
2*ln(-1/2*c/x+1/2)*ln(1/2*c/x+1/2)-a*b/c^2*x^2*arctanh(c/x)+1/2*a*b*ln(1+c/
x)-1/2*a*b*ln(c/x-1)-a*b*x/c)
```

Maxima [A]
time = 0.26, size = 136, normalized size = 1.64

$$\frac{1}{2}\,b^2x^2\,\operatorname{artanh}\left(\frac{c}{x}\right)^2+\frac{1}{2}\,a^2x^2+\frac{1}{2}\left(2\,x^2\,\operatorname{artanh}\left(\frac{c}{x}\right)-\left(c\log\left(c+x\right)-c\log\left(-c+x\right)-2\,x\right)c\right)ab+\frac{1}{8}\left(\left(\log\left(c+x\right)^2-2\left(\log\left(c+x\right)-2\right)\log\left(-c+x\right)+\log\left(-c+x\right)^2+4\,\log\left(c+x\right)\right)c^2-4\left(c\log\left(c+x\right)-c\log\left(-c+x\right)-2\,x\right)c\,\operatorname{artanh}\left(\frac{c}{x}\right)\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arctanh(c/x)^2 + 1/2*a^2*x^2 + 1/2*(2*x^2*arctanh(c/x) - (c*log
(c + x) - c*log(-c + x) - 2*x)*c)*a*b + 1/8*((log(c + x)^2 - 2*(log(c + x)
- 2)*log(-c + x) + log(-c + x)^2 + 4*log(c + x))*c^2 - 4*(c*log(c + x) - c*
log(-c + x) - 2*x)*c*arctanh(c/x))*b^2
```

Fricas [A]
time = 0.43, size = 111, normalized size = 1.34

$$abcx+\frac{1}{2}\,a^2x^2-\frac{1}{2}\left(ab-b^2\right)c^2\log\left(c+x\right)+\frac{1}{2}\left(ab+b^2\right)c^2\log\left(-c+x\right)-\frac{1}{8}\left(b^2c^2-b^2x^2\right)\log\left(-\frac{c+x}{c-x}\right)^2+\frac{1}{2}\left(b^2cx+abx^2\right)\log\left(-\frac{c+x}{c-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^2,x, algorithm="fricas")
```

```
[Out] a*b*c*x + 1/2*a^2*x^2 - 1/2*(a*b - b^2)*c^2*log(c + x) + 1/2*(a*b + b^2)*c^
2*log(-c + x) - 1/8*(b^2*c^2 - b^2*x^2)*log(-(c + x)/(c - x))^2 + 1/2*(b^2*
c*x + a*b*x^2)*log(-(c + x)/(c - x))
```

Sympy [A]
time = 0.16, size = 104, normalized size = 1.25

$$\frac{a^2x^2}{2} - abc^2 \operatorname{atanh}\left(\frac{c}{x}\right) + abcx + abx^2 \operatorname{atanh}\left(\frac{c}{x}\right) + b^2c^2 \log\left(-c+x\right) - \frac{b^2c^2 \operatorname{atanh}^2\left(\frac{c}{x}\right)}{2} + b^2c^2 \operatorname{atanh}\left(\frac{c}{x}\right) + b^2cx \operatorname{atanh}\left(\frac{c}{x}\right) + \frac{b^2x^2 \operatorname{atanh}^2\left(\frac{c}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c/x))**2,x)
```

```
[Out] a**2*x**2/2 - a*b*c**2*atanh(c/x) + a*b*c*x + a*b*x**2*atanh(c/x) + b**2*c*
*2*log(-c + x) - b**2*c**2*atanh(c/x)**2/2 + b**2*c**2*atanh(c/x) + b**2*c*
x*atanh(c/x) + b**2*x**2*atanh(c/x)**2/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(77) = 154.
time = 0.42, size = 268, normalized size = 3.23

$$-\frac{2\,b^2c^3\log\left(-\frac{c+x}{c-x}-1\right) - 2\,b^2c^3\log\left(-\frac{c+x}{c-x}\right) + \frac{b^2(c+x)c^3\log\left(-\frac{c+x}{c-x}\right)^2}{(c-x)\left(\frac{(c+x)^2}{(c-x)^2}+\frac{2\,(c+x)}{c-x}+1\right)} + \frac{2\left(b^2c^3+\frac{2\,ab(c+x)c^3}{c-x}+\frac{b^2(c+x)c^3}{c-x}\right)\log\left(-\frac{c+x}{c-x}\right)}{\frac{(c+x)^2}{(c-x)^2}+\frac{2\,(c+x)}{c-x}+1} + \frac{4\left(abc^3+\frac{a^2(c+x)c^3}{c-x}+\frac{ab(c+x)c^3}{c-x}\right)}{\frac{(c+x)^2}{(c-x)^2}+\frac{2\,(c+x)}{c-x}+1}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*c^3*log(-(c + x)/(c - x) - 1) - 2*b^2*c^3*log(-(c + x)/(c - x))
 + b^2*(c + x)*c^3*log(-(c + x)/(c - x))^2/((c - x)*((c + x)^2/(c - x)^2 +
2*(c + x)/(c - x) + 1)) + 2*(b^2*c^3 + 2*a*b*(c + x)*c^3/(c - x) + b^2*(c +
 x)*c^3/(c - x))*log(-(c + x)/(c - x))/((c + x)^2/(c - x)^2 + 2*(c + x)/(c
- x) + 1) + 4*(a*b*c^3 + a^2*(c + x)*c^3/(c - x) + a*b*(c + x)*c^3/(c - x))
/((c + x)^2/(c - x)^2 + 2*(c + x)/(c - x) + 1))/c
```

Mupad [B]
time = 0.78, size = 101, normalized size = 1.22

$$\frac{a^2\,x^2}{2} - \frac{b^2\,c^2\,\operatorname{atanh}\left(\frac{c}{x}\right)^2}{2} + \frac{b^2\,x^2\,\operatorname{atanh}\left(\frac{c}{x}\right)^2}{2} + \frac{b^2\,c^2\,\ln\left(x^2-c^2\right)}{2} - a\,b\,c^2\,\operatorname{atanh}\left(\frac{c}{x}\right) + a\,b\,x^2\,\operatorname{atanh}\left(\frac{c}{x}\right) + b^2\,c\,x\,\operatorname{atanh}\left(\frac{c}{x}\right) + a\,b\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c/x))^2,x)
```

```
[Out] (a^2*x^2)/2 - (b^2*c^2*atanh(c/x)^2)/2 + (b^2*x^2*atanh(c/x)^2)/2 + (b^2*c^
2*log(x^2 - c^2))/2 - a*b*c^2*atanh(c/x) + a*b*x^2*atanh(c/x) + b^2*c*x*ata
nh(c/x) + a*b*c*x
```

3.146 $\int \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal. Leaf size=74

$$c\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-2bc\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2c}{c-x}\right)-b^2c\text{PolyLog}\left(2,-\frac{c+x}{c-x}\right)$$

```
[Out] c*(a+b*arccoth(x/c))^2+x*(a+b*arccoth(x/c))^2-2*b*c*(a+b*arccoth(x/c))*ln(2
*c/(c-x))-b^2*c*polylog(2,(-c-x)/(c-x))
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6025, 6022, 6132, 6056, 2449, 2352}

$$c\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-2bc\log\left(\frac{2c}{c-x}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+b^2(-c)\text{Li}_2\left(-\frac{c+x}{c-x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] c*(a + b*ArcCoth[x/c])^2 + x*(a + b*ArcCoth[x/c])^2 - 2*b*c*(a + b*ArcCoth[
x/c])*Log[(2*c)/(c - x)] - b^2*c*PolyLog[2, -((c + x)/(c - x))]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6025

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[(a + b*
ArcCoth[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && ILtQ[n, 0
```

```
]
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2\,dx = \int \left(a^2 - ab\log\left(1-\frac{c}{x}\right)+\frac{1}{4}b^2\log^2\left(1-\frac{c}{x}\right)+ab\log\left(1+\frac{c}{x}\right)-\frac{1}{2}b^2\log\left(1\right.\right.$$

$$= a^2x-(ab)\int\log\left(1-\frac{c}{x}\right)\,dx+(ab)\int\log\left(1+\frac{c}{x}\right)\,dx+\frac{1}{4}b^2\int\log^2\left(1-\frac{c}{x}\right.$$

$$= a^2x-abx\log\left(1-\frac{c}{x}\right)-\frac{1}{4}b^2(c-x)\log^2\left(1-\frac{c}{x}\right)+abx\log\left(1+\frac{c}{x}\right)-\frac{1}{2}b^2x$$

$$= a^2x-abx\log\left(1-\frac{c}{x}\right)-\frac{1}{4}b^2(c-x)\log^2\left(1-\frac{c}{x}\right)+abx\log\left(1+\frac{c}{x}\right)-\frac{1}{2}b^2x$$

$$= a^2x-abx\log\left(1-\frac{c}{x}\right)-\frac{1}{4}b^2(c-x)\log^2\left(1-\frac{c}{x}\right)+abx\log\left(1+\frac{c}{x}\right)-\frac{1}{2}b^2x$$

$$= a^2x-abx\log\left(1-\frac{c}{x}\right)-\frac{1}{4}b^2(c-x)\log^2\left(1-\frac{c}{x}\right)+abx\log\left(1+\frac{c}{x}\right)-\frac{1}{2}b^2x$$

$$= a^2x-abx\log\left(1-\frac{c}{x}\right)-\frac{1}{4}b^2(c-x)\log^2\left(1-\frac{c}{x}\right)+abx\log\left(1+\frac{c}{x}\right)-\frac{1}{2}b^2x$$

$$= a^2x-abx\log\left(1-\frac{c}{x}\right)-\frac{1}{4}b^2(c-x)\log^2\left(1-\frac{c}{x}\right)+abx\log\left(1+\frac{c}{x}\right)-\frac{1}{2}b^2x$$

$$= a^2x-abx\log\left(1-\frac{c}{x}\right)-\frac{1}{4}b^2(c-x)\log^2\left(1-\frac{c}{x}\right)+abx\log\left(1+\frac{c}{x}\right)-\frac{1}{2}b^2x$$

$$= a^2x-abx\log\left(1-\frac{c}{x}\right)-\frac{1}{4}b^2(c-x)\log^2\left(1-\frac{c}{x}\right)+abx\log\left(1+\frac{c}{x}\right)-\frac{1}{2}b^2x$$

Mathematica [A]
time = 0.08, size = 97, normalized size = 1.31

$$b^2(-c+x)\tanh^{-1}\left(\frac{c}{x}\right)^2+2b\tanh^{-1}\left(\frac{c}{x}\right)\left(ax-bc\log\left(1-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)+a\left(ax+bc\log\left(1-\frac{c^2}{x^2}\right)-2bc\log\left(\frac{c}{x}\right)\right)+b^2c\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] b^2*(-c + x)*ArcTanh[c/x]^2 + 2*b*ArcTanh[c/x]*(a*x - b*c*Log[1 - E^(-2*Arc
Tanh[c/x])]) + a*(a*x + b*c*Log[1 - c^2/x^2] - 2*b*c*Log[c/x]) + b^2*c*Poly
Log[2, E^(-2*ArcTanh[c/x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(77) = 154.
time = 0.22, size = 287, normalized size = 3.88

method	result
derivativedivides	$-c\left(-\frac{a^2x}{c}-\frac{b^2x\operatorname{arctanh}\left(\frac{c}{x}\right)^2}{c}-b^2\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)+2b^2\ln\left(\frac{c}{x}\right)\operatorname{arctanh}\left(\frac{c}{x}\right)-b^2\operatorname{arcta}\right.$
default	$-c\left(-\frac{a^2x}{c}-\frac{b^2x\operatorname{arctanh}\left(\frac{c}{x}\right)^2}{c}-b^2\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)+2b^2\ln\left(\frac{c}{x}\right)\operatorname{arctanh}\left(\frac{c}{x}\right)-b^2\operatorname{arcta}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -c*(-a^2/c*x-b^2/c*x*arctanh(c/x)^2-b^2*arctanh(c/x)*ln(1+c/x)+2*b^2*ln(c/x
)*arctanh(c/x)-b^2*arctanh(c/x)*ln(c/x-1)-1/4*b^2*ln(c/x-1)^2+b^2*dilog(1/2
*c/x+1/2)+1/2*b^2*ln(c/x-1)*ln(1/2*c/x+1/2)+1/4*b^2*ln(1+c/x)^2+1/2*b^2*ln(
-1/2*c/x+1/2)*ln(1/2*c/x+1/2)-1/2*b^2*ln(-1/2*c/x+1/2)*ln(1+c/x)-b^2*dilog(
1+c/x)-b^2*ln(c/x)*ln(1+c/x)-b^2*dilog(c/x)-2*a*b/c*x*arctanh(c/x)-a*b*ln(1
+c/x)+2*a*b*ln(c/x)-a*b*ln(c/x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2,x, algorithm="maxima")
```

```
[Out] (2*x*arctanh(c/x) + c*log(-c^2 + x^2))*a*b + 1/4*(x*log(c + x)^2 - 2*(c + x
)*log(c + x)*log(-c + x) - (c - x)*log(-c + x)^2 + integrate(-2*(c^2 + 3*c*
x)*log(c + x)/(c^2 - x^2), x))*b^2 + a^2*x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2,x, algorithm="fricas")
[Out] integral(b^2*arctanh(c/x)^2 + 2*a*b*arctanh(c/x) + a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{atanh}\left(\frac{c}{x}\right)\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**2,x)
[Out] Integral((a + b*atanh(c/x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c/x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))^2,x)
[Out] int((a + b*atanh(c/x))^2, x)
```

3.147 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$

Optimal. Leaf size=133

$$-2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)+b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)-b\left(a+b\coth^{-1}\right.$$

```
[Out] 2*(a+b*arccoth(x/c))^2*arctanh(-1+2/(1-c/x))+b*(a+b*arccoth(x/c))*polylog(2
,1-2/(1-c/x))-b*(a+b*arccoth(x/c))*polylog(2,-1+2/(1-c/x))-1/2*b^2*polylog(
3,1-2/(1-c/x))+1/2*b^2*polylog(3,-1+2/(1-c/x))
```

Rubi [A]
time = 0.21, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6035, 6033, 6199, 6095, 6205, 6745}

$$b\text{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-b\text{Li}_2\left(\frac{2}{1-\frac{c}{x}}-1\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{2}b^2\text{Li}_3\left(1-\frac{2}{1-\frac{c}{x}}\right)+\frac{1}{2}b^2\text{Li}_3\left(\frac{2}{1-\frac{c}{x}}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])^2/x,x]
```

```
[Out] -2*(a + b*ArcCoth[x/c])^2*ArcTanh[1 - 2/(1 - c/x)] + b*(a + b*ArcCoth[x/c])
*PolyLog[2, 1 - 2/(1 - c/x)] - b*(a + b*ArcCoth[x/c])*PolyLog[2, -1 + 2/(1
- c/x)] - (b^2*PolyLog[3, 1 - 2/(1 - c/x)])/2 + (b^2*PolyLog[3, -1 + 2/(1 -
 c/x)])/2
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x}\,dx = -\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right) + (4bc)\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}\right.}{}\right.$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right) - (2bc)\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}\right.}{1-}\right.$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right) + b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\text{Li}_2\left(1-\right.$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right) + b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\text{Li}_2\left(1-\right.$$

Mathematica [A]
time = 0.07, size = 114, normalized size = 0.86

$$-2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2\tanh^{-1}\left(\frac{c+x}{c-x}\right)+\frac{1}{2}b\left(2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)\text{PolyLog}\left(2,\frac{c+x}{c-x}\right)-2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)\text{PolyLog}\left(2,\frac{c+x}{-c+x}\right)+b\left(-\text{PolyLog}\left(3,\frac{c+x}{c-x}\right)+\text{PolyLog}\left(3,\frac{c+x}{-c+x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])^2/x,x]
```

```
[Out] -2*(a + b*ArcTanh[c/x])^2*ArcTanh[(c + x)/(c - x)] + (b*(2*(a + b*ArcTanh[c
/x])*PolyLog[2, (c + x)/(c - x)] - 2*(a + b*ArcTanh[c/x])*PolyLog[2, (c + x
)/(-c + x)] + b*(-PolyLog[3, (c + x)/(c - x)] + PolyLog[3, (c + x)/(-c + x)
])))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.45, size = 780, normalized size = 5.86 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2*ln(c/x)-b^2*ln(c/x)*arctanh(c/x)^2+b^2*arctanh(c/x)*polylog(2,-(1+c/x)
^2/(1-c^2/x^2))-1/2*b^2*polylog(3,-(1+c/x)^2/(1-c^2/x^2))+b^2*arctanh(c/x)^
2*ln((1+c/x)^2/(1-c^2/x^2)-1)-b^2*arctanh(c/x)^2*ln(1+(1+c/x)/(1-c^2/x^2)^(
1/2))-2*b^2*arctanh(c/x)*polylog(2,-(1+c/x)/(1-c^2/x^2)^(1/2))+2*b^2*polylo
g(3,-(1+c/x)/(1-c^2/x^2)^(1/2))-b^2*arctanh(c/x)^2*ln(1-(1+c/x)/(1-c^2/x^2)
^(1/2))-2*b^2*arctanh(c/x)*polylog(2,(1+c/x)/(1-c^2/x^2)^(1/2))+2*b^2*polyl
og(3,(1+c/x)/(1-c^2/x^2)^(1/2))+1/2*I*b^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-
1))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^2*arctanh(c
/x)^2-1/2*I*b^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2
)))^3*arctanh(c/x)^2-1/2*I*b^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*csgn(I/
(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1
-c^2/x^2)))*arctanh(c/x)^2+1/2*I*b^2*Pi*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*c
sgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^2*arctanh(c/x)^2
-2*a*b*ln(c/x)*arctanh(c/x)+a*b*ln(c/x)*ln(1+c/x)+a*b*dilog(1+c/x)+a*b*dilo
g(c/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + integrate(1/4*b^2*(log(c/x + 1) - log(-c/x + 1))^2/x + a*b*(lo
g(c/x + 1) - log(-c/x + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x)^2 + 2*a*b*arctanh(c/x) + a^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x}\right)\right)^{2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c/x))**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)}^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))^2/x,x)
```

```
[Out] int((a + b*atanh(c/x))^2/x, x)
```

3.148 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=87

$$-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{c}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{x}+\frac{2b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2}{1-\frac{c}{x}}\right)}{c}+\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)}{c}$$

```
[Out] -(a+b*arccoth(x/c))^2/c-(a+b*arccoth(x/c))^2/x+2*b*(a+b*arccoth(x/c))*ln(2/
(1-c/x))/c+b^2*polylog(2,1-2/(1-c/x))/c
```

Rubi [A]
time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6039, 6021, 6131, 6055, 2449, 2352}

$$-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{c}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{x}+\frac{2b\log\left(\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)}{c}+\frac{b^2\text{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])^2/x^2,x]
```

```
[Out] -((a + b*ArcCoth[x/c])^2/c) - (a + b*ArcCoth[x/c])^2/x + (2*b*(a + b*ArcCot
h[x/c])*Log[2/(1 - c/x)])/c + (b^2*PolyLog[2, 1 - 2/(1 - c/x)])/c
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
```

```
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4x^2}+\frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)}{2x^2}+\frac{b^2\log^2\left(1+\right.}{4x^2}\right.$$

$$=\frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{x^2}\,dx+\frac{1}{2}b\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)}{x^2}\,dx+\frac{1}{4}$$

$$=-\left(\frac{1}{4}\text{Subst}\left(\int (2a-b\log(1-cx))^2\,dx,x,\frac{1}{x}\right)\right)-\frac{1}{2}b\text{Subst}\left(\int (2a-b\log(1-\right.$$

$$=-\frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(\frac{c+x}{x}\right)}{2x}+\frac{\text{Subst}\left(\int (2a-b\log(x))^2\,dx,x,1-\frac{c}{x}\right)}{4c}-\frac{b}{}$$

$$=\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c}-\frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(\frac{c+x}{x}\right)}{2x}-\frac{b^2\left(1+\frac{c}{x}\right)l}{4c}$$

$$=-\frac{ab}{x}-\frac{b^2}{2x}+\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c}+\frac{b^2\left(1+\frac{c}{x}\right)\log\left(\frac{c+x}{x}\right)}{2c}-\frac{b\left(2a-b\right.}{}$$

$$=-\frac{b^2}{x}-\frac{b^2\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{2c}+\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c}-\frac{b\left(2a-b\log\left(\right.\right.}{}$$

$$=-\frac{b^2}{2x}-\frac{b^2\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{2c}+\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c}-\frac{b\left(2a-b\log\right.}{}$$

$$=\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c}-\frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(\frac{c+x}{2x}\right)}{2c}-\frac{b\left(2a-b\log\right.}{}$$

Mathematica [A]
time = 0.13, size = 101, normalized size = 1.16

$$\frac{b^2(-c+x)\tanh^{-1}\left(\frac{c}{x}\right)^2+2b\tanh^{-1}\left(\frac{c}{x}\right)\left(-ac+bx\log\left(1+e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)+a\left(-ac+2bx\log\left(\frac{1}{\sqrt{1-\frac{c^2}{x^2}}}\right)\right)-b^2x\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)}{cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])^2/x^2,x]
```

```
[Out] (b^2*(-c + x)*ArcTanh[c/x]^2 + 2*b*ArcTanh[c/x]*(-(a*c) + b*x*Log[1 + E^(-2
*ArcTanh[c/x])]) + a*(-(a*c) + 2*b*x*Log[1/Sqrt[1 - c^2/x^2]]) - b^2*x*Poly
Log[2, -E^(-2*ArcTanh[c/x])])/(c*x)
```

Maple [A]
time = 0.58, size = 137, normalized size = 1.57

method	result
derivativedivides	$-\frac{\frac{c\,a^2}{x}+\frac{\operatorname{arctanh}\left(\frac{c}{x}\right)^2 b^2 c}{x}-2\ln\left(1+\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right)\operatorname{arctanh}\left(\frac{c}{x}\right)b^2+b^2\operatorname{arctanh}\left(\frac{c}{x}\right)^2-\operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right)b^2+\frac{2abc\operatorname{arctanh}\left(\frac{c}{x}\right)}{x}}{c}$
default	$-\frac{\frac{c\,a^2}{x}+\frac{\operatorname{arctanh}\left(\frac{c}{x}\right)^2 b^2 c}{x}-2\ln\left(1+\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right)\operatorname{arctanh}\left(\frac{c}{x}\right)b^2+b^2\operatorname{arctanh}\left(\frac{c}{x}\right)^2-\operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right)b^2+\frac{2abc\operatorname{arctanh}\left(\frac{c}{x}\right)}{x}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c*(c/x*a^2+arctanh(c/x)^2*b^2*c/x-2*ln(1+(1+c/x)^2/(1-c^2/x^2))*arctanh(
c/x)*b^2+b^2*arctanh(c/x)^2-polylog(2,-(1+c/x)^2/(1-c^2/x^2))*b^2+2*a*b*c/x
*arctanh(c/x)+a*b*ln(1-c^2/x^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c^3*integrate(-log(x)^2/(c^3*x^2 - c*x^4), x) + c^2*(log(-c^2 + x^2)/c
^3 - log(x^2)/c^3) - 4*c^2*integrate(-x*log(c + x)/(c^3*x^2 - c*x^4), x) +
2*c^2*integrate(-x*log(x)/(c^3*x^2 - c*x^4), x) + 2*c*(log(-c + x)/c^2 - lo
g(x)/c^2 + 1/(c*x))*log(-c/x + 1) - c*(log(c + x)/c^2 - log(-c + x)/c^2) -
c*integrate(-x^2*log(x)^2/(c^3*x^2 - c*x^4), x) - 2*c*integrate(-x^2*log(c
+ x)/(c^3*x^2 - c*x^4), x) + 4*c*integrate(-x^2*log(x)/(c^3*x^2 - c*x^4), x
```

```
) - log(-c/x + 1)^2/x - (c*log(c + x)^2 - 2*((c + x)*log(c + x) - (c + x)*l
og(x) - c)*log(-c + x))/(c*x) - (x*log(-c + x)^2 + x*log(x)^2 - 2*(x*log(x)
 - x)*log(-c + x) - 2*x*log(x) + 2*c)/(c*x) - 2*integrate(-x^3*log(c + x)/(
c^3*x^2 - c*x^4), x) + 2*integrate(-x^3*log(x)/(c^3*x^2 - c*x^4), x))*b^2 -
 a*b*(2*c*arctanh(c/x)/x + log(-c^2/x^2 + 1))/c - a^2/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*arctanh(c/x)^2 + 2*a*b*arctanh(c/x) + a^2)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}{\left(\frac{c}{x} \right)}\right)^{2}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**2/x**2,x)
[Out] Integral((a + b*atanh(c/x))**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c/x) + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)}^2}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))^2/x^2,x)
[Out] int((a + b*atanh(c/x))^2/x^2, x)
```

3.149 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=87

$$-\frac{ab}{cx}-\frac{b^2\coth^{-1}\left(\frac{x}{c}\right)}{cx}+\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2}-\frac{b^2\log\left(1-\frac{c^2}{x^2}\right)}{2c^2}$$

```
[Out] -a*b/c/x-b^2*arccoth(x/c)/c/x+1/2*(a+b*arccoth(x/c))^2/c^2-1/2*(a+b*arccoth
(x/c))^2/x^2-1/2*b^2*ln(1-c^2/x^2)/c^2
```

Rubi [A]
time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6039, 6037, 6127, 6021, 266, 6095}

$$\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2}-\frac{ab}{cx}-\frac{b^2\log\left(1-\frac{c^2}{x^2}\right)}{2c^2}-\frac{b^2\coth^{-1}\left(\frac{x}{c}\right)}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])^2/x^3,x]
```

```
[Out] -((a*b)/(c*x)) - (b^2*ArcCoth[x/c])/(c*x) + (a + b*ArcCoth[x/c])^2/(2*c^2)
- (a + b*ArcCoth[x/c])^2/(2*x^2) - (b^2*Log[1 - c^2/x^2])/(2*c^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4x^3}+\frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)}{2x^3}+\frac{b^2\log^2\left(1+\frac{c}{x}\right)}{4x^3}\right.$$

$$=\frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{x^3}\,dx+\frac{1}{2}b\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)}{x^3}\,dx+\frac{1}{4}b^2$$

$$=-\left(\frac{1}{4}\text{Subst}\left(\int x(2a-b\log(1-cx))^2\,dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int \left(\frac{2a\log\left(1+\frac{c}{x}\right)}{x^3}-\right.$$

$$=-\left(\frac{1}{4}\text{Subst}\left(\int \left(\frac{(2a-b\log(1-cx))^2}{c}-\frac{(1-cx)(2a-b\log(1-cx))^2}{c}\right)\,dx,x,\right.\right.$$

$$=\frac{b^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)}{4x^2}-(ab)\text{Subst}\left(\int x\log(1+cx)\,dx,x,\frac{1}{x}\right)+\frac{1}{2}b^2\int \frac{c\log}{2x}$$

$$=\frac{b^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)}{4x^2}-\frac{ab\log\left(\frac{c+x}{x}\right)}{2x^2}+\frac{\text{Subst}\left(\int(2a-b\log(x))^2\,dx,x,1-\frac{c}{x}\right)}{4c^2}$$

$$=\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c^2}-\frac{\left(1-\frac{c}{x}\right)^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{8c^2}+\frac{b^2\log\left(1-\frac{c}{x}\right)}{4x}$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}-\frac{3ab}{2cx}+\frac{b^2}{2cx}-\frac{b\left(1-\frac{c}{x}\right)^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)}{8c^2}$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}-\frac{3ab}{2cx}-\frac{b^2\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{2c^2}-\frac{b\left(1-\frac{c}{x}\right)^2}{}$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}-\frac{3ab}{2cx}-\frac{b^2\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{2c^2}-\frac{b^2\log\left(1-\right.}{8x^2}$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}-\frac{3ab}{2cx}-\frac{3b^2\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{4c^2}-\frac{b^2\log\left(1-\right.}{8x^2}$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}+\frac{b^2}{8x^2}-\frac{3ab}{2cx}+\frac{b^2\log\left(1-\frac{c}{x}\right)}{8c^2}-\frac{3b^2\left(1-\frac{c}{x}\right)}{4c}$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}+\frac{b^2}{8x^2}-\frac{3ab}{2cx}+\frac{b^2\log\left(1-\frac{c}{x}\right)}{8c^2}-\frac{3b^2\left(1-\frac{c}{x}\right)}{4c}$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}+\frac{b^2}{8x^2}-\frac{3ab}{2cx}+\frac{b^2\log\left(1-\frac{c}{x}\right)}{8c^2}-\frac{3b^2\left(1-\frac{c}{x}\right)}{4c}$$

Mathematica [A]

time = 0.04, size = 119, normalized size = 1.37

$$-\frac{a^2c^2+2abcx+2bc(ac+bx)\tanh^{-1}\left(\frac{c}{x}\right)+b^2(c^2-x^2)\tanh^{-1}\left(\frac{c}{x}\right)^2-2b^2x^2\log(x)+abx^2\log(-c+x)+b^2x^2\log(-c+x)-abx^2\log(c+x)+b^2x^2\log(c+x)}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])^2/x^3,x]
```

```
[Out] -1/2*(a^2*c^2 + 2*a*b*c*x + 2*b*c*(a*c + b*x)*ArcTanh[c/x] + b^2*(c^2 - x^2
)*ArcTanh[c/x]^2 - 2*b^2*x^2*Log[x] + a*b*x^2*Log[-c + x] + b^2*x^2*Log[-c
+ x] - a*b*x^2*Log[c + x] + b^2*x^2*Log[c + x])/(c^2*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(81) = 162$.
time = 1.92, size = 258, normalized size = 2.97

method	result
derivativedivides	$-\frac{\frac{a^2c^2}{2x^2}+\frac{b^2c^2\,\mathrm{arctanh}\left(\frac{c}{x}\right)^2}{2x^2}+\frac{b^2\,\mathrm{arctanh}\left(\frac{c}{x}\right)c}{x}+\frac{b^2\,\mathrm{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)}{2}-\frac{b^2\,\mathrm{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{2}+\frac{b^2\ln\left(\frac{c}{x}-1\right)^2}{8}-\frac{b^2\ln\left(\frac{c}{x}-1\right.}{}}{}$
default	$-\frac{\frac{a^2c^2}{2x^2}+\frac{b^2c^2\,\mathrm{arctanh}\left(\frac{c}{x}\right)^2}{2x^2}+\frac{b^2\,\mathrm{arctanh}\left(\frac{c}{x}\right)c}{x}+\frac{b^2\,\mathrm{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)}{2}-\frac{b^2\,\mathrm{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{2}+\frac{b^2\ln\left(\frac{c}{x}-1\right)^2}{8}-\frac{b^2\ln\left(\frac{c}{x}-1\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2*(1/2*a^2*c^2/x^2+1/2*b^2*c^2/x^2*arctanh(c/x)^2+b^2*arctanh(c/x)*c/x
+1/2*b^2*arctanh(c/x)*ln(c/x-1)-1/2*b^2*arctanh(c/x)*ln(1+c/x)+1/8*b^2*ln(c
/x-1)^2-1/4*b^2*ln(c/x-1)*ln(1/2*c/x+1/2)+1/2*b^2*ln(c/x-1)+1/2*b^2*ln(1+c/
x)+1/8*b^2*ln(1+c/x)^2+1/4*b^2*ln(-1/2*c/x+1/2)*ln(1/2*c/x+1/2)-1/4*b^2*ln(
-1/2*c/x+1/2)*ln(1+c/x)+a*b*c^2/x^2*arctanh(c/x)+a*b*c/x+1/2*a*b*ln(c/x-1)-
1/2*a*b*ln(1+c/x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(81) = 162$.
time = 0.26, size = 165, normalized size = 1.90

$$\frac{1}{2}\left(c\left(\frac{\log\left(c+x\right)}{c^3}-\frac{\log\left(-c+x\right)}{c^3}-\frac{2}{c^2x}\right)-\frac{2\,\mathrm{artanh}\left(\frac{c}{x}\right)}{x^2}\right)ab-\frac{1}{8}\left(c^2\left(\frac{\log\left(c+x\right)^2-2\left(\log\left(c+x\right)-2\right)\log\left(-c+x\right)+\log\left(-c+x\right)^2+4\log\left(c+x\right)}{c^4}-\frac{8\log\left(x\right)}{c^4}\right)-4c\left(\frac{\log\left(c+x\right)}{c^3}-\frac{\log\left(-c+x\right)}{c^3}-\frac{2}{c^2x}\right)\mathrm{artanh}\left(\frac{c}{x}\right)\right)b^2-\frac{b^2\,\mathrm{artanh}\left(\frac{c}{x}\right)^2}{2x^2}-\frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(c*(log(c + x)/c^3 - log(-c + x)/c^3 - 2/(c^2*x)) - 2*arctanh(c/x)/x^2)
*a*b - 1/8*(c^2*((log(c + x)^2 - 2*(log(c + x) - 2)*log(-c + x) + log(-c +
x)^2 + 4*log(c + x))/c^4 - 8*log(x)/c^4) - 4*c*(log(c + x)/c^3 - log(-c + x
)/c^3 - 2/(c^2*x))*arctanh(c/x))*b^2 - 1/2*b^2*arctanh(c/x)^2/x^2 - 1/2*a^2
/x^2
```

Fricas [A]
time = 0.35, size = 130, normalized size = 1.49

$$\frac{8\,b^2x^2\log\left(x\right)-4\,a^2c^2-8\,abcx+4\,(ab-b^2)x^2\log\left(c+x\right)-4\,(ab+b^2)x^2\log\left(-c+x\right)-(b^2c^2-b^2x^2)\log\left(-\frac{c+x}{c-x}\right)^2-4\,(abc^2+b^2cx)\log\left(-\frac{c+x}{c-x}\right)}{8\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b^2*x^2*log(x) - 4*a^2*c^2 - 8*a*b*c*x + 4*(a*b - b^2)*x^2*log(c + x
) - 4*(a*b + b^2)*x^2*log(-c + x) - (b^2*c^2 - b^2*x^2)*log(-(c + x)/(c - x
))^2 - 4*(a*b*c^2 + b^2*c*x)*log(-(c + x)/(c - x)))/(c^2*x^2)
```

Sympy [A]
time = 0.38, size = 124, normalized size = 1.43

$$\begin{cases} -\frac{a^2}{2x^2} - \frac{ab \operatorname{atanh}\left(\frac{c}{x}\right)}{x^2} - \frac{ab}{cx} + \frac{ab \operatorname{atanh}\left(\frac{c}{x}\right)}{c^2} - \frac{b^2 \operatorname{atanh}^2\left(\frac{c}{x}\right)}{2x^2} - \frac{b^2 \operatorname{atanh}\left(\frac{c}{x}\right)}{cx} + \frac{b^2 \log(x)}{c^2} - \frac{b^2 \log(-c+x)}{c^2} + \frac{b^2 \operatorname{atanh}^2\left(\frac{c}{x}\right)}{2c^2} - \frac{b^2 \operatorname{atanh}\left(\frac{c}{x}\right)}{c^2} & \text{for}\ c \neq 0 \\ -\frac{a^2}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**2/x**3,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - a*b*atanh(c/x)/x**2 - a*b/(c*x) + a*b*atanh(c/x
)/c**2 - b**2*atanh(c/x)**2/(2*x**2) - b**2*atanh(c/x)/(c*x) + b**2*log(x)/
c**2 - b**2*log(-c + x)/c**2 + b**2*atanh(c/x)**2/(2*c**2) - b**2*atanh(c/x
)/c**2, Ne(c, 0)), (-a**2/(2*x**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(81) = 162.
time = 0.43, size = 255, normalized size = 2.93

$$-\frac{\frac{b^2(c+x)\log\left(-\frac{c+x}{c-x}\right)^2}{\left(\frac{(c+x)^2c}{(c-x)^2}-\frac{2\,(c+x)c}{c-x}+c\right)(c-x)} - \frac{2\,b^2\log\left(-\frac{c+x}{c-x}+1\right)}{c} + \frac{2\,b^2\log\left(-\frac{c+x}{c-x}\right)}{c} - \frac{2\left(b^2-\frac{2\,ab(c+x)}{c-x}-\frac{b^2(c+x)}{c-x}\right)\log\left(-\frac{c+x}{c-x}\right)}{\frac{(c+x)^2c}{(c-x)^2}-\frac{2\,(c+x)c}{c-x}+c} - \frac{4\left(ab-\frac{a^2(c+x)}{c-x}-\frac{ab(c+x)}{c-x}\right)}{\frac{(c+x)^2c}{(c-x)^2}-\frac{2\,(c+x)c}{c-x}+c}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*(c + x)*log(-(c + x)/(c - x))^2/(((c + x)^2*c/(c - x)^2 - 2*(c +
x)*c/(c - x) + c)*(c - x)) - 2*b^2*log(-(c + x)/(c - x) + 1)/c + 2*b^2*log(
-(c + x)/(c - x))/c - 2*(b^2 - 2*a*b*(c + x)/(c - x) - b^2*(c + x)/(c - x))
*log(-(c + x)/(c - x))/((c + x)^2*c/(c - x)^2 - 2*(c + x)*c/(c - x) + c) -
4*(a*b - a^2*(c + x)/(c - x) - a*b*(c + x)/(c - x))/((c + x)^2*c/(c - x)^2
- 2*(c + x)*c/(c - x) + c))/c
```

Mupad [B]
time = 1.28, size = 235, normalized size = 2.70

$$\ln\left(1-\frac{c}{x}\right)\left(\frac{a\,b}{2\,x^2}-\ln\left(\frac{c}{x}+1\right)\left(\frac{b^2}{4\,c^2}-\frac{b^2}{4\,x^2}\right)+\frac{b^2\,\left(2\,c\,x-c^2\right)}{8\,c^2\,x^2}+\frac{b^2\,\left(2\,c^2+4\,x\,c\right)}{16\,c^2\,x^2}\right)-\frac{\frac{a^2}{2}+\frac{a\,b\,x}{c}}{x^2}+\ln\left(\frac{c}{x}+1\right)^2\left(\frac{b^2}{8\,c^2}-\frac{b^2}{8\,x^2}\right)+\ln\left(1-\frac{c}{x}\right)^2\left(\frac{b^2}{8\,c^2}-\frac{b^2}{8\,x^2}\right)-\frac{\ln\left(x-c\right)\,\left(b^2+a\,b\right)}{2\,c^2}+\frac{\ln\left(c+x\right)\,\left(a\,b-b^2\right)}{2\,c^2}-\frac{\ln\left(\frac{c}{x}+1\right)\,\left(\frac{a\,b}{2}+\frac{b^2\,x}{2\,c}\right)}{x^2}+\frac{b^2\,\ln\left(x\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))^2/x^3,x)
```

```
[Out] log(1 - c/x)*((a*b)/(2*x^2) - log(c/x + 1)*(b^2/(4*c^2) - b^2/(4*x^2)) + (b
^2*(2*c*x - c^2))/(8*c^2*x^2) + (b^2*(4*c*x + 2*c^2))/(16*c^2*x^2)) - (a^2/
2 + (a*b*x)/c)/x^2 + log(c/x + 1)^2*(b^2/(8*c^2) - b^2/(8*x^2)) + log(1 - c
/x)^2*(b^2/(8*c^2) - b^2/(8*x^2)) - (log(x - c)*(a*b + b^2))/(2*c^2) + (log
(c + x)*(a*b - b^2))/(2*c^2) - (log(c/x + 1)*((a*b)/2 + (b^2*x)/(2*c)))/x^2
 + (b^2*log(x))/c^2
```

3.150 $\int x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3\,dx$

Optimal. Leaf size=203

$$\frac{1}{4}b^3c^3x-\frac{1}{4}b^3c^4\coth^{-1}\left(\frac{x}{c}\right)+\frac{1}{4}b^2c^2x^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-bc^4\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{3}{4}bc^3x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right.$$

```
[Out] 1/4*b^3*c^3*x-1/4*b^3*c^4*arccoth(x/c)+1/4*b^2*c^2*x^2*(a+b*arccoth(x/c))-b
*c^4*(a+b*arccoth(x/c))^2+3/4*b*c^3*x*(a+b*arccoth(x/c))^2+1/4*b*c*x^3*(a+b
*arccoth(x/c))^2-1/4*c^4*(a+b*arccoth(x/c))^3+1/4*x^4*(a+b*arccoth(x/c))^3-
2*b^2*c^4*(a+b*arccoth(x/c))*ln(2-2/(1+c/x))+b^3*c^4*polylog(2,-1+2/(1+c/x)
)
```

Rubi [A]
time = 0.45, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$,
Rules used = {6039, 6037, 6129, 331, 212, 6135, 6079, 2497, 6095}

$$-2b^2c^4\log\left(2-\frac{2}{\frac{c}{x}+1}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{4}b^2c^2x^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{4}c^4\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3-bc^4\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{3}{4}bc^3x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{4}x^4\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{4}bcx^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+b^3c^4\text{Li}_2\left(\frac{2}{\frac{c}{x}+1}-1\right)-\frac{1}{4}b^3c^4\coth^{-1}\left(\frac{x}{c}\right)+\frac{1}{4}b^3c^3x$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] (b^3*c^3*x)/4 - (b^3*c^4*ArcCoth[x/c])/4 + (b^2*c^2*x^2*(a + b*ArcCoth[x/c]
))/4 - b*c^4*(a + b*ArcCoth[x/c])^2 + (3*b*c^3*x*(a + b*ArcCoth[x/c])^2)/4
+ (b*c*x^3*(a + b*ArcCoth[x/c])^2)/4 - (c^4*(a + b*ArcCoth[x/c])^3)/4 + (x^
4*(a + b*ArcCoth[x/c])^3)/4 - 2*b^2*c^4*(a + b*ArcCoth[x/c])*Log[2 - 2/(1 +
 c/x)] + b^3*c^4*PolyLog[2, -1 + 2/(1 + c/x)]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3\,dx = \int \Bigg(\frac{1}{8}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{8}bx^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)$$

$$= \frac{1}{8}\int x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3 dx+\frac{1}{8}(3b)\int x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\Big($$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^3}{x^5}\,dx,x,\frac{1}{x}\right)\right)+\frac{1}{8}(3b)\int \Big(4a^2x^3\log\Big(1+$$

$$= \frac{1}{32}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{1}{32}b^3x^4\log^3\left(\frac{c+x}{x}\right)+\frac{1}{2}(3a^2b)\int x^3\log\Big(1$$

$$= \frac{1}{32}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{8}a^2bx^4\log\left(1+\frac{c}{x}\right)-\frac{3}{8}ab^2x^4\log\left(1-\frac{c}{x}\right)\mathrm{lo}$$

$$= \frac{1}{32}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{8}a^2bx^4\log\left(1+\frac{c}{x}\right)-\frac{3}{8}ab^2x^4\log\left(1-\frac{c}{x}\right)\mathrm{lo}$$

$$= \frac{1}{32}bcx^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{32}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{8}a^2bx^4\log\Big($$

$$= \frac{3}{8}a^2bc^3x-\frac{3}{16}a^2bc^2x^2+\frac{1}{8}a^2bcx^3+\frac{3}{64}bc^2x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{32}bcx$$

$$= \frac{3}{8}a^2bc^3x-\frac{3}{16}a^2bc^2x^2+\frac{1}{8}a^2bcx^3-\frac{3}{8}ab^2c^3x\log\left(1-\frac{c}{x}\right)+\frac{3}{16}ab^2c^2x^2\log\Big(1$$

$$= \frac{3}{8}a^2bc^3x-\frac{3}{16}a^2bc^2x^2+\frac{1}{8}a^2bcx^3-\frac{3}{8}ab^2c^3x\log\left(1-\frac{c}{x}\right)+\frac{3}{16}ab^2c^2x^2\log\Big(1$$

$$= \frac{3}{8}a^2bc^3x-\frac{5}{16}ab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}a^2bc^2x^2+\frac{1}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3+\frac{1}{32}b$$

$$= \frac{3}{8}a^2bc^3x-\frac{5}{16}ab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}a^2bc^2x^2+\frac{3}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3+\frac{1}{32}b$$

$$= \frac{3}{8}a^2bc^3x-\frac{5}{16}ab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}a^2bc^2x^2+\frac{3}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3+\frac{1}{32}b$$

$$= \frac{3}{8}a^2bc^3x-\frac{5}{16}ab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}a^2bc^2x^2+\frac{3}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3+\frac{1}{32}b$$

Mathematica [A]
time = 0.45, size = 286, normalized size = 1.41

$$\frac{1}{8}\left(-2ab^2c^4+6a^2bc^3x+2b^3c^3x+2ab^2c^2x^2+2a^2bcx^3+2a^3x^4+2b^2\left(bc\left(-4c^3+3c^2x+x^3\right)+3a\left(-c^4+x^4\right)\right)\tanh^{-1}\left(\frac{c}{x}\right)^2+2b^3\left(-c^4+x^4\right)\tanh^{-1}\left(\frac{c}{x}\right)^3+2b\tanh^{-1}\left(\frac{c}{x}\right)\left(3a^2x^4+2abcx\left(3c^2+x^2\right)+b^2\left(-c^4+c^2x^2\right)-8b^2c^4\log\left(1-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)+3a^2bc^4\log\left(1-\frac{c}{x}\right)-16ab^2c^4\log\left(\frac{c}{\sqrt{1-\frac{c^2}{x^2}}\,x}\right)-3a^2bc^4\log\left(\frac{c+x}{x}\right)+8b^3c^4\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] (-2*a*b^2*c^4 + 6*a^2*b*c^3*x + 2*b^3*c^3*x + 2*a*b^2*c^2*x^2 + 2*a^2*b*c*x
^3 + 2*a^3*x^4 + 2*b^2*(b*c*(-4*c^3 + 3*c^2*x + x^3) + 3*a*(-c^4 + x^4))*Ar
cTanh[c/x]^2 + 2*b^3*(-c^4 + x^4)*ArcTanh[c/x]^3 + 2*b*ArcTanh[c/x]*(3*a^2*
x^4 + 2*a*b*c*x*(3*c^2 + x^2) + b^2*(-c^4 + c^2*x^2) - 8*b^2*c^4*Log[1 - E^
(-2*ArcTanh[c/x])]) + 3*a^2*b*c^4*Log[1 - c/x] - 16*a*b^2*c^4*Log[c/(Sqrt[1
 - c^2/x^2]*x)] - 3*a^2*b*c^4*Log[(c + x)/x] + 8*b^3*c^4*PolyLog[2, E^(-2*A
rcTanh[c/x])])/8
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 8.02, size = 1337, normalized size = 6.59

method	result	size
derivativedivides	Expression too large to display	1337
default	Expression too large to display	1337
risch	Expression too large to display	46929

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c/x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -c^4*(-3/16*I*b^3*arctanh(c/x)^2*Pi*csgn(I*(1+c/x)/(1-c^2/x^2)^(1/2))^2*csg
n(I*(1+c/x)^2/(-1+c^2/x^2))-3/4/c*a^2*b*x-3/8*a*b^2*ln(-1/2*c/x+1/2)*ln(1/2
*c/x+1/2)+3/8*a*b^2*ln(-1/2*c/x+1/2)*ln(1+c/x)+3/4*a*b^2*arctanh(c/x)*ln(1+
c/x)-3/4*a*b^2*arctanh(c/x)*ln(c/x-1)+3/8*a*b^2*ln(c/x-1)*ln(1/2*c/x+1/2)+3
/8*I*b^3*arctanh(c/x)^2*Pi-1/4*b^3*arctanh(c/x)/c^2*x^2-3/4*b^3/c*x*arctanh
(c/x)^2-1/4*b^3/c^4*x^4*arctanh(c/x)^3-1/4*b^3/c^3*x^3*arctanh(c/x)^2+1/4*b
^3*arctanh(c/x)-2*b^3*dilog((1+c/x)/(1-c^2/x^2)^(1/2))-b^3*arctanh(c/x)^2+2
*b^3*dilog(1+(1+c/x)/(1-c^2/x^2)^(1/2))+1/4*b^3*arctanh(c/x)^3-3/8*I*b^3*ar
ctanh(c/x)^2*Pi*csgn(I*(1+c/x)/(1-c^2/x^2)^(1/2))*csgn(I*(1+c/x)^2/(-1+c^2/
x^2))^2-3/16*I*b^3*arctanh(c/x)^2*Pi*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn
(I*(1+c/x)^2/(-1+c^2/x^2)/(1+(1+c/x)^2/(1-c^2/x^2)))^2+3/16*I*b^3*arctanh(c
/x)^2*Pi*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/(1+(1
+c/x)^2/(1-c^2/x^2)))^2-3/16*a*b^2*ln(c/x-1)^2-3/16*a*b^2*ln(1+c/x)^2-a*b^2
*ln(1+c/x)+2*a*b^2*ln(c/x)-a*b^2*ln(c/x-1)-1/4*a^3/c^4*x^4+3/8*b^3*arctanh(
c/x)^2*ln(1+c/x)-3/8*b^3*arctanh(c/x)^2*ln(c/x-1)-1/4*b^3/(-(1-c^2/x^2)^(1/
2)+c/x+1)*(1-c^2/x^2)^(1/2)+2*b^3*arctanh(c/x)*ln(1+(1+c/x)/(1-c^2/x^2)^(1/
2))-3/4*b^3*arctanh(c/x)^2*ln((1+c/x)/(1-c^2/x^2)^(1/2))+1/4*b^3/((1-c^2/x^
2)^(1/2)+c/x+1)*(1-c^2/x^2)^(1/2)+3/8*a^2*b*ln(1+c/x)-3/8*a^2*b*ln(c/x-1)-3
/4*a^2*b/c^4*x^4*arctanh(c/x)-3/4*a*b^2/c^4*x^4*arctanh(c/x)^2-1/2*a*b^2/c^
3*x^3*arctanh(c/x)-3/2*a*b^2/c*x*arctanh(c/x)-3/16*I*b^3*arctanh(c/x)^2*Pi*
csgn(I*(1+c/x)^2/(-1+c^2/x^2))^3-3/16*I*b^3*arctanh(c/x)^2*Pi*csgn(I*(1+c/x
)^2/(-1+c^2/x^2)/(1+(1+c/x)^2/(1-c^2/x^2)))^3-3/8*I*b^3*arctanh(c/x)^2*Pi*c
```

```
sgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))^2+3/8*I*b^3*arctanh(c/x)^2*Pi*csgn(I/(1+(1
+c/x)^2/(1-c^2/x^2)))^3-1/4/c^3*a^2*b*x^3-1/4/c^2*a*b^2*x^2+3/16*I*b^3*arct
anh(c/x)^2*Pi*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I*(1+c/x)^2/(-1+c^2/x^
2))*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/(1+(1+c/x)^2/(1-c^2/x^2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arctanh(c/x)^2 + 1/4*a^3*x^4 + 1/8*(6*x^4*arctanh(c/x) - (3*c
^3*log(c + x) - 3*c^3*log(-c + x) - 6*c^2*x - 2*x^3)*c)*a^2*b + 1/16*((3*c^
2*log(c + x)^2 + 3*c^2*log(-c + x)^2 + 16*c^2*log(c + x) + 4*x^2 - 2*(3*c^2
*log(c + x) - 8*c^2)*log(-c + x))*c^2 - 4*(3*c^3*log(c + x) - 3*c^3*log(-c
+ x) - 6*c^2*x - 2*x^3)*c*arctanh(c/x))*a*b^2 + 1/32*(16*c^5*integrate(-log
(c + x)/(c^2 - x^2), x) + 40*c^4*integrate(-x*log(c + x)/(c^2 - x^2), x) -
2*(c*log(c + x) - c*log(-c + x) - 2*x)*c^3 - (c^4 - x^4)*log(c + x)^3 + (c^
4 - x^4)*log(-c + x)^3 + 2*(c^2*log(-c^2 + x^2) + x^2)*c^2 + 8*c^2*integrat
e(-x^3*log(c + x)/(c^2 - x^2), x) + 2*(3*c^3*x + c*x^3)*log(c + x)^2 - (8*c
^4 - 6*c^3*x - 2*c*x^3 + 3*(c^4 - x^4)*log(c + x))*log(-c + x)^2 - (4*c^2*x
^2 - 3*(c^4 - x^4)*log(c + x)^2 + 4*(4*c^4 + 3*c^3*x + c*x^3)*log(c + x))*l
og(-c + x))*b^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctanh(c/x)^3 + 3*a*b^2*x^3*arctanh(c/x)^2 + 3*a^2*b*x^3*
arctanh(c/x) + a^3*x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + b \operatorname{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c/x))**3,x)
```

```
[Out] Integral(x**3*(a + b*atanh(c/x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^3,x, algorithm="giac")
[Out] integrate((b*arctanh(c/x) + a)^3*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c/x))^3,x)
[Out] int(x^3*(a + b*atanh(c/x))^3, x)
```

3.151 $\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3\,dx$

Optimal. Leaf size=217

$$b^2c^2x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{2}bc^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}bcx^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{3}c^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3$$

```
[Out] b^2*c^2*x*(a+b*arccoth(x/c))-1/2*b*c^3*(a+b*arccoth(x/c))^2+1/2*b*c*x^2*(a+
b*arccoth(x/c))^2-1/3*c^3*(a+b*arccoth(x/c))^3+1/3*x^3*(a+b*arccoth(x/c))^3
-b*c^3*(a+b*arccoth(x/c))^2*ln(2-2/(1+c/x))+1/2*b^3*c^3*ln(1-c^2/x^2)+b^3*c
^3*ln(x)+b^2*c^3*(a+b*arccoth(x/c))*polylog(2,-1+2/(1+c/x))+1/2*b^3*c^3*pol
ylog(3,-1+2/(1+c/x))
```

Rubi [A]
time = 0.37, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6039, 6037, 6129, 272, 36, 29, 31, 6095, 6135, 6079, 6203, 6745}

$$b^2c^3\text{Li}_2\left(\frac{2}{\frac{c}{x}+1}-1\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+b^2c^2x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{3}c^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3-\frac{1}{2}bc^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-bc^3\log\left(2-\frac{2}{\frac{c}{x}+1}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{3}x^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{2}bcx^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}b^3c^3\text{Li}_3\left(\frac{2}{\frac{c}{x}+1}-1\right)+b^3c^3\log(x)+\frac{1}{2}b^3c^3\log\left(1-\frac{c^2}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] b^2*c^2*x*(a + b*ArcCoth[x/c]) - (b*c^3*(a + b*ArcCoth[x/c])^2)/2 + (b*c*x^
2*(a + b*ArcCoth[x/c])^2)/2 - (c^3*(a + b*ArcCoth[x/c])^3)/3 + (x^3*(a + b*
ArcCoth[x/c])^3)/3 - b*c^3*(a + b*ArcCoth[x/c])^2*Log[2 - 2/(1 + c/x)] + (b
^3*c^3*Log[1 - c^2/x^2])/2 + b^3*c^3*Log[x] + b^2*c^3*(a + b*ArcCoth[x/c])*
PolyLog[2, -1 + 2/(1 + c/x)] + (b^3*c^3*PolyLog[3, -1 + 2/(1 + c/x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3\,dx &= \int \Bigl(\frac{1}{8}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{8}bx^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\Bigl(1+\frac{c}{x}\\
&= \frac{1}{8}\int x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3\,dx+\frac{1}{8}(3b)\int x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\\
&= -\left(\frac{1}{8}\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^3}{x^4}\,dx,x,\frac{1}{x}\right)\right)+\frac{1}{8}(3b)\int\Bigl(4a^2x^2\log\Bigl(1\\
&= \frac{1}{24}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{1}{24}b^3x^3\log^3\left(\frac{c+x}{x}\right)+\frac{1}{2}(3a^2b)\int x^2\log\Bigl(\\
&= \frac{1}{24}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{1}{2}a^2bx^3\log\left(1+\frac{c}{x}\right)-\frac{1}{2}ab^2x^3\log\left(1-\frac{c}{x}\right)\\
&= \frac{1}{24}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{1}{2}a^2bx^3\log\left(1+\frac{c}{x}\right)-\frac{1}{2}ab^2x^3\log\left(1-\frac{c}{x}\right)\\
&= \frac{1}{16}bcx^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{24}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{1}{2}a^2bx^3\log\\
&= -\frac{1}{2}a^2bc^2x+\frac{1}{4}a^2bcx^2+\frac{1}{8}bc^2\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{16}bcx^2\Bigl(2\\
&= -\frac{1}{2}a^2bc^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{4}ab^2cx^2\log\left(1-\frac{c}{x}\right)+\frac{1}{8}b\\
&= -\frac{1}{2}a^2bc^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{4}ab^2cx^2\log\left(1-\frac{c}{x}\right)+\frac{1}{8}b\\
&= -\frac{1}{2}a^2bc^2x+\frac{1}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{4}ab^2cx^2\log\Bigl(1\\
&= -\frac{1}{2}a^2bc^2x+\frac{3}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{4}ab^2cx^2\log\Bigl(1\\
&= -\frac{1}{2}a^2bc^2x+\frac{3}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{4}ab^2cx^2\log\Bigl(1\\
&= -\frac{1}{2}a^2bc^2x+\frac{3}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{4}ab^2cx^2\log\Bigl(1
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.53, size = 316, normalized size = 1.46

$$\frac{1}{6}\left(3a^2bcx^2+2a^3x^3+6a^2bx^3\tanh^{-1}\left(\frac{c}{x}\right)+3a^2bc^3\log\left(-c^2+x^2\right)+6ab^2\left(c^2x+\left(-c^3+x^3\right)\tanh^{-1}\left(\frac{c}{x}\right)^2+c\tanh^{-1}\left(\frac{c}{x}\right)\left(-c^2+x^2-2c^2\log\left(1-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)+c^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)+\frac{1}{4}b^3\left(-ic^3\pi^3+24c^2x\tanh^{-1}\left(\frac{c}{x}\right)-12c^3\tanh^{-1}\left(\frac{c}{x}\right)^2+12cx^2\tanh^{-1}\left(\frac{c}{x}\right)^2+8c^3\tanh^{-1}\left(\frac{c}{x}\right)^3+8x^3\tanh^{-1}\left(\frac{c}{x}\right)^3-24c^3\tanh^{-1}\left(\frac{c}{x}\right)^2\log\left(1-e^{2\tanh^{-1}\left(\frac{c}{x}\right)}\right)-24c^3\log\left(\frac{c}{\sqrt{1-\frac{c^2}{x^2}}\,x}\right)-24c^3\tanh^{-1}\left(\frac{c}{x}\right)\text{PolyLog}\left(2,e^{2\tanh^{-1}\left(\frac{c}{x}\right)}\right)+12c^3\text{PolyLog}\left(3,e^{2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] (3*a^2*b*c*x^2 + 2*a^3*x^3 + 6*a^2*b*x^3*ArcTanh[c/x] + 3*a^2*b*c^3*Log[-c^
2 + x^2] + 6*a*b^2*(c^2*x + (-c^3 + x^3)*ArcTanh[c/x]^2 + c*ArcTanh[c/x]*(-
c^2 + x^2 - 2*c^2*Log[1 - E^(-2*ArcTanh[c/x])]) + c^3*PolyLog[2, E^(-2*ArcT
anh[c/x])]) + (b^3*((-I)*c^3*Pi^3 + 24*c^2*x*ArcTanh[c/x] - 12*c^3*ArcTanh[
c/x]^2 + 12*c*x^2*ArcTanh[c/x]^2 + 8*c^3*ArcTanh[c/x]^3 + 8*x^3*ArcTanh[c/x
]^3 - 24*c^3*ArcTanh[c/x]^2*Log[1 - E^(2*ArcTanh[c/x])] - 24*c^3*Log[c/(Sqr
t[1 - c^2/x^2]*x)] - 24*c^3*ArcTanh[c/x]*PolyLog[2, E^(2*ArcTanh[c/x])] + 1
2*c^3*PolyLog[3, E^(2*ArcTanh[c/x])]))/4)/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.79, size = 1915, normalized size = 8.82

method	result	size
derivativedivides	Expression too large to display	1915
default	Expression too large to display	1915

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c/x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -c^3*(1/2*I*b^3*arctanh(c/x)^2*csgn(I*(1+c/x)^2/(-1+c^2/x^2))^2*csgn(I*(1+c
/x)/(1-c^2/x^2)^(1/2))*Pi+1/2*a*b^2*ln(-1/2*c/x+1/2)*ln(1/2*c/x+1/2)-1/2*a*
b^2*ln(-1/2*c/x+1/2)*ln(1+c/x)-a*b^2*arctanh(c/x)*ln(1+c/x)-a*b^2*arctanh(c
/x)*ln(c/x-1)+1/2*a*b^2*ln(c/x-1)*ln(1/2*c/x+1/2)-a^2*b/c^3*x^3*arctanh(c/x
)-a*b^2/c^3*x^3*arctanh(c/x)^2-a*b^2/c^2*x^2*arctanh(c/x)+1/4*I*b^3*arctanh
(c/x)^2*csgn(I*(1+c/x)^2/(-1+c^2/x^2))^3*Pi+1/4*I*b^3*arctanh(c/x)^2*csgn(I
*(1+c/x)^2/(-1+c^2/x^2)/(1+(1+c/x)^2/(1-c^2/x^2)))^3*Pi+1/2*I*b^3*arctanh(c
/x)^2*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^3*Pi+1/2*
I*b^3*arctanh(c/x)^2*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))^3*Pi-1/2*I*b^3*arcta
nh(c/x)^2*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))^2*Pi-b^3*arctanh(c/x)+1/2*b^3*a
rctanh(c/x)^2-1/3*b^3*arctanh(c/x)^3+2*a*b^2*ln(c/x)*arctanh(c/x)-a*b^2*ln(
c/x)*ln(1+c/x)-1/4*a*b^2*ln(c/x-1)^2+1/4*a*b^2*ln(1+c/x)^2+1/2*a*b^2*ln(1+c
/x)-1/2*a*b^2*ln(c/x-1)-1/2*b^3*arctanh(c/x)^2*ln(1+c/x)-1/2*b^3*arctanh(c/
x)^2*ln(c/x-1)+b^3*arctanh(c/x)^2*ln((1+c/x)/(1-c^2/x^2)^(1/2))-1/2*a^2*b*l
n(1+c/x)-1/2*a^2*b*ln(c/x-1)+b^3*ln(1+(1+c/x)/(1-c^2/x^2)^(1/2))-2*b^3*poly
log(3,(1+c/x)/(1-c^2/x^2)^(1/2))+b^3*ln((1+c/x)/(1-c^2/x^2)^(1/2)-1)-2*b^3*
polylog(3,-(1+c/x)/(1-c^2/x^2)^(1/2))+1/2*I*b^3*arctanh(c/x)^2*Pi-1/2*a^2*b
/c^2*x^2-a*b^2/c*x-1/3*b^3/c^3*x^3*arctanh(c/x)^3-1/2*b^3/c^2*x^2*arctanh(c
/x)^2-b^3*arctanh(c/x)/c*x-1/4*I*b^3*arctanh(c/x)^2*csgn(I*(1+c/x)^2/(-1+c^
2/x^2))*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/(1+(1+c/x)^2/(1-c^2/x^2)))^2*Pi+1/4*I
*b^3*arctanh(c/x)^2*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*csgn(I*(1+c/x)/(1-c^2/x^
2)^(1/2))^2*Pi+1/4*I*b^3*arctanh(c/x)^2*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/(1+(1
+c/x)^2/(1-c^2/x^2)))^2*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*Pi-1/2*I*b^3*arct
anh(c/x)^2*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^2*cs
gn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*Pi-1/2*I*b^3*arctanh(c/x)^2*csgn(I*((1+c/x)
^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^2*csgn(I*((1+c/x)^2/(1-c^2/x^2
```

```
)-1))*Pi-1/3*a^3/c^3*x^3+a^2*b*ln(c/x)+a*b^2*dilog(1/2*c/x+1/2)-a*b^2*dilog
(1+c/x)-a*b^2*dilog(c/x)-b^3*arctanh(c/x)^2*ln((1+c/x)^2/(1-c^2/x^2)-1)+b^3
*arctanh(c/x)^2*ln(1-(1+c/x)/(1-c^2/x^2)^(1/2))+2*b^3*arctanh(c/x)*polylog(
2,(1+c/x)/(1-c^2/x^2)^(1/2))+b^3*arctanh(c/x)^2*ln(1+(1+c/x)/(1-c^2/x^2)^(1
/2))+2*b^3*arctanh(c/x)*polylog(2,-(1+c/x)/(1-c^2/x^2)^(1/2))+b^3*ln(c/x)*a
rctanh(c/x)^2+b^3*arctanh(c/x)^2*ln(2)-1/4*I*b^3*arctanh(c/x)^2*csgn(I*(1+c
/x)^2/(-1+c^2/x^2))*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/(1+(1+c/x)^2/(1-c^2/x^2))
)*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*Pi+1/2*I*b^3*arctanh(c/x)^2*csgn(I*((1+
c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I/(1+(1+c/x)^2/(1-c^2
/x^2)))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*Pi)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*x^3 + 1/2*(2*x^3*arctanh(c/x) + (c^2*log(-c^2 + x^2) + x^2)*c)*a^2*
b + 1/24*(b^3*c^3 - b^3*x^3)*log(-c + x)^3 + 1/8*(b^3*c*x^2 + 2*a*b^2*x^3 +
 (b^3*c^3 + b^3*x^3)*log(c + x))*log(-c + x)^2 - integrate(-1/8*((b^3*c*x^2
 - b^3*x^3)*log(c + x)^3 + 6*(a*b^2*c*x^2 - a*b^2*x^3)*log(c + x)^2 + (2*b^
3*c*x^2 + 4*a*b^2*x^3 - 3*(b^3*c*x^2 - b^3*x^3)*log(c + x)^2 + 2*(b^3*c^3 -
 6*a*b^2*c*x^2 + (6*a*b^2 + b^3)*x^3)*log(c + x))*log(-c + x))/(c - x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctanh(c/x)^3 + 3*a*b^2*x^2*arctanh(c/x)^2 + 3*a^2*b*x^2*
arctanh(c/x) + a^3*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a + b\operatorname{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c/x))**3,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c/x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^3,x, algorithm="giac")
[Out] integrate((b*arctanh(c/x) + a)^3*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c/x))^3,x)
[Out] int(x^2*(a + b*atanh(c/x))^3, x)
```

3.152 $\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3\,dx$

Optimal. Leaf size=135

$$-\frac{3}{2}bc^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{3}{2}bcx\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{2}c^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{2}x^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right.$$

```
[Out] -3/2*b*c^2*(a+b*arccoth(x/c))^2+3/2*b*c*x*(a+b*arccoth(x/c))^2-1/2*c^2*(a+b
*arccoth(x/c))^3+1/2*x^2*(a+b*arccoth(x/c))^3-3*b^2*c^2*(a+b*arccoth(x/c))*
ln(2-2/(1+c/x))+3/2*b^3*c^2*polylog(2,-1+2/(1+c/x))
```

Rubi [A]
time = 0.25, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6039, 6037, 6129, 6135, 6079, 2497, 6095}

$$-3b^2c^2\log\left(2-\frac{2}{\frac{c}{x}+1}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-\frac{3}{2}bc^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{2}c^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{2}x^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{3}{2}bcx\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{3}{2}b^3c^2\text{Li}_2\left(\frac{2}{\frac{c}{x}+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] (-3*b*c^2*(a + b*ArcCoth[x/c])^2)/2 + (3*b*c*x*(a + b*ArcCoth[x/c])^2)/2 -
(c^2*(a + b*ArcCoth[x/c])^3)/2 + (x^2*(a + b*ArcCoth[x/c])^3)/2 - 3*b^2*c^2
*(a + b*ArcCoth[x/c])*Log[2 - 2/(1 + c/x)] + (3*b^3*c^2*PolyLog[2, -1 + 2/(
1 + c/x)])/2
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
```

fy[(m + 1)/n]]

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx = \int \left(\frac{1}{8}x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{8}bx\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)+\cdots\right.$$

$$=\frac{1}{8}\int x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3 dx+\frac{1}{8}(3b)\int x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1\cdots\right.$$

$$=-\left(\frac{1}{8}\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^3}{x^3}\,dx,x,\frac{1}{x}\right)\right)+\frac{1}{8}(3b)\int\left(4a^2x\log\left(1+\cdots\right.\right.$$

$$=\frac{1}{16}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{1}{16}b^3x^2\log^3\left(\frac{c+x}{x}\right)+\frac{1}{2}(3a^2b)\int x\log\left(1\cdots\right.$$

$$=\frac{1}{16}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{4}a^2bx^2\log\left(1+\frac{c}{x}\right)-\frac{3}{4}ab^2x^2\log\left(1-\frac{c}{x}\right)\cdots$$

$$=\frac{1}{16}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{4}a^2bx^2\log\left(1+\frac{c}{x}\right)-\frac{3}{4}ab^2x^2\log\left(1-\frac{c}{x}\right)\cdots$$

$$=\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{16}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{4}a\cdots$$

$$=\frac{3}{4}a^2bcx+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{16}x^2\left(2a-b\log\left(1-\frac{c}{x}\cdots\right.\right.$$

$$=\frac{3}{4}a^2bcx-\frac{3}{4}ab^2cx\log\left(1-\frac{c}{x}\right)+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2-\cdots$$

$$=\frac{3}{4}a^2bcx-\frac{3}{4}ab^2cx\log\left(1-\frac{c}{x}\right)+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2-\cdots$$

$$=\frac{3}{4}a^2bcx-\frac{3}{4}ab^2cx\log\left(1-\frac{c}{x}\right)+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2-\cdots$$

$$=\frac{3}{4}a^2bcx-\frac{3}{4}ab^2cx\log\left(1-\frac{c}{x}\right)+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2-\cdots$$

$$=\frac{3}{4}a^2bcx-\frac{3}{4}ab^2cx\log\left(1-\frac{c}{x}\right)+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2-\cdots$$

$$=\frac{3}{4}a^2bcx-\frac{3}{4}ab^2cx\log\left(1-\frac{c}{x}\right)+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2-\cdots$$

Mathematica [A]
time = 0.21, size = 193, normalized size = 1.43

$$\frac{1}{4}\left(6b^2(-c+x)(bc+a(c+x))\tanh^{-1}\left(\frac{c}{x}\right)^2+2b^3(-c^2+x^2)\tanh^{-1}\left(\frac{c}{x}\right)^3+6b\tanh^{-1}\left(\frac{c}{x}\right)\left(ax(2bc+ax)-2b^2c^2\log\left(1-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)+a\left(3abc^2\log\left(1-\frac{c}{x}\right)-12b^2c^2\log\left(\frac{c}{\sqrt{1-\frac{c^2}{x^2}}\,x}\right)+a\left(6bcx+2ax^2-3bc^2\log\left(\frac{c+x}{x}\right)\right)\right)+6b^3c^2\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] (6*b^2*(-c + x)*(b*c + a*(c + x))*ArcTanh[c/x]^2 + 2*b^3*(-c^2 + x^2)*ArcTa
nh[c/x]^3 + 6*b*ArcTanh[c/x]*(a*x*(2*b*c + a*x) - 2*b^2*c^2*Log[1 - E^(-2*A
rcTanh[c/x])]) + a*(3*a*b*c^2*Log[1 - c/x] - 12*b^2*c^2*Log[c/(Sqrt[1 - c^2
/x^2]*x)] + a*(6*b*c*x + 2*a*x^2 - 3*b*c^2*Log[(c + x)/x])) + 6*b^3*c^2*Pol
yLog[2, E^(-2*ArcTanh[c/x])])/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.07, size = 5310, normalized size = 39.33

method	result	size
derivativedivides	Expression too large to display	5310
default	Expression too large to display	5310
risch	Expression too large to display	41689

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c/x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a*b^2*x^2*arctanh(c/x)^2 + 1/2*a^3*x^2 + 3/4*(2*x^2*arctanh(c/x) - (c*l
og(c + x) - c*log(-c + x) - 2*x)*c)*a^2*b + 3/8*((log(c + x)^2 - 2*(log(c +
 x) - 2)*log(-c + x) + log(-c + x)^2 + 4*log(c + x))*c^2 - 4*(c*log(c + x)
- c*log(-c + x) - 2*x)*c*arctanh(c/x))*a*b^2 + 1/16*(6*c*x*log(c + x)^2 - (
c^2 - x^2)*log(c + x)^3 + (c^2 - x^2)*log(-c + x)^3 - 3*(2*c^2 - 2*c*x + (c
^2 - x^2)*log(c + x))*log(-c + x)^2 + 3*((c^2 - x^2)*log(c + x)^2 - 4*(c^2
+ c*x)*log(c + x))*log(-c + x) + 2*integrate(-6*(c^3 + 3*c^2*x)*log(c + x)/
(c^2 - x^2), x))*b^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctanh(c/x)^3 + 3*a*b^2*x*arctanh(c/x)^2 + 3*a^2*b*x*arctan
h(c/x) + a^3*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(a + b \operatorname{atanh}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c/x))**3,x)
```

```
[Out] Integral(x*(a + b*atanh(c/x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \left(a + b \operatorname{atanh}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c/x))^3,x)
```

```
[Out] int(x*(a + b*atanh(c/x))^3, x)
```

3.153 $\int \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 \, dx$

Optimal. Leaf size=108

$$c\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3+x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3-3bc\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(\frac{2c}{c-x}\right)-3b^2c\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right.$$

```
[Out] c*(a+b*arccoth(x/c))^3+x*(a+b*arccoth(x/c))^3-3*b*c*(a+b*arccoth(x/c))^2*ln
(2*c/(c-x))-3*b^2*c*(a+b*arccoth(x/c))*polylog(2,1-2*c/(c-x))+3/2*b^3*c*pol
ylog(3,1-2*c/(c-x))
```

Rubi [A]
time = 0.18, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6025, 6022, 6132, 6056, 6096, 6206, 6745}

$$-3b^2c\text{Li}_2\left(1-\frac{2c}{c-x}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+c\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3+x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3-3bc\log\left(\frac{2c}{c-x}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{3}{2}b^3c\text{Li}_3\left(1-\frac{2c}{c-x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] c*(a + b*ArcCoth[x/c])^3 + x*(a + b*ArcCoth[x/c])^3 - 3*b*c*(a + b*ArcCoth[
x/c])^2*Log[(2*c)/(c - x)] - 3*b^2*c*(a + b*ArcCoth[x/c])*PolyLog[2, 1 - (2
*c)/(c - x)] + (3*b^3*c*PolyLog[3, 1 - (2*c)/(c - x)])/2
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6025

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[(a + b*
ArcCoth[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && ILtQ[n, 0
]
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6206

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx &= \int \left(a^3-\frac{3}{2}a^2b\log\left(1-\frac{c}{x}\right)+\frac{3}{4}ab^2\log^2\left(1-\frac{c}{x}\right)-\frac{1}{8}b^3\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a^2b\,\mathrm{lo}\right.\\
&= a^3x-\frac{1}{2}(3a^2b)\int \log\left(1-\frac{c}{x}\right)dx+\frac{1}{2}(3a^2b)\int \log\left(1+\frac{c}{x}\right)dx+\frac{1}{4}(3ab^2)\int\\
&= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\right.\\
&= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\right.\\
&= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\right.\\
&= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\right.\\
&= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\right.\\
&= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\right.\\
&= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\right.\\
&= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\right.
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 198, normalized size = 1.83

$$a^3x+3a^2bx\tanh^{-1}\left(\frac{c}{x}\right)+\frac{3}{2}a^2bc\log\left(-c^2+x^2\right)-3ab^2\left(\tanh^{-1}\left(\frac{c}{x}\right)\left((c-x)\tanh^{-1}\left(\frac{c}{x}\right)+2c\log\left(1-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)-c\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)+\frac{1}{8}b^3\left(-ic\pi^3+8c\tanh^{-1}\left(\frac{c}{x}\right)^3+8x\tanh^{-1}\left(\frac{c}{x}\right)^3-24c\tanh^{-1}\left(\frac{c}{x}\right)^2\log\left(1-e^{2\tanh^{-1}\left(\frac{c}{x}\right)}\right)-24c\tanh^{-1}\left(\frac{c}{x}\right)\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}\left(\frac{c}{x}\right)}\right)+12c\mathrm{PolyLog}\left(3,e^{2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] a^3*x + 3*a^2*b*x*ArcTanh[c/x] + (3*a^2*b*c*Log[-c^2 + x^2])/2 - 3*a*b^2*(A
rcTanh[c/x]*((c - x)*ArcTanh[c/x] + 2*c*Log[1 - E^(-2*ArcTanh[c/x])]) - c*P
olyLog[2, E^(-2*ArcTanh[c/x])]) + (b^3*((-I)*c*Pi^3 + 8*c*ArcTanh[c/x]^3 +
8*x*ArcTanh[c/x]^3 - 24*c*ArcTanh[c/x]^2*Log[1 - E^(2*ArcTanh[c/x])] - 24*c
*ArcTanh[c/x]*PolyLog[2, E^(2*ArcTanh[c/x])] + 12*c*PolyLog[3, E^(2*ArcTanh
[c/x])]))/8
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.71, size = 1732, normalized size = 16.04

method	result	size
derivativedivides	Expression too large to display	1732
default	Expression too large to display	1732

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -c*(-a^3/c*x-3/2*I*b^3*arctanh(c/x)^2*Pi*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*
csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^2+3/2*a*b^2*ln(
-1/2*c/x+1/2)*ln(1/2*c/x+1/2)-3/2*a*b^2*ln(-1/2*c/x+1/2)*ln(1+c/x)-3*a*b^2*
arctanh(c/x)*ln(1+c/x)-3*a*b^2*arctanh(c/x)*ln(c/x-1)+3/2*a*b^2*ln(c/x-1)*l
n(1/2*c/x+1/2)-3/2*I*b^3*arctanh(c/x)^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)
)*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^2+3/4*I*b^3*a
rctanh(c/x)^2*Pi*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I*(1+c/x)^2/(-1+c^2
/x^2)/(1+(1+c/x)^2/(1-c^2/x^2)))^2+3/2*I*b^3*arctanh(c/x)^2*Pi*csgn(I*(1+c/
x)/(1-c^2/x^2)^(1/2))*csgn(I*(1+c/x)^2/(-1+c^2/x^2))^2+3/4*I*b^3*arctanh(c/
x)^2*Pi*csgn(I*(1+c/x)/(1-c^2/x^2)^(1/2))^2*csgn(I*(1+c/x)^2/(-1+c^2/x^2))-
3/4*I*b^3*arctanh(c/x)^2*Pi*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*csgn(I*(1+c/x)^2
/(-1+c^2/x^2)/(1+(1+c/x)^2/(1-c^2/x^2)))^2-b^3*arctanh(c/x)^3+3/2*I*b^3*arc
tanh(c/x)^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*csgn(I/(1+(1+c/x)^2/(1-c^2
/x^2)))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))-3/4*I*b
^3*arctanh(c/x)^2*Pi*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I*(1+c/x)^2/(-1
+c^2/x^2))*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/(1+(1+c/x)^2/(1-c^2/x^2)))+3/2*I*b
^3*arctanh(c/x)^2*Pi-b^3/c*x*arctanh(c/x)^3+6*a*b^2*ln(c/x)*arctanh(c/x)-3*
a*b^2*ln(c/x)*ln(1+c/x)-3/4*a*b^2*ln(c/x-1)^2+3/4*a*b^2*ln(1+c/x)^2-3/2*b^3
*arctanh(c/x)^2*ln(1+c/x)-3/2*b^3*arctanh(c/x)^2*ln(c/x-1)+3*b^3*arctanh(c/
x)^2*ln((1+c/x)/(1-c^2/x^2)^(1/2))-3/2*a^2*b*ln(1+c/x)-3/2*a^2*b*ln(c/x-1)-
6*b^3*polylog(3,(1+c/x)/(1-c^2/x^2)^(1/2))-6*b^3*polylog(3,-(1+c/x)/(1-c^2/
x^2)^(1/2))+3/2*I*b^3*arctanh(c/x)^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1
+(1+c/x)^2/(1-c^2/x^2)))^3-3/2*I*b^3*arctanh(c/x)^2*Pi*csgn(I/(1+(1+c/x)^2/
(1-c^2/x^2)))^2+3/4*I*b^3*arctanh(c/x)^2*Pi*csgn(I*(1+c/x)^2/(-1+c^2/x^2))^
3+3/4*I*b^3*arctanh(c/x)^2*Pi*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/(1+(1+c/x)^2/(1
-c^2/x^2)))^3+3/2*I*b^3*arctanh(c/x)^2*Pi*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))
^3-3*a*b^2/c*x*arctanh(c/x)^2-3*a^2*b/c*x*arctanh(c/x)+3*a^2*b*ln(c/x)+3*a*
b^2*dilog(1/2*c/x+1/2)-3*a*b^2*dilog(1+c/x)-3*a*b^2*dilog(c/x)-3*b^3*arctan
h(c/x)^2*ln((1+c/x)^2/(1-c^2/x^2)-1)+3*b^3*arctanh(c/x)^2*ln(1-(1+c/x)/(1-c
^2/x^2)^(1/2))+6*b^3*arctanh(c/x)*polylog(2,(1+c/x)/(1-c^2/x^2)^(1/2))+3*b^
3*arctanh(c/x)^2*ln(1+(1+c/x)/(1-c^2/x^2)^(1/2))+6*b^3*arctanh(c/x)*polylog
(2,-(1+c/x)/(1-c^2/x^2)^(1/2))+3*b^3*ln(c/x)*arctanh(c/x)^2+3*b^3*arctanh(c
/x)^2*ln(2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*(2*x*arctanh(c/x) + c*log(-c^2 + x^2))*a^2*b + a^3*x + 1/8*(b^3*c - b^3
*x)*log(-c + x)^3 + 3/8*(2*a*b^2*x + (b^3*c + b^3*x)*log(c + x))*log(-c + x
)^2 - integrate(-1/8*((b^3*c - b^3*x)*log(c + x)^3 + 6*(a*b^2*c - a*b^2*x)*
log(c + x)^2 + 3*(4*a*b^2*x - (b^3*c - b^3*x)*log(c + x)^2 - 2*(2*a*b^2*c -
 b^3*c - (2*a*b^2 + b^3)*x)*log(c + x))*log(-c + x))/(c - x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctanh(c/x)^3 + 3*a*b^2*arctanh(c/x)^2 + 3*a^2*b*arctanh(c/x)
 + a^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{atanh}\left(\frac{c}{x}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**3,x)
```

```
[Out] Integral((a + b*atanh(c/x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))^3,x)
```

```
[Out] int((a + b*atanh(c/x))^3, x)
```

3.154 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$

Optimal. Leaf size=208

$$-2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)+\frac{3}{2}b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)-\frac{3}{2}b\left(a+b\,\text{c}\right.$$

```
[Out] 2*(a+b*arccoth(x/c))^3*arctanh(-1+2/(1-c/x))+3/2*b*(a+b*arccoth(x/c))^2*pol
ylog(2,1-2/(1-c/x))-3/2*b*(a+b*arccoth(x/c))^2*polylog(2,-1+2/(1-c/x))-3/2*
b^2*(a+b*arccoth(x/c))*polylog(3,1-2/(1-c/x))+3/2*b^2*(a+b*arccoth(x/c))*po
lylog(3,-1+2/(1-c/x))+3/4*b^3*polylog(4,1-2/(1-c/x))-3/4*b^3*polylog(4,-1+2
/(1-c/x))
```

Rubi [A]
time = 0.34, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6035, 6033, 6199, 6095, 6205, 6209, 6745}

$$-\frac{3}{2}b^2\text{Li}_3\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}b^2\text{Li}_3\left(\frac{2}{1-\frac{c}{x}}-1\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}b\text{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{3}{2}b\text{Li}_2\left(\frac{2}{1-\frac{c}{x}}-1\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{3}{4}b^3\text{Li}_4\left(1-\frac{2}{1-\frac{c}{x}}\right)-\frac{3}{4}b^3\text{Li}_4\left(\frac{2}{1-\frac{c}{x}}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])^3/x,x]
```

```
[Out] -2*(a + b*ArcCoth[x/c])^3*ArcTanh[1 - 2/(1 - c/x)] + (3*b*(a + b*ArcCoth[x/
c])^2*PolyLog[2, 1 - 2/(1 - c/x)])/2 - (3*b*(a + b*ArcCoth[x/c])^2*PolyLog[
2, -1 + 2/(1 - c/x)])/2 - (3*b^2*(a + b*ArcCoth[x/c])*PolyLog[3, 1 - 2/(1 -
 c/x)])/2 + (3*b^2*(a + b*ArcCoth[x/c])*PolyLog[3, -1 + 2/(1 - c/x)])/2 + (
3*b^3*PolyLog[4, 1 - 2/(1 - c/x)])/4 - (3*b^3*PolyLog[4, -1 + 2/(1 - c/x)])
/4
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6209

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x}\,dx = -\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right) + (6bc)\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(\right.}{}\right.$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right) - (3bc)\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(\right.}{1-}\right.$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right) + \frac{3}{2}b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\text{Li}_2\left(1\right.$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right) + \frac{3}{2}b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\text{Li}_2\left(1\right.$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right) + \frac{3}{2}b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\text{Li}_2\left(1\right.$$

Mathematica [A]
time = 0.13, size = 171, normalized size = 0.82

$$-2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3\tanh^{-1}\left(\frac{c+x}{c-x}\right)+\frac{3}{4}b\left(2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2\text{PolyLog}\left(2,\frac{c+x}{c-x}\right)-2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2\text{PolyLog}\left(2,\frac{c+x}{-c+x}\right)+b\left(-2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)\text{PolyLog}\left(3,\frac{c+x}{c-x}\right)+2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)\text{PolyLog}\left(3,\frac{c+x}{-c+x}\right)+b\left(\text{PolyLog}\left(4,\frac{c+x}{c-x}\right)-\text{PolyLog}\left(4,\frac{c+x}{-c+x}\right)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])^3/x,x]
```

```
[Out] -2*(a + b*ArcTanh[c/x])^3*ArcTanh[(c + x)/(c - x)] + (3*b*(2*(a + b*ArcTanh
[c/x])^2*PolyLog[2, (c + x)/(c - x)] - 2*(a + b*ArcTanh[c/x])^2*PolyLog[2,
(c + x)/(-c + x)] + b*(-2*(a + b*ArcTanh[c/x])*PolyLog[3, (c + x)/(c - x)]
+ 2*(a + b*ArcTanh[c/x])*PolyLog[3, (c + x)/(-c + x)] + b*(PolyLog[4, (c +
x)/(c - x)] - PolyLog[4, (c + x)/(-c + x)]))))/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.71, size = 1631, normalized size = 7.84

method	result	size
derivativedivides	Expression too large to display	1631
default	Expression too large to display	1631

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*b^3*Pi*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I*((1+c/x)^2/(1-c^2/x^2
)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^2*arctanh(c/x)^3+1/2*I*b^3*Pi*csgn(I*((1+c/
x)^2/(1-c^2/x^2)-1))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x
^2)))^2*arctanh(c/x)^3-3/2*I*a*b^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(
1+c/x)^2/(1-c^2/x^2)))^3*arctanh(c/x)^2-1/2*I*b^3*Pi*csgn(I*((1+c/x)^2/(1-c
^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^3*arctanh(c/x)^3-a^3*ln(c/x)+3/4*b^3*
polylog(4,-(1+c/x)^2/(1-c^2/x^2))-6*b^3*polylog(4,(1+c/x)/(1-c^2/x^2)^(1/2)
)-6*b^3*polylog(4,-(1+c/x)/(1-c^2/x^2)^(1/2))-3/2*I*a*b^2*Pi*csgn(I*((1+c/x
)^2/(1-c^2/x^2)-1))*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I*((1+c/x)^2/(1-
c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))*arctanh(c/x)^2-b^3*ln(c/x)*arctanh(c
/x)^3+b^3*arctanh(c/x)^3*ln((1+c/x)^2/(1-c^2/x^2)-1)+3/2*b^3*arctanh(c/x)^2
*polylog(2,-(1+c/x)^2/(1-c^2/x^2))-3/2*b^3*arctanh(c/x)*polylog(3,-(1+c/x)^
2/(1-c^2/x^2))-b^3*arctanh(c/x)^3*ln(1-(1+c/x)/(1-c^2/x^2)^(1/2))-3*b^3*arc
tanh(c/x)^2*polylog(2,(1+c/x)/(1-c^2/x^2)^(1/2))+6*b^3*arctanh(c/x)*polylog
(3,(1+c/x)/(1-c^2/x^2)^(1/2))-b^3*arctanh(c/x)^3*ln(1+(1+c/x)/(1-c^2/x^2)^(
1/2))-3*b^3*arctanh(c/x)^2*polylog(2,-(1+c/x)/(1-c^2/x^2)^(1/2))+6*b^3*arct
anh(c/x)*polylog(3,-(1+c/x)/(1-c^2/x^2)^(1/2))-3/2*a*b^2*polylog(3,-(1+c/x)
^2/(1-c^2/x^2))+6*a*b^2*polylog(3,-(1+c/x)/(1-c^2/x^2)^(1/2))+6*a*b^2*polyl
og(3,(1+c/x)/(1-c^2/x^2)^(1/2))+3/2*a^2*b*dilog(1+c/x)+3/2*a^2*b*dilog(c/x)
-1/2*I*b^3*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*csgn(I/(1+(1+c/x)^2/(1-c^2/
x^2)))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))*arctanh(
c/x)^3+3/2*I*a*b^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*csgn(I*((1+c/x)^2/(
1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^2*arctanh(c/x)^2+3/2*I*a*b^2*Pi*cs
gn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)
^2/(1-c^2/x^2)))^2*arctanh(c/x)^2-3*a*b^2*ln(c/x)*arctanh(c/x)^2+3*a*b^2*ar
ctanh(c/x)*polylog(2,-(1+c/x)^2/(1-c^2/x^2))+3*a*b^2*arctanh(c/x)^2*ln((1+c
/x)^2/(1-c^2/x^2)-1)-3*a*b^2*arctanh(c/x)^2*ln(1+(1+c/x)/(1-c^2/x^2)^(1/2))
-6*a*b^2*arctanh(c/x)*polylog(2,-(1+c/x)/(1-c^2/x^2)^(1/2))-3*a*b^2*arctanh
(c/x)^2*ln(1-(1+c/x)/(1-c^2/x^2)^(1/2))-6*a*b^2*arctanh(c/x)*polylog(2,(1+c
/x)/(1-c^2/x^2)^(1/2))-3*a^2*b*ln(c/x)*arctanh(c/x)+3/2*a^2*b*ln(c/x)*ln(1+
c/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + integrate(1/8*b^3*(log(c/x + 1) - log(-c/x + 1))^3/x + 3/4*a*b
^2*(log(c/x + 1) - log(-c/x + 1))^2/x + 3/2*a^2*b*(log(c/x + 1) - log(-c/x
+ 1))/x, x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c/x)^3 + 3*a*b^2*arctanh(c/x)^2 + 3*a^2*b*arctanh(c/x
) + a^3)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\operatorname{atanh}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**3/x,x)
```

```
[Out] Integral((a + b*atanh(c/x))**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))^3/x,x)
```

```
[Out] int((a + b*atanh(c/x))^3/x, x)
```

3.155 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=126

$$-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{c}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{x}+\frac{3b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(\frac{2}{1-\frac{c}{x}}\right)}{c}+\frac{3b^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\text{Poly}}{c}$$

```
[Out] -(a+b*arccoth(x/c))^3/c-(a+b*arccoth(x/c))^3/x+3*b*(a+b*arccoth(x/c))^2*ln(
2/(1-c/x))/c+3*b^2*(a+b*arccoth(x/c))*polylog(2,1-2/(1-c/x))/c-3/2*b^3*poly
log(3,1-2/(1-c/x))/c
```

Rubi [A]
time = 0.18, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6039, 6021, 6131, 6055, 6095, 6205, 6745}

$$\frac{3b^2\text{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)}{c}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{c}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{x}+\frac{3b\log\left(\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{c}-\frac{3b^3\text{Li}_3\left(1-\frac{2}{1-\frac{c}{x}}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])^3/x^2,x]
```

```
[Out] -((a + b*ArcCoth[x/c])^3/c) - (a + b*ArcCoth[x/c])^3/x + (3*b*(a + b*ArcCot
h[x/c])^2*Log[2/(1 - c/x)])/c + (3*b^2*(a + b*ArcCoth[x/c])*PolyLog[2, 1 -
2/(1 - c/x)])/c - (3*b^3*PolyLog[3, 1 - 2/(1 - c/x)])/(2*c)
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
```

```
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8x^2}+\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)}{8x^2}+\frac{3b^2\left(2a-b\right.}{}\right.$$

$$= \frac{1}{8}\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{x^2}\,dx+\frac{1}{8}(3b)\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)}{x^2}\,dx$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int (2a-b\log(1-cx))^3\,dx,x,\frac{1}{x}\right)\right)-\frac{1}{8}(3b)\text{Subst}\left(\int (2a-b\log(\right.\right.$$

$$= -\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(\frac{c+x}{x}\right)}{8x}-\frac{3b^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log^2\left(\frac{c+x}{x}\right)}{8x}+\text{Su}$$

$$= \frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(\frac{c+x}{x}\right)}{8x}-\frac{3b^2\left(2a-\right.}{}$$

$$= \frac{3b\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{8c}+\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a-b\right.}{}$$

$$= -\frac{3ab^2}{2x}+\frac{3b^3}{4x}+\frac{3b\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{8c}+\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right.\right.}{8c}$$

$$= -\frac{3ab^2}{2x}-\frac{3b^3\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{4c}+\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a-b\right.}{}$$

$$= -\frac{3b^3}{4x}-\frac{3b^3\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{4c}+\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a-b\,\mathrm{l}\right.}{}$$

$$= \frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(\frac{c+x}{2x}\right)}{4c}+\frac{3b\left(2a-\right.}{}$$

$$= \frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(\frac{c+x}{2x}\right)}{4c}+\frac{3b\left(2a-\right.}{}$$

<u>**Mathematica**</u> [A]
time = 0.18, size = 215, normalized size = 1.71

$$-\frac{a^3}{x}-\frac{3a^2b\tanh^{-1}\left(\frac{c}{x}\right)}{x}+\frac{3a^2b\log(x)}{c}-\frac{3a^2b\log\left(-c^2+x^2\right)}{2c}-\frac{3ab^2\left(\tanh^{-1}\left(\frac{c}{x}\right)\left(-\tanh^{-1}\left(\frac{c}{x}\right)+\frac{c\tanh^{-1}\left(\frac{c}{x}\right)}{x}-2\log\left(1+e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)+\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)}{c}-\frac{b^3\left(\tanh^{-1}\left(\frac{c}{x}\right)^2\left(-\tanh^{-1}\left(\frac{c}{x}\right)+\frac{c\tanh^{-1}\left(\frac{c}{x}\right)}{x}-3\log\left(1+e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)+3\tanh^{-1}\left(\frac{c}{x}\right)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)+\frac{3}{2}\text{PolyLog}\left(3,-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])^3/x^2,x]
```

```
[Out] -(a^3/x) - (3*a^2*b*ArcTanh[c/x])/x + (3*a^2*b*Log[x])/c - (3*a^2*b*Log[-c^
2 + x^2])/(2*c) - (3*a*b^2*(ArcTanh[c/x]*(-ArcTanh[c/x] + (c*ArcTanh[c/x])/
x - 2*Log[1 + E^(-2*ArcTanh[c/x])]) + PolyLog[2, -E^(-2*ArcTanh[c/x])]))/c
- (b^3*(ArcTanh[c/x]^2*(-ArcTanh[c/x] + (c*ArcTanh[c/x])/x - 3*Log[1 + E^(-
```

```
2*ArcTanh[c/x])]) + 3*ArcTanh[c/x]*PolyLog[2, -E^(-2*ArcTanh[c/x])] + (3*Po
lyLog[3, -E^(-2*ArcTanh[c/x])])/2))/c
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(124) = 248$.
time = 1.40, size = 280, normalized size = 2.22

method	result
derivativedivides	$-\frac{\frac{c\,a^3}{x}+\frac{b^3 \operatorname{arctanh}\left(\frac{c}{x}\right)^3 c}{x}+b^3 \operatorname{arctanh}\left(\frac{c}{x}\right)^3-3 b^3 \operatorname{arctanh}\left(\frac{c}{x}\right)^2 \ln\left(1+\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right)-3 b^3 \operatorname{arctanh}\left(\frac{c}{x}\right) \operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right)+}{}$
default	$-\frac{\frac{c\,a^3}{x}+\frac{b^3 \operatorname{arctanh}\left(\frac{c}{x}\right)^3 c}{x}+b^3 \operatorname{arctanh}\left(\frac{c}{x}\right)^3-3 b^3 \operatorname{arctanh}\left(\frac{c}{x}\right)^2 \ln\left(1+\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right)-3 b^3 \operatorname{arctanh}\left(\frac{c}{x}\right) \operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right)+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c*(c/x*a^3+b^3*arctanh(c/x)^3*c/x+b^3*arctanh(c/x)^3-3*b^3*arctanh(c/x)^
2*ln(1+(1+c/x)^2/(1-c^2/x^2))-3*b^3*arctanh(c/x)*polylog(2,-(1+c/x)^2/(1-c^
2/x^2))+3/2*b^3*polylog(3,-(1+c/x)^2/(1-c^2/x^2))+3*arctanh(c/x)^2*a*b^2*c/
x+3*a*b^2*arctanh(c/x)^2-6*arctanh(c/x)*ln(1+(1+c/x)^2/(1-c^2/x^2))*a*b^2-3
*polylog(2,-(1+c/x)^2/(1-c^2/x^2))*a*b^2+3*a^2*b*c/x*arctanh(c/x)+3/2*a^2*b
*ln(1-c^2/x^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^2,x, algorithm="maxima")
```

```
[Out] -3/2*a^2*b*(2*c*arctanh(c/x)/x + log(-c^2/x^2 + 1))/c - a^3/x + 1/8*((b^3*c
 - b^3*x)*log(-c + x)^3 - 3*(2*a*b^2*c + (b^3*c + b^3*x)*log(c + x))*log(-c
 + x)^2)/(c*x) - integrate(-1/8*((b^3*c^2 - b^3*c*x)*log(c + x)^3 + 6*(a*b^
2*c^2 - a*b^2*c*x)*log(c + x)^2 - 3*(4*a*b^2*c*x + (b^3*c^2 - b^3*c*x)*log(
c + x)^2 + 2*(2*a*b^2*c^2 + b^3*x^2 - (2*a*b^2*c - b^3*c)*x)*log(c + x))*lo
g(-c + x))/(c^2*x^2 - c*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c/x)^3 + 3*a*b^2*arctanh(c/x)^2 + 3*a^2*b*arctanh(c/x
) + a^3)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x}\right)\right)^{3}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**3/x**2,x)
```

```
[Out] Integral((a + b*atanh(c/x))**3/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)}^3}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))^3/x^2,x)
```

```
[Out] int((a + b*atanh(c/x))^3/x^2, x)
```

3.156 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=139

$$-\frac{3b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}-\frac{3b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2cx}+\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{2x^2}+\frac{3b^2\left(a+b\coth\right.}{}$$

```
[Out] -3/2*b*(a+b*arccoth(x/c))^2/c^2-3/2*b*(a+b*arccoth(x/c))^2/c/x+1/2*(a+b*arc
coth(x/c))^3/c^2-1/2*(a+b*arccoth(x/c))^3/x^2+3*b^2*(a+b*arccoth(x/c))*ln(2
/(1-c/x))/c^2+3/2*b^3*polylog(2,1-2/(1-c/x))/c^2
```

Rubi [A]
time = 0.20, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {6039, 6037, 6127, 6021, 6131, 6055, 2449, 2352, 6095}

$$\frac{3b^2\log\left(\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)}{c^2}-\frac{3b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}+\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{2x^2}-\frac{3b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2cx}+\frac{3b^3\text{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])^3/x^3,x]
```

```
[Out] (-3*b*(a + b*ArcCoth[x/c])^2)/(2*c^2) - (3*b*(a + b*ArcCoth[x/c])^2)/(2*c*x
) + (a + b*ArcCoth[x/c])^3/(2*c^2) - (a + b*ArcCoth[x/c])^3/(2*x^2) + (3*b^
2*(a + b*ArcCoth[x/c])*Log[2/(1 - c/x)])/c^2 + (3*b^3*PolyLog[2, 1 - 2/(1 -
 c/x)])/(2*c^2)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8x^3} + \frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)}{8x^3} + \frac{3b^2\left(2a-b\,\mathrm{l}\right.}{}\right.$$

$$= \frac{1}{8}\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{x^3}\,dx + \frac{1}{8}(3b)\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)}{x^3}\,dx +$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int x(2a-b\log(1-cx))^3\,dx, x, \frac{1}{x}\right)\right) + \frac{1}{8}(3b)\int\left(\frac{4a^2\log\left(1+\frac{c}{x}\right)}{x^3}\right.$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int\left(\frac{(2a-b\log(1-cx))^3}{c} - \frac{(1-cx)(2a-b\log(1-cx))^3}{c}\right)dx, x,\right.\right.$$

$$= \frac{3ab^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)}{4x^2} - \frac{1}{2}\left(3a^2b\right)\text{Subst}\left(\int x\log(1+cx)\,dx, x, \frac{1}{x}\right) - \frac{1}{4}(3$$

$$= \frac{3ab^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)}{4x^2} - \frac{3a^2b\log\left(\frac{c+x}{x}\right)}{4x^2} - \frac{1}{4}\left(3ab^2\right)\text{Subst}\left(\int\left(-\frac{\log^2(1+}{c}\right.\right.$$

$$= \frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c^2} - \frac{\left(1-\frac{c}{x}\right)^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{16c^2} + \frac{3ab^2\log\left(1-\right.}{4}$$

$$= \frac{3a^2b}{8x^2} - \frac{3a^2b}{4cx} + \frac{3b\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{8c^2} - \frac{3b\left(1-\frac{c}{x}\right)^2\left(2a-b\log\left(1-\frac{c}{x}\right.\right.}{32c^2}$$

$$= -\frac{3b^3\left(1-\frac{c}{x}\right)^2}{64c^2} + \frac{3b^3\left(1+\frac{c}{x}\right)^2}{64c^2} + \frac{3a^2b}{8x^2} - \frac{3a^2b}{4cx} - \frac{3ab^2}{2cx} - \frac{3b^3}{4cx} - \frac{3b^2\left(1-\frac{c}{x}\right)^2(2a}{32c}$$

$$= -\frac{3b^3\left(1-\frac{c}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{c}{x}\right)^2}{16c^2} + \frac{3b^3\left(1+\frac{c}{x}\right)^2}{64c^2} + \frac{3a^2b}{8x^2} - \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} - \frac{3b^3\left(1-\right.}{}$$

$$= -\frac{3b^3\left(1-\frac{c}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{c}{x}\right)^2}{16c^2} + \frac{3b^3\left(1+\frac{c}{x}\right)^2}{64c^2} + \frac{3a^2b}{8x^2} - \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} - \frac{3ab^2\left(1-\right.}{}$$

$$= -\frac{3b^3\left(1-\frac{c}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{c}{x}\right)^2}{16c^2} + \frac{3b^3\left(1+\frac{c}{x}\right)^2}{64c^2} + \frac{3a^2b}{8x^2} + \frac{3ab^2}{8x^2} - \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} +$$

$$= -\frac{3b^3\left(1-\frac{c}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{c}{x}\right)^2}{16c^2} + \frac{3b^3\left(1+\frac{c}{x}\right)^2}{64c^2} + \frac{3a^2b}{8x^2} + \frac{3ab^2}{8x^2} - \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} +$$

$$= -\frac{3b^3\left(1-\frac{c}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{c}{x}\right)^2}{16c^2} + \frac{3b^3\left(1+\frac{c}{x}\right)^2}{64c^2} + \frac{3a^2b}{8x^2} + \frac{3ab^2}{8x^2} - \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} +$$

Mathematica [A]
time = 0.22, size = 195, normalized size = 1.40

$$\frac{6b^2(-c+x)(bx+a(c+x))\tanh^{-1}\left(\frac{c}{x}\right)^2 + 2b^3(-c^2+x^2)\tanh^{-1}\left(\frac{c}{x}\right)^3 + 6b\tanh^{-1}\left(\frac{c}{x}\right)\left(-ac(ac+2bx) + 2b^2x^2\log\left(1+e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right) + a\left(12b^2x^2\log\left(\frac{1}{\sqrt{1-\frac{c^2}{x^2}}}\right) - a\left(2ac^2 + 6bcx + 3bx^2\log\left(1-\frac{c}{x}\right) - 3bx^2\log\left(\frac{c+x}{x}\right)\right)\right) - 6b^3x^2\text{PolyLog}\left(2, -e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)}{4c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])^3/x^3,x]
```

```
[Out] (6*b^2*(-c + x)*(b*x + a*(c + x))*ArcTanh[c/x]^2 + 2*b^3*(-c^2 + x^2)*ArcTa
nh[c/x]^3 + 6*b*ArcTanh[c/x]*(-(a*c*(a*c + 2*b*x)) + 2*b^2*x^2*Log[1 + E^(-
2*ArcTanh[c/x])]) + a*(12*b^2*x^2*Log[1/Sqrt[1 - c^2/x^2]] - a*(2*a*c^2 + 6
*b*c*x + 3*b*x^2*Log[1 - c/x] - 3*b*x^2*Log[(c + x)/x])) - 6*b^3*x^2*PolyLo
g[2, -E^(-2*ArcTanh[c/x])])/(4*c^2*x^2)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.17, size = 6380, normalized size = 45.90

method	result	size
derivativedivides	Expression too large to display	6380
default	Expression too large to display	6380

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*(c*(log(c + x)/c^3 - log(-c + x)/c^3 - 2/(c^2*x)) - 2*arctanh(c/x)/x^2)
*a^2*b - 3/8*(c^2*((log(c + x)^2 - 2*(log(c + x) - 2)*log(-c + x) + log(-c
+ x)^2 + 4*log(c + x))/c^4 - 8*log(x)/c^4) - 4*c*(log(c + x)/c^3 - log(-c +
 x)/c^3 - 2/(c^2*x))*arctanh(c/x))*a*b^2 + 1/64*(32*c^4*integrate(-1/4*log(
x)^3/(c^4*x^3 - c^2*x^5), x) - 3*c^3*(log(c + x)/c^5 - log(-c + x)/c^5 - 2/
(c^4*x)) + 48*c^3*integrate(-1/4*x*log(x)^2/(c^4*x^3 - c^2*x^5), x) + 48*c^
3*integrate(-1/4*x*log(x)/(c^4*x^3 - c^2*x^5), x) - 6*c*(2*log(-c + x)/c^3
- 2*log(x)/c^3 + (c + 2*x)/(c^2*x^2))*log(-c/x + 1)^2 + 21*c^2*(log(c + x)/
c^4 + log(-c + x)/c^4 - 2*log(x)/c^4) - 32*c^2*integrate(-1/4*x^2*log(x)^3/
(c^4*x^3 - c^2*x^5), x) + 48*c^2*integrate(-1/4*x^2*log(x)^2/(c^4*x^3 - c^2
*x^5), x) - 384*c^2*integrate(-1/4*x^2*log(c + x)/(c^4*x^3 - c^2*x^5), x) +
 144*c^2*integrate(-1/4*x^2*log(x)/(c^4*x^3 - c^2*x^5), x) - 18*c*(log(c +
x)/c^3 - log(-c + x)/c^3) + c*(6*(2*x^2*log(-c + x)^2 + 2*x^2*log(x)^2 - 6*
x^2*log(x) + c^2 + 6*c*x - 2*(2*x^2*log(x) - 3*x^2)*log(-c + x))*log(-c/x +
 1)/(c^3*x^2) - (4*x^2*log(-c + x)^3 - 4*x^2*log(x)^3 + 18*x^2*log(x)^2 - 6
*(2*x^2*log(x) - 3*x^2)*log(-c + x)^2 - 42*x^2*log(x) + 3*c^2 + 42*c*x + 6*
```

```
(2*x^2*log(x)^2 - 6*x^2*log(x) + 7*x^2)*log(-c + x))/(c^3*x^2)) - 48*c*inte
grate(-1/4*x^3*log(x)^2/(c^4*x^3 - c^2*x^5), x) - 192*c*integrate(-1/4*x^3*
log(c + x)/(c^4*x^3 - c^2*x^5), x) + 336*c*integrate(-1/4*x^3*log(x)/(c^4*x
^3 - c^2*x^5), x) + 4*log(-c/x + 1)^3/x^2 - 2*(12*c*x*log(c + x)^2 + 2*(c^2
 - x^2)*log(c + x)^3 - 3*(c^2 - 2*c*x + x^2 - 2*(c^2 - x^2)*log(c + x) + 2*
(c^2 - x^2)*log(x))*log(-c + x)^2 - 3*(2*(c^2 - x^2)*log(c + x)^2 - 2*(c^2
- x^2)*log(x)^2 - c^2 - 6*c*x + 8*(c*x + x^2)*log(c + x) - 2*(c^2 + 2*c*x +
 5*x^2)*log(x))*log(-c + x))/(c^2*x^2) - 48*integrate(-1/4*x^4*log(x)^2/(c^
4*x^3 - c^2*x^5), x) - 192*integrate(-1/4*x^4*log(c + x)/(c^4*x^3 - c^2*x^5
), x) + 240*integrate(-1/4*x^4*log(x)/(c^4*x^3 - c^2*x^5), x))*b^3 - 3/2*a*
b^2*arctanh(c/x)^2/x^2 - 1/2*a^3/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^3,x, algorithm="fricas")
[Out] integral((b^3*arctanh(c/x)^3 + 3*a*b^2*arctanh(c/x)^2 + 3*a^2*b*arctanh(c/x
) + a^3)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x}\right)\right)^{3}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**3/x**3,x)
[Out] Integral((a + b*atanh(c/x))**3/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^3,x, algorithm="giac")
[Out] integrate((b*arctanh(c/x) + a)^3/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)}^3}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))^3/x^3,x)
```

```
[Out] int((a + b*atanh(c/x))^3/x^3, x)
```

3.157 $\int x^7 \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)\,dx$

Optimal. Leaf size=54

$$\frac{1}{8}bc^3x^2+\frac{1}{24}bcx^6+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{8}bc^4\tanh^{-1}\left(\frac{x^2}{c}\right)$$

[Out] 1/8*b*c^3*x^2+1/24*b*c*x^6+1/8*x^8*(a+b*arctanh(c/x^2))-1/8*b*c^4*arctanh(x^2/c)

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6037, 269, 281, 308, 213}

$$\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{8}bc^4\tanh^{-1}\left(\frac{x^2}{c}\right)+\frac{1}{8}bc^3x^2+\frac{1}{24}bcx^6$$

Antiderivative was successfully verified.

[In] Int[x^7*(a + b*ArcTanh[c/x^2]),x]

[Out] (b*c^3*x^2)/8 + (b*c*x^6)/24 + (x^8*(a + b*ArcTanh[c/x^2]))/8 - (b*c^4*ArcTanh[x^2/c])/8

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)\,dx &= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4}(bc)\int\frac{x^5}{1-\frac{c^2}{x^4}}\,dx\\ &= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4}(bc)\int\frac{x^9}{-c^2+x^4}\,dx\\ &= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{8}(bc)\text{Subst}\left(\int\frac{x^4}{-c^2+x^2}\,dx,x,x^2\right)\\ &= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{8}(bc)\text{Subst}\left(\int\left(c^2+x^2+\frac{c^4}{-c^2+x^2}\right)\,dx,x,\right.\\ &= \frac{1}{8}bc^3x^2+\frac{1}{24}bcx^6+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{8}\left(bc^5\right)\text{Subst}\left(\int\frac{1}{-c^2+x}\right.\\ &= \frac{1}{8}bc^3x^2+\frac{1}{24}bcx^6+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{8}bc^4\tanh^{-1}\left(\frac{x^2}{c}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 73, normalized size = 1.35

$$\frac{1}{8}bc^3x^2+\frac{1}{24}bcx^6+\frac{ax^8}{8}+\frac{1}{8}bx^8\tanh^{-1}\left(\frac{c}{x^2}\right)+\frac{1}{16}bc^4\log\left(-c+x^2\right)-\frac{1}{16}bc^4\log\left(c+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c^3*x^2)/8 + (b*c*x^6)/24 + (a*x^8)/8 + (b*x^8*ArcTanh[c/x^2])/8 + (b*c^
4*Log[-c + x^2])/16 - (b*c^4*Log[c + x^2])/16
```

Maple [A]
time = 0.16, size = 64, normalized size = 1.19

method	result
derivativedivides	$\frac{x^8a}{8}+\frac{b\,x^8\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{8}+\frac{bc\,x^6}{24}+\frac{b\,c^3x^2}{8}-\frac{b\,c^4\ln\left(1+\frac{c}{x^2}\right)}{16}+\frac{b\,c^4\ln\left(\frac{c}{x^2}-1\right)}{16}$

default	$\frac{x^8 a}{8}+\frac{b\,x^8 \operatorname{arctanh}\left(\frac{c}{x^2}\right)}{8}+\frac{bc\,x^6}{24}+\frac{b\,c^3 x^2}{8}-\frac{b\,c^4 \ln\left(1+\frac{c}{x^2}\right)}{16}+\frac{b\,c^4 \ln\left(\frac{c}{x^2}-1\right)}{16}$
risch	$\frac{x^8 b \ln(x^2+c)}{16}-\frac{x^8 b \ln(-x^2+c)}{16}+\frac{i\pi b\,x^8 \operatorname{csgn}\left(\frac{i}{x^2}\right)\operatorname{csgn}\left(\frac{i(x^2+c)}{x^2}\right)^2}{32}-\frac{i\pi b\,x^8 \operatorname{csgn}\left(\frac{i}{x^2}\right)\operatorname{csgn}(i(x^2+c))\operatorname{csgn}\left(\frac{i(x^2+c)}{x^2}\right)}{32}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctanh(c/x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^8*a+1/8*b*x^8*arctanh(c/x^2)+1/24*b*c*x^6+1/8*b*c^3*x^2-1/16*b*c^4*ln
(1+c/x^2)+1/16*b*c^4*ln(c/x^2-1)
```

Maxima [A]
time = 0.26, size = 62, normalized size = 1.15

$$\frac{1}{8}\,ax^8+\frac{1}{48}\left(6\,x^8 \operatorname{artanh}\left(\frac{c}{x^2}\right)+\left(2\,x^6+6\,c^2x^2-3\,c^3\log\left(x^2+c\right)+3\,c^3\log\left(x^2-c\right)\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8 + 1/48*(6*x^8*arctanh(c/x^2) + (2*x^6 + 6*c^2*x^2 - 3*c^3*log(x^2
 + c) + 3*c^3*log(x^2 - c))*c)*b
```

Fricas [A]
time = 0.33, size = 53, normalized size = 0.98

$$\frac{1}{8}\,ax^8+\frac{1}{24}\,bcx^6+\frac{1}{8}\,bc^3x^2+\frac{1}{16}\left(bx^8-bc^4\right)\log\left(\frac{x^2+c}{x^2-c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] 1/8*a*x^8 + 1/24*b*c*x^6 + 1/8*b*c^3*x^2 + 1/16*(b*x^8 - b*c^4)*log((x^2 +
c)/(x^2 - c))
```

Sympy [A]
time = 3.10, size = 51, normalized size = 0.94

$$\frac{ax^8}{8}-\frac{bc^4\operatorname{atanh}\left(\frac{c}{x^2}\right)}{8}+\frac{bc^3x^2}{8}+\frac{bcx^6}{24}+\frac{bx^8\operatorname{atanh}\left(\frac{c}{x^2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atanh(c/x**2)),x)
```

```
[Out] a*x**8/8 - b*c**4*atanh(c/x**2)/8 + b*c**3*x**2/8 + b*c*x**6/24 + b*x**8*at
anh(c/x**2)/8
```

Giac [A]
time = 0.44, size = 71, normalized size = 1.31

$$\frac{1}{16}bx^8\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{8}ax^8+\frac{1}{24}bcx^6+\frac{1}{8}bc^3x^2-\frac{1}{16}bc^4\log\left(x^2+c\right)+\frac{1}{16}bc^4\log\left(-x^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] 1/16*b*x^8*log((x^2 + c)/(x^2 - c)) + 1/8*a*x^8 + 1/24*b*c*x^6 + 1/8*b*c^3*
x^2 - 1/16*b*c^4*log(x^2 + c) + 1/16*b*c^4*log(-x^2 + c)
```

Mupad [B]
time = 1.01, size = 66, normalized size = 1.22

$$\frac{a\,x^8}{8}+\frac{b\,c^3\,x^2}{8}+\frac{b\,x^8\,\ln\left(x^2+c\right)}{16}+\frac{b\,c\,x^6}{24}-\frac{b\,x^8\,\ln\left(x^2-c\right)}{16}+\frac{b\,c^4\,\mathrm{atan}\left(\frac{x^2\,1{}\mathrm{i}}{c}\right)\,1{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*atanh(c/x^2)),x)
```

```
[Out] (a*x^8)/8 + (b*c^3*x^2)/8 + (b*x^8*log(c + x^2))/16 + (b*c^4*atan((x^2*1i)/
c)*1i)/8 + (b*c*x^6)/24 - (b*x^8*log(x^2 - c))/16
```

3.158 $\int x^5 \left(a+b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) \, dx$

Optimal. Leaf size=45

$$\frac{1}{12}bcx^4+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{12}bc^3\log\left(c^2-x^4\right)$$

```
[Out] 1/12*b*c*x^4+1/6*x^6*(a+b*arctanh(c/x^2))+1/12*b*c^3*ln(-x^4+c^2)
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6037, 269, 272, 45}

$$\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{12}bc^3\log\left(c^2-x^4\right)+\frac{1}{12}bcx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c*x^4)/12 + (x^6*(a + b*ArcTanh[c/x^2]))/6 + (b*c^3*Log[c^2 - x^4])/12
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)\,dx &= \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{3}(bc)\int\frac{x^3}{1-\frac{c^2}{x^4}}\,dx\\ &= \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{3}(bc)\int\frac{x^7}{-c^2+x^4}\,dx\\ &= \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{12}(bc)\text{Subst}\left(\int\frac{x}{-c^2+x}\,dx,x,x^4\right)\\ &= \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{12}(bc)\text{Subst}\left(\int\left(1-\frac{c^2}{c^2-x}\right)\,dx,x,x^4\right)\\ &= \frac{1}{12}bcx^4+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{12}bc^3\log\left(c^2-x^4\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.11

$$\frac{1}{12}bcx^4+\frac{ax^6}{6}+\frac{1}{6}bx^6\tanh^{-1}\left(\frac{c}{x^2}\right)+\frac{1}{12}bc^3\log\left(-c^2+x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c*x^4)/12 + (a*x^6)/6 + (b*x^6*ArcTanh[c/x^2])/6 + (b*c^3*Log[-c^2 + x^4
])/12
```

Maple [A]
time = 0.39, size = 65, normalized size = 1.44

method	result
derivativedivides	$\frac{x^6 a}{6}+\frac{b\,x^6\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{6}+\frac{bc\,x^4}{12}-\frac{b\,c^3\ln\left(\frac{1}{x}\right)}{3}+\frac{b\,c^3\ln\left(1+\frac{c}{x^2}\right)}{12}+\frac{b\,c^3\ln\left(\frac{c}{x^2}-1\right)}{12}$
default	$\frac{x^6 a}{6}+\frac{b\,x^6\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{6}+\frac{bc\,x^4}{12}-\frac{b\,c^3\ln\left(\frac{1}{x}\right)}{3}+\frac{b\,c^3\ln\left(1+\frac{c}{x^2}\right)}{12}+\frac{b\,c^3\ln\left(\frac{c}{x^2}-1\right)}{12}$
risch	$\frac{x^6 b\ln\left(x^2+c\right)}{12}-\frac{x^6 b\ln\left(-x^2+c\right)}{12}-\frac{i\pi b\,x^6\mathrm{csgn}\left(\frac{i\left(-x^2+c\right)}{x^2}\right)^3}{24}+\frac{i\pi b\,x^6\mathrm{csgn}\left(i\left(x^2+c\right)\right)\mathrm{csgn}\left(\frac{i\left(x^2+c\right)}{x^2}\right)^2}{24}-\frac{i\pi b\,x^6\mathrm{c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c/x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^6*a+1/6*b*x^6*arctanh(c/x^2)+1/12*b*c*x^4-1/3*b*c^3*ln(1/x)+1/12*b*c^
3*ln(1+c/x^2)+1/12*b*c^3*ln(c/x^2-1)
```

Maxima [A]
time = 0.25, size = 42, normalized size = 0.93

$$\frac{1}{6}\,ax^{6}+\frac{1}{12}\left(2\,x^{6}\,\mathrm{artanh}\left(\frac{c}{x^{2}}\right)+\left(x^{4}+c^{2}\log\left(x^{4}-c^{2}\right)\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/12*(2*x^6*arctanh(c/x^2) + (x^4 + c^2*log(x^4 - c^2))*c)*b
```

Fricas [A]
time = 0.34, size = 52, normalized size = 1.16

$$\frac{1}{12}\,bx^{6}\log\left(\frac{x^{2}+c}{x^{2}-c}\right)+\frac{1}{6}\,ax^{6}+\frac{1}{12}\,bcx^{4}+\frac{1}{12}\,bc^{3}\log\left(x^{4}-c^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] 1/12*b*x^6*log((x^2 + c)/(x^2 - c)) + 1/6*a*x^6 + 1/12*b*c*x^4 + 1/12*b*c^3
*log(x^4 - c^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(37) = 74$.
time = 2.26, size = 75, normalized size = 1.67

$$\frac{ax^{6}}{6}+\frac{bc^{3}\log\left(x-\sqrt{-c}\right)}{6}+\frac{bc^{3}\log\left(x+\sqrt{-c}\right)}{6}-\frac{bc^{3}\,\mathrm{atanh}\left(\frac{c}{x^{2}}\right)}{6}+\frac{bcx^{4}}{12}+\frac{bx^{6}\,\mathrm{atanh}\left(\frac{c}{x^{2}}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c/x**2)),x)
```

```
[Out] a*x**6/6 + b*c**3*log(x - sqrt(-c))/6 + b*c**3*log(x + sqrt(-c))/6 - b*c**3
*atanh(c/x**2)/6 + b*c*x**4/12 + b*x**6*atanh(c/x**2)/6
```

Giac [A]
time = 0.42, size = 52, normalized size = 1.16

$$\frac{1}{12}\,bx^{6}\log\left(\frac{x^{2}+c}{x^{2}-c}\right)+\frac{1}{6}\,ax^{6}+\frac{1}{12}\,bcx^{4}+\frac{1}{12}\,bc^{3}\log\left(x^{4}-c^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] 1/12*b*x^6*log((x^2 + c)/(x^2 - c)) + 1/6*a*x^6 + 1/12*b*c*x^4 + 1/12*b*c^3
*log(x^4 - c^2)
```

Mupad [B]
time = 0.81, size = 56, normalized size = 1.24

$$\frac{a\,x^6}{6}+\frac{b\,c^3\,\ln\left(x^4-c^2\right)}{12}+\frac{b\,x^6\,\ln\left(x^2+c\right)}{12}+\frac{b\,c\,x^4}{12}-\frac{b\,x^6\,\ln\left(x^2-c\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atanh(c/x^2)),x)
```

```
[Out] (a*x^6)/6 + (b*c^3*log(x^4 - c^2))/12 + (b*x^6*log(c + x^2))/12 + (b*c*x^4)
/12 - (b*x^6*log(x^2 - c))/12
```

3.159 $\int x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)\,dx$

Optimal. Leaf size=43

$$\frac{1}{4}bcx^2+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{4}bc^2\tanh^{-1}\left(\frac{x^2}{c}\right)$$

```
[Out] 1/4*b*c*x^2+1/4*x^4*(a+b*arctanh(c/x^2))-1/4*b*c^2*arctanh(x^2/c)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6037, 269, 281, 327, 213}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{4}bc^2\tanh^{-1}\left(\frac{x^2}{c}\right)+\frac{1}{4}bcx^2$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c*x^2)/4 + (x^4*(a + b*ArcTanh[c/x^2]))/4 - (b*c^2*ArcTanh[x^2/c])/4
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)\,dx &= \frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{2}(bc)\int\frac{x}{1-\frac{c^2}{x^4}}\,dx\\
&= \frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{2}(bc)\int\frac{x^5}{-c^2+x^4}\,dx\\
&= \frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4}(bc)\text{Subst}\left(\int\frac{x^2}{-c^2+x^2}\,dx,x,x^2\right)\\
&= \frac{1}{4}bcx^2+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4}\left(bc^3\right)\text{Subst}\left(\int\frac{1}{-c^2+x^2}\,dx,x,x^2\right)\\
&= \frac{1}{4}bcx^2+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{4}bc^2\tanh^{-1}\left(\frac{x^2}{c}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 62, normalized size = 1.44

$$\frac{1}{4}bcx^2+\frac{ax^4}{4}+\frac{1}{4}bx^4\tanh^{-1}\left(\frac{c}{x^2}\right)+\frac{1}{8}bc^2\log\left(-c+x^2\right)-\frac{1}{8}bc^2\log\left(c+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c*x^2)/4 + (a*x^4)/4 + (b*x^4*ArcTanh[c/x^2])/4 + (b*c^2*Log[-c + x^2])/
8 - (b*c^2*Log[c + x^2])/8
```

Maple [A]
time = 0.21, size = 55, normalized size = 1.28

method	result	size
derivativedivides	$\frac{x^4 a}{4}+\frac{\operatorname{arctanh}\left(\frac{c}{x^2}\right) b\,x^4}{4}-\frac{b\,c^2\ln\left(1+\frac{c}{x^2}\right)}{8}+\frac{b\,c^2\ln\left(\frac{c}{x^2}-1\right)}{8}+\frac{bc\,x^2}{4}$	55
default	$\frac{x^4 a}{4}+\frac{\operatorname{arctanh}\left(\frac{c}{x^2}\right) b\,x^4}{4}-\frac{b\,c^2\ln\left(1+\frac{c}{x^2}\right)}{8}+\frac{b\,c^2\ln\left(\frac{c}{x^2}-1\right)}{8}+\frac{bc\,x^2}{4}$	55

risch	Expression too large to display	4322

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c/x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*a+1/4*arctanh(c/x^2)*b*x^4-1/8*b*c^2*ln(1+c/x^2)+1/8*b*c^2*ln(c/x^2
-1)+1/4*b*c*x^2
```

Maxima [A]
time = 0.26, size = 49, normalized size = 1.14

$$\frac{1}{4}\,ax^4+\frac{1}{8}\left(2\,x^4\,\mathrm{artanh}\left(\frac{c}{x^2}\right)+\left(2\,x^2-c\log\left(x^2+c\right)+c\log\left(x^2-c\right)\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/8*(2*x^4*arctanh(c/x^2) + (2*x^2 - c*log(x^2 + c) + c*log(x^2
 - c))*c)*b
```

Fricas [A]
time = 0.34, size = 44, normalized size = 1.02

$$\frac{1}{4}\,ax^4+\frac{1}{4}\,bcx^2+\frac{1}{8}\left(bx^4-bc^2\right)\log\left(\frac{x^2+c}{x^2-c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] 1/4*a*x^4 + 1/4*b*c*x^2 + 1/8*(b*x^4 - b*c^2)*log((x^2 + c)/(x^2 - c))
```

Sympy [A]
time = 1.68, size = 41, normalized size = 0.95

$$\frac{ax^4}{4}-\frac{bc^2\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{4}+\frac{bcx^2}{4}+\frac{bx^4\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c/x**2)),x)
```

```
[Out] a*x**4/4 - b*c**2*atanh(c/x**2)/4 + b*c*x**2/4 + b*x**4*atanh(c/x**2)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(37) = 74.
time = 0.43, size = 162, normalized size = 3.77

$$\frac{\frac{(x^2+c)bc^3\log\left(\frac{x^2+c}{x^2-c}\right)}{(x^2-c)\left(\frac{\left(x^2+c\right)^2}{\left(x^2-c\right)^2}-\frac{2\left(x^2+c\right)}{x^2-c}+1\right)}+\frac{\frac{2\left(x^2+c\right)ac^3}{x^2-c}+\frac{\left(x^2+c\right)bc^3}{x^2-c}-bc^3}{\frac{\left(x^2+c\right)^2}{\left(x^2-c\right)^2}-\frac{2\left(x^2+c\right)}{x^2-c}+1}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] 1/2*((x^2 + c)*b*c^3*log((x^2 + c)/(x^2 - c))/((x^2 - c)*((x^2 + c)^2/(x^2
- c)^2 - 2*(x^2 + c)/(x^2 - c) + 1)) + (2*(x^2 + c)*a*c^3/(x^2 - c) + (x^2
+ c)*b*c^3/(x^2 - c) - b*c^3)/((x^2 + c)^2/(x^2 - c)^2 - 2*(x^2 + c)/(x^2 -
 c) + 1))/c
```

Mupad [B]
time = 0.86, size = 57, normalized size = 1.33

$$\frac{a\,x^4}{4}+\frac{b\,x^4\,\ln\left(x^2+c\right)}{8}+\frac{b\,c\,x^2}{4}-\frac{b\,x^4\,\ln\left(x^2-c\right)}{8}+\frac{b\,c^2\,\mathrm{atan}\left(\frac{x^2\,1{}\mathrm{i}}{c}\right)\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c/x^2)),x)
```

```
[Out] (a*x^4)/4 + (b*x^4*log(c + x^2))/8 + (b*c^2*atan((x^2*1i)/c)*1i)/4 + (b*c*x
^2)/4 - (b*x^4*log(x^2 - c))/8
```

3.160 $\int x\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)\,dx$

Optimal. Leaf size=34

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4}bc\log\left(c^2-x^4\right)$$

```
[Out] 1/2*x^2*(a+b*arctanh(c/x^2))+1/4*b*c*ln(-x^4+c^2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6037, 269, 266}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4}bc\log\left(c^2-x^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c/x^2]),x]
[Out] (x^2*(a + b*ArcTanh[c/x^2]))/2 + (b*c*Log[c^2 - x^4])/4
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)\,dx &= \frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+(bc)\int\frac{1}{\left(1-\frac{c^2}{x^4}\right)x}\,dx\\ &= \frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+(bc)\int\frac{x^3}{-c^2+x^4}\,dx\\ &= \frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4}bc\log\left(c^2-x^4\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.15

$$\frac{ax^2}{2}+\frac{1}{2}bx^2\tanh^{-1}\left(\frac{c}{x^2}\right)+\frac{1}{4}bc\log\left(-c^2+x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*ArcTanh[c/x^2])/2 + (b*c*Log[-c^2 + x^4])/4
```

Maple [A]
time = 0.10, size = 52, normalized size = 1.53

method	result
derivativedivides	$\frac{a\,x^2}{2}+\frac{b\,x^2\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{2}-bc\ln\left(\frac{1}{x}\right)+\frac{bc\ln\left(1+\frac{c}{x^2}\right)}{4}+\frac{bc\ln\left(\frac{c}{x^2}-1\right)}{4}$
default	$\frac{a\,x^2}{2}+\frac{b\,x^2\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{2}-bc\ln\left(\frac{1}{x}\right)+\frac{bc\ln\left(1+\frac{c}{x^2}\right)}{4}+\frac{bc\ln\left(\frac{c}{x^2}-1\right)}{4}$
risch	$\frac{b\,x^2\ln\left(x^2+c\right)}{4}-\frac{b\,x^2\ln\left(-x^2+c\right)}{4}+\frac{i\pi b\,x^2\mathrm{csgn}\left(\frac{i}{x^2}\right)\mathrm{csgn}\left(i\left(-x^2+c\right)\right)\mathrm{csgn}\left(\frac{i\left(-x^2+c\right)}{x^2}\right)}{8}+\frac{i\pi b\,x^2\mathrm{csgn}\left(i\left(x^2+c\right)\right)\mathrm{csg}}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c/x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctanh(c/x^2)-b*c*ln(1/x)+1/4*b*c*ln(1+c/x^2)+1/4*b*c*
ln(c/x^2-1)
```

Maxima [A]
time = 0.25, size = 34, normalized size = 1.00

$$\frac{1}{2}\,ax^2+\frac{1}{4}\left(2\,x^2\,\mathrm{artanh}\left(\frac{c}{x^2}\right)+c\log\left(x^4-c^2\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(2*x^2*arctanh(c/x^2) + c*log(x^4 - c^2))*b
```

Fricas [A]
time = 0.34, size = 43, normalized size = 1.26

$$\frac{1}{4}\,bx^2\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{2}\,ax^2+\frac{1}{4}\,bc\log\left(x^4-c^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] 1/4*b*x^2*log((x^2 + c)/(x^2 - c)) + 1/2*a*x^2 + 1/4*b*c*log(x^4 - c^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(27) = 54$.
time = 1.35, size = 61, normalized size = 1.79

$$\frac{ax^2}{2} + \frac{bc\log\left(x - \sqrt{-c}\right)}{2} + \frac{bc\log\left(x + \sqrt{-c}\right)}{2} - \frac{bc\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2} + \frac{bx^2\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c/x**2)),x)
```

```
[Out] a*x**2/2 + b*c*log(x - sqrt(-c))/2 + b*c*log(x + sqrt(-c))/2 - b*c*atanh(c/
x**2)/2 + b*x**2*atanh(c/x**2)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(30) = 60$.
time = 0.43, size = 184, normalized size = 5.41

$$\frac{1}{2}\,ax^2 + \frac{\left(c^2\left(\log\left(\frac{|-x^2-c|}{|-x^2+c|}\right) - \log\left(\left|\frac{x^2+c}{x^2-c} - 1\right|\right)\right) + \frac{c^2\log\left(-\frac{\frac{c\left(\frac{x^2+c}{(x^2-c)c} - \frac{1}{c}\right)}{\frac{x^2+c}{x^2-c}+1} + 1}{\frac{c\left(\frac{x^2+c}{(x^2-c)c} - \frac{1}{c}\right)}{\frac{x^2+c}{x^2-c}+1} - 1}\right)}{\frac{x^2+c}{x^2-c} - 1}\right)b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] 1/2*a*x^2 + 1/2*(c^2*(log(abs(-x^2 - c)/abs(-x^2 + c)) - log(abs((x^2 + c)/
(x^2 - c) - 1))) + c^2*log(-(c*((x^2 + c)/((x^2 - c)*c) - 1/c)/((x^2 + c)/(
x^2 - c) + 1) + 1)/(c*((x^2 + c)/((x^2 - c)*c) - 1/c)/((x^2 + c)/(x^2 - c)
+ 1) - 1))/((x^2 + c)/(x^2 - c) - 1))*b/c
```

Mupad [B]
time = 0.79, size = 47, normalized size = 1.38

$$\frac{a\,x^2}{2} + \frac{b\,x^2\,\ln\left(x^2+c\right)}{4} + \frac{b\,c\,\ln\left(x^4-c^2\right)}{4} - \frac{b\,x^2\,\ln\left(x^2-c\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c/x^2)),x)
```

```
[Out] (a*x^2)/2 + (b*x^2*log(c + x^2))/4 + (b*c*log(x^4 - c^2))/4 - (b*x^2*log(x^
2 - c))/4
```

3.161 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}\,dx$

Optimal. Leaf size=30

$$a\log(x)+\frac{1}{4}b\text{PolyLog}\left(2,-\frac{c}{x^2}\right)-\frac{1}{4}b\text{PolyLog}\left(2,\frac{c}{x^2}\right)$$

```
[Out] a*ln(x)+1/4*b*polylog(2,-c/x^2)-1/4*b*polylog(2,c/x^2)
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6035, 6031}

$$a\log(x)+\frac{1}{4}b\text{Li}_2\left(-\frac{c}{x^2}\right)-\frac{1}{4}b\text{Li}_2\left(\frac{c}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x,x]
```

```
[Out] a*Log[x] + (b*PolyLog[2, -(c/x^2)])/4 - (b*PolyLog[2, c/x^2])/4
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx,x,\frac{1}{x^2}\right)\right)\\ &= a\log(x)+\frac{1}{4}b\text{Li}_2\left(-\frac{c}{x^2}\right)-\frac{1}{4}b\text{Li}_2\left(\frac{c}{x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.93

$$a\log(x)+\frac{1}{4}b\left(\text{PolyLog}\left(2,-\frac{c}{x^2}\right)-\text{PolyLog}\left(2,\frac{c}{x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x,x]
```

```
[Out] a*Log[x] + (b*(PolyLog[2, -(c/x^2)] - PolyLog[2, c/x^2]))/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(26) = 52$.
time = 0.13, size = 154, normalized size = 5.13

method	result
derivativedivides	$-a\ln\left(\frac{1}{x}\right)-b\ln\left(\frac{1}{x}\right)\operatorname{arctanh}\left(\frac{c}{x^2}\right)+\frac{b\ln\left(\frac{1}{x}\right)\ln\left(1+\frac{\sqrt{-c}}{x}\right)}{2}+\frac{b\ln\left(\frac{1}{x}\right)\ln\left(1-\frac{\sqrt{-c}}{x}\right)}{2}+\frac{b\,\mathrm{dilog}\left(1\right.}{}$
default	$-a\ln\left(\frac{1}{x}\right)-b\ln\left(\frac{1}{x}\right)\operatorname{arctanh}\left(\frac{c}{x^2}\right)+\frac{b\ln\left(\frac{1}{x}\right)\ln\left(1+\frac{\sqrt{-c}}{x}\right)}{2}+\frac{b\ln\left(\frac{1}{x}\right)\ln\left(1-\frac{\sqrt{-c}}{x}\right)}{2}+\frac{b\,\mathrm{dilog}\left(1\right.}{}$
risch	$\frac{b\ln(x)\ln(x^2+c)}{2}-\frac{b\ln(x)\ln(-x^2+c)}{2}-\frac{i\pi\ln(x)b\,\mathrm{csgn}\left(\frac{i(-x^2+c)}{x^2}\right)^3}{4}+\frac{i\pi\ln(x)b\,\mathrm{csgn}\left(\frac{i}{x^2}\right)\mathrm{csgn}\left(\frac{i(x^2+c)}{x^2}\right)^2}{4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] -a*ln(1/x)-b*ln(1/x)*arctanh(c/x^2)+1/2*b*ln(1/x)*ln(1+(-c)^(1/2)/x)+1/2*b*
ln(1/x)*ln(1-(-c)^(1/2)/x)+1/2*b*dilog(1+(-c)^(1/2)/x)+1/2*b*dilog(1-(-c)^(
1/2)/x)-1/2*b*ln(1/x)*ln(1-1/x*c^(1/2))-1/2*b*ln(1/x)*ln(1+1/x*c^(1/2))-1/2
*b*dilog(1-1/x*c^(1/2))-1/2*b*dilog(1+1/x*c^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(c/x^2 + 1) - log(-c/x^2 + 1))/x, x) + a*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c/x^2) + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x,x)
```

```
[Out] Integral((a + b*atanh(c/x**2))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))/x,x)
```

```
[Out] int((a + b*atanh(c/x^2))/x, x)
```

3.162 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^3} dx$

Optimal. Leaf size=37

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2}-\frac{b\log\left(1-\frac{c^2}{x^4}\right)}{4c}$$

```
[Out] 1/2*(-a-b*arctanh(c/x^2))/x^2-1/4*b*ln(1-c^2/x^4)/c
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6037, 266}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2}-\frac{b\log\left(1-\frac{c^2}{x^4}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x^3,x]
```

```
[Out] -1/2*(a + b*ArcTanh[c/x^2])/x^2 - (b*Log[1 - c^2/x^4])/(4*c)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^3}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2}-(bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^5}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2}-\frac{b\log\left(1-\frac{c^2}{x^4}\right)}{4c}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.14

$$-\frac{a}{2x^2}-\frac{b\tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2}-\frac{b\log\left(1-\frac{c^2}{x^4}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^3,x]
[Out] -1/2*a/x^2 - (b*ArcTanh[c/x^2])/(2*x^2) - (b*Log[1 - c^2/x^4])/(4*c)
```

Maple [A]
time = 0.10, size = 39, normalized size = 1.05

method	result
derivativedivides	$-\frac{\frac{ca}{x^2}+\frac{bc\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{x^2}+\frac{b\ln\left(1-\frac{c^2}{x^4}\right)}{2}}{2c}$
default	$-\frac{\frac{ca}{x^2}+\frac{bc\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{x^2}+\frac{b\ln\left(1-\frac{c^2}{x^4}\right)}{2}}{2c}$
risch	$-\frac{b\ln\left(x^2+c\right)}{4x^2}-\frac{-i\pi bc\,\mathrm{csgn}\left(i\left(-x^2+c\right)\right)\mathrm{csgn}\left(\frac{i\left(-x^2+c\right)}{x^2}\right)^2-i\pi bc\,\mathrm{csgn}\left(\frac{i}{x^2}\right)\mathrm{csgn}\left(i\left(x^2+c\right)\right)\mathrm{csgn}\left(\frac{i\left(x^2+c\right)}{x^2}\right)+2i\pi bc\mathrm{csgn}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2/c*(c/x^2*a+b*c/x^2*arctanh(c/x^2)+1/2*b*ln(1-c^2/x^4))
```

Maxima [A]
time = 0.25, size = 37, normalized size = 1.00

$$-\frac{b\left(\frac{2\,c\,\mathrm{artanh}\left(\frac{c}{x^2}\right)}{x^2}+\log\left(-\frac{c^2}{x^4}+1\right)\right)}{4\,c}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^3,x, algorithm="maxima")
[Out] -1/4*b*(2*c*arctanh(c/x^2)/x^2 + log(-c^2/x^4 + 1))/c - 1/2*a/x^2
```

Fricas [A]
time = 0.36, size = 55, normalized size = 1.49

$$-\frac{bx^2\log\left(x^4-c^2\right)-4\,bx^2\log\left(x\right)+bc\log\left(\frac{x^2+c}{x^2-c}\right)+2\,ac}{4\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*x^2*log(x^4 - c^2) - 4*b*x^2*log(x) + b*c*log((x^2 + c)/(x^2 - c))
+ 2*a*c)/(c*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(32) = 64$.
time = 4.29, size = 76, normalized size = 2.05

$$\begin{cases} -\frac{a}{2x^2} - \frac{b\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2x^2} + \frac{b\log(x)}{c} - \frac{b\log\left(x-\sqrt{-c}\right)}{2c} - \frac{b\log\left(x+\sqrt{-c}\right)}{2c} + \frac{b\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2c} & \text{for } c \neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) - b*atanh(c/x**2)/(2*x**2) + b*log(x)/c - b*log(x -
sqrt(-c))/(2*c) - b*log(x + sqrt(-c))/(2*c) + b*atanh(c/x**2)/(2*c), Ne(c,
0)), (-a/(2*x**2), True))
```

Giac [A]
time = 0.41, size = 52, normalized size = 1.41

$$-\frac{b\log\left(x^4-c^2\right)}{4\,c} + \frac{b\log\left(x\right)}{c} - \frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{4\,x^2} - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*b*log(x^4 - c^2)/c + b*log(x)/c - 1/4*b*log((x^2 + c)/(x^2 - c))/x^2 -
 1/2*a/x^2
```

Mupad [B]
time = 0.84, size = 56, normalized size = 1.51

$$\frac{b\,\ln\left(x\right)}{c} - \frac{b\,\ln\left(x^4-c^2\right)}{4\,c} - \frac{a}{2\,x^2} - \frac{b\,\ln\left(x^2+c\right)}{4\,x^2} + \frac{b\,\ln\left(x^2-c\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))/x^3,x)
```

```
[Out] (b*log(x))/c - (b*log(x^4 - c^2))/(4*c) - a/(2*x^2) - (b*log(c + x^2))/(4*x
^2) + (b*log(x^2 - c))/(4*x^2)
```

3.163 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^5}\,dx$

Optimal. Leaf size=45

$$-\frac{b}{4cx^2}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}+\frac{b\tanh^{-1}\left(\frac{x^2}{c}\right)}{4c^2}$$

```
[Out] -1/4*b/c/x^2+1/4*(-a-b*arctanh(c/x^2))/x^4+1/4*b*arctanh(x^2/c)/c^2
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6037, 269, 281, 331, 213}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}+\frac{b\tanh^{-1}\left(\frac{x^2}{c}\right)}{4c^2}-\frac{b}{4cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x^5,x]
```

```
[Out] -1/4*b/(c*x^2) - (a + b*ArcTanh[c/x^2])/(4*x^4) + (b*ArcTanh[x^2/c])/(4*c^2
)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^5}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{1}{2}(bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^7}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{1}{2}(bc)\int \frac{1}{x^3\left(-c^2+x^4\right)}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{1}{4}(bc)\text{Subst}\left(\int \frac{1}{x^2\left(-c^2+x^2\right)}\,dx,x,x^2\right)\\ &= -\frac{b}{4cx^2}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{b\text{Subst}\left(\int \frac{1}{-c^2+x^2}\,dx,x,x^2\right)}{4c}\\ &= -\frac{b}{4cx^2}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}+\frac{b\tanh^{-1}\left(\frac{x^2}{c}\right)}{4c^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 64, normalized size = 1.42

$$-\frac{a}{4x^4}-\frac{b}{4cx^2}-\frac{b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{b\log\left(-c+x^2\right)}{8c^2}+\frac{b\log\left(c+x^2\right)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^5,x]
```

```
[Out] -1/4*a/x^4 - b/(4*c*x^2) - (b*ArcTanh[c/x^2])/(4*x^4) - (b*Log[-c + x^2])/(
8*c^2) + (b*Log[c + x^2])/(8*c^2)
```

Maple [A]
time = 0.11, size = 57, normalized size = 1.27

method	result

derivativedivides	$-\frac{a}{4x^4}-\frac{b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{b}{4c\,x^2}-\frac{b\ln\left(\frac{c}{x^2}-1\right)}{8c^2}+\frac{b\ln\left(1+\frac{c}{x^2}\right)}{8c^2}$
default	$-\frac{a}{4x^4}-\frac{b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{b}{4c\,x^2}-\frac{b\ln\left(\frac{c}{x^2}-1\right)}{8c^2}+\frac{b\ln\left(1+\frac{c}{x^2}\right)}{8c^2}$
risch	$-\frac{b\ln(x^2+c)}{8x^4}-\frac{2i\pi b\,c^2\mathrm{csgn}\left(\frac{i(-x^2+c)}{x^2}\right)^2+i\pi b\,c^2\mathrm{csgn}\left(\frac{i}{x^2}\right)\mathrm{csgn}(i(-x^2+c))\mathrm{csgn}\left(\frac{i(-x^2+c)}{x^2}\right)-2i\pi b\,c^2-i\pi b\,c^2\mathrm{csgn}\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctanh(c/x^2)-1/4*b/c/x^2-1/8*b/c^2*ln(c/x^2-1)+1/8*b
/c^2*ln(1+c/x^2)
```

Maxima [A]
time = 0.26, size = 56, normalized size = 1.24

$$\frac{1}{8}\left(c\left(\frac{\log\left(x^2+c\right)}{c^3}-\frac{\log\left(x^2-c\right)}{c^3}-\frac{2}{c^2x^2}\right)-\frac{2\,\mathrm{artanh}\left(\frac{c}{x^2}\right)}{x^4}\right)b-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*(c*(log(x^2 + c)/c^3 - log(x^2 - c)/c^3 - 2/(c^2*x^2)) - 2*arctanh(c/x^
2)/x^4)*b - 1/4*a/x^4
```

Fricas [A]
time = 0.36, size = 52, normalized size = 1.16

$$-\frac{2\,bcx^2+2\,ac^2-(bx^4-bc^2)\log\left(\frac{x^2+c}{x^2-c}\right)}{8\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*c*x^2 + 2*a*c^2 - (b*x^4 - b*c^2)*log((x^2 + c)/(x^2 - c)))/(c^2*
x^4)
```

Sympy [A]
time = 5.82, size = 49, normalized size = 1.09

$$\begin{cases}-\frac{a}{4x^4}-\frac{b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{b}{4cx^2}+\frac{b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{4c^2} & \text{for}\ c\neq 0\\ -\frac{a}{4x^4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x**5,x)
```

```
[Out] Piecewise((-a/(4*x**4) - b*atanh(c/x**2)/(4*x**4) - b/(4*c*x**2) + b*atanh(
c/x**2)/(4*c**2), Ne(c, 0)), (-a/(4*x**4), True))
```

Giac [A]
time = 0.42, size = 66, normalized size = 1.47

$$\frac{b\log\left(x^2+c\right)}{8\,c^2}-\frac{b\log\left(-x^2+c\right)}{8\,c^2}-\frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{8\,x^4}-\frac{bx^2+ac}{4\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b*log(x^2 + c)/c^2 - 1/8*b*log(-x^2 + c)/c^2 - 1/8*b*log((x^2 + c)/(x^2
 - c))/x^4 - 1/4*(b*x^2 + a*c)/(c*x^4)
```

Mupad [B]
time = 1.00, size = 59, normalized size = 1.31

$$\frac{\frac{b\,x^4\,\mathrm{atanh}\left(\frac{x^2}{c}\right)}{4}-\frac{b\,c\,x^2}{4}}{c^2\,x^4}-\frac{\frac{a}{4}-\frac{b\,\ln\left(x^2-c\right)}{8}+\frac{b\,\ln\left(x^2+c\right)}{8}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))/x^5,x)
```

```
[Out] ((b*x^4*atanh(x^2/c))/4 - (b*c*x^2)/4)/(c^2*x^4) - (a/4 - (b*log(x^2 - c))/
8 + (b*log(c + x^2))/8)/x^4
```

3.164 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^7}\,dx$

Optimal. Leaf size=57

$$-\frac{b}{12cx^4}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2-x^4\right)}{12c^3}$$

```
[Out] -1/12*b/c/x^4+1/6*(-a-b*arctanh(c/x^2))/x^6+1/3*b*ln(x)/c^3-1/12*b*ln(-x^4+
c^2)/c^3
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6037, 269, 272, 46}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2-x^4\right)}{12c^3}-\frac{b}{12cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x^7,x]
```

```
[Out] -1/12*b/(c*x^4) - (a + b*ArcTanh[c/x^2])/(6*x^6) + (b*Log[x])/(3*c^3) - (b*
Log[c^2 - x^4])/(12*c^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^7}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6}-\frac{1}{3}(bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^9}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6}-\frac{1}{3}(bc)\int \frac{1}{x^5\left(-c^2+x^4\right)}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6}-\frac{1}{12}(bc)\text{Subst}\left(\int \frac{1}{x^2\left(-c^2+x\right)}\,dx,x,x^4\right)\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6}-\frac{1}{12}(bc)\text{Subst}\left(\int \left(-\frac{1}{c^4\left(c^2-x\right)}-\frac{1}{c^2x^2}-\frac{1}{c^4x}\right)dx,x,x^4\right)\\ &= -\frac{b}{12cx^4}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2-x^4\right)}{12c^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 62, normalized size = 1.09

$$-\frac{a}{6x^6}-\frac{b}{12cx^4}-\frac{b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(-c^2+x^4\right)}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^7,x]
```

```
[Out] -1/6*a/x^6 - b/(12*c*x^4) - (b*ArcTanh[c/x^2])/(6*x^6) + (b*Log[x])/(3*c^3)
 - (b*Log[-c^2 + x^4])/(12*c^3)
```

Maple [A]
time = 0.11, size = 45, normalized size = 0.79

method	result
derivativedivides	$-\frac{a}{6x^6}-\frac{b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{6x^6}-\frac{b}{12c\,x^4}-\frac{b\ln\left(\frac{c^2}{x^4}-1\right)}{12c^3}$
default	$-\frac{a}{6x^6}-\frac{b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{6x^6}-\frac{b}{12c\,x^4}-\frac{b\ln\left(\frac{c^2}{x^4}-1\right)}{12c^3}$
risch	$-\frac{b\ln\left(x^2+c\right)}{12x^6}-\frac{i\pi b\,c^3\mathrm{csgn}\left(\frac{i}{x^2}\right)\mathrm{csgn}\left(i\left(-x^2+c\right)\right)\mathrm{csgn}\left(\frac{i\left(-x^2+c\right)}{x^2}\right)-i\pi b\,c^3\mathrm{csgn}\left(\frac{i}{x^2}\right)\mathrm{csgn}\left(\frac{i\left(-x^2+c\right)}{x^2}\right)^2-2i\pi b\,c^3-i\pi}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a/x^6-1/6*b/x^6*arctanh(c/x^2)-1/12*b/c/x^4-1/12*b/c^3*ln(c^2/x^4-1)
```

Maxima [A]
time = 0.26, size = 55, normalized size = 0.96

$$-\frac{1}{12}\left(c\left(\frac{\log\left(x^4-c^2\right)}{c^4}-\frac{\log\left(x^4\right)}{c^4}+\frac{1}{c^2x^4}\right)+\frac{2\,\operatorname{artanh}\left(\frac{c}{x^2}\right)}{x^6}\right)b-\frac{a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*(c*(log(x^4 - c^2)/c^4 - log(x^4)/c^4 + 1/(c^2*x^4)) + 2*arctanh(c/x^
2)/x^6)*b - 1/6*a/x^6
```

Fricas [A]
time = 0.41, size = 67, normalized size = 1.18

$$-\frac{bx^6\log\left(x^4-c^2\right)-4\,bx^6\log\left(x\right)+bc^2x^2+bc^3\log\left(\frac{x^2+c}{x^2-c}\right)+2\,ac^3}{12\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(b*x^6*log(x^4 - c^2) - 4*b*x^6*log(x) + b*c^2*x^2 + b*c^3*log((x^2 +
 c)/(x^2 - c)) + 2*a*c^3)/(c^3*x^6)
```

Sympy [A]
time = 8.24, size = 94, normalized size = 1.65

$$\begin{cases}-\frac{a}{6x^6}-\frac{b\operatorname{atanh}\left(\frac{c}{x^2}\right)}{6x^6}-\frac{b}{12cx^4}+\frac{b\log\left(x\right)}{3c^3}-\frac{b\log\left(x-\sqrt{-c}\right)}{6c^3}-\frac{b\log\left(x+\sqrt{-c}\right)}{6c^3}+\frac{b\operatorname{atanh}\left(\frac{c}{x^2}\right)}{6c^3} & \text{for}\ c\neq 0\\ -\frac{a}{6x^6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x**7,x)
```

```
[Out] Piecewise((-a/(6*x**6) - b*atanh(c/x**2)/(6*x**6) - b/(12*c*x**4) + b*log(x
)/(3*c**3) - b*log(x - sqrt(-c))/(6*c**3) - b*log(x + sqrt(-c))/(6*c**3) +
b*atanh(c/x**2)/(6*c**3), Ne(c, 0)), (-a/(6*x**6), True))
```

Giac [A]
time = 0.41, size = 65, normalized size = 1.14

$$-\frac{b\log\left(x^4-c^2\right)}{12\,c^3}+\frac{b\log\left(x\right)}{3\,c^3}-\frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{12\,x^6}-\frac{bx^2+2\,ac}{12\,cx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^7,x, algorithm="giac")
```

```
[Out] -1/12*b*log(x^4 - c^2)/c^3 + 1/3*b*log(x)/c^3 - 1/12*b*log((x^2 + c)/(x^2 -
 c))/x^6 - 1/12*(b*x^2 + 2*a*c)/(c*x^6)
```

Mupad [B]
time = 0.89, size = 66, normalized size = 1.16

$$\frac{b\,\ln\left(x\right)}{3\,c^3}-\frac{b\,\ln\left(x^4-c^2\right)}{12\,c^3}-\frac{b}{12\,c\,x^4}-\frac{a}{6\,x^6}-\frac{b\,\ln\left(x^2+c\right)}{12\,x^6}+\frac{b\,\ln\left(x^2-c\right)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))/x^7,x)
```

```
[Out] (b*log(x))/(3*c^3) - (b*log(x^4 - c^2))/(12*c^3) - b/(12*c*x^4) - a/(6*x^6)
 - (b*log(c + x^2))/(12*x^6) + (b*log(x^2 - c))/(12*x^6)
```

3.165 $\int x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)\,dx$

Optimal. Leaf size=63

$$\frac{2}{15}bcx^3+\frac{1}{5}bc^{5/2}\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

```
[Out] 2/15*b*c*x^3+1/5*b*c^(5/2)*arctan(x/c^(1/2))+1/5*x^5*(a+b*arctanh(c/x^2))-1
/5*b*c^(5/2)*arctanh(x/c^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6037, 269, 327, 304, 209, 212}

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{5}bc^{5/2}\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{2}{15}bcx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (2*b*c*x^3)/15 + (b*c^(5/2)*ArcTan[x/Sqrt[c]])/5 + (x^5*(a + b*ArcTanh[c/x^
2]))/5 - (b*c^(5/2)*ArcTanh[x/Sqrt[c]])/5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
```

```
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)\,dx&=\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{5}(2bc)\int\frac{x^2}{1-\frac{c^2}{x^4}}\,dx\\&=\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{5}(2bc)\int\frac{x^6}{-c^2+x^4}\,dx\\&=\frac{2}{15}bcx^3+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{5}\left(2bc^3\right)\int\frac{x^2}{-c^2+x^4}\,dx\\&=\frac{2}{15}bcx^3+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{5}\left(bc^3\right)\int\frac{1}{c-x^2}\,dx+\frac{1}{5}\left(bc^3\right)\int\frac{1}{c+\cdots}\\&=\frac{2}{15}bcx^3+\frac{1}{5}bc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{5}bc^{5/2}\tanh^{\cdots}\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 88, normalized size = 1.40

$$\frac{2}{15}bcx^3+\frac{ax^5}{5}+\frac{1}{5}bc^{5/2}\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{5}bx^5\tanh^{-1}\left(\frac{c}{x^2}\right)+\frac{1}{10}bc^{5/2}\log\left(\sqrt{c}-x\right)-\frac{1}{10}bc^{5/2}\log\left(\sqrt{c}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (2*b*c*x^3)/15 + (a*x^5)/5 + (b*c^(5/2)*ArcTan[x/Sqrt[c]])/5 + (b*x^5*ArcTa
nh[c/x^2])/5 + (b*c^(5/2)*Log[Sqrt[c] - x])/10 - (b*c^(5/2)*Log[Sqrt[c] + x
])/10
```

Maple [A]
time = 0.23, size = 55, normalized size = 0.87

method	result
derivativedivides	$\frac{a\,x^{5}}{5}+\frac{x^{5} b \,\mathrm{arctanh}\left(\frac{c}{x^{2}}\right)}{5}-\frac{b\,c^{\frac{5}{2}} \arctan\left(\frac{\sqrt{c}}{x}\right)}{5}+\frac{2 b c\,x^{3}}{15}-\frac{b\,c^{\frac{5}{2}} \mathrm{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{5}$
default	$\frac{a\,x^{5}}{5}+\frac{x^{5} b \,\mathrm{arctanh}\left(\frac{c}{x^{2}}\right)}{5}-\frac{b\,c^{\frac{5}{2}} \arctan\left(\frac{\sqrt{c}}{x}\right)}{5}+\frac{2 b c\,x^{3}}{15}-\frac{b\,c^{\frac{5}{2}} \mathrm{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{5}$
risch	$\frac{x^{5} b \ln\left(x^{2}+c\right)}{10}-\frac{x^{5} b \ln\left(-x^{2}+c\right)}{10}+\frac{i \pi b\,x^{5} \mathrm{csgn}\left(\frac{i}{x^{2}}\right) \mathrm{csgn}\left(i \left(-x^{2}+c\right)\right) \mathrm{csgn}\left(\frac{i \left(-x^{2}+c\right)}{x^{2}}\right)}{20}-\frac{i \pi b\,x^{5} \mathrm{csgn}\left(\frac{i}{x^{2}}\right) \mathrm{csgn}\left(i (x^{2}\right.}{20}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c/x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*x^5+1/5*x^5*b*arctanh(c/x^2)-1/5*b*c^(5/2)*arctan(1/x*c^(1/2))+2/15*b
*c*x^3-1/5*b*c^(5/2)*arctanh(1/x*c^(1/2))
```

Maxima [A]
time = 0.46, size = 62, normalized size = 0.98

$$\frac{1}{5}\,ax^{5}+\frac{1}{30}\left(6\,x^{5}\,\mathrm{artanh}\left(\frac{c}{x^{2}}\right)+\left(4\,x^{3}+6\,c^{\frac{3}{2}}\arctan\left(\frac{x}{\sqrt{c}}\right)+3\,c^{\frac{3}{2}}\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/30*(6*x^5*arctanh(c/x^2) + (4*x^3 + 6*c^(3/2)*arctan(x/sqrt(c
)) + 3*c^(3/2)*log((x - sqrt(c))/(x + sqrt(c))))*c)*b
```

Fricas [A]
time = 0.35, size = 170, normalized size = 2.70

$$\left[\frac{1}{10}\,bx^{5}\log\left(\frac{x^{2}+c}{x^{2}-c}\right)+\frac{1}{5}\,ax^{5}+\frac{2}{15}\,bcx^{3}+\frac{1}{5}\,bc^{\frac{5}{2}}\arctan\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{10}\,bc^{\frac{5}{2}}\log\left(\frac{x^{2}-2\,\sqrt{c}\,x+c}{x^{2}-c}\right),\ \frac{1}{10}\,bx^{5}\log\left(\frac{x^{2}+c}{x^{2}-c}\right)+\frac{1}{5}\,ax^{5}+\frac{2}{15}\,bcx^{3}+\frac{1}{5}\,b\sqrt{-c}\,c^{2}\arctan\left(\frac{\sqrt{-c}\,x}{c}\right)+\frac{1}{10}\,b\sqrt{-c}\,c^{2}\log\left(\frac{x^{2}+2\,\sqrt{-c}\,x-c}{x^{2}+c}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] [1/10*b*x^5*log((x^2 + c)/(x^2 - c)) + 1/5*a*x^5 + 2/15*b*c*x^3 + 1/5*b*c^(
5/2)*arctan(x/sqrt(c)) + 1/10*b*c^(5/2)*log((x^2 - 2*sqrt(c)*x + c)/(x^2 -
c)), 1/10*b*x^5*log((x^2 + c)/(x^2 - c)) + 1/5*a*x^5 + 2/15*b*c*x^3 + 1/5*b
*sqrt(-c)*c^2*arctan(sqrt(-c)*x/c) + 1/10*b*sqrt(-c)*c^2*log((x^2 + 2*sqrt(
-c)*x - c)/(x^2 + c))]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 845 vs. $2(58) = 116$.

time = 3.78, size = 845, normalized size = 13.41

$$\begin{cases} \frac{ax^5}{5} & \text{for } c = 0 \\ \frac{x^5(a-\infty b)}{5} & \text{for } c = -x^2 \\ \frac{x^5(a+\infty b)}{5} & \text{for } c = x^2 \\ -\frac{6ac^2x^5\sqrt{-c}}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} + \frac{6ax^9\sqrt{-c}}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} - \frac{6bc^{\frac{9}{2}}\sqrt{-c}\log\left(-\sqrt{c}+x\right)}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} + \frac{3bc^{\frac{9}{2}}\sqrt{-c}\log\left(x-\sqrt{-c}\right)}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} + \frac{3bc^{\frac{9}{2}}\sqrt{-c}\log\left(x+\sqrt{-c}\right)}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} - \frac{6bc^{\frac{9}{2}}\sqrt{-c}\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} + \frac{6bc^{\frac{5}{2}}x^4\sqrt{-c}\log\left(-\sqrt{c}+x\right)}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} - \frac{3bc^{\frac{5}{2}}x^4\sqrt{-c}\log\left(x-\sqrt{-c}\right)}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} - \frac{3bc^{\frac{5}{2}}x^4\sqrt{-c}\log\left(x+\sqrt{-c}\right)}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} + \frac{6bc^{\frac{5}{2}}x^4\sqrt{-c}\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} - \frac{3bc^5\log\left(x-\sqrt{-c}\right)}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} + \frac{3bc^5\log\left(x+\sqrt{-c}\right)}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} + \frac{3bc^3x^4\log\left(x-\sqrt{-c}\right)}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} - \frac{3bc^3x^4\log\left(x+\sqrt{-c}\right)}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} - \frac{4bc^3x^3\sqrt{-c}}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} - \frac{6bc^2x^5\sqrt{-c}\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} + \frac{4bcx^7\sqrt{-c}}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} + \frac{6bx^9\sqrt{-c}\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c/x**2)),x)
```

```
[Out] Piecewise((a*x**5/5, Eq(c, 0)), (x**5*(a - oo*b)/5, Eq(c, -x**2)), (x**5*(a
 + oo*b)/5, Eq(c, x**2)), (-6*a*c**2*x**5*sqrt(-c)/(-30*c**2*sqrt(-c) + 30*
x**4*sqrt(-c)) + 6*a*x**9*sqrt(-c)/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) -
 6*b*c**(9/2)*sqrt(-c)*log(-sqrt(c) + x)/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(
-c)) + 3*b*c**(9/2)*sqrt(-c)*log(x - sqrt(-c))/(-30*c**2*sqrt(-c) + 30*x**4
*sqrt(-c)) + 3*b*c**(9/2)*sqrt(-c)*log(x + sqrt(-c))/(-30*c**2*sqrt(-c) + 3
0*x**4*sqrt(-c)) - 6*b*c**(9/2)*sqrt(-c)*atanh(c/x**2)/(-30*c**2*sqrt(-c) +
 30*x**4*sqrt(-c)) + 6*b*c**(5/2)*x**4*sqrt(-c)*log(-sqrt(c) + x)/(-30*c**2
*sqrt(-c) + 30*x**4*sqrt(-c)) - 3*b*c**(5/2)*x**4*sqrt(-c)*log(x - sqrt(-c)
)/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) - 3*b*c**(5/2)*x**4*sqrt(-c)*log(x
 + sqrt(-c))/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) + 6*b*c**(5/2)*x**4*sqr
t(-c)*atanh(c/x**2)/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) - 3*b*c**5*log(x
 - sqrt(-c))/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) + 3*b*c**5*log(x + sqrt
(-c))/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) + 3*b*c**3*x**4*log(x - sqrt(-
c))/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) - 3*b*c**3*x**4*log(x + sqrt(-c)
)/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) - 4*b*c**3*x**3*sqrt(-c)/(-30*c**2
*sqrt(-c) + 30*x**4*sqrt(-c)) - 6*b*c**2*x**5*sqrt(-c)*atanh(c/x**2)/(-30*c
**2*sqrt(-c) + 30*x**4*sqrt(-c)) + 4*b*c*x**7*sqrt(-c)/(-30*c**2*sqrt(-c) +
 30*x**4*sqrt(-c)) + 6*b*x**9*sqrt(-c)*atanh(c/x**2)/(-30*c**2*sqrt(-c) + 3
0*x**4*sqrt(-c)), True))
```

Giac [A]

time = 0.44, size = 67, normalized size = 1.06

$$\frac{1}{10}\,bx^5\log\left(\frac{x^2+c}{x^2-c}\right) + \frac{1}{5}\,ax^5 + \frac{2}{15}\,bcx^3 + \frac{bc^3\arctan\left(\frac{x}{\sqrt{-c}}\right)}{5\,\sqrt{-c}} + \frac{1}{5}\,bc^{\frac{5}{2}}\arctan\left(\frac{x}{\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] 1/10*b*x^5*log((x^2 + c)/(x^2 - c)) + 1/5*a*x^5 + 2/15*b*c*x^3 + 1/5*b*c^3*
arctan(x/sqrt(-c))/sqrt(-c) + 1/5*b*c^(5/2)*arctan(x/sqrt(c))
```

Mupad [B]

time = 0.98, size = 67, normalized size = 1.06

$$\frac{a\,x^5}{5}+\frac{b\,c^{5/2}\,\mathrm{atan}\left(\frac{x}{\sqrt{c}}\right)}{5}+\frac{b\,x^5\,\ln\left(x^2+c\right)}{10}+\frac{2\,b\,c\,x^3}{15}-\frac{b\,x^5\,\ln\left(x^2-c\right)}{10}+\frac{b\,c^{5/2}\,\mathrm{atan}\left(\frac{x\,1{}\mathrm{i}}{\sqrt{c}}\right)\,1{}\mathrm{i}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atanh(c/x^2)),x)
```

```
[Out] (a*x^5)/5 + (b*c^(5/2)*atan(x/c^(1/2)))/5 + (b*c^(5/2)*atan((x*1i)/c^(1/2))
*1i)/5 + (b*x^5*log(c + x^2))/10 + (2*b*c*x^3)/15 - (b*x^5*log(x^2 - c))/10
```

3.166 $\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)\,dx$

Optimal. Leaf size=61

$$\frac{2bcx}{3}-\frac{1}{3}bc^{3/2}\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{3}bc^{3/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

```
[Out] 2/3*b*c*x-1/3*b*c^(3/2)*arctan(x/c^(1/2))+1/3*x^3*(a+b*arctanh(c/x^2))-1/3*
b*c^(3/2)*arctanh(x/c^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.429$, Rules used = {6037, 199, 327, 218, 212, 209}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{3}bc^{3/2}\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{3}bc^{3/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{2bcx}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (2*b*c*x)/3 - (b*c^(3/2)*ArcTan[x/Sqrt[c]])/3 + (x^3*(a + b*ArcTanh[c/x^2])
)/3 - (b*c^(3/2)*ArcTanh[x/Sqrt[c]])/3
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)dx &= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{3}(2bc)\int\frac{1}{1-\frac{c^2}{x^4}}\,dx\\ &= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{3}(2bc)\int\frac{x^4}{-c^2+x^4}\,dx\\ &= \frac{2bcx}{3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{3}\left(2bc^3\right)\int\frac{1}{-c^2+x^4}\,dx\\ &= \frac{2bcx}{3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{3}\left(bc^2\right)\int\frac{1}{c-x^2}\,dx-\frac{1}{3}\left(bc^2\right)\int\frac{1}{c+x^2}\\ &= \frac{2bcx}{3}-\frac{1}{3}bc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{3}bc^{3/2}\tanh^{-1}\left(\right.\end{aligned}$$

Mathematica [A]
time = 0.01, size = 86, normalized size = 1.41

$$\frac{2bcx}{3}+\frac{ax^3}{3}-\frac{1}{3}bc^{3/2}\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{3}bx^3\tanh^{-1}\left(\frac{c}{x^2}\right)+\frac{1}{6}bc^{3/2}\log\left(\sqrt{c}-x\right)-\frac{1}{6}bc^{3/2}\log\left(\sqrt{c}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (2*b*c*x)/3 + (a*x^3)/3 - (b*c^(3/2)*ArcTan[x/Sqrt[c]])/3 + (b*x^3*ArcTanh[
c/x^2])/3 + (b*c^(3/2)*Log[Sqrt[c] - x])/6 - (b*c^(3/2)*Log[Sqrt[c] + x])/6
```

Maple [A]
time = 0.16, size = 53, normalized size = 0.87

method	result
derivativedivides	$\frac{x^3 a}{3}+\frac{x^3 b \operatorname{arctanh}\left(\frac{c}{x^2}\right)}{3}+\frac{2bcx}{3}+\frac{b\,c^{\frac{3}{2}} \arctan\left(\frac{\sqrt{c}}{x}\right)}{3}-\frac{b\,c^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{3}$
default	$\frac{x^3 a}{3}+\frac{x^3 b \operatorname{arctanh}\left(\frac{c}{x^2}\right)}{3}+\frac{2bcx}{3}+\frac{b\,c^{\frac{3}{2}} \arctan\left(\frac{\sqrt{c}}{x}\right)}{3}-\frac{b\,c^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{3}$
risch	$\frac{x^3 b \ln(x^2+c)}{6}-\frac{x^3 b \ln(-x^2+c)}{6}-\frac{i\pi b\,x^3 \operatorname{csgn}\left(\frac{i}{x^2}\right)\operatorname{csgn}\left(\frac{i(-x^2+c)}{x^2}\right)^2}{12}-\frac{i\pi b\,x^3 \operatorname{csgn}\left(\frac{i}{x^2}\right)\operatorname{csgn}(i(x^2+c))\operatorname{csgn}\left(\frac{i(x}{}\right.}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c/x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*a+1/3*x^3*b*arctanh(c/x^2)+2/3*b*c*x+1/3*b*c^(3/2)*arctan(1/x*c^(1/
2))-1/3*b*c^(3/2)*arctanh(1/x*c^(1/2))
```

Maxima [A]
time = 0.47, size = 61, normalized size = 1.00

$$\frac{1}{3}\,ax^3+\frac{1}{6}\left(2\,x^3\,\operatorname{artanh}\left(\frac{c}{x^2}\right)-\left(2\,\sqrt{c}\,\arctan\left(\frac{x}{\sqrt{c}}\right)-\sqrt{c}\,\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)-4\,x\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctanh(c/x^2) - (2*sqrt(c)*arctan(x/sqrt(c)) - sqrt
(c)*log((x - sqrt(c))/(x + sqrt(c))) - 4*x)*c)*b
```

Fricas [A]
time = 0.35, size = 162, normalized size = 2.66

$$\left[\frac{1}{6}\,bx^3\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{3}\,ax^3-\frac{1}{3}\,bc^{\frac{3}{2}}\arctan\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{6}\,bc^{\frac{3}{2}}\log\left(\frac{x^2-2\,\sqrt{c}\,x+c}{x^2-c}\right)+\frac{2}{3}\,bcx,\ \frac{1}{6}\,bx^3\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{3}\,ax^3+\frac{1}{3}\,b\sqrt{-c}\,c\arctan\left(\frac{\sqrt{-c}\,x}{c}\right)+\frac{1}{6}\,b\sqrt{-c}\,c\log\left(\frac{x^2-2\,\sqrt{-c}\,x-c}{x^2+c}\right)+\frac{2}{3}\,bcx\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] [1/6*b*x^3*log((x^2 + c)/(x^2 - c)) + 1/3*a*x^3 - 1/3*b*c^(3/2)*arctan(x/sq
rt(c)) + 1/6*b*c^(3/2)*log((x^2 - 2*sqrt(c)*x + c)/(x^2 - c)) + 2/3*b*c*x,
1/6*b*x^3*log((x^2 + c)/(x^2 - c)) + 1/3*a*x^3 + 1/3*b*sqrt(-c)*c*arctan(sq
rt(-c)*x/c) + 1/6*b*sqrt(-c)*c*log((x^2 - 2*sqrt(-c)*x - c)/(x^2 + c)) + 2/
3*b*c*x]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(56) = 112$.
time = 2.84, size = 830, normalized size = 13.61

$$\begin{cases} \frac{ax^3}{3} & \text{for } c = 0 \\ \frac{x^3(a-\infty b)}{3} & \text{for } c = -x^2 \\ \frac{x^3(a+\infty b)}{3} & \text{for } c = x^2 \\ -\frac{2ac^2x^3\sqrt{-c}}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}} + \frac{2ax^7\sqrt{-c}}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}} - \frac{2bc^{\frac{7}{2}}\sqrt{-c}\log\left(-\sqrt{c}+x\right)}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}} + \frac{bc^{\frac{7}{2}}\sqrt{-c}\log\left(x-\sqrt{-c}\right)}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}} + \frac{bc^{\frac{7}{2}}\sqrt{-c}\log\left(x+\sqrt{-c}\right)}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}} - \frac{2bc^{\frac{7}{2}}\sqrt{-c}\,\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}} + \frac{2bc^{\frac{3}{2}}x^4\sqrt{-c}\log\left(-\sqrt{c}+x\right)}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}} - \frac{bc^{\frac{3}{2}}x^4\sqrt{-c}\log\left(x-\sqrt{-c}\right)}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}} - \frac{bc^{\frac{3}{2}}x^4\sqrt{-c}\log\left(x+\sqrt{-c}\right)}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}} + \frac{2bc^{\frac{3}{2}}x^4\sqrt{-c}\,\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}} + \frac{bc^4\log\left(x-\sqrt{-c}\right)}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}} - \frac{bc^4\log\left(x+\sqrt{-c}\right)}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}} - \frac{4bc^3x\sqrt{-c}}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}} - \frac{bc^2x^4\log\left(x-\sqrt{-c}\right)}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}} + \frac{bc^2x^4\log\left(x+\sqrt{-c}\right)}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}} - \frac{2bc^2x^3\sqrt{-c}\,\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}} + \frac{4bcx^5\sqrt{-c}}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}} + \frac{2bx^7\sqrt{-c}\,\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c/x**2)),x)
```

```
[Out] Piecewise((a*x**3/3, Eq(c, 0)), (x**3*(a - oo*b)/3, Eq(c, -x**2)), (x**3*(a
 + oo*b)/3, Eq(c, x**2)), (-2*a*c**2*x**3*sqrt(-c)/(-6*c**2*sqrt(-c) + 6*x*
*4*sqrt(-c)) + 2*a*x**7*sqrt(-c)/(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c)) - 2*b
*c**(7/2)*sqrt(-c)*log(-sqrt(c) + x)/(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c)) +
 b*c**(7/2)*sqrt(-c)*log(x - sqrt(-c))/(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c))
 + b*c**(7/2)*sqrt(-c)*log(x + sqrt(-c))/(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c
)) - 2*b*c**(7/2)*sqrt(-c)*atanh(c/x**2)/(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c
)) + 2*b*c**(3/2)*x**4*sqrt(-c)*log(-sqrt(c) + x)/(-6*c**2*sqrt(-c) + 6*x**
4*sqrt(-c)) - b*c**(3/2)*x**4*sqrt(-c)*log(x - sqrt(-c))/(-6*c**2*sqrt(-c)
+ 6*x**4*sqrt(-c)) - b*c**(3/2)*x**4*sqrt(-c)*log(x + sqrt(-c))/(-6*c**2*sq
rt(-c) + 6*x**4*sqrt(-c)) + 2*b*c**(3/2)*x**4*sqrt(-c)*atanh(c/x**2)/(-6*c*
*2*sqrt(-c) + 6*x**4*sqrt(-c)) + b*c**4*log(x - sqrt(-c))/(-6*c**2*sqrt(-c)
 + 6*x**4*sqrt(-c)) - b*c**4*log(x + sqrt(-c))/(-6*c**2*sqrt(-c) + 6*x**4*s
qrt(-c)) - 4*b*c**3*x*sqrt(-c)/(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c)) - b*c**
2*x**4*log(x - sqrt(-c))/(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c)) + b*c**2*x**4
*log(x + sqrt(-c))/(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c)) - 2*b*c**2*x**3*sqr
t(-c)*atanh(c/x**2)/(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c)) + 4*b*c*x**5*sqrt(
-c)/(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c)) + 2*b*x**7*sqrt(-c)*atanh(c/x**2)/
(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c)), True))
```

Giac [**A**]
time = 0.43, size = 69, normalized size = 1.13

$$\frac{1}{3}\,bc^3\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}\,c} - \frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right) + \frac{1}{6}\,bx^3\log\left(\frac{x^2+c}{x^2-c}\right) + \frac{1}{3}\,ax^3 + \frac{2}{3}\,bcx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] 1/3*b*c^3*(arctan(x/sqrt(-c))/(sqrt(-c)*c) - arctan(x/sqrt(c))/c^(3/2)) + 1
/6*b*x^3*log((x^2 + c)/(x^2 - c)) + 1/3*a*x^3 + 2/3*b*c*x
```

Mupad [**B**]

time = 0.90, size = 65, normalized size = 1.07

$$\frac{a\,x^3}{3}-\frac{b\,c^{3/2}\,\mathrm{atan}\left(\frac{x}{\sqrt{c}}\right)}{3}+\frac{2\,b\,c\,x}{3}+\frac{b\,x^3\,\ln\left(x^2+c\right)}{6}-\frac{b\,x^3\,\ln\left(x^2-c\right)}{6}+\frac{b\,c^{3/2}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c/x^2)),x)
```

```
[Out] (a*x^3)/3 - (b*c^(3/2)*atan(x/c^(1/2)))/3 + (b*c^(3/2)*atan((x*1i)/c^(1/2))
*1i)/3 + (2*b*c*x)/3 + (b*x^3*log(c + x^2))/6 - (b*x^3*log(x^2 - c))/6
```

3.167 $\int \left(a+b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx$

Optimal. Leaf size=44

$$ax+b\sqrt{c}\,\mathrm{ArcTan}\left(\frac{x}{\sqrt{c}}\right)+bx\tanh^{-1}\left(\frac{c}{x^2}\right)-b\sqrt{c}\,\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

```
[Out] a*x+b*x*arctanh(c/x^2)+b*arctan(x/c^(1/2))*c^(1/2)-b*arctanh(x/c^(1/2))*c^(
1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6021, 269, 304, 209, 212}

$$ax+b\sqrt{c}\,\mathrm{ArcTan}\left(\frac{x}{\sqrt{c}}\right)+bx\tanh^{-1}\left(\frac{c}{x^2}\right)-b\sqrt{c}\,\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c/x^2],x]
```

```
[Out] a*x + b*Sqrt[c]*ArcTan[x/Sqrt[c]] + b*x*ArcTanh[c/x^2] - b*Sqrt[c]*ArcTanh[
x/Sqrt[c]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
```

```
/b, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)\,dx &= ax+b\int \tanh^{-1}\left(\frac{c}{x^2}\right)\,dx\\ &= ax+bx\tanh^{-1}\left(\frac{c}{x^2}\right)+(2bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^2}\,dx\\ &= ax+bx\tanh^{-1}\left(\frac{c}{x^2}\right)+(2bc)\int \frac{x^2}{-c^2+x^4}\,dx\\ &= ax+bx\tanh^{-1}\left(\frac{c}{x^2}\right)-(bc)\int \frac{1}{c-x^2}\,dx+(bc)\int \frac{1}{c+x^2}\,dx\\ &= ax+b\sqrt{c}\,\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+bx\tanh^{-1}\left(\frac{c}{x^2}\right)-b\sqrt{c}\,\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 1.23

$$ax+bx\tanh^{-1}\left(\frac{c}{x^2}\right)+\frac{1}{2}b\sqrt{c}\left(2\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)+\log\left(\sqrt{c}-x\right)-\log\left(\sqrt{c}+x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c/x^2],x]
```

```
[Out] a*x + b*x*ArcTanh[c/x^2] + (b*Sqrt[c]*(2*ArcTan[x/Sqrt[c]] + Log[Sqrt[c] -
x] - Log[Sqrt[c] + x]))/2
```

Maple [A]
time = 0.12, size = 39, normalized size = 0.89

method	result
default	$ax+bx\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)+b\arctan\left(\frac{x}{\sqrt{c}}\right)\sqrt{c}-b\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c}}{x}\right)$

derivativedivides	$ax+bx\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)-b\sqrt{c}\,\arctan\left(\frac{\sqrt{c}}{x}\right)-b\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c}}{x}\right)$
risch	$ax+\frac{bx\ln(x^2+c)}{2}-\frac{bx\ln(-x^2+c)}{2}+\frac{ib\pi x\,\mathrm{csgn}\left(\frac{i}{x^2}\right)\mathrm{csgn}\left(\frac{i(x^2+c)}{x^2}\right)^2}{4}-\frac{ib\pi x\mathrm{csgn}\left(\frac{i(-x^2+c)}{x^2}\right)^3}{4}+\frac{ib\pi x\,\mathrm{csgn}(i$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*x*arctanh(c/x^2)+b*arctan(x/c^(1/2))*c^(1/2)-b*c^(1/2)*arctanh(1/x*c^
(1/2))
```

Maxima [A]
time = 0.46, size = 51, normalized size = 1.16

$$\frac{1}{2}\left(c\left(\frac{2\arctan\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)}{\sqrt{c}}\right)+2\,x\,\mathrm{artanh}\left(\frac{c}{x^2}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c/x^2),x, algorithm="maxima")
```

```
[Out] 1/2*(c*(2*arctan(x/sqrt(c))/sqrt(c) + log((x - sqrt(c))/(x + sqrt(c)))/sqrt
(c)) + 2*x*arctanh(c/x^2))*b + a*x
```

Fricas [A] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(36) = 72$.
time = 0.49, size = 138, normalized size = 3.14

$$\left[\frac{1}{2}\,bx\log\left(\frac{x^2+c}{x^2-c}\right)+b\sqrt{c}\,\arctan\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{2}\,b\sqrt{c}\,\log\left(\frac{x^2-2\,\sqrt{c}\,x+c}{x^2-c}\right)+ax,\ \frac{1}{2}\,bx\log\left(\frac{x^2+c}{x^2-c}\right)+b\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{c}\right)+\frac{1}{2}\,b\sqrt{-c}\,\log\left(\frac{x^2+2\,\sqrt{-c}\,x-c}{x^2+c}\right)+ax\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c/x^2),x, algorithm="fricas")
```

```
[Out] [1/2*b*x*log((x^2 + c)/(x^2 - c)) + b*sqrt(c)*arctan(x/sqrt(c)) + 1/2*b*sqr
t(c)*log((x^2 - 2*sqrt(c)*x + c)/(x^2 - c)) + a*x, 1/2*b*x*log((x^2 + c)/(x
^2 - c)) + b*sqrt(-c)*arctan(sqrt(-c)*x/c) + 1/2*b*sqrt(-c)*log((x^2 + 2*sq
rt(-c)*x - c)/(x^2 + c)) + a*x]
```

Sympy [A]
time = 2.10, size = 632, normalized size = 14.36

$$ax+b\left(\begin{cases}0 & \text{for}\ c=0\\ -\infty x & \text{for}\ c=-x^2\\ \infty x & \text{for}\ c=x^2\\ -\frac{2c^{\frac{5}{2}}\sqrt{-c}\log\left(-\sqrt{c}+x\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}}+\frac{c^{\frac{5}{2}}\sqrt{-c}\log\left(x-\sqrt{-c}\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}}+\frac{c^{\frac{5}{2}}\sqrt{-c}\log\left(x+\sqrt{-c}\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}}-\frac{2c^{\frac{5}{2}}\sqrt{-c}\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}}+\frac{2\sqrt{c}\,x^4\sqrt{-c}\log\left(-\sqrt{c}+x\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}}-\frac{\sqrt{c}\,x^4\sqrt{-c}\log\left(x-\sqrt{-c}\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}}-\frac{\sqrt{c}\,x^4\sqrt{-c}\log\left(x+\sqrt{-c}\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}}+\frac{2\sqrt{c}\,x^4\sqrt{-c}\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}}-\frac{c^3\log\left(x-\sqrt{-c}\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}}+\frac{c^3\log\left(x+\sqrt{-c}\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}}-\frac{2c^2x\sqrt{-c}\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}}+\frac{cx^4\log\left(x-\sqrt{-c}\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}}-\frac{cx^4\log\left(x+\sqrt{-c}\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}}+\frac{2x^5\sqrt{-c}\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c/x**2),x)
```

```
[Out] a*x + b*Piecewise((0, Eq(c, 0)), (-oo*x, Eq(c, -x**2)), (oo*x, Eq(c, x**2))
, (-2*c**(5/2)*sqrt(-c)*log(-sqrt(c) + x)/(-2*c**2*sqrt(-c) + 2*x**4*sqrt(-
c)) + c**(5/2)*sqrt(-c)*log(x - sqrt(-c))/(-2*c**2*sqrt(-c) + 2*x**4*sqrt(-
c)) + c**(5/2)*sqrt(-c)*log(x + sqrt(-c))/(-2*c**2*sqrt(-c) + 2*x**4*sqrt(-
c)) - 2*c**(5/2)*sqrt(-c)*atanh(c/x**2)/(-2*c**2*sqrt(-c) + 2*x**4*sqrt(-c)
) + 2*sqrt(c)*x**4*sqrt(-c)*log(-sqrt(c) + x)/(-2*c**2*sqrt(-c) + 2*x**4*sq
rt(-c)) - sqrt(c)*x**4*sqrt(-c)*log(x - sqrt(-c))/(-2*c**2*sqrt(-c) + 2*x**
4*sqrt(-c)) - sqrt(c)*x**4*sqrt(-c)*log(x + sqrt(-c))/(-2*c**2*sqrt(-c) + 2
*x**4*sqrt(-c)) + 2*sqrt(c)*x**4*sqrt(-c)*atanh(c/x**2)/(-2*c**2*sqrt(-c) +
 2*x**4*sqrt(-c)) - c**3*log(x - sqrt(-c))/(-2*c**2*sqrt(-c) + 2*x**4*sqrt(
-c)) + c**3*log(x + sqrt(-c))/(-2*c**2*sqrt(-c) + 2*x**4*sqrt(-c)) - 2*c**2
*x*sqrt(-c)*atanh(c/x**2)/(-2*c**2*sqrt(-c) + 2*x**4*sqrt(-c)) + c*x**4*log
(x - sqrt(-c))/(-2*c**2*sqrt(-c) + 2*x**4*sqrt(-c)) - c*x**4*log(x + sqrt(-
c))/(-2*c**2*sqrt(-c) + 2*x**4*sqrt(-c)) + 2*x**5*sqrt(-c)*atanh(c/x**2)/(-
2*c**2*sqrt(-c) + 2*x**4*sqrt(-c)), True))
```

Giac [**A**]
time = 0.42, size = 57, normalized size = 1.30

$$\frac{1}{2}\left(2\,c\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}\right)+x\log\left(-\frac{\frac{c}{x^2}+1}{\frac{c}{x^2}-1}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c/x^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*c*(arctan(x/sqrt(-c))/sqrt(-c) + arctan(x/sqrt(c))/sqrt(c)) + x*log(
-(c/x^2 + 1)/(c/x^2 - 1)))*b + a*x
```

Mupad [**B**]
time = 0.82, size = 52, normalized size = 1.18

$$a\,x+\frac{b\,x\,\ln\left(x^2+c\right)}{2}+b\,\sqrt{c}\,\mathrm{atan}\left(\frac{x}{\sqrt{c}}\right)-\frac{b\,x\,\ln\left(x^2-c\right)}{2}+b\,\sqrt{c}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atanh(c/x^2),x)
```

```
[Out] a*x + (b*x*log(c + x^2))/2 + b*c^(1/2)*atan(x/c^(1/2)) + b*c^(1/2)*atan((x*
1i)/c^(1/2))*1i - (b*x*log(x^2 - c))/2
```

3.168 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^2}\,dx$

Optimal. Leaf size=46

$$\frac{b\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] (-a-b*arctanh(c/x^2))/x+b*arctan(x/c^(1/2))/c^(1/2)+b*arctanh(x/c^(1/2))/c^
(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6037, 269, 218, 212, 209}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}+\frac{b\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x^2,x]
```

```
[Out] (b*ArcTan[x/Sqrt[c]])/Sqrt[c] - (a + b*ArcTanh[c/x^2])/x + (b*ArcTanh[x/Sqr
t[c]])/Sqrt[c]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^2}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}-(2bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^4}\,dx\\
&= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}-(2bc)\int \frac{1}{-c^2+x^4}\,dx\\
&= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}+b\int \frac{1}{c-x^2}\,dx+b\int \frac{1}{c+x^2}\,dx\\
&= \frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}
\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 72, normalized size = 1.57

$$-\frac{a}{x}+\frac{b\mathrm{ArcTan}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}-\frac{b\log\left(\sqrt{c}-x\right)}{2\sqrt{c}}+\frac{b\log\left(\sqrt{c}+x\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^2,x]
```

```
[Out] -(a/x) + (b*ArcTan[x/Sqrt[c]])/Sqrt[c] - (b*ArcTanh[c/x^2])/x - (b*Log[Sqrt
[c] - x])/(2*Sqrt[c]) + (b*Log[Sqrt[c] + x])/(2*Sqrt[c])
```

Maple [**A**]
time = 0.12, size = 47, normalized size = 1.02

method	result
derivativedivides	$-\frac{a}{x}-\frac{b\,\mathrm{arctanh}\left(\frac{c}{x^{2}}\right)}{x}-\frac{b\arctan\left(\frac{\sqrt{c}}{x}\right)}{\sqrt{c}}+\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{\sqrt{c}}$
default	$-\frac{a}{x}-\frac{b\,\mathrm{arctanh}\left(\frac{c}{x^{2}}\right)}{x}-\frac{b\arctan\left(\frac{\sqrt{c}}{x}\right)}{\sqrt{c}}+\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{\sqrt{c}}$
risch	$-\frac{b\ln\left(x^{2}+c\right)}{2x}-\frac{-i\pi bc\,\mathrm{csgn}\left(i\left(-x^{2}+c\right)\right)\mathrm{csgn}\left(\frac{i\left(-x^{2}+c\right)}{x^{2}}\right)^{2}+i\pi bc\,\mathrm{csgn}\left(\frac{i}{x^{2}}\right)\mathrm{csgn}\left(i\left(-x^{2}+c\right)\right)\mathrm{csgn}\left(\frac{i\left(-x^{2}+c\right)}{x^{2}}\right)-i\pi bc\,\mathrm{cs}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/x-b/x*arctanh(c/x^2)-b/c^(1/2)*arctan(1/x*c^(1/2))+b/c^(1/2)*arctanh(1/x
*c^(1/2))
```

Maxima [A]
time = 0.46, size = 57, normalized size = 1.24

$$\frac{1}{2}\left(c\left(\frac{2\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)-\frac{2\,\mathrm{artanh}\left(\frac{c}{x^{2}}\right)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c*(2*arctan(x/sqrt(c))/c^(3/2) - log((x - sqrt(c))/(x + sqrt(c)))/c^(3
/2)) - 2*arctanh(c/x^2)/x)*b - a/x
```

Fricas [A] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(38) = 76$.
time = 0.37, size = 159, normalized size = 3.46

$$\left[\frac{2\,b\sqrt{c}\,x\arctan\left(\frac{x}{\sqrt{c}}\right)+b\sqrt{c}\,x\log\left(\frac{x^{2}+2\,\sqrt{c}\,x+c}{x^{2}-c}\right)-bc\log\left(\frac{x^{2}+c}{x^{2}-c}\right)-2\,ac}{2\,cx},-\frac{2\,b\sqrt{-c}\,x\arctan\left(\frac{\sqrt{-c}\,x}{c}\right)+b\sqrt{-c}\,x\log\left(\frac{x^{2}-2\,\sqrt{-c}\,x-c}{x^{2}+c}\right)+bc\log\left(\frac{x^{2}+c}{x^{2}-c}\right)+2\,ac}{2\,cx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*sqrt(c)*x*arctan(x/sqrt(c)) + b*sqrt(c)*x*log((x^2 + 2*sqrt(c)*x
+ c)/(x^2 - c)) - b*c*log((x^2 + c)/(x^2 - c)) - 2*a*c)/(c*x), -1/2*(2*b*sq
rt(-c)*x*arctan(sqrt(-c)*x/c) + b*sqrt(-c)*x*log((x^2 - 2*sqrt(-c)*x - c)/(
x^2 + c)) + b*c*log((x^2 + c)/(x^2 - c)) + 2*a*c)/(c*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 886 vs. $2(42) = 84$.

time = 3.66, size = 886, normalized size = 19.26

$$\begin{cases} -\frac{a}{x} & \text{for } c = 0 \\ -\frac{a-\infty b}{x} & \text{for } c = -x^2 \\ -\frac{a+\infty b}{x} & \text{for } c = x^2 \\ \frac{2ac^{\frac{7}{2}}\sqrt{-c}}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} - \frac{2ac^{\frac{3}{2}}x^4\sqrt{-c}}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} - \frac{bc^{\frac{7}{2}}x\log\left(x-\sqrt{-c}\right)}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} + \frac{bc^{\frac{7}{2}}x\log\left(x+\sqrt{-c}\right)}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} + \frac{2bc^{\frac{7}{2}}\sqrt{-c}\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} + \frac{bc^{\frac{3}{2}}x^5\log\left(x-\sqrt{-c}\right)}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} - \frac{bc^{\frac{3}{2}}x^5\log\left(x+\sqrt{-c}\right)}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} - \frac{2bc^{\frac{3}{2}}x^4\sqrt{-c}\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} + \frac{2bc^3x\sqrt{-c}\log\left(-\sqrt{c}+x\right)}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} - \frac{bc^3x\sqrt{-c}\log\left(x-\sqrt{-c}\right)}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} - \frac{bc^3x\sqrt{-c}\log\left(x+\sqrt{-c}\right)}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} + \frac{2bc^3x\sqrt{-c}\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} - \frac{2bcx^5\sqrt{-c}\log\left(-\sqrt{c}+x\right)}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} + \frac{bcx^5\sqrt{-c}\log\left(x-\sqrt{-c}\right)}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} + \frac{bcx^5\sqrt{-c}\log\left(x+\sqrt{-c}\right)}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} - \frac{2bcx^5\sqrt{-c}\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x**2,x)
```

```
[Out] Piecewise((-a/x, Eq(c, 0)), (-(a - oo*b)/x, Eq(c, -x**2)), (-(a + oo*b)/x,
Eq(c, x**2)), (2*a*c**(7/2)*sqrt(-c)/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3/2)*x
**5*sqrt(-c)) - 2*a*c**(3/2)*x**4*sqrt(-c)/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(
3/2)*x**5*sqrt(-c)) - b*c**(7/2)*x*log(x - sqrt(-c))/(-2*c**(7/2)*x*sqrt(-c
) + 2*c**(3/2)*x**5*sqrt(-c)) + b*c**(7/2)*x*log(x + sqrt(-c))/(-2*c**(7/2)
*x*sqrt(-c) + 2*c**(3/2)*x**5*sqrt(-c)) + 2*b*c**(7/2)*sqrt(-c)*atanh(c/x**
2)/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3/2)*x**5*sqrt(-c)) + b*c**(3/2)*x**5*lo
g(x - sqrt(-c))/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3/2)*x**5*sqrt(-c)) - b*c**
(3/2)*x**5*log(x + sqrt(-c))/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3/2)*x**5*sqrt
(-c)) - 2*b*c**(3/2)*x**4*sqrt(-c)*atanh(c/x**2)/(-2*c**(7/2)*x*sqrt(-c) +
2*c**(3/2)*x**5*sqrt(-c)) + 2*b*c**3*x*sqrt(-c)*log(-sqrt(c) + x)/(-2*c**(7
/2)*x*sqrt(-c) + 2*c**(3/2)*x**5*sqrt(-c)) - b*c**3*x*sqrt(-c)*log(x - sqrt
(-c))/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3/2)*x**5*sqrt(-c)) - b*c**3*x*sqrt(-
c)*log(x + sqrt(-c))/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3/2)*x**5*sqrt(-c)) +
2*b*c**3*x*sqrt(-c)*atanh(c/x**2)/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3/2)*x**5
*sqrt(-c)) - 2*b*c*x**5*sqrt(-c)*log(-sqrt(c) + x)/(-2*c**(7/2)*x*sqrt(-c)
+ 2*c**(3/2)*x**5*sqrt(-c)) + b*c*x**5*sqrt(-c)*log(x - sqrt(-c))/(-2*c**(7
/2)*x*sqrt(-c) + 2*c**(3/2)*x**5*sqrt(-c)) + b*c*x**5*sqrt(-c)*log(x + sqrt
(-c))/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3/2)*x**5*sqrt(-c)) - 2*b*c*x**5*sqrt
(-c)*atanh(c/x**2)/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3/2)*x**5*sqrt(-c)), Tru
e))
```

Giac [A]

time = 0.43, size = 62, normalized size = 1.35

$$-bc\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}\,c} - \frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right) - \frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{2\,x} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^2,x, algorithm="giac")
```

```
[Out] -b*c*(arctan(x/sqrt(-c))/(sqrt(-c)*c) - arctan(x/sqrt(c))/c^(3/2)) - 1/2*b*
log((x^2 + c)/(x^2 - c))/x - a/x
```

Mupad [B]
time = 0.96, size = 59, normalized size = 1.28

$$\frac{b\,\mathrm{atan}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{a}{x}-\frac{b\,\ln\left(x^2+c\right)}{2\,x}+\frac{b\,\ln\left(x^2-c\right)}{2\,x}-\frac{b\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))/x^2,x)
```

```
[Out] (b*atan(x/c^(1/2)))/c^(1/2) - a/x - (b*atan((x*1i)/c^(1/2))*1i)/c^(1/2) - (
b*log(c + x^2))/(2*x) + (b*log(x^2 - c))/(2*x)
```

3.169 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^4}\,dx$

Optimal. Leaf size=65

$$-\frac{2b}{3cx}-\frac{b\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}$$

```
[Out] -2/3*b/c/x-1/3*b*arctan(x/c^(1/2))/c^(3/2)+1/3*(-a-b*arctanh(c/x^2))/x^3+1/
3*b*arctanh(x/c^(1/2))/c^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.429$, Rules used = {6037, 269, 331, 304, 209, 212}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{b\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}-\frac{2b}{3cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x^4,x]
```

```
[Out] (-2*b)/(3*c*x) - (b*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) - (a + b*ArcTanh[c/x^2])
/(3*x^3) + (b*ArcTanh[x/Sqrt[c]])/(3*c^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^4}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{1}{3}(2bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^6}\,dx\\
&= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{1}{3}(2bc)\int \frac{1}{x^2\left(-c^2+x^4\right)}\,dx\\
&= -\frac{2b}{3cx}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{(2b)\int \frac{x^2}{-c^2+x^4}\,dx}{3c}\\
&= -\frac{2b}{3cx}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}+\frac{b\int \frac{1}{c-x^2}\,dx}{3c}-\frac{b\int \frac{1}{c+x^2}\,dx}{3c}\\
&= -\frac{2b}{3cx}-\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.02, size = 90, normalized size = 1.38

$$
-\frac{a}{3x^3}-\frac{2b}{3cx}-\frac{b\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}-\frac{b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{b\log\left(\sqrt{c}-x\right)}{6c^{3/2}}+\frac{b\log\left(\sqrt{c}+x\right)}{6c^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (2*b)/(3*c*x) - (b*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) - (b*ArcTanh
[c/x^2])/(3*x^3) - (b*Log[Sqrt[c] - x])/(6*c^(3/2)) + (b*Log[Sqrt[c] + x])/
(6*c^(3/2))
```

Maple [A]
time = 0.13, size = 57, normalized size = 0.88

method	result
derivativedivides	$-\frac{a}{3x^3}-\frac{b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{2b}{3cx}+\frac{b\arctan\left(\frac{\sqrt{c}}{x}\right)}{3c^{\frac{3}{2}}}+\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{3c^{\frac{3}{2}}}$
default	$-\frac{a}{3x^3}-\frac{b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{2b}{3cx}+\frac{b\arctan\left(\frac{\sqrt{c}}{x}\right)}{3c^{\frac{3}{2}}}+\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{3c^{\frac{3}{2}}}$
risch	$-\frac{b\ln\left(x^2+c\right)}{6x^3}-\frac{-2i\pi b\,c^2-i\pi b\,c^2\,\mathrm{csgn}\left(\frac{i}{x^2}\right)\mathrm{csgn}\left(i\left(x^2+c\right)\right)\mathrm{csgn}\left(\frac{i\left(x^2+c\right)}{x^2}\right)+2i\pi b\,c^2\,\mathrm{csgn}\left(\frac{i\left(-x^2+c\right)}{x^2}\right)^2-i\pi b\,c^2\,\mathrm{csgn}(i}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctanh(c/x^2)-2/3*b/c/x+1/3*b/c^(3/2)*arctan(1/x*c^(1
/2))+1/3*b/c^(3/2)*arctanh(1/x*c^(1/2))
```

Maxima [A]
time = 0.46, size = 64, normalized size = 0.98

$$-\frac{1}{6}\left(c\left(\frac{2\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{4}{c^2x}\right)+\frac{2\,\mathrm{artanh}\left(\frac{c}{x^2}\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(c*(2*arctan(x/sqrt(c))/c^(5/2) + log((x - sqrt(c))/(x + sqrt(c)))/c^(
5/2) + 4/(c^2*x)) + 2*arctanh(c/x^2)/x^3)*b - 1/3*a/x^3
```

Fricas [A] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(49) = 98.
time = 0.43, size = 189, normalized size = 2.91

$$\left[-\frac{2\,b\sqrt{c}\,x^3\arctan\left(\frac{x}{\sqrt{c}}\right)-b\sqrt{c}\,x^3\log\left(\frac{x^2+2\,\sqrt{c}\,x+c}{x^2-c}\right)+4\,bcx^2+bc^2\log\left(\frac{x^2+c}{x^2-c}\right)+2\,ac^2}{6\,c^2x^3},\,-\frac{2\,b\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{-c}\,x}{c}\right)+b\sqrt{-c}\,x^3\log\left(\frac{x^2+2\,\sqrt{-c}\,x-c}{x^2+c}\right)+4\,bcx^2+bc^2\log\left(\frac{x^2+c}{x^2-c}\right)+2\,ac^2}{6\,c^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^4,x, algorithm="fricas")
```

```
[Out] [-1/6*(2*b*sqrt(c)*x^3*arctan(x/sqrt(c)) - b*sqrt(c)*x^3*log((x^2 + 2*sqrt(
c)*x + c)/(x^2 - c)) + 4*b*c*x^2 + b*c^2*log((x^2 + c)/(x^2 - c)) + 2*a*c^2
)/(c^2*x^3), -1/6*(2*b*sqrt(-c)*x^3*arctan(sqrt(-c)*x/c) + b*sqrt(-c)*x^3*l
og((x^2 + 2*sqrt(-c)*x - c)/(x^2 + c)) + 4*b*c*x^2 + b*c^2*log((x^2 + c)/(x
^2 - c)) + 2*a*c^2)/(c^2*x^3)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. $2(60) = 120$.
time = 4.99, size = 1046, normalized size = 16.09

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3), Eq(c, 0)), (-(a - oo*b)/(3*x**3), Eq(c, -x**2)), (-
(a + oo*b)/(3*x**3), Eq(c, x**2)), (2*a*c**(17/2)*sqrt(-c)/(-6*c**(17/2)*x*
*3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) - 2*a*c**(13/2)*x**4*sqrt(-c)/(-6*
c**(17/2)*x**3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) + 2*b*c**(17/2)*sqrt(-
c)*atanh(c/x**2)/(-6*c**(17/2)*x**3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) +
 b*c**(15/2)*x**3*log(x - sqrt(-c))/(-6*c**(17/2)*x**3*sqrt(-c) + 6*c**(13/
2)*x**7*sqrt(-c)) - b*c**(15/2)*x**3*log(x + sqrt(-c))/(-6*c**(17/2)*x**3*s
qrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) + 4*b*c**(15/2)*x**2*sqrt(-c)/(-6*c**(
17/2)*x**3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) - 2*b*c**(13/2)*x**4*sqrt(
-c)*atanh(c/x**2)/(-6*c**(17/2)*x**3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c))
- b*c**(11/2)*x**7*log(x - sqrt(-c))/(-6*c**(17/2)*x**3*sqrt(-c) + 6*c**(13
/2)*x**7*sqrt(-c)) + b*c**(11/2)*x**7*log(x + sqrt(-c))/(-6*c**(17/2)*x**3*
sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) - 4*b*c**(11/2)*x**6*sqrt(-c)/(-6*c**
(17/2)*x**3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) + 2*b*c**7*x**3*sqrt(-c)*
log(-sqrt(c) + x)/(-6*c**(17/2)*x**3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c))
- b*c**7*x**3*sqrt(-c)*log(x - sqrt(-c))/(-6*c**(17/2)*x**3*sqrt(-c) + 6*c*
*(13/2)*x**7*sqrt(-c)) - b*c**7*x**3*sqrt(-c)*log(x + sqrt(-c))/(-6*c**(17/
2)*x**3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) + 2*b*c**7*x**3*sqrt(-c)*atan
h(c/x**2)/(-6*c**(17/2)*x**3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) - 2*b*c*
*5*x**7*sqrt(-c)*log(-sqrt(c) + x)/(-6*c**(17/2)*x**3*sqrt(-c) + 6*c**(13/2
)*x**7*sqrt(-c)) + b*c**5*x**7*sqrt(-c)*log(x - sqrt(-c))/(-6*c**(17/2)*x**
3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) + b*c**5*x**7*sqrt(-c)*log(x + sqrt
(-c))/(-6*c**(17/2)*x**3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) - 2*b*c**5*x
**7*sqrt(-c)*atanh(c/x**2)/(-6*c**(17/2)*x**3*sqrt(-c) + 6*c**(13/2)*x**7*s
qrt(-c)), True))
```

Giac [**A**]

time = 0.45, size = 72, normalized size = 1.11

$$-\frac{b\arctan\left(\frac{x}{\sqrt{-c}}\right)}{3\sqrt{-c}\,c}-\frac{b\arctan\left(\frac{x}{\sqrt{c}}\right)}{3\,c^{\frac{3}{2}}}-\frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{6\,x^3}-\frac{2\,bx^2+ac}{3\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^4,x, algorithm="giac")
```

```
[Out] -1/3*b*arctan(x/sqrt(-c))/(sqrt(-c)*c) - 1/3*b*arctan(x/sqrt(c))/c^(3/2) -
1/6*b*log((x^2 + c)/(x^2 - c))/x^3 - 1/3*(2*b*x^2 + a*c)/(c*x^3)
```

Mupad [B]
time = 0.99, size = 69, normalized size = 1.06

$$\frac{b\,\ln\left(x^2-c\right)}{6\,x^3}-\frac{2\,b}{3\,c\,x}-\frac{b\,\mathrm{atan}\left(\frac{x}{\sqrt{c}}\right)}{3\,c^{3/2}}-\frac{b\,\ln\left(x^2+c\right)}{6\,x^3}-\frac{a}{3\,x^3}-\frac{b\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}}{3\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))/x^4,x)
```

```
[Out] (b*log(x^2 - c))/(6*x^3) - (2*b)/(3*c*x) - (b*atan(x/c^(1/2)))/(3*c^(3/2))
- (b*atan((x*1i)/c^(1/2))*1i)/(3*c^(3/2)) - (b*log(c + x^2))/(6*x^3) - a/(3
*x^3)
```

3.170 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^6}\,dx$

Optimal. Leaf size=65

$$-\frac{2b}{15cx^3}+\frac{b\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}$$

```
[Out] -2/15*b/c/x^3+1/5*b*arctan(x/c^(1/2))/c^(5/2)+1/5*(-a-b*arctanh(c/x^2))/x^5
+1/5*b*arctanh(x/c^(1/2))/c^(5/2)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6037, 269, 331, 218, 212, 209}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}+\frac{b\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}-\frac{2b}{15cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x^6,x]
```

```
[Out] (-2*b)/(15*c*x^3) + (b*ArcTan[x/Sqrt[c]])/(5*c^(5/2)) - (a + b*ArcTanh[c/x^
2])/(5*x^5) + (b*ArcTanh[x/Sqrt[c]])/(5*c^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^6}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}-\frac{1}{5}(2bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^8}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}-\frac{1}{5}(2bc)\int \frac{1}{x^4\left(-c^2+x^4\right)}\,dx\\ &= -\frac{2b}{15cx^3}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}-\frac{(2b)\int \frac{1}{-c^2+x^4}\,dx}{5c}\\ &= -\frac{2b}{15cx^3}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}+\frac{b\int \frac{1}{c-x^2}\,dx}{5c^2}+\frac{b\int \frac{1}{c+x^2}\,dx}{5c^2}\\ &= -\frac{2b}{15cx^3}+\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 90, normalized size = 1.38

$$-\frac{a}{5x^5}-\frac{2b}{15cx^3}+\frac{b\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}-\frac{b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}-\frac{b\log\left(\sqrt{c}-x\right)}{10c^{5/2}}+\frac{b\log\left(\sqrt{c}+x\right)}{10c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^6,x]
[Out] -1/5*a/x^5 - (2*b)/(15*c*x^3) + (b*ArcTan[x/Sqrt[c]])/(5*c^(5/2)) - (b*ArcT
anh[c/x^2])/(5*x^5) - (b*Log[Sqrt[c] - x])/(10*c^(5/2)) + (b*Log[Sqrt[c] +
x])/(10*c^(5/2))
```

Maple [A]
time = 0.13, size = 57, normalized size = 0.88

method	result
derivativedivides	$-\frac{a}{5x^{5}}-\frac{b\,\mathrm{arctanh}\left(\frac{c}{x^{2}}\right)}{5x^{5}}-\frac{2b}{15c\,x^{3}}-\frac{b\arctan\left(\frac{\sqrt{c}}{x}\right)}{5c^{\frac{5}{2}}}+\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{5c^{\frac{5}{2}}}$
default	$-\frac{a}{5x^{5}}-\frac{b\,\mathrm{arctanh}\left(\frac{c}{x^{2}}\right)}{5x^{5}}-\frac{2b}{15c\,x^{3}}-\frac{b\arctan\left(\frac{\sqrt{c}}{x}\right)}{5c^{\frac{5}{2}}}+\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{5c^{\frac{5}{2}}}$
risch	$-\frac{b\ln\left(x^{2}+c\right)}{10x^{5}}+\frac{ib\pi\,\mathrm{csgn}\left(i\left(-x^{2}+c\right)\right)\mathrm{csgn}\left(\frac{i\left(-x^{2}+c\right)}{x^{2}}\right)^{2}}{20x^{5}}+\frac{ib\pi\,\mathrm{csgn}\left(\frac{i}{x^{2}}\right)\mathrm{csgn}\left(i\left(x^{2}+c\right)\right)\mathrm{csgn}\left(\frac{i\left(x^{2}+c\right)}{x^{2}}\right)}{20x^{5}}+\frac{ib\pi\mathrm{csgn}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x^6,x,method=_RETURNVERBOSE)
[Out] -1/5*a/x^5-1/5*b/x^5*arctanh(c/x^2)-2/15*b/c/x^3-1/5*b/c^(5/2)*arctan(1/x*c
^(1/2))+1/5*b/c^(5/2)*arctanh(1/x*c^(1/2))
```

Maxima [A]
time = 0.47, size = 65, normalized size = 1.00

$$\frac{1}{30}\left(c\left(\frac{6\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{7}{2}}}-\frac{3\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)}{c^{\frac{7}{2}}}-\frac{4}{c^{2}x^{3}}\right)-\frac{6\,\mathrm{artanh}\left(\frac{c}{x^{2}}\right)}{x^{5}}\right)b-\frac{a}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^6,x, algorithm="maxima")
[Out] 1/30*(c*(6*arctan(x/sqrt(c))/c^(7/2) - 3*log((x - sqrt(c))/(x + sqrt(c)))/c
^(7/2) - 4/(c^2*x^3)) - 6*arctanh(c/x^2)/x^5)*b - 1/5*a/x^5
```

Fricas [A] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(49) = 98.
time = 0.38, size = 196, normalized size = 3.02

$$\left[\frac{6\,b\sqrt{c}\,x^{5}\arctan\left(\frac{x}{\sqrt{c}}\right)+3\,b\sqrt{c}\,x^{5}\log\left(\frac{x^{2}+2\,\sqrt{c}\,x+c}{x^{2}-c}\right)-4\,bc^{2}x^{2}-3\,bc^{3}\log\left(\frac{x^{2}+c}{x^{2}-c}\right)-6\,ac^{3}}{30\,c^{3}x^{5}},\ -\frac{6\,b\sqrt{-c}\,x^{5}\arctan\left(\frac{\sqrt{-c}\,x}{c}\right)+3\,b\sqrt{-c}\,x^{5}\log\left(\frac{x^{2}-2\,\sqrt{-c}\,x-c}{x^{2}+c}\right)+4\,bc^{2}x^{2}+3\,bc^{3}\log\left(\frac{x^{2}+c}{x^{2}-c}\right)+6\,ac^{3}}{30\,c^{3}x^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^6,x, algorithm="fricas")
```

```
[Out] [1/30*(6*b*sqrt(c)*x^5*arctan(x/sqrt(c)) + 3*b*sqrt(c)*x^5*log((x^2 + 2*sqr
t(c)*x + c)/(x^2 - c)) - 4*b*c^2*x^2 - 3*b*c^3*log((x^2 + c)/(x^2 - c)) - 6
*a*c^3)/(c^3*x^5), -1/30*(6*b*sqrt(-c)*x^5*arctan(sqrt(-c)*x/c) + 3*b*sqrt(
-c)*x^5*log((x^2 - 2*sqrt(-c)*x - c)/(x^2 + c)) + 4*b*c^2*x^2 + 3*b*c^3*log
((x^2 + c)/(x^2 - c)) + 6*a*c^3)/(c^3*x^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 994 vs. $2(61) = 122$.
time = 7.13, size = 994, normalized size = 15.29

$$\begin{cases} -\frac{a}{5x^5} & \text{for}\: c = 0 \\ -\frac{a - \infty b}{5x^5} & \text{for}\: c = -x^2 \\ -\frac{a + \infty b}{5x^5} & \text{for}\: c = x^2 \\ \frac{6ac^{13}\sqrt{-c}}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} - \frac{6ac^{11}x^4\sqrt{-c}}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} + \frac{6bc^{\frac{21}{2}}x^5\sqrt{-c}\log\left(-\sqrt{c}+x\right)}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} - \frac{3bc^{\frac{21}{2}}x^5\sqrt{-c}\log\left(x-\sqrt{-c}\right)}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} - \frac{3bc^{\frac{21}{2}}x^5\sqrt{-c}\log\left(x+\sqrt{-c}\right)}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} + \frac{6bc^{\frac{21}{2}}x^5\sqrt{-c}\,\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} - \frac{6bc^{\frac{17}{2}}x^9\sqrt{-c}\log\left(-\sqrt{c}+x\right)}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} + \frac{3bc^{\frac{17}{2}}x^9\sqrt{-c}\log\left(x-\sqrt{-c}\right)}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} + \frac{3bc^{\frac{17}{2}}x^9\sqrt{-c}\log\left(x+\sqrt{-c}\right)}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} - \frac{6bc^{\frac{17}{2}}x^9\sqrt{-c}\,\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} + \frac{6bc^{13}\sqrt{-c}\,\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} + \frac{4bc^{12}x^2\sqrt{-c}}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} - \frac{3bc^{11}x^5\log\left(x-\sqrt{-c}\right)}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} + \frac{3bc^{11}x^5\log\left(x+\sqrt{-c}\right)}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} - \frac{6bc^{11}x^4\sqrt{-c}\,\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} - \frac{4bc^{10}x^6\sqrt{-c}}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} + \frac{3bc^9x^9\log\left(x-\sqrt{-c}\right)}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} - \frac{3bc^9x^9\log\left(x+\sqrt{-c}\right)}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x**6,x)
```

```
[Out] Piecewise((-a/(5*x**5), Eq(c, 0)), (-(a - oo*b)/(5*x**5), Eq(c, -x**2)), (-
(a + oo*b)/(5*x**5), Eq(c, x**2)), (6*a*c**13*sqrt(-c)/(-30*c**13*x**5*sqrt
(-c) + 30*c**11*x**9*sqrt(-c)) - 6*a*c**11*x**4*sqrt(-c)/(-30*c**13*x**5*sq
rt(-c) + 30*c**11*x**9*sqrt(-c)) + 6*b*c**(21/2)*x**5*sqrt(-c)*log(-sqrt(c)
 + x)/(-30*c**13*x**5*sqrt(-c) + 30*c**11*x**9*sqrt(-c)) - 3*b*c**(21/2)*x*
*5*sqrt(-c)*log(x - sqrt(-c))/(-30*c**13*x**5*sqrt(-c) + 30*c**11*x**9*sqrt
(-c)) - 3*b*c**(21/2)*x**5*sqrt(-c)*log(x + sqrt(-c))/(-30*c**13*x**5*sqrt(
-c) + 30*c**11*x**9*sqrt(-c)) + 6*b*c**(21/2)*x**5*sqrt(-c)*atanh(c/x**2)/(
-30*c**13*x**5*sqrt(-c) + 30*c**11*x**9*sqrt(-c)) - 6*b*c**(17/2)*x**9*sqrt
(-c)*log(-sqrt(c) + x)/(-30*c**13*x**5*sqrt(-c) + 30*c**11*x**9*sqrt(-c)) +
 3*b*c**(17/2)*x**9*sqrt(-c)*log(x - sqrt(-c))/(-30*c**13*x**5*sqrt(-c) + 3
0*c**11*x**9*sqrt(-c)) + 3*b*c**(17/2)*x**9*sqrt(-c)*log(x + sqrt(-c))/(-30
*c**13*x**5*sqrt(-c) + 30*c**11*x**9*sqrt(-c)) - 6*b*c**(17/2)*x**9*sqrt(-c
)*atanh(c/x**2)/(-30*c**13*x**5*sqrt(-c) + 30*c**11*x**9*sqrt(-c)) + 6*b*c*
*13*sqrt(-c)*atanh(c/x**2)/(-30*c**13*x**5*sqrt(-c) + 30*c**11*x**9*sqrt(-c
)) + 4*b*c**12*x**2*sqrt(-c)/(-30*c**13*x**5*sqrt(-c) + 30*c**11*x**9*sqrt(
-c)) - 3*b*c**11*x**5*log(x - sqrt(-c))/(-30*c**13*x**5*sqrt(-c) + 30*c**11
*x**9*sqrt(-c)) + 3*b*c**11*x**5*log(x + sqrt(-c))/(-30*c**13*x**5*sqrt(-c)
 + 30*c**11*x**9*sqrt(-c)) - 6*b*c**11*x**4*sqrt(-c)*atanh(c/x**2)/(-30*c**
13*x**5*sqrt(-c) + 30*c**11*x**9*sqrt(-c)) - 4*b*c**10*x**6*sqrt(-c)/(-30*c
**13*x**5*sqrt(-c) + 30*c**11*x**9*sqrt(-c)) + 3*b*c**9*x**9*log(x - sqrt(-
c))/(-30*c**13*x**5*sqrt(-c) + 30*c**11*x**9*sqrt(-c)) - 3*b*c**9*x**9*log(
x + sqrt(-c))/(-30*c**13*x**5*sqrt(-c) + 30*c**11*x**9*sqrt(-c)), True))
```

Giac [A]
time = 0.49, size = 74, normalized size = 1.14

$$-\frac{1}{5}\,b\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}\,c^2} - \frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right) - \frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{10\,x^5} - \frac{2\,bx^2 + 3\,ac}{15\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^6,x, algorithm="giac")
```

```
[Out] -1/5*b*(arctan(x/sqrt(-c))/(sqrt(-c)*c^2) - arctan(x/sqrt(c))/c^(5/2)) - 1/
10*b*log((x^2 + c)/(x^2 - c))/x^5 - 1/15*(2*b*x^2 + 3*a*c)/(c*x^5)
```

Mupad [B]
time = 1.00, size = 69, normalized size = 1.06

$$\frac{b\,\mathrm{atan}\left(\frac{x}{\sqrt{c}}\right)}{5\,c^{5/2}}-\frac{2\,b}{15\,c\,x^3}-\frac{a}{5\,x^5}-\frac{b\,\ln\left(x^2+c\right)}{10\,x^5}+\frac{b\,\ln\left(x^2-c\right)}{10\,x^5}-\frac{b\,\mathrm{atan}\left(\frac{x\,1{}\mathrm{i}}{\sqrt{c}}\right)\,1{}\mathrm{i}}{5\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))/x^6,x)
```

```
[Out] (b*atan(x/c^(1/2)))/(5*c^(5/2)) - (2*b)/(15*c*x^3) - a/(5*x^5) - (b*atan((x
*1i)/c^(1/2))*1i)/(5*c^(5/2)) - (b*log(c + x^2))/(10*x^5) + (b*log(x^2 - c)
)/(10*x^5)
```

3.171 $\int x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2\,dx$

Optimal. Leaf size=94

$$\frac{1}{2}bcx^2\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)-\frac{1}{4}c^2\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2+\frac{1}{4}x^4\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2+\frac{1}{4}b^2c^2\log\left(1-\frac{c^2}{x^4}\right)$$

```
[Out] 1/2*b*c*x^2*(a+b*arccoth(x^2/c))-1/4*c^2*(a+b*arccoth(x^2/c))^2+1/4*x^4*(a+
b*arccoth(x^2/c))^2+1/4*b^2*c^2*ln(1-c^2/x^4)+b^2*c^2*ln(x)
```

Rubi [A]
time = 0.12, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6039, 6037, 6129, 272, 36, 29, 31, 6095}

$$-\frac{1}{4}c^2\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2+\frac{1}{2}bcx^2\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)+\frac{1}{4}x^4\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2+\frac{1}{4}b^2c^2\log\left(1-\frac{c^2}{x^4}\right)+b^2c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] (b*c*x^2*(a + b*ArcCoth[x^2/c]))/2 - (c^2*(a + b*ArcCoth[x^2/c])^2)/4 + (x^
4*(a + b*ArcCoth[x^2/c])^2)/4 + (b^2*c^2*Log[1 - c^2/x^4])/4 + b^2*c^2*Log[
x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int x^3\Big(a+b\tanh^{-1}\Big(\frac{c}{x^2}\Big)\Big)^2\,dx = \int \Big(\frac{1}{4}x^3\Big(2a-b\log\Big(1-\frac{c}{x^2}\Big)\Big)^2-\frac{1}{2}bx^3\Big(-2a+b\log\Big(1-\frac{c}{x^2}\Big)\Big)\log\Big(1+$$

$$=\frac{1}{4}\int x^3\Big(2a-b\log\Big(1-\frac{c}{x^2}\Big)\Big)^2\,dx-\frac{1}{2}b\int x^3\Big(-2a+b\log\Big(1-\frac{c}{x^2}\Big)\Big)\log$$

$$=-\Big(\frac{1}{8}\mathrm{Subst}\Big(\int\frac{(2a-b\log(1-cx))^2}{x^3}\,dx,x,\frac{1}{x^2}\Big)\Big)-\frac{1}{4}b\mathrm{Subst}\Big(\int x\Big(-2a$$

$$=\frac{1}{16}x^4\Big(2a-b\log\Big(1-\frac{c}{x^2}\Big)\Big)^2+\frac{1}{16}b^2x^4\log^2\Big(1+\frac{c}{x^2}\Big)-\frac{1}{4}b\mathrm{Subst}\Big(\int\Big(-2$$

$$=\frac{1}{16}x^4\Big(2a-b\log\Big(1-\frac{c}{x^2}\Big)\Big)^2+\frac{1}{16}b^2x^4\log^2\Big(1+\frac{c}{x^2}\Big)+\frac{1}{8}b\mathrm{Subst}\Big(\int\frac{2a}{x}$$

$$=\frac{1}{16}x^4\Big(2a-b\log\Big(1-\frac{c}{x^2}\Big)\Big)^2+\frac{1}{4}abx^4\log\Big(1+\frac{c}{x^2}\Big)-\frac{1}{8}b^2x^4\log\Big(1-\frac{c}{x^2}$$

$$=\frac{1}{8}bc\Big(1-\frac{c}{x^2}\Big)x^2\Big(2a-b\log\Big(1-\frac{c}{x^2}\Big)\Big)+\frac{1}{16}x^4\Big(2a-b\log\Big(1-\frac{c}{x^2}\Big)\Big)^2+$$

$$=\frac{1}{8}bc\Big(1-\frac{c}{x^2}\Big)x^2\Big(2a-b\log\Big(1-\frac{c}{x^2}\Big)\Big)-\frac{1}{16}c^2\Big(2a-b\log\Big(1-\frac{c}{x^2}\Big)\Big)^2+$$

$$=\frac{1}{4}abcx^2+\frac{1}{8}bc\Big(1-\frac{c}{x^2}\Big)x^2\Big(2a-b\log\Big(1-\frac{c}{x^2}\Big)\Big)-\frac{1}{16}c^2\Big(2a-b\log\Big(1-$$

$$=\frac{1}{4}abcx^2-\frac{1}{8}b^2cx^2\log\Big(1-\frac{c}{x^2}\Big)+\frac{1}{8}bc\Big(1-\frac{c}{x^2}\Big)x^2\Big(2a-b\log\Big(1-\frac{c}{x^2}\Big)\Big)$$

$$=\frac{1}{4}abcx^2-\frac{1}{8}b^2cx^2\log\Big(1-\frac{c}{x^2}\Big)+\frac{1}{8}bc\Big(1-\frac{c}{x^2}\Big)x^2\Big(2a-b\log\Big(1-\frac{c}{x^2}\Big)\Big)$$

$$=\frac{1}{4}abcx^2-\frac{1}{8}b^2cx^2\log\Big(1-\frac{c}{x^2}\Big)+\frac{1}{8}bc\Big(1-\frac{c}{x^2}\Big)x^2\Big(2a-b\log\Big(1-\frac{c}{x^2}\Big)\Big)$$

$$=\frac{1}{4}abcx^2-\frac{1}{8}b^2cx^2\log\Big(1-\frac{c}{x^2}\Big)+\frac{1}{8}bc\Big(1-\frac{c}{x^2}\Big)x^2\Big(2a-b\log\Big(1-\frac{c}{x^2}\Big)\Big)$$

$$=\frac{1}{4}abcx^2-\frac{1}{8}b^2cx^2\log\Big(1-\frac{c}{x^2}\Big)+\frac{1}{8}bc\Big(1-\frac{c}{x^2}\Big)x^2\Big(2a-b\log\Big(1-\frac{c}{x^2}\Big)\Big)$$

$$=\frac{1}{4}abcx^2-\frac{1}{8}b^2cx^2\log\Big(1-\frac{c}{x^2}\Big)+\frac{1}{8}bc\Big(1-\frac{c}{x^2}\Big)x^2\Big(2a-b\log\Big(1-\frac{c}{x^2}\Big)\Big)$$

Mathematica [A]
time = 0.04, size = 104, normalized size = 1.11

$$\frac{1}{4}\Big(2abcx^2+a^2x^4+2bx^2(bc+ax^2)\tanh^{-1}\Big(\frac{c}{x^2}\Big)+b^2(-c^2+x^4)\tanh^{-1}\Big(\frac{c}{x^2}\Big)^2+b(a+b)c^2\log(-c+x^2)-abc^2\log(c+x^2)+b^2c^2\log(c+x^2)\Big)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] (2*a*b*c*x^2 + a^2*x^4 + 2*b*x^2*(b*c + a*x^2)*ArcTanh[c/x^2] + b^2*(-c^2 +
 x^4)*ArcTanh[c/x^2]^2 + b*(a + b)*c^2*Log[-c + x^2] - a*b*c^2*Log[c + x^2]
 + b^2*c^2*Log[c + x^2])/4
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^{3} \left(a+b\,\mathrm{arctanh}\left(\frac{c}{x^{2}}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c/x^2))^2,x)
```

```
[Out] int(x^3*(a+b*arctanh(c/x^2))^2,x)
```

Maxima [A]
time = 0.26, size = 157, normalized size = 1.67

$$\frac{1}{4}\,b^{2}x^{4}\,\mathrm{artanh}\left(\frac{c}{x^{2}}\right)^{2}+\frac{1}{4}\,a^{2}x^{4}+\frac{1}{4}\left(2\,x^{4}\,\mathrm{artanh}\left(\frac{c}{x^{2}}\right)+\left(2\,x^{2}-c\log\left(x^{2}+c\right)+c\log\left(x^{2}-c\right)\right)c\right)ab+\frac{1}{16}\left(\left(\log\left(x^{2}+c\right)^{2}-2\left(\log\left(x^{2}+c\right)-2\right)\log\left(x^{2}-c\right)+\log\left(x^{2}-c\right)^{2}+4\,\log\left(x^{2}+c\right)\right)c^{2}+4\left(2\,x^{2}-c\log\left(x^{2}+c\right)+c\log\left(x^{2}-c\right)\right)c\,\mathrm{artanh}\left(\frac{c}{x^{2}}\right)\right)b^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctanh(c/x^2)^2 + 1/4*a^2*x^4 + 1/4*(2*x^4*arctanh(c/x^2) + (2
*x^2 - c*log(x^2 + c) + c*log(x^2 - c))*c)*a*b + 1/16*((log(x^2 + c)^2 - 2*
(log(x^2 + c) - 2)*log(x^2 - c) + log(x^2 - c)^2 + 4*log(x^2 + c))*c^2 + 4*
(2*x^2 - c*log(x^2 + c) + c*log(x^2 - c))*c*arctanh(c/x^2))*b^2
```

Fricas [A]
time = 0.42, size = 126, normalized size = 1.34

$$\frac{1}{4}\,a^{2}x^{4}+\frac{1}{2}\,abcx^{2}-\frac{1}{4}\left(ab-b^{2}\right)c^{2}\log\left(x^{2}+c\right)+\frac{1}{4}\left(ab+b^{2}\right)c^{2}\log\left(x^{2}-c\right)+\frac{1}{16}\left(b^{2}x^{4}-b^{2}c^{2}\right)\log\left(\frac{x^{2}+c}{x^{2}-c}\right)^{2}+\frac{1}{4}\left(abx^{4}+b^{2}cx^{2}\right)\log\left(\frac{x^{2}+c}{x^{2}-c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*a^2*x^4 + 1/2*a*b*c*x^2 - 1/4*(a*b - b^2)*c^2*log(x^2 + c) + 1/4*(a*b +
 b^2)*c^2*log(x^2 - c) + 1/16*(b^2*x^4 - b^2*c^2)*log((x^2 + c)/(x^2 - c))^
2 + 1/4*(a*b*x^4 + b^2*c*x^2)*log((x^2 + c)/(x^2 - c))
```

Sympy [A]
time = 2.30, size = 151, normalized size = 1.61

$$\frac{a^{2}x^{4}}{4}-\frac{abc^{2}\,\mathrm{atanh}\left(\frac{c}{x^{2}}\right)}{2}+\frac{abcx^{2}}{2}+\frac{abx^{4}\,\mathrm{atanh}\left(\frac{c}{x^{2}}\right)}{2}+\frac{b^{2}c^{2}\log\left(x-\sqrt{-c}\right)}{2}+\frac{b^{2}c^{2}\log\left(x+\sqrt{-c}\right)}{2}-\frac{b^{2}c^{2}\,\mathrm{atanh}^{2}\left(\frac{c}{x^{2}}\right)}{4}-\frac{b^{2}c^{2}\,\mathrm{atanh}\left(\frac{c}{x^{2}}\right)}{2}+\frac{b^{2}cx^{2}\,\mathrm{atanh}\left(\frac{c}{x^{2}}\right)}{2}+\frac{b^{2}x^{4}\,\mathrm{atanh}^{2}\left(\frac{c}{x^{2}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c/x**2))**2,x)
```

```
[Out] a**2*x**4/4 - a*b*c**2*atanh(c/x**2)/2 + a*b*c*x**2/2 + a*b*x**4*atanh(c/x*
*2)/2 + b**2*c**2*log(x - sqrt(-c))/2 + b**2*c**2*log(x + sqrt(-c))/2 - b**
2*c**2*atanh(c/x**2)**2/4 - b**2*c**2*atanh(c/x**2)/2 + b**2*c*x**2*atanh(c
/x**2)/2 + b**2*x**4*atanh(c/x**2)**2/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(86) = 172$.
time = 0.42, size = 327, normalized size = 3.48

$$-\frac{2\,b^2c^3\log\left(\frac{x^2+c}{x^2-c}-1\right)-2\,b^2c^3\log\left(\frac{x^2+c}{x^2-c}\right)-\frac{(x^2+c)b^2c^3\log\left(\frac{x^2+c}{x^2-c}\right)^2}{(x^2-c)\left(\frac{(x^2+c)^2}{(x^2-c)^2}-\frac{2\,(x^2+c)}{x^2-c}+1\right)}-\frac{2\left(\frac{2\,(x^2+c)abc^3}{x^2-c}+\frac{(x^2+c)b^2c^3}{x^2-c}-b^2c^3\right)\log\left(\frac{x^2+c}{x^2-c}\right)}{\frac{(x^2+c)^2}{(x^2-c)^2}-\frac{2\,(x^2+c)}{x^2-c}+1}-\frac{4\left(\frac{(x^2+c)a^2c^3}{x^2-c}+\frac{(x^2+c)abc^3}{x^2-c}-abc^3\right)}{\frac{(x^2+c)^2}{(x^2-c)^2}-\frac{2\,(x^2+c)}{x^2-c}+1}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*b^2*c^3*log((x^2 + c)/(x^2 - c) - 1) - 2*b^2*c^3*log((x^2 + c)/(x^2
 - c)) - (x^2 + c)*b^2*c^3*log((x^2 + c)/(x^2 - c))^2/((x^2 - c)*((x^2 + c)
^2/(x^2 - c)^2 - 2*(x^2 + c)/(x^2 - c) + 1)) - 2*(2*(x^2 + c)*a*b*c^3/(x^2
- c) + (x^2 + c)*b^2*c^3/(x^2 - c) - b^2*c^3)*log((x^2 + c)/(x^2 - c))/((x^
2 + c)^2/(x^2 - c)^2 - 2*(x^2 + c)/(x^2 - c) + 1) - 4*((x^2 + c)*a^2*c^3/(x
^2 - c) + (x^2 + c)*a*b*c^3/(x^2 - c) - a*b*c^3)/((x^2 + c)^2/(x^2 - c)^2 -
 2*(x^2 + c)/(x^2 - c) + 1))/c
```

Mupad [B]
time = 1.21, size = 247, normalized size = 2.63

$$\frac{a^2\,x^4}{4}-\frac{a\,b\,c^2\,\ln\left(x^2+c\right)}{4}+\frac{a\,b\,c^2\,\ln\left(x^2-c\right)}{4}+\frac{a\,b\,c\,x^2}{2}+\frac{a\,b\,x^4\,\ln\left(x^2+c\right)}{4}-\frac{a\,b\,x^4\,\ln\left(x^2-c\right)}{4}-\frac{b^2\,c^2\ln\left(x^2+c\right)^2}{16}+\frac{b^2\,c^2\,\ln\left(x^2+c\right)\,\ln\left(x^2-c\right)}{8}+\frac{b^2\,c^2\,\ln\left(x^2+c\right)}{4}-\frac{b^2\,c^2\ln\left(x^2-c\right)^2}{16}+\frac{b^2\,c^2\,\ln\left(x^2-c\right)}{4}+\frac{b^2\,c\,x^2\,\ln\left(x^2+c\right)}{4}-\frac{b^2\,c\,x^2\,\ln\left(x^2-c\right)}{4}+\frac{b^2\,x^4\ln\left(x^2+c\right)^2}{16}-\frac{b^2\,x^4\,\ln\left(x^2+c\right)\,\ln\left(x^2-c\right)}{8}+\frac{b^2\,x^4\ln\left(x^2-c\right)^2}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c/x^2))^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*c^2*log(x^2 - c))/4 - (b^2*c^2*log(c + x^2)^2)/16 + (b^2
*x^4*log(c + x^2)^2)/16 - (b^2*c^2*log(x^2 - c)^2)/16 + (b^2*x^4*log(x^2 -
c)^2)/16 + (b^2*c^2*log(c + x^2))/4 + (a*b*x^4*log(c + x^2))/4 + (a*b*c^2*l
og(x^2 - c))/4 + (b^2*c^2*log(c + x^2)*log(x^2 - c))/8 + (a*b*c*x^2)/2 - (a
*b*x^4*log(x^2 - c))/4 + (b^2*c*x^2*log(c + x^2))/4 - (b^2*x^4*log(c + x^2)
*log(x^2 - c))/8 - (b^2*c*x^2*log(x^2 - c))/4 - (a*b*c^2*log(c + x^2))/4
```

3.172 $\int x\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2\,dx$

Optimal. Leaf size=94

$$-\frac{1}{2}c\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2+\frac{1}{2}x^2\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2-bc\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)\log\left(2-\frac{2}{1+\frac{c}{x^2}}\right)+\frac{1}{2}b^2c$$

```
[Out] -1/2*c*(a+b*arccoth(x^2/c))^2+1/2*x^2*(a+b*arccoth(x^2/c))^2-b*c*(a+b*arcco
th(x^2/c))*ln(2-2/(1+c/x^2))+1/2*b^2*c*polylog(2,-1+2/(1+c/x^2))
```

Rubi [A]
time = 0.12, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6039, 6037, 6135, 6079, 2497}

$$\frac{1}{2}x^2\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2-\frac{1}{2}c\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2-bc\log\left(2-\frac{2}{\frac{c}{x^2}+1}\right)\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)+\frac{1}{2}b^2c\,\text{Li}_2\left(\frac{2}{\frac{c}{x^2}+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] -1/2*(c*(a + b*ArcCoth[x^2/c])^2) + (x^2*(a + b*ArcCoth[x^2/c])^2)/2 - b*c*
(a + b*ArcCoth[x^2/c])*Log[2 - 2/(1 + c/x^2)] + (b^2*c*PolyLog[2, -1 + 2/(1
 + c/x^2)])/2
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx &= \int \left(\frac{1}{4}x\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2-\frac{1}{2}bx\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right.\right.\\
&= \frac{1}{4}\int x\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2 dx-\frac{1}{2}b\int x\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1\right.\\
&= -\left(\frac{1}{8}\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^2}{x^2}dx,x,\frac{1}{x^2}\right)\right)-\frac{1}{4}b\text{Subst}\left(\int\left(-2a+b\right.\right.\\
&= \frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{8}b^2\left(1+\frac{c}{x^2}\right)x^2\log^2\left(1+\frac{c}{x^2}\right)-\frac{1}{4}\\
&= \frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{8}b^2\left(1+\frac{c}{x^2}\right)x^2\log^2\left(1+\frac{c}{x^2}\right)+a\\
&= \frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1\right.\\
&= \frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1\right.\\
&= \frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1\right.\\
&= \frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1\right.\\
&= \frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1\right.\\
&= \frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1\right.\\
&= \frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1\right.\\
&= \frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1\right.
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 107, normalized size = 1.14

$$\frac{1}{2}\left(b^2\left(-c+x^2\right)\tanh^{-1}\left(\frac{c}{x^2}\right)^2+2b\tanh^{-1}\left(\frac{c}{x^2}\right)\left(ax^2-bc\log\left(1-e^{-2\tanh^{-1}\left(\frac{c}{x^2}\right)}\right)\right)+a\left(ax^2+bc\log\left(1-\frac{c^2}{x^4}\right)-2bc\log\left(\frac{c}{x^2}\right)\right)+b^2c\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(\frac{c}{x^2}\right)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] (b^2*(-c + x^2)*ArcTanh[c/x^2]^2 + 2*b*ArcTanh[c/x^2]*(a*x^2 - b*c*Log[1 -
E^(-2*ArcTanh[c/x^2])]) + a*(a*x^2 + b*c*Log[1 - c^2/x^4] - 2*b*c*Log[c/x^2
]) + b^2*c*PolyLog[2, E^(-2*ArcTanh[c/x^2])])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 6869, normalized size = 73.07

method	result	size
risch	Expression too large to display	6869

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c/x^2))^2,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
[Out] 1/2*a^2*x^2 + 1/2*(2*x^2*arctanh(c/x^2) + c*log(x^4 - c^2))*a*b + 1/8*(x^2*
log(x^2 + c)^2 - 2*(x^2 + c)*log(x^2 + c)*log(x^2 - c) + (x^2 - c)*log(x^2
- c)^2 + 2*integrate(2*(3*c*x^3 + c^2*x)*log(x^2 + c)/(x^4 - c^2), x))*b^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
[Out] integral(b^2*x*arctanh(c/x^2)^2 + 2*a*b*x*arctanh(c/x^2) + a^2*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a + b\operatorname{atanh}\left(\frac{c}{x^{2}}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c/x**2))**2,x)
[Out] Integral(x*(a + b*atanh(c/x**2))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c/x^2))^2,x)
```

```
[Out] int(x*(a + b*atanh(c/x^2))^2, x)
```

3.173 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x}\,dx$

Optimal. Leaf size=144

$$-\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x^2}}\right)+\frac{1}{2}b\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x^2}}\right)-\frac{1}{2}b\Big(a$$

```
[Out] (a+b*arccoth(x^2/c))^2*arctanh(-1+2/(1-c/x^2))+1/2*b*(a+b*arccoth(x^2/c))*p
olylog(2,1-2/(1-c/x^2))-1/2*b*(a+b*arccoth(x^2/c))*polylog(2,-1+2/(1-c/x^2)
)-1/4*b^2*polylog(3,1-2/(1-c/x^2))+1/4*b^2*polylog(3,-1+2/(1-c/x^2))
```

Rubi [A]
time = 0.22, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6035, 6033, 6199, 6095, 6205, 6745}

$$\frac{1}{2}b\text{Li}_2\left(1-\frac{2}{1-\frac{c}{x^2}}\right)\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)-\frac{1}{2}b\text{Li}_2\left(\frac{2}{1-\frac{c}{x^2}}-1\right)\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)-\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x^2}}\right)\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2-\frac{1}{4}b^2\text{Li}_3\left(1-\frac{2}{1-\frac{c}{x^2}}\right)+\frac{1}{4}b^2\text{Li}_3\left(\frac{2}{1-\frac{c}{x^2}}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2/x,x]
```

```
[Out] -((a + b*ArcCoth[x^2/c])^2*ArcTanh[1 - 2/(1 - c/x^2)]) + (b*(a + b*ArcCoth[
x^2/c])*PolyLog[2, 1 - 2/(1 - c/x^2)])/2 - (b*(a + b*ArcCoth[x^2/c])*PolyLo
g[2, -1 + 2/(1 - c/x^2)])/2 - (b^2*PolyLog[3, 1 - 2/(1 - c/x^2)])/4 + (b^2*
PolyLog[3, -1 + 2/(1 - c/x^2)])/4
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x^2}}\right)+(2bc)\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}\right.}{}\right.\\
&= -\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x^2}}\right)-(bc)\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}\right.}{1-}\right.\\
&= -\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x^2}}\right)+\frac{1}{2}b\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)\text{Li}\\
&= -\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x^2}}\right)+\frac{1}{2}b\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)\text{Li}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 183, normalized size = 1.27

$$
\frac{1}{2}\left(-2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x^2}}\right)\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2+4bc\left(\frac{1}{2}\left(\frac{\left(-a-b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)\text{PolyLog}\left(2,\frac{-1-\frac{c}{x^2}}{-1+\frac{c}{x^2}}\right)}{2c}+\frac{b\text{PolyLog}\left(3,\frac{-1-\frac{c}{x^2}}{-1+\frac{c}{x^2}}\right)}{4c}\right)+\frac{1}{2}\left(-\frac{\left(-a-b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)\text{PolyLog}\left(2,\frac{1+\frac{c}{x^2}}{-1+\frac{c}{x^2}}\right)}{2c}-\frac{b\text{PolyLog}\left(3,\frac{1+\frac{c}{x^2}}{-1+\frac{c}{x^2}}\right)}{4c}\right)\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2/x,x]
```

```
[Out] (-2*ArcTanh[1 - 2/(1 - c/x^2)]*(a + b*ArcTanh[c/x^2])^2 + 4*b*c*((((-a - b*
ArcTanh[c/x^2])*PolyLog[2, (-1 - c/x^2)/(-1 + c/x^2)])/(2*c) + (b*PolyLog[3
, (-1 - c/x^2)/(-1 + c/x^2)])/(4*c))/2 + (-1/2*((-a - b*ArcTanh[c/x^2])*Pol
yLog[2, (1 + c/x^2)/(-1 + c/x^2)])/c - (b*PolyLog[3, (1 + c/x^2)/(-1 + c/x^
2)])/(4*c))/2))/2
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2/x,x)
```

```
[Out] int((a+b*arctanh(c/x^2))^2/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + integrate(1/4*b^2*(log(c/x^2 + 1) - log(-c/x^2 + 1))^2/x + a*b
*(log(c/x^2 + 1) - log(-c/x^2 + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c/x**2))**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))^2/x,x)
```

```
[Out] int((a + b*atanh(c/x^2))^2/x, x)
```

3.174 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=99

$$-\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{2c}-\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{2x^2}+\frac{b\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)\log\left(\frac{2}{1-\frac{c}{x^2}}\right)}{c}+\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x^2}}\right)}{2c}$$

```
[Out] -1/2*(a+b*arccoth(x^2/c))^2/c-1/2*(a+b*arccoth(x^2/c))^2/x^2+b*(a+b*arccoth
(x^2/c))*ln(2/(1-c/x^2))/c+1/2*b^2*polylog(2,1-2/(1-c/x^2))/c
```

Rubi [A]
time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6039, 6021, 6131, 6055, 2449, 2352}

$$-\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{2c}-\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{2x^2}+\frac{b\log\left(\frac{2}{1-\frac{c}{x^2}}\right)\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)}{c}+\frac{b^2\text{Li}_2\left(1-\frac{2}{1-\frac{c}{x^2}}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2/x^3,x]
```

```
[Out] -1/2*(a + b*ArcCoth[x^2/c])^2/c - (a + b*ArcCoth[x^2/c])^2/(2*x^2) + (b*(a
+ b*ArcCoth[x^2/c])*Log[2/(1 - c/x^2)])/c + (b^2*PolyLog[2, 1 - 2/(1 - c/x^
2)])/(2*c)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x^3} - \frac{b\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{2x^3} + \frac{b^2\log^2(\ldots}{4\ldots}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{x^3}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{x^3}\,dx\ldots$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int (2a-b\log(1-cx))^2\,dx, x, \frac{1}{x^2}\right)\right) + \frac{1}{4}b\text{Subst}\left(\int (-2a+b\log(\ldots\right.$$

$$= -\frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{4x^2} + \frac{\text{Subst}\left(\int (2a-b\log(x))^2\,dx, x, 1-\frac{c}{x^2}\ldots\right.}{8c}$$

$$= \frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{8c} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{4x^2} - \frac{b^2(1\ldots}{\ldots}$$

$$= -\frac{ab}{2x^2} - \frac{b^2}{4x^2} + \frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{8c} + \frac{b^2\left(1+\frac{c}{x^2}\right)\log\left(1+\frac{c}{x^2}\right)}{4c} - \ldots$$

$$= -\frac{b^2}{2x^2} - \frac{b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\frac{c}{x^2}\right)}{4c} + \frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{8c} - \frac{b(2a-\ldots}{\ldots}$$

$$= -\frac{b^2}{4x^2} - \frac{b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\frac{c}{x^2}\right)}{4c} + \frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{8c} - \frac{b(2a-\ldots}{\ldots}$$

$$= \frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{8c} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(\frac{1}{2}\left(1+\frac{c}{x^2}\right)\right)}{4c} - \frac{b^2\ldots}{\ldots}$$

Mathematica [A]
time = 0.12, size = 114, normalized size = 1.15

$$-\frac{a^2}{2x^2} - \frac{ab\left(\frac{c\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^2} - \log\left(\frac{1}{\sqrt{1-\frac{c^2}{x^4}}}\right)\right)}{c} - \frac{b^2\left(\tanh^{-1}\left(\frac{c}{x^2}\right)\left(-\tanh^{-1}\left(\frac{c}{x^2}\right) + \frac{c\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^2} - 2\log\left(1+e^{-2\tanh^{-1}\left(\frac{c}{x^2}\right)}\right)\right) + \text{PolyLog}\left(2, -e^{-2\tanh^{-1}\left(\frac{c}{x^2}\right)}\right)\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 - (a*b*((c*ArcTanh[c/x^2])/x^2 - Log[1/Sqrt[1 - c^2/x^4]]))/c
- (b^2*(ArcTanh[c/x^2]*(-ArcTanh[c/x^2] + (c*ArcTanh[c/x^2])/x^2 - 2*Log[1
+ E^(-2*ArcTanh[c/x^2])]) + PolyLog[2, -E^(-2*ArcTanh[c/x^2])]))/(2*c)
```

Maple [A]
time = 0.64, size = 137, normalized size = 1.38

method	result

derivativedivides	$-\frac{\frac{c\,a^2}{x^2}+\frac{\operatorname{arctanh}\left(\frac{c}{x^2}\right)^2 b^2 c}{x^2}+b^2\operatorname{arctanh}\left(\frac{c}{x^2}\right)^2-2\operatorname{arctanh}\left(\frac{c}{x^2}\right)\ln\left(1+\frac{\left(1+\frac{c}{x^2}\right)^2}{1-\frac{c^2}{x^4}}\right)b^2-\operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x^2}\right)^2}{1-\frac{c^2}{x^4}}\right)b^2+\frac{2abc\,a}{}}{2c}$
default	$-\frac{\frac{c\,a^2}{x^2}+\frac{\operatorname{arctanh}\left(\frac{c}{x^2}\right)^2 b^2 c}{x^2}+b^2\operatorname{arctanh}\left(\frac{c}{x^2}\right)^2-2\operatorname{arctanh}\left(\frac{c}{x^2}\right)\ln\left(1+\frac{\left(1+\frac{c}{x^2}\right)^2}{1-\frac{c^2}{x^4}}\right)b^2-\operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x^2}\right)^2}{1-\frac{c^2}{x^4}}\right)b^2+\frac{2abc\,a}{}}{2c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c*(c/x^2*a^2+arctanh(c/x^2)^2*b^2*c/x^2+b^2*arctanh(c/x^2)^2-2*arctanh
(c/x^2)*ln(1+(1+c/x^2)^2/(1-c^2/x^4))*b^2-polylog(2,-(1+c/x^2)^2/(1-c^2/x^4
))*b^2+2*a*b*c/x^2*arctanh(c/x^2)+a*b*ln(1-c^2/x^4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(8*c^3*integrate(log(x)^2/(c*x^7 - c^3*x^3), x) + c^2*(log(x^2 + c)/c^3
 + log(x^2 - c)/c^3 - 4*log(x)/c^3) - 8*c^2*integrate(x^2*log(x^2 + c)/(c*x
^7 - c^3*x^3), x) + 8*c^2*integrate(x^2*log(x)/(c*x^7 - c^3*x^3), x) + 2*c*
(log(x^2 - c)/c^2 - log(x^2)/c^2 + 1/(c*x^2))*log(-c/x^2 + 1) - c*(log(x^2
+ c)/c^2 - log(x^2 - c)/c^2) - 8*c*integrate(x^4*log(x)^2/(c*x^7 - c^3*x^3)
, x) - 4*c*integrate(x^4*log(x^2 + c)/(c*x^7 - c^3*x^3), x) + 16*c*integrat
e(x^4*log(x)/(c*x^7 - c^3*x^3), x) - log(-c/x^2 + 1)^2/x^2 - (x^2*log(x^2 -
 c)^2 + 4*x^2*log(x)^2 - 4*x^2*log(x) - 2*(2*x^2*log(x) - x^2)*log(x^2 - c)
 + 2*c)/(c*x^2) - (c*log(x^2 + c)^2 - 2*((x^2 + c)*log(x^2 + c) - 2*(x^2 +
c)*log(x) - c)*log(x^2 - c))/(c*x^2) - 4*integrate(x^6*log(x^2 + c)/(c*x^7
- c^3*x^3), x) + 8*integrate(x^6*log(x)/(c*x^7 - c^3*x^3), x))*b^2 - 1/2*a*
b*(2*c*arctanh(c/x^2)/x^2 + log(-c^2/x^4 + 1))/c - 1/2*a^2/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x**3,x)
[Out] Integral((a + b*atanh(c/x**2))**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^3,x, algorithm="giac")
[Out] integrate((b*arctanh(c/x^2) + a)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))^2/x^3,x)
[Out] int((a + b*atanh(c/x^2))^2/x^3, x)
```

3.175 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^5} dx$

Optimal. Leaf size=97

$$-\frac{ab}{2cx^2}-\frac{b^2\coth^{-1}\left(\frac{x^2}{c}\right)}{2cx^2}+\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{4c^2}-\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{4x^4}-\frac{b^2\log\left(1-\frac{c^2}{x^4}\right)}{4c^2}$$

```
[Out] -1/2*a*b/c/x^2-1/2*b^2*arccoth(x^2/c)/c/x^2+1/4*(a+b*arccoth(x^2/c))^2/c^2-
1/4*(a+b*arccoth(x^2/c))^2/x^4-1/4*b^2*ln(1-c^2/x^4)/c^2
```

Rubi [A]
time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6039, 6037, 6127, 6021, 266, 6095}

$$\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{4c^2}-\frac{ab}{2cx^2}-\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{4x^4}-\frac{b^2\log\left(1-\frac{c^2}{x^4}\right)}{4c^2}-\frac{b^2\coth^{-1}\left(\frac{x^2}{c}\right)}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2/x^5,x]
```

```
[Out] -1/2*(a*b)/(c*x^2) - (b^2*ArcCoth[x^2/c])/(2*c*x^2) + (a + b*ArcCoth[x^2/c]
)^2/(4*c^2) - (a + b*ArcCoth[x^2/c])^2/(4*x^4) - (b^2*Log[1 - c^2/x^4])/(4*
c^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^5}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x^5} - \frac{b\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{2x^5} + \frac{b^2\log^2\left(1\right.}{4x^5}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{x^5}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{x^5}\,dx +$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int x(2a-b\log(1-cx))^2\,dx, x, \frac{1}{x^2}\right)\right) - \frac{1}{4}b\text{Subst}\left(\int \frac{(-2a+b\log}{}\right.$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int \left(\frac{(2a-b\log(1-cx))^2}{c} - \frac{(1-cx)(2a-b\log(1-cx))^2}{c}\right)dx, x\right.\right.$$

$$= \frac{1}{2}(ab)\text{Subst}\left(\int \frac{\log\left(1+\frac{c}{x}\right)}{x^3}\,dx, x, x^2\right) - \frac{1}{4}b^2\text{Subst}\left(\int \frac{\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)}{x^3}\right.$$

$$= \frac{b^2\log\left(1-\frac{c}{x^2}\right)\log\left(1+\frac{c}{x^2}\right)}{8x^4} - \frac{1}{2}(ab)\text{Subst}\left(\int x\log(1+cx)\,dx, x, \frac{1}{x^2}\right) + \frac{1}{4}b^2\text{Su}$$

$$= \frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{8c^2} - \frac{\left(1-\frac{c}{x^2}\right)^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{16c^2} - \frac{ab\log\left(1\right.}{4x^4}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} - \frac{ab}{2cx^2} + \frac{b^2}{4cx^2} - \frac{b\left(1-\frac{c}{x^2}\right)^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right.}{16c^2}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} - \frac{3ab}{4cx^2} - \frac{b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\frac{c}{x^2}\right)}{4c^2} - \frac{b\left(1-\right.}{}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} - \frac{3ab}{4cx^2} - \frac{b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\frac{c}{x^2}\right)}{4c^2} - \frac{b\left(1-\right.}{}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} - \frac{3ab}{4cx^2} - \frac{b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\frac{c}{x^2}\right)}{4c^2} - \frac{b^2\log}{1}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} - \frac{3ab}{4cx^2} - \frac{3b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\frac{c}{x^2}\right)}{8c^2} - \frac{b^2\log}{}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} + \frac{b^2}{16x^4} - \frac{3ab}{4cx^2} + \frac{b^2\log\left(1-\frac{c}{x^2}\right)}{16c^2} - \frac{3b^2(1}{}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} + \frac{b^2}{16x^4} - \frac{3ab}{4cx^2} + \frac{b^2\log\left(1-\frac{c}{x^2}\right)}{16c^2} - \frac{3b^2(1}{}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} + \frac{b^2}{16x^4} - \frac{3ab}{4cx^2} + \frac{b^2\log\left(1-\frac{c}{x^2}\right)}{16c^2} - \frac{3b^2(1}{}$$

Mathematica [A]
time = 0.05, size = 131, normalized size = 1.35

$$-\frac{a^2c^2+2abcx^2+2bc(ac+bx^2)\tanh^{-1}\left(\frac{c}{x^2}\right)+b^2(c^2-x^4)\tanh^{-1}\left(\frac{c}{x^2}\right)^2-4b^2x^4\log(x)+abx^4\log\left(-c+x^2\right)+b^2x^4\log\left(-c+x^2\right)-abx^4\log\left(c+x^2\right)+b^2x^4\log\left(c+x^2\right)}{4c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2/x^5,x]
```

```
[Out] -1/4*(a^2*c^2 + 2*a*b*c*x^2 + 2*b*c*(a*c + b*x^2)*ArcTanh[c/x^2] + b^2*(c^2
 - x^4)*ArcTanh[c/x^2]^2 - 4*b^2*x^4*Log[x] + a*b*x^4*Log[-c + x^2] + b^2*x
^4*Log[-c + x^2] - a*b*x^4*Log[c + x^2] + b^2*x^4*Log[c + x^2])/(c^2*x^4)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2/x^5,x)
```

```
[Out] int((a+b*arctanh(c/x^2))^2/x^5,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(87) = 174$.
time = 0.26, size = 183, normalized size = 1.89

$$\frac{1}{4}\left(c\left(\frac{\log\left(x^2+c\right)}{c^3}-\frac{\log\left(x^2-c\right)}{c^3}-\frac{2}{c^2x^2}\right)-\frac{2\,\mathrm{artanh}\left(\frac{c}{x^2}\right)}{x^4}\right)ab-\frac{1}{16}\left(c^2\left(\frac{\log\left(x^2+c\right)^2-2\left(\log\left(x^2+c\right)-2\right)\log\left(x^2-c\right)+\log\left(x^2-c\right)^2+4\,\log\left(x^2+c\right)}{c^4}-\frac{16\,\log\left(x\right)}{c^4}\right)-4\,c\left(\frac{\log\left(x^2+c\right)}{c^3}-\frac{\log\left(x^2-c\right)}{c^3}-\frac{2}{c^2x^2}\right)\mathrm{artanh}\left(\frac{c}{x^2}\right)\right)b^2-\frac{b^2\,\mathrm{artanh}\left(\frac{c}{x^2}\right)^2}{4\,x^4}-\frac{a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(c*(log(x^2 + c)/c^3 - log(x^2 - c)/c^3 - 2/(c^2*x^2)) - 2*arctanh(c/x^
2)/x^4)*a*b - 1/16*(c^2*((log(x^2 + c)^2 - 2*(log(x^2 + c) - 2)*log(x^2 - c
) + log(x^2 - c)^2 + 4*log(x^2 + c))/c^4 - 16*log(x)/c^4) - 4*c*(log(x^2 +
c)/c^3 - log(x^2 - c)/c^3 - 2/(c^2*x^2))*arctanh(c/x^2))*b^2 - 1/4*b^2*arct
anh(c/x^2)^2/x^4 - 1/4*a^2/x^4
```

Fricas [A]
time = 0.38, size = 143, normalized size = 1.47

$$\frac{16\,b^2x^4\log\left(x\right)+4\,(ab-b^2)x^4\log\left(x^2+c\right)-4\,(ab+b^2)x^4\log\left(x^2-c\right)-8\,abcx^2-4\,a^2c^2+(b^2x^4-b^2c^2)\log\left(\frac{x^2+c}{x^2-c}\right)^2-4\,(b^2cx^2+abc^2)\log\left(\frac{x^2+c}{x^2-c}\right)}{16\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/16*(16*b^2*x^4*log(x) + 4*(a*b - b^2)*x^4*log(x^2 + c) - 4*(a*b + b^2)*x^
4*log(x^2 - c) - 8*a*b*c*x^2 - 4*a^2*c^2 + (b^2*x^4 - b^2*c^2)*log((x^2 + c
)/(x^2 - c))^2 - 4*(b^2*c*x^2 + a*b*c^2)*log((x^2 + c)/(x^2 - c)))/(c^2*x^4
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(78) = 156$.
time = 6.08, size = 172, normalized size = 1.77

$$\begin{cases} -\frac{a^2}{4x^4} - \frac{ab\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2x^4} - \frac{ab}{2cx^2} + \frac{ab\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2c^2} - \frac{b^2\operatorname{atanh}^2\left(\frac{c}{x^2}\right)}{4x^4} - \frac{b^2\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2cx^2} + \frac{b^2\log(x)}{c^2} - \frac{b^2\log\left(x-\sqrt{-c}\right)}{2c^2} - \frac{b^2\log\left(x+\sqrt{-c}\right)}{2c^2} + \frac{b^2\operatorname{atanh}^2\left(\frac{c}{x^2}\right)}{4c^2} + \frac{b^2\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2c^2} & \text{for } c \neq 0 \\ -\frac{a^2}{4x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x**5,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) - a*b*atanh(c/x**2)/(2*x**4) - a*b/(2*c*x**2) + a
*b*atanh(c/x**2)/(2*c**2) - b**2*atanh(c/x**2)**2/(4*x**4) - b**2*atanh(c/x
**2)/(2*c*x**2) + b**2*log(x)/c**2 - b**2*log(x - sqrt(-c))/(2*c**2) - b**2
*log(x + sqrt(-c))/(2*c**2) + b**2*atanh(c/x**2)**2/(4*c**2) + b**2*atanh(c
/x**2)/(2*c**2), Ne(c, 0)), (-a**2/(4*x**4), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2/x^5, x)
```

Mupad [B]
time = 1.54, size = 262, normalized size = 2.70

$$\frac{b^2\ln(x^2+c)^2}{16c^2} - \frac{b^2\ln(x^2-c)}{4c^2} - \frac{a^2}{4x^4} - \frac{b^2\ln(x^2+c)^2}{16x^4} + \frac{b^2\ln(x^2-c)^2}{16c^2} - \frac{b^2\ln(x^2-c)^2}{16x^4} + \frac{b^2\ln(x)}{c^2} - \frac{b^2\ln(x^2+c)}{4c^2} - \frac{ab\ln(x^2+c)}{4x^4} + \frac{b^2\ln(x^2-c)}{4cx^2} - \frac{ab\ln(x^2-c)}{4c^2} - \frac{b^2\ln(x^2+c)\ln(x^2-c)}{8c^2} + \frac{ab\ln(x^2-c)}{4x^4} + \frac{b^2\ln(x^2+c)\ln(x^2-c)}{8x^4} - \frac{ab}{2cx^2} - \frac{b^2\ln(x^2+c)}{4cx^2} + \frac{ab\ln(x^2+c)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))^2/x^5,x)
```

```
[Out] (b^2*log(c + x^2)^2)/(16*c^2) - (b^2*log(x^2 - c))/(4*c^2) - a^2/(4*x^4) -
(b^2*log(c + x^2)^2)/(16*x^4) + (b^2*log(x^2 - c)^2)/(16*c^2) - (b^2*log(x^
2 - c)^2)/(16*x^4) + (b^2*log(x))/c^2 - (b^2*log(c + x^2))/(4*c^2) - (a*b*l
og(c + x^2))/(4*x^4) + (b^2*log(x^2 - c))/(4*c*x^2) - (a*b*log(x^2 - c))/(4
*c^2) - (b^2*log(c + x^2)*log(x^2 - c))/(8*c^2) + (a*b*log(x^2 - c))/(4*x^4
) + (b^2*log(c + x^2)*log(x^2 - c))/(8*x^4) - (a*b)/(2*c*x^2) - (b^2*log(c
+ x^2))/(4*c*x^2) + (a*b*log(c + x^2))/(4*c^2)
```

3.176 $\int x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$

Optimal. Leaf size=1214

$$\frac{8}{15}b^2c^2x+\frac{2}{15}abcx^3+\frac{2}{5}abc^{5/2}\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)-\frac{4}{15}b^2c^{5/2}\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{5}ib^2c^{5/2}\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)^2-\frac{4}{15}b^2c^{5/2}$$

```
[Out] 1/20*x^5*(2*a-b*ln(1-c/x^2))^2-1/15*b^2*c*x^3*ln(1-c/x^2)-1/5*b^2*c^(5/2)*a
rctan(x/c^(1/2))*ln(1-c/x^2)+1/15*b*c*x^3*(2*a-b*ln(1-c/x^2))-1/5*b*c^(5/2)
*arctanh(x/c^(1/2))*(2*a-b*ln(1-c/x^2))+2/15*b^2*c*x^3*ln(1+c/x^2)+1/5*a*b*
x^5*ln(1+c/x^2)+1/5*b^2*c^(5/2)*arctan(x/c^(1/2))*ln(1+c/x^2)-1/5*b^2*c^(5/
2)*arctanh(x/c^(1/2))*ln(1+c/x^2)-1/10*b^2*x^5*ln(1-c/x^2)*ln(1+c/x^2)-2/5*
b^2*c^(5/2)*arctan(x/c^(1/2))*ln(2*c^(1/2)/(-I*x+c^(1/2)))+1/5*b^2*c^(5/2)*
arctan(x/c^(1/2))*ln((1+I)*(-x+c^(1/2))/(-I*x+c^(1/2)))-2/5*b^2*c^(5/2)*arc
tanh(x/c^(1/2))*ln(2*c^(1/2)/(x+c^(1/2)))+1/5*b^2*c^(5/2)*arctanh(x/c^(1/2)
)*ln(2*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/2)))+1/5*b^2*c^
(5/2)*arctan(x/c^(1/2))*ln((1-I)*(x+c^(1/2))/(-I*x+c^(1/2)))+1/5*b^2*c^(5/2
)*arctanh(x/c^(1/2))*ln(2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(1/2)+c^
(1/2)))+2/5*b^2*c^(5/2)*arctan(x/c^(1/2))*ln(2-2*c^(1/2)/(-I*x+c^(1/2)))+2/
5*b^2*c^(5/2)*arctanh(x/c^(1/2))*ln(2-2*c^(1/2)/(x+c^(1/2)))-1/5*I*b^2*c^(5
/2)*arctan(x/c^(1/2))^2-1/5*I*b^2*c^(5/2)*polylog(2,-I*x/c^(1/2))-1/5*I*b^2
*c^(5/2)*polylog(2,-1+2*c^(1/2)/(-I*x+c^(1/2)))-1/10*I*b^2*c^(5/2)*polylog(
2,1-(1+I)*(-x+c^(1/2))/(-I*x+c^(1/2)))-1/10*I*b^2*c^(5/2)*polylog(2,1+(-1+I
)*(x+c^(1/2))/(-I*x+c^(1/2)))+2/5*a*b*c^(5/2)*arctan(x/c^(1/2))+8/15*b^2*c^
2*x+2/15*a*b*c*x^3+1/5*I*b^2*c^(5/2)*polylog(2,I*x/c^(1/2))+1/5*I*b^2*c^(5/
2)*polylog(2,1-2*c^(1/2)/(-I*x+c^(1/2)))-4/15*b^2*c^(5/2)*arctan(x/c^(1/2))
-4/15*b^2*c^(5/2)*arctanh(x/c^(1/2))+1/5*b^2*c^(5/2)*arctanh(x/c^(1/2))^2+1
/20*b^2*x^5*ln(1+c/x^2)^2+1/5*b^2*c^(5/2)*polylog(2,-x/c^(1/2))-1/5*b^2*c^(
5/2)*polylog(2,x/c^(1/2))+1/5*b^2*c^(5/2)*polylog(2,1-2*c^(1/2)/(x+c^(1/2))
)-1/5*b^2*c^(5/2)*polylog(2,-1+2*c^(1/2)/(x+c^(1/2)))-1/10*b^2*c^(5/2)*poly
log(2,1-2*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/2)))-1/10*b^
2*c^(5/2)*polylog(2,1-2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(1/2)+c^(1
/2)))
```

Rubi [A]
time = 1.93, antiderivative size = 1214, normalized size of antiderivative = 1.00, number of steps used = 98, number of rules used = 34, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 2.125$, Rules used = {6045, 6042, 2507, 2526, 2498, 269, 213, 2505, 199, 327, 2520, 12, 266, 6820, 6135, 6079, 2497, 308, 6874, 209, 30, 2637, 6139, 6031, 6057, 2449, 2352, 210, 5048, 4940, 2438, 4966, 5044, 4988}

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] (8*b^2*c^2*x)/15 + (2*a*b*c*x^3)/15 + (2*a*b*c^(5/2)*ArcTan[x/Sqrt[c]])/5 -
 (4*b^2*c^(5/2)*ArcTan[x/Sqrt[c]])/15 - (I/5)*b^2*c^(5/2)*ArcTan[x/Sqrt[c]]
^2 - (4*b^2*c^(5/2)*ArcTanh[x/Sqrt[c]])/15 + (b^2*c^(5/2)*ArcTanh[x/Sqrt[c]
]^2)/5 + (2*b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*
x)])/5 - (b^2*c*x^3*Log[1 - c/x^2])/15 - (b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log
[1 - c/x^2])/5 + (b*c*x^3*(2*a - b*Log[1 - c/x^2]))/15 - (b*c^(5/2)*ArcTanh
[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/5 + (x^5*(2*a - b*Log[1 - c/x^2])^2)/
20 + (2*b^2*c*x^3*Log[1 + c/x^2])/15 + (a*b*x^5*Log[1 + c/x^2])/5 + (b^2*c^
(5/2)*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/5 - (b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]
*Log[1 + c/x^2])/5 - (b^2*x^5*Log[1 - c/x^2]*Log[1 + c/x^2])/10 + (b^2*x^5*
Log[1 + c/x^2]^2)/20 - (2*b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sq
rt[c] - I*x)])/5 + (b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[c] - x
))/(Sqrt[c] - I*x)])/5 - (2*b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c])/
(Sqrt[c] + x)])/5 + (b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c
] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/5 + (b^2*c^(5/2)*ArcTanh[x/S
qrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))
])/5 + (b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c]
- I*x)])/5 + (2*b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c]
 + x)])/5 + (I/5)*b^2*c^(5/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)] -
 (I/5)*b^2*c^(5/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)] - (I/10)*b^
2*c^(5/2)*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] + (b^2*c^
(5/2)*PolyLog[2, -(x/Sqrt[c])])/5 - (I/5)*b^2*c^(5/2)*PolyLog[2, ((-I)*x)/S
qrt[c]] + (I/5)*b^2*c^(5/2)*PolyLog[2, (I*x)/Sqrt[c]] - (b^2*c^(5/2)*PolyLo
g[2, x/Sqrt[c]])/5 + (b^2*c^(5/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] + x)]
)/5 - (b^2*c^(5/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/5 - (b^2*c^(
5/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[
c] + x))])/10 - (b^2*c^(5/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sq
rt[-c] + Sqrt[c])*(Sqrt[c] + x))])/10 - (I/10)*b^2*c^(5/2)*PolyLog[2, 1 - (
(1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6042

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + 1/(x^n*c)]/2) - b*(Log[1 - 1/(x^n*c
)]/2))^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rule 6045

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[x^m*(a + b*ArcCoth[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c, m}, x] && IGtQ[
p, 1] && ILtQ[n, 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(\frac{1}{4}x^4\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2 - \frac{1}{2}bx^4\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\right.\right.$$

$$= \frac{1}{4}\int x^4\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2 dx - \frac{1}{2}b\int x^4\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(\right.$$

$$= \frac{1}{20}x^5\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2 + \frac{1}{20}b^2x^5\log^2\left(1+\frac{c}{x^2}\right) - \frac{1}{2}b\int\left(-2ax^4\log\left(\right.\right.$$

$$= \frac{1}{20}x^5\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2 + \frac{1}{20}b^2x^5\log^2\left(1+\frac{c}{x^2}\right) + (ab)\int x^4\log\left(1+\right.$$

$$= \frac{1}{20}x^5\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2 + \frac{1}{5}abx^5\log\left(1+\frac{c}{x^2}\right) - \frac{1}{10}b^2x^5\log\left(1-\frac{c}{x^2}\right)$$

$$= \frac{2}{5}abc^2x + \frac{1}{15}bcx^3\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right) - \frac{1}{5}bc^{5/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\,\mathrm{l}\right.$$

$$= \frac{2}{5}abc^2x - \frac{1}{5}b^2c^2x\log\left(1-\frac{c}{x^2}\right) + \frac{1}{15}bcx^3\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right) - \frac{1}{5}bc^{5/2}\,\mathrm{ta}$$

$$= \frac{4}{15}b^2c^2x + \frac{2}{15}abcx^3 - \frac{2}{5}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{5}b^2c^2x\log\left(1-\frac{c}{x^2}\right) + \frac{1}{15}bc$$

$$= \frac{4}{15}b^2c^2x + \frac{2}{15}abcx^3 + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{8}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{5}$$

$$= \frac{4}{15}b^2c^2x + \frac{2}{15}abcx^3 + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{8}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{5}$$

$$= \frac{8}{15}b^2c^2x + \frac{2}{15}abcx^3 + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{2}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{5}$$

$$= \frac{8}{15}b^2c^2x + \frac{2}{15}abcx^3 + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{5}$$

$$= \frac{8}{15}b^2c^2x + \frac{2}{15}abcx^3 + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{5}$$

$$= \frac{8}{15}b^2c^2x + \frac{2}{15}abcx^3 + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{5}$$

$$\frac{8}{15}b^2c^2x + \frac{2}{15}abcx^3 + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{}$$

Mathematica [F]
time = 9.13, size = 0, normalized size = 0.00

$$\int x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2\,dx$$

Verification is not applicable to the result.

[In] Integrate[x^4*(a + b*ArcTanh[c/x^2])^2,x]

[Out] Integrate[x^4*(a + b*ArcTanh[c/x^2])^2, x]

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int x^4\left(a+b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(a+b*arctanh(c/x^2))^2,x)

[Out] int(x^4*(a+b*arctanh(c/x^2))^2,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")

[Out] 1/5*a^2*x^5 + 1/15*(6*x^5*arctanh(c/x^2) + (4*x^3 + 6*c^(3/2)*arctan(x/sqrt(c)) + 3*c^(3/2)*log((x - sqrt(c))/(x + sqrt(c))))*c)*a*b + 1/20*(x^5*log(x^2 - c)^2 - 5*integrate(-1/5*(5*(x^6 - c*x^4)*log(x^2 + c)^2 - 2*(2*x^6 + 5*(x^6 - c*x^4)*log(x^2 + c))*log(x^2 - c))/(x^2 - c), x))*b^2

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")

[Out] integral(b^2*x^4*arctanh(c/x^2)^2 + 2*a*b*x^4*arctanh(c/x^2) + a^2*x^4, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(a+b\operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c/x**2))**2,x)
```

```
[Out] Integral(x**4*(a + b*atanh(c/x**2))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atanh(c/x^2))^2,x)
```

```
[Out] int(x^4*(a + b*atanh(c/x^2))^2, x)
```

3.177 $\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$

Optimal. Leaf size=1172

$$\frac{4}{3}abcx-\frac{2}{3}abc^{3/2}\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)+\frac{4}{3}b^2c^{3/2}\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{3}ib^2c^{3/2}\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)^2-\frac{4}{3}b^2c^{3/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

```
[Out] 4/3*a*b*c*x-2/3*a*b*c^(3/2)*arctan(x/c^(1/2))-2/3*b^2*c*x*ln(1-c/x^2)+1/3*b
^2*c^(3/2)*arctan(x/c^(1/2))*ln(1-c/x^2)-1/3*b*c^(3/2)*arctanh(x/c^(1/2))*(
2*a-b*ln(1-c/x^2))+2/3*b^2*c*x*ln(1+c/x^2)+1/3*a*b*x^3*ln(1+c/x^2)-1/3*b^2*
c^(3/2)*arctan(x/c^(1/2))*ln(1+c/x^2)-1/3*b^2*c^(3/2)*arctanh(x/c^(1/2))*ln
(1+c/x^2)-1/6*b^2*x^3*ln(1-c/x^2)*ln(1+c/x^2)+2/3*b^2*c^(3/2)*arctan(x/c^(1
/2))*ln(2*c^(1/2)/(-I*x+c^(1/2)))-1/3*b^2*c^(3/2)*arctan(x/c^(1/2))*ln((1+I
)*(-x+c^(1/2))/(-I*x+c^(1/2)))-2/3*b^2*c^(3/2)*arctanh(x/c^(1/2))*ln(2*c^(1
/2)/(x+c^(1/2)))+1/3*b^2*c^(3/2)*arctanh(x/c^(1/2))*ln(2*(-x+(-c)^(1/2))*c^
(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/2)))-1/3*b^2*c^(3/2)*arctan(x/c^(1/2))*l
n((1-I)*(x+c^(1/2))/(-I*x+c^(1/2)))+1/3*b^2*c^(3/2)*arctanh(x/c^(1/2))*ln(2
*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(1/2)+c^(1/2)))-2/3*b^2*c^(3/2)*a
rctan(x/c^(1/2))*ln(2-2*c^(1/2)/(-I*x+c^(1/2)))+2/3*b^2*c^(3/2)*arctanh(x/c
^(1/2))*ln(2-2*c^(1/2)/(x+c^(1/2)))+1/3*I*b^2*c^(3/2)*polylog(2,-I*x/c^(1/2
))+1/3*I*b^2*c^(3/2)*polylog(2,-1+2*c^(1/2)/(-I*x+c^(1/2)))+1/6*I*b^2*c^(3/
2)*polylog(2,1-(1+I)*(-x+c^(1/2))/(-I*x+c^(1/2)))+1/6*I*b^2*c^(3/2)*polylog
(2,1+(-1+I)*(x+c^(1/2))/(-I*x+c^(1/2)))-1/3*I*b^2*c^(3/2)*polylog(2,I*x/c^(
1/2))-1/3*I*b^2*c^(3/2)*polylog(2,1-2*c^(1/2)/(-I*x+c^(1/2)))+1/3*I*b^2*c^(
3/2)*arctan(x/c^(1/2))^2+1/12*x^3*(2*a-b*ln(1-c/x^2))^2+4/3*b^2*c^(3/2)*arc
tan(x/c^(1/2))-4/3*b^2*c^(3/2)*arctanh(x/c^(1/2))+1/3*b^2*c^(3/2)*arctanh(x
/c^(1/2))^2+1/12*b^2*x^3*ln(1+c/x^2)^2+1/3*b^2*c^(3/2)*polylog(2,-x/c^(1/2)
)-1/3*b^2*c^(3/2)*polylog(2,x/c^(1/2))+1/3*b^2*c^(3/2)*polylog(2,1-2*c^(1/2
)/(x+c^(1/2)))-1/3*b^2*c^(3/2)*polylog(2,-1+2*c^(1/2)/(x+c^(1/2)))-1/6*b^2*
c^(3/2)*polylog(2,1-2*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/
2)))-1/6*b^2*c^(3/2)*polylog(2,1-2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)
^(1/2)+c^(1/2)))
```

Rubi [A]
time = 1.56, antiderivative size = 1172, normalized size of antiderivative = 1.00, number of steps used = 80, number of rules used = 34, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 2.125$, Rules used = {6045, 6042, 2507, 2521, 2498, 269, 213, 2520, 12, 266, 6820, 6135, 6079, 2497, 2505, 199, 327, 6874, 209, 30, 2637, 6139, 6031, 6057, 2449, 2352, 2526, 210, 5048, 4940, 2438, 4966, 5044, 4988}

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] (4*a*b*c*x)/3 - (2*a*b*c^(3/2)*ArcTan[x/Sqrt[c]])/3 + (4*b^2*c^(3/2)*ArcTan
[x/Sqrt[c]])/3 + (I/3)*b^2*c^(3/2)*ArcTan[x/Sqrt[c]]^2 - (4*b^2*c^(3/2)*Arc
Tanh[x/Sqrt[c]])/3 + (b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]^2)/3 - (2*b^2*c^(3/2)*
ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/3 - (2*b^2*c*x*Log[
1 - c/x^2])/3 + (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/3 - (b*c^(3/
2)*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/3 + (x^3*(2*a - b*Log[1 - c
/x^2])^2)/12 + (2*b^2*c*x*Log[1 + c/x^2])/3 + (a*b*x^3*Log[1 + c/x^2])/3 -
(b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/3 - (b^2*c^(3/2)*ArcTanh[x/S
qrt[c]]*Log[1 + c/x^2])/3 - (b^2*x^3*Log[1 - c/x^2]*Log[1 + c/x^2])/6 + (b^
2*x^3*Log[1 + c/x^2]^2)/12 + (2*b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c
])/(Sqrt[c] - I*x)])/3 - (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[
c] - x))/(Sqrt[c] - I*x)])/3 - (2*b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqr
t[c])/(Sqrt[c] + x)])/3 + (b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(S
qrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/3 + (b^2*c^(3/2)*ArcTa
nh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c]
 + x))])/3 - (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sq
rt[c] - I*x)])/3 + (2*b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(S
qrt[c] + x)])/3 - (I/3)*b^2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I
*x)] + (I/3)*b^2*c^(3/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)] + (I/
6)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] + (b
^2*c^(3/2)*PolyLog[2, -(x/Sqrt[c])])/3 + (I/3)*b^2*c^(3/2)*PolyLog[2, ((-I)
*x)/Sqrt[c]] - (I/3)*b^2*c^(3/2)*PolyLog[2, (I*x)/Sqrt[c]] - (b^2*c^(3/2)*P
olyLog[2, x/Sqrt[c]])/3 + (b^2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c]
+ x)])/3 - (b^2*c^(3/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/3 - (b^
2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(
Sqrt[c] + x))])/6 - (b^2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/
((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/6 + (I/6)*b^2*c^(3/2)*PolyLog[2, 1 -
 ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
```

```
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6042

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + 1/(x^n*c)]/2) - b*(Log[1 - 1/(x^n*c
)]/2))^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rule 6045

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[x^m*(a + b*ArcCoth[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c, m}, x] && IGtQ[
p, 1] && ILtQ[n, 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(\frac{1}{4}x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2-\frac{1}{2}bx^2\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\right.\right.$$

$$=\frac{1}{4}\int x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2 dx-\frac{1}{2}b\int x^2\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(\right.$$

$$=\frac{1}{12}x^3\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{12}b^2x^3\log^2\left(1+\frac{c}{x^2}\right)-\frac{1}{2}b\int\left(-2ax^2\log\right.$$

$$=\frac{1}{12}x^3\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{12}b^2x^3\log^2\left(1+\frac{c}{x^2}\right)+(ab)\int x^2\log\left(1+\right.$$

$$=\frac{2}{3}abcx+\frac{1}{12}x^3\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{3}abx^3\log\left(1+\frac{c}{x^2}\right)-\frac{1}{6}b^2x^3\log\left(\right.$$

$$=\frac{2}{3}abcx-\frac{1}{3}b^2cx\log\left(1-\frac{c}{x^2}\right)-\frac{1}{3}bc^{3/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right.\right.$$

$$=\frac{4}{3}abcx-\frac{1}{3}b^2cx\log\left(1-\frac{c}{x^2}\right)-\frac{1}{3}bc^{3/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right.\right.$$

$$=\frac{4}{3}abcx-\frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{2}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{2}{3}b^2c^{3/2}\tanh^{-1}\left(\right.$$

$$=\frac{4}{3}abcx-\frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{2}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\right.$$

$$=\frac{4}{3}abcx-\frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{2}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\right.$$

$$=\frac{4}{3}abcx-\frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\right.$$

$$=\frac{4}{3}abcx-\frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\right.$$

$$=\frac{4}{3}abcx-\frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\right.$$

$$=\frac{4}{3}abcx-\frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\right.$$

$$=\frac{4}{3}abcx-\frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\right.$$

Mathematica [F]
time = 7.33, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] Integrate[x^2*(a + b*ArcTanh[c/x^2])^2, x]
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c/x^2))^2,x)
```

```
[Out] int(x^2*(a+b*arctanh(c/x^2))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arctanh(c/x^2) - (2*sqrt(c)*arctan(x/sqrt(c)) - sq
rt(c)*log((x - sqrt(c))/(x + sqrt(c))) - 4*x)*c)*a*b + 1/12*(x^3*log(x^2 -
c)^2 - 3*integrate(-1/3*(3*(x^4 - c*x^2)*log(x^2 + c)^2 - 2*(2*x^4 + 3*(x^4
 - c*x^2)*log(x^2 + c))*log(x^2 - c))/(x^2 - c), x))*b^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctanh(c/x^2)^2 + 2*a*b*x^2*arctanh(c/x^2) + a^2*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c/x**2))**2,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c/x**2))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c/x^2))^2,x)
```

```
[Out] int(x^2*(a + b*atanh(c/x^2))^2, x)
```

3.178 $\int \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$

Optimal. Leaf size=1549

result too large to display

```
[Out] -1/2*b^2*ln(1+c/x^2)*ln(-x+(-c)^(1/2))*(-c)^(1/2)+1/2*b^2*ln(1+c/x^2)*ln(x+
(-c)^(1/2))*(-c)^(1/2)-1/2*b^2*ln(1/2*(-x+(-c)^(1/2))/(-c)^(1/2))*ln(x+(-c)
^(1/2))*(-c)^(1/2)+1/2*b^2*ln(-x+(-c)^(1/2))*ln(1/2*(x+(-c)^(1/2))/(-c)^(1/
2))*(-c)^(1/2)+2*a*b*arctan(x/c^(1/2))*c^(1/2)-2*a*b*arctanh(x/c^(1/2))*c^(
1/2)-1/2*b^2*ln(1-c/x^2)*ln(-x+c^(1/2))*c^(1/2)-2*b^2*arctan(x/c^(1/2))*ln(
2*c^(1/2)/(-I*x+c^(1/2)))*c^(1/2)-2*b^2*arctanh(x/c^(1/2))*ln(2*c^(1/2)/(x+
c^(1/2)))*c^(1/2)+1/2*b^2*ln(1-c/x^2)*ln(x+c^(1/2))*c^(1/2)-1/2*b^2*ln(1/2*
(-x+c^(1/2))/c^(1/2))*ln(x+c^(1/2))*c^(1/2)+1/2*b^2*ln(-x+c^(1/2))*ln(1/2*(
x+c^(1/2))/c^(1/2))*c^(1/2)-I*b^2*polylog(2,-I*x/c^(1/2))*c^(1/2)-1/2*I*b^2
*polylog(2,1-(1+I)*(-x+c^(1/2))/(-I*x+c^(1/2)))*c^(1/2)-1/2*I*b^2*polylog(2
,1+(-1+I)*(x+c^(1/2))/(-I*x+c^(1/2)))*c^(1/2)-a*b*x*ln(1-c/x^2)-b^2*ln(x/(-
c)^(1/2))*ln(-x+(-c)^(1/2))*(-c)^(1/2)+a*b*x*ln(1+c/x^2)+b^2*ln(-x/(-c)^(1/
2))*ln(x+(-c)^(1/2))*(-c)^(1/2)-b^2*arctan(x/c^(1/2))*ln(1-c/x^2)*c^(1/2)-b
^2*arctanh(x/c^(1/2))*ln(1+c/x^2)*c^(1/2)-b^2*ln(x/c^(1/2))*ln(-x+c^(1/2))*
c^(1/2)+b^2*arctan(x/c^(1/2))*ln((1+I)*(-x+c^(1/2))/(-I*x+c^(1/2)))*c^(1/2)
+b^2*arctanh(x/c^(1/2))*ln(2*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(
x+c^(1/2)))*c^(1/2)+b^2*ln(-x/c^(1/2))*ln(x+c^(1/2))*c^(1/2)+b^2*arctan(x/c
^(1/2))*ln((1-I)*(x+c^(1/2))/(-I*x+c^(1/2)))*c^(1/2)+b^2*arctanh(x/c^(1/2))
*ln(2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(1/2)+c^(1/2)))*c^(1/2)+I*b^
2*polylog(2,I*x/c^(1/2))*c^(1/2)+I*b^2*polylog(2,1-2*c^(1/2)/(-I*x+c^(1/2))
)*c^(1/2)-1/2*b^2*x*ln(1-c/x^2)*ln(1+c/x^2)+1/4*b^2*ln(-x+c^(1/2))^2*c^(1/2
)-1/4*b^2*ln(x+c^(1/2))^2*c^(1/2)+1/2*b^2*polylog(2,1/2-1/2*x/c^(1/2))*c^(1
/2)-1/2*b^2*polylog(2,1/2*(x+c^(1/2))/c^(1/2))*c^(1/2)-1/2*b^2*polylog(2,1-
2*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/2)))*c^(1/2)-1/2*b^2
*polylog(2,1-2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(1/2)+c^(1/2)))*c^(
1/2)-b^2*polylog(2,1-x/(-c)^(1/2))*(-c)^(1/2)+b^2*polylog(2,1+x/(-c)^(1/2))
*(-c)^(1/2)-b^2*polylog(2,1-x/c^(1/2))*c^(1/2)+b^2*polylog(2,1+x/c^(1/2))*c
^(1/2)+b^2*polylog(2,-x/c^(1/2))*c^(1/2)-b^2*polylog(2,x/c^(1/2))*c^(1/2)+b
^2*polylog(2,1-2*c^(1/2)/(x+c^(1/2)))*c^(1/2)+1/4*b^2*x*ln(1-c/x^2)^2+1/4*b
^2*x*ln(1+c/x^2)^2+1/4*b^2*ln(-x+(-c)^(1/2))^2*(-c)^(1/2)-1/4*b^2*ln(x+(-c)
^(1/2))^2*(-c)^(1/2)+1/2*b^2*polylog(2,1/2-1/2*x/(-c)^(1/2))*(-c)^(1/2)-1/2
*b^2*polylog(2,1/2*(c-x*(-c)^(1/2))/c)*(-c)^(1/2)+a^2*x
```

Rubi [A]

time = 1.61, antiderivative size = 1549, normalized size of antiderivative = 1.00, number of steps used = 100, number of rules used = 30, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.500, Rules used = {6025, 6024, 2498, 269, 213, 2500, 2526, 2512, 266, 2463, 2441, 2352, 2437, 2338, 2440, 2438, 209, 2636, 12, 2520, 6820, 6139, 6031, 6057, 2449, 2497, 210, 5048, 4940, 4966}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] a^2*x + 2*a*b*Sqrt[c]*ArcTan[x/Sqrt[c]] - 2*a*b*Sqrt[c]*ArcTanh[x/Sqrt[c]]
- a*b*x*Log[1 - c/x^2] - b^2*Sqrt[c]*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2] + (b^
2*x*Log[1 - c/x^2]^2)/4 + a*b*x*Log[1 + c/x^2] - b^2*Sqrt[c]*ArcTanh[x/Sqrt
[c]]*Log[1 + c/x^2] - (b^2*x*Log[1 - c/x^2]*Log[1 + c/x^2])/2 + (b^2*x*Log[
1 + c/x^2]^2)/4 - (b^2*Sqrt[-c]*Log[1 + c/x^2]*Log[Sqrt[-c] - x])/2 + (b^2*
Sqrt[-c]*Log[Sqrt[-c] - x]^2)/4 - (b^2*Sqrt[c]*Log[1 - c/x^2]*Log[Sqrt[c] -
 x])/2 + (b^2*Sqrt[c]*Log[Sqrt[c] - x]^2)/4 - 2*b^2*Sqrt[c]*ArcTan[x/Sqrt[c
]]*Log[(2*Sqrt[c])/(Sqrt[c] - I*x)] + b^2*Sqrt[c]*ArcTan[x/Sqrt[c]]*Log[((1
 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] - b^2*Sqrt[-c]*Log[Sqrt[-c] - x]*Log[
x/Sqrt[-c]] - b^2*Sqrt[c]*Log[Sqrt[c] - x]*Log[x/Sqrt[c]] + (b^2*Sqrt[-c]*L
og[1 + c/x^2]*Log[Sqrt[-c] + x])/2 - (b^2*Sqrt[-c]*Log[(Sqrt[-c] - x)/(2*Sq
rt[-c])]*Log[Sqrt[-c] + x])/2 + b^2*Sqrt[-c]*Log[-(x/Sqrt[-c])]*Log[Sqrt[-c
] + x] - (b^2*Sqrt[-c]*Log[Sqrt[-c] + x]^2)/4 + (b^2*Sqrt[-c]*Log[Sqrt[-c]
- x]*Log[(Sqrt[-c] + x)/(2*Sqrt[-c])])/2 - 2*b^2*Sqrt[c]*ArcTanh[x/Sqrt[c]]
*Log[(2*Sqrt[c])/(Sqrt[c] + x)] + b^2*Sqrt[c]*ArcTanh[x/Sqrt[c]]*Log[(2*Sqr
t[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))] + b^2*Sqrt[c]*Ar
cTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt
[c] + x))] + (b^2*Sqrt[c]*Log[1 - c/x^2]*Log[Sqrt[c] + x])/2 - (b^2*Sqrt[c]
*Log[(Sqrt[c] - x)/(2*Sqrt[c])]*Log[Sqrt[c] + x])/2 + b^2*Sqrt[c]*Log[-(x/S
qrt[c])]*Log[Sqrt[c] + x] - (b^2*Sqrt[c]*Log[Sqrt[c] + x]^2)/4 + (b^2*Sqrt[
c]*Log[Sqrt[c] - x]*Log[(Sqrt[c] + x)/(2*Sqrt[c])])/2 + b^2*Sqrt[c]*ArcTan[
x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)] + I*b^2*Sqrt[c]*Pol
yLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)] - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 -
 ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] + b^2*Sqrt[c]*PolyLog[2, -(x/Sqrt
[c])] - I*b^2*Sqrt[c]*PolyLog[2, ((-I)*x)/Sqrt[c]] + I*b^2*Sqrt[c]*PolyLog[
2, (I*x)/Sqrt[c]] - b^2*Sqrt[c]*PolyLog[2, x/Sqrt[c]] - (b^2*Sqrt[c]*PolyLo
g[2, (Sqrt[c] + x)/(2*Sqrt[c])])/2 + (b^2*Sqrt[-c]*PolyLog[2, (1 - x/Sqrt[-
c])/2])/2 - b^2*Sqrt[-c]*PolyLog[2, 1 - x/Sqrt[-c]] + b^2*Sqrt[-c]*PolyLog[
2, 1 + x/Sqrt[-c]] - (b^2*Sqrt[-c]*PolyLog[2, (c - Sqrt[-c]*x)/(2*c)])/2 -
b^2*Sqrt[c]*PolyLog[2, 1 - x/Sqrt[c]] + (b^2*Sqrt[c]*PolyLog[2, 1/2 - x/(2*
Sqrt[c])])/2 + b^2*Sqrt[c]*PolyLog[2, 1 + x/Sqrt[c]] + b^2*Sqrt[c]*PolyLog[
2, 1 - (2*Sqrt[c])/(Sqrt[c] + x)] - (b^2*Sqrt[c]*PolyLog[2, 1 - (2*Sqrt[c]*
(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/2 - (b^2*Sqrt[c]*Pol
yLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))
])/2 - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] -
I*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
```

```
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2636

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[x*Log[w]*(D[v, x]/v), x], x] - Int[SimplifyIntegrand[x*Log[v]*(D[
w, x]/w), x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
```

```
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6024

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[ExpandI
ntegrand[(a + b*(Log[1 + 1/(x^n*c)]/2) - b*(Log[1 - 1/(x^n*c)]/2))^p, x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6025

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[(a + b*
ArcCoth[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && ILtQ[n, 0
]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \left(a + b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(a^2 - ab\log\left(1 - \frac{c}{x^2}\right) + \frac{1}{4}b^2\log^2\left(1 - \frac{c}{x^2}\right) + ab\log\left(1 + \frac{c}{x^2}\right) - \frac{1}{2}b^2\log\left(\right.\right.$$

$$= a^2x - (ab)\int \log\left(1 - \frac{c}{x^2}\right) dx + (ab)\int \log\left(1 + \frac{c}{x^2}\right) dx + \frac{1}{4}b^2\int \log^2\left(1 -\right.$$

$$= a^2x - abx\log\left(1 - \frac{c}{x^2}\right) + \frac{1}{4}b^2x\log^2\left(1 - \frac{c}{x^2}\right) + abx\log\left(1 + \frac{c}{x^2}\right) - \frac{1}{2}b^2x\log$$

$$= a^2x - abx\log\left(1 - \frac{c}{x^2}\right) + \frac{1}{4}b^2x\log^2\left(1 - \frac{c}{x^2}\right) + abx\log\left(1 + \frac{c}{x^2}\right) - \frac{1}{2}b^2x\log$$

$$= a^2x + 2ab\sqrt{c}\,\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\,\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1 - \frac{c}{x^2}\right) -$$

$$= a^2x + 2ab\sqrt{c}\,\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\,\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1 - \frac{c}{x^2}\right) -$$

$$= a^2x + 2ab\sqrt{c}\,\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\,\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1 - \frac{c}{x^2}\right) -$$

$$= a^2x + 2ab\sqrt{c}\,\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\,\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1 - \frac{c}{x^2}\right) -$$

$$= a^2x + 2ab\sqrt{c}\,\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\,\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1 - \frac{c}{x^2}\right) -$$

$$= a^2x + 2ab\sqrt{c}\,\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\,\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1 - \frac{c}{x^2}\right) -$$

$$= a^2x + 2ab\sqrt{c}\,\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\,\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1 - \frac{c}{x^2}\right) -$$

$$= a^2x + 2ab\sqrt{c}\,\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\,\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1 - \frac{c}{x^2}\right) -$$

$$= a^2x + 2ab\sqrt{c}\,\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\,\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1 - \frac{c}{x^2}\right) -$$

$$= a^2x + 2ab\sqrt{c}\,\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\,\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1 - \frac{c}{x^2}\right) -$$

Mathematica [A]
time = 2.76, size = 565, normalized size = 0.36

$$a^2x-2ab\sqrt{\frac{c}{x^2}}\,x\left(\text{ArcTan}\left(\sqrt{\frac{c}{x^2}}\right)+\tanh^{-1}\left(\sqrt{\frac{c}{x^2}}\right)\right)+2abx\tanh^{-1}\left(\frac{c}{x^2}\right)-\frac{1}{2}b^2\sqrt{\frac{c}{x^2}}\,x\left(-2i\text{ArcTan}\left(\sqrt{\frac{c}{x^2}}\right)^2+4\text{ArcTan}\left(\sqrt{\frac{c}{x^2}}\right)\tanh^{-1}\left(\frac{c}{x^2}\right)-\frac{2\tanh^{-1}\left(\frac{c}{x^2}\right)^2}{\sqrt{\frac{c}{x^2}}}+2\text{ArcTan}\left(\sqrt{\frac{c}{x^2}}\right)\log\left(1+e^{4i\text{ArcTan}\left(\sqrt{\frac{c}{x^2}}\right)}\right)-2\tanh^{-1}\left(\frac{c}{x^2}\right)\log\left(1-\sqrt{\frac{c}{x^2}}\right)+\log(2)\log\left(1-\sqrt{\frac{c}{x^2}}\right)-\frac{1}{2}\log^2\left(1-\sqrt{\frac{c}{x^2}}\right)+\log\left(1-\sqrt{\frac{c}{x^2}}\right)\log\left(\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i+\sqrt{\frac{c}{x^2}}\right)\right)+2\tanh^{-1}\left(\frac{c}{x^2}\right)\log\left(1+\sqrt{\frac{c}{x^2}}\right)-\log(2)\log\left(1+\sqrt{\frac{c}{x^2}}\right)-\log\left(\frac{1}{2}\left((1+i)-(1-i)\sqrt{\frac{c}{x^2}}\right)\right)\log\left(1+\sqrt{\frac{c}{x^2}}\right)-\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)\left(i+\sqrt{\frac{c}{x^2}}\right)\right)\log\left(1+\sqrt{\frac{c}{x^2}}\right)+\frac{1}{2}\log^2\left(1+\sqrt{\frac{c}{x^2}}\right)+\log\left(1-\sqrt{\frac{c}{x^2}}\right)\log\left(\frac{1}{2}\left((1+i)+(1-i)\sqrt{\frac{c}{x^2}}\right)\right)-\frac{i}{2}\text{PolyLog}\left(2,-e^{4i\text{ArcTan}\left(\sqrt{\frac{c}{x^2}}\right)}\right)-\text{PolyLog}\left(2,\frac{1}{2}\left(1-\sqrt{\frac{c}{x^2}}\right)\right)+\text{PolyLog}\left(2,\left(-\frac{1}{2}-\frac{i}{2}\right)\left(-1+\sqrt{\frac{c}{x^2}}\right)\right)+\text{PolyLog}\left(2,\left(-\frac{1}{2}+\frac{i}{2}\right)\left(-1+\sqrt{\frac{c}{x^2}}\right)\right)+\text{PolyLog}\left(2,\frac{1}{2}\left(1+\sqrt{\frac{c}{x^2}}\right)\right)-\text{PolyLog}\left(2,\left(\frac{1}{2}-\frac{i}{2}\right)\left(1+\sqrt{\frac{c}{x^2}}\right)\right)-\text{PolyLog}\left(2,\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+\sqrt{\frac{c}{x^2}}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] a^2*x - 2*a*b*Sqrt[c/x^2]*x*(ArcTan[Sqrt[c/x^2]] + ArcTanh[Sqrt[c/x^2]]) +
2*a*b*x*ArcTanh[c/x^2] - (b^2*Sqrt[c/x^2]*x*((-2*I)*ArcTan[Sqrt[c/x^2]]^2 +
 4*ArcTan[Sqrt[c/x^2]]*ArcTanh[c/x^2] - (2*ArcTanh[c/x^2]^2)/Sqrt[c/x^2] +
2*ArcTan[Sqrt[c/x^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c/x^2]])] - 2*ArcTanh[c/
x^2]*Log[1 - Sqrt[c/x^2]] + Log[2]*Log[1 - Sqrt[c/x^2]] - Log[1 - Sqrt[c/x^
2]]^2/2 + Log[1 - Sqrt[c/x^2]]*Log[(1/2 + I/2)*(-I + Sqrt[c/x^2])] + 2*ArcT
anh[c/x^2]*Log[1 + Sqrt[c/x^2]] - Log[2]*Log[1 + Sqrt[c/x^2]] - Log[((1 + I
) - (1 - I)*Sqrt[c/x^2])/2]*Log[1 + Sqrt[c/x^2]] - Log[(-1/2 - I/2)*(I + Sq
rt[c/x^2])]*Log[1 + Sqrt[c/x^2]] + Log[1 + Sqrt[c/x^2]]^2/2 + Log[1 - Sqrt[
c/x^2]]*Log[((1 + I) + (1 - I)*Sqrt[c/x^2])/2] - (I/2)*PolyLog[2, -E^((4*I)
*ArcTan[Sqrt[c/x^2]])] - PolyLog[2, (1 - Sqrt[c/x^2])/2] + PolyLog[2, (-1/2
 - I/2)*(-1 + Sqrt[c/x^2])] + PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[c/x^2])] +
 PolyLog[2, (1 + Sqrt[c/x^2])/2] - PolyLog[2, (1/2 - I/2)*(1 + Sqrt[c/x^2])
] - PolyLog[2, (1/2 + I/2)*(1 + Sqrt[c/x^2])]))/2
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2,x)
```

```
[Out] int((a+b*arctanh(c/x^2))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] (c*(2*arctan(x/sqrt(c))/sqrt(c) + log((x - sqrt(c))/(x + sqrt(c)))/sqrt(c))
 + 2*x*arctanh(c/x^2))*a*b + 1/4*(x*log(x^2 - c)^2 - integrate(-((x^2 - c)*
log(x^2 + c)^2 - 2*(2*x^2 + (x^2 - c)*log(x^2 + c))*log(x^2 - c))/(x^2 - c)
, x))*b^2 + a^2*x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
[Out] integral(b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2,x)
[Out] Integral((a + b*atanh(c/x**2))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c/x^2) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))^2,x)
[Out] int((a + b*atanh(c/x^2))^2, x)
```

3.179 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=1117

$$\frac{2ab}{x}-\frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{2b^2\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{2b^2\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{2b^2\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{ib^2\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)^2}{\sqrt{c}}$$

```
[Out] -a*b*ln(1+c/x^2)/x+b^2*arccot(x/c^(1/2))*ln(1-c/x^2)/c^(1/2)+1/2*b^2*ln(1-c
/x^2)*ln(1+c/x^2)/x-2*a*b*arccot(x/c^(1/2))/c^(1/2)+2*b^2*arccot(x/c^(1/2))
*ln(2/(1-I*c^(1/2)/x))/c^(1/2)+2*b^2*arccoth(x/c^(1/2))*ln(2/(1+1/x*c^(1/2)
))/c^(1/2)+2*b^2*arctan(x/c^(1/2))*ln(2-2*c^(1/2)/(-I*x+c^(1/2)))/c^(1/2)-2
*b^2*arctanh(x/c^(1/2))*ln(2-2*c^(1/2)/(x+c^(1/2)))/c^(1/2)-I*b^2*arctan(x/
c^(1/2))^2/c^(1/2)-I*b^2*polylog(2,-1+2*c^(1/2)/(-I*x+c^(1/2)))/c^(1/2)-I*b
^2*polylog(2,1-2/(1-I*c^(1/2)/x))/c^(1/2)+b*arctanh(x/c^(1/2))*(2*a-b*ln(1-
c/x^2))/c^(1/2)+b^2*arccoth(x/c^(1/2))*ln(1+c/x^2)/c^(1/2)+b^2*arctan(x/c^(
1/2))*ln(1+c/x^2)/c^(1/2)-b^2*arccot(x/c^(1/2))*ln((1+I)*(1-1/x*c^(1/2))/(1
-I*c^(1/2)/x))/c^(1/2)-b^2*arccoth(x/c^(1/2))*ln(-2*(1-(-c)^(1/2)/x)*c^(1/2
)/((-c)^(1/2)-c^(1/2))/(1+1/x*c^(1/2)))/c^(1/2)-b^2*arccoth(x/c^(1/2))*ln(2
*(1+(-c)^(1/2)/x)*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+1/x*c^(1/2)))/c^(1/2)-b^2
*arccot(x/c^(1/2))*ln((1-I)*(1+1/x*c^(1/2))/(1-I*c^(1/2)/x))/c^(1/2)+1/2*I*
b^2*polylog(2,1-(1+I)*(1-1/x*c^(1/2))/(1-I*c^(1/2)/x))/c^(1/2)+1/2*I*b^2*po
lylog(2,1+(-1+I)*(1+1/x*c^(1/2))/(1-I*c^(1/2)/x))/c^(1/2)+b^2*polylog(2,-1+
2*c^(1/2)/(x+c^(1/2)))/c^(1/2)-b^2*polylog(2,1-2/(1+1/x*c^(1/2)))/c^(1/2)+2
*a*b/x-1/4*b^2*ln(1+c/x^2)^2/x-2*b^2*arccot(x/c^(1/2))/c^(1/2)-2*b^2*arccot
h(x/c^(1/2))/c^(1/2)-2*b^2*arctan(x/c^(1/2))/c^(1/2)+2*b^2*arctanh(x/c^(1/2
))/c^(1/2)+1/2*b^2*polylog(2,1+2*(1-(-c)^(1/2)/x)*c^(1/2)/((-c)^(1/2)-c^(1/
2))/(1+1/x*c^(1/2)))/c^(1/2)+1/2*b^2*polylog(2,1-2*(1+(-c)^(1/2)/x)*c^(1/2)
/((-c)^(1/2)+c^(1/2))/(1+1/x*c^(1/2)))/c^(1/2)-b^2*ln(1-c/x^2)/x-b*(2*a-b*l
n(1-c/x^2))/x-b^2*arctanh(x/c^(1/2))^2/c^(1/2)-1/4*(2*a-b*ln(1-c/x^2))^2/x
```

Rubi [A]
time = 1.53, antiderivative size = 1117, normalized size of antiderivative = 1.00, number of steps used = 72, number of rules used = 30, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.875, Rules used = {6045, 6042, 2507, 2526, 2505, 269, 331, 213, 212, 2520, 12, 266, 6820, 6135, 6079, 2497, 6847, 2498, 327, 6874, 209, 2636, 6139, 6057, 2449, 2352, 5048, 4966, 5044, 4988}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2/x^2,x]
```

```
[Out] (2*a*b)/x - (2*a*b*ArcCot[x/Sqrt[c]])/Sqrt[c] - (2*b^2*ArcCot[x/Sqrt[c]])/S
qrt[c] - (2*b^2*ArcCoth[x/Sqrt[c]])/Sqrt[c] - (2*b^2*ArcTan[x/Sqrt[c]])/Sqr
t[c] - (I*b^2*ArcTan[x/Sqrt[c]]^2)/Sqrt[c] + (2*b^2*ArcTanh[x/Sqrt[c]])/Sqr
t[c] - (b^2*ArcTanh[x/Sqrt[c]]^2)/Sqrt[c] + (2*b^2*ArcTan[x/Sqrt[c]]*Log[2
- (2*Sqrt[c])/(Sqrt[c] - I*x)])/Sqrt[c] - (b^2*Log[1 - c/x^2])/x + (b^2*Arc
Cot[x/Sqrt[c]]*Log[1 - c/x^2])/Sqrt[c] - (b*(2*a - b*Log[1 - c/x^2]))/x + (
b*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/Sqrt[c] - (2*a - b*Log[1 - c
/x^2])^2/(4*x) - (a*b*Log[1 + c/x^2])/x + (b^2*ArcCoth[x/Sqrt[c]]*Log[1 + c
/x^2])/Sqrt[c] + (b^2*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/Sqrt[c] + (b^2*Log[
1 - c/x^2]*Log[1 + c/x^2])/(2*x) - (b^2*Log[1 + c/x^2]^2)/(4*x) + (2*b^2*Ar
cCot[x/Sqrt[c]]*Log[2/(1 - (I*Sqrt[c])/x)])/Sqrt[c] - (b^2*ArcCot[x/Sqrt[c]
]*Log[((1 + I)*(1 - Sqrt[c]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[c] + (2*b^2*ArcC
oth[x/Sqrt[c]]*Log[2/(1 + Sqrt[c]/x)])/Sqrt[c] - (b^2*ArcCoth[x/Sqrt[c]]*Lo
g[(-2*Sqrt[c]*(1 - Sqrt[-c]/x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]/x))])/Sq
rt[c] - (b^2*ArcCoth[x/Sqrt[c]]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]/x))/((Sqrt[-c]
 + Sqrt[c])*(1 + Sqrt[c]/x))])/Sqrt[c] - (b^2*ArcCot[x/Sqrt[c]]*Log[((1 - I
)*(1 + Sqrt[c]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[c] - (2*b^2*ArcTanh[x/Sqrt[c]
]*Log[2 - (2*Sqrt[c])/(Sqrt[c] + x)])/Sqrt[c] - (I*b^2*PolyLog[2, 1 - 2/(1
- (I*Sqrt[c])/x)])/Sqrt[c] + ((I/2)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c
]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[c] - (b^2*PolyLog[2, 1 - 2/(1 + Sqrt[c]/x)
])/Sqrt[c] + (b^2*PolyLog[2, 1 + (2*Sqrt[c]*(1 - Sqrt[-c]/x))/((Sqrt[-c] -
Sqrt[c])*(1 + Sqrt[c]/x))])/(2*Sqrt[c]) + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1
 + Sqrt[-c]/x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]/x))])/(2*Sqrt[c]) + ((I/
2)*b^2*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt[c]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[
c] - (I*b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)])/Sqrt[c] + (b^2*Po
lyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/Sqrt[c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
```

```
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2636

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[x*Log[w]*(D[v, x]/v), x], x] - Int[SimplifyIntegrand[x*Log[v]*(D[
w, x]/w), x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
```

```
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6042

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + 1/(x^n*c)]/2) - b*(Log[1 - 1/(x^n*c
)]/2))^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rule 6045

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[x^m*(a + b*ArcCoth[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c, m}, x] && IGtQ[
p, 1] && ILtQ[n, 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
```

```
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x^2} - \frac{b\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{2x^2} + \frac{b^2\log^2\left(\right.}{4}$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{x^2}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{x^2}\,dx$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{4x} + \frac{1}{2}b\text{Subst}\left(\int \left(-2a+b\log\left(1-\right.\right.\right.$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{4x} + \frac{1}{2}b\text{Subst}\left(\int \left(-2a\log\left(1+cx^2\right)\right.\right.$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{4x} + b\int \frac{2a-b\log\left(1-\frac{c}{x^2}\right)}{x^2}\,dx + b$$

$$= -\frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{x} + \frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{\sqrt{c}} - \frac{\left(2a-b\right.}{}$$

$$= \frac{2ab}{x} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{x} + \frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{\sqrt{c}} - \frac{\left(2a\right.}{}$$

$$= \frac{2ab}{x} - \frac{4b^2}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{x} + \frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{}$$

$$= \frac{2ab}{x} - \frac{4b^2}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{\sqrt{c}} +$$

$$= \frac{2ab}{x} - \frac{4b^2}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{\sqrt{c}} +$$

$$= \frac{2ab}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{\sqrt{c}} + \frac{2b^2\tan}{}$$

$$= \frac{2ab}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan}{}$$

$$= \frac{2ab}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan}{}$$

$$\frac{2ab}{} \quad \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{} \quad \frac{2b^2\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{} \quad \frac{2b^2\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)}{} \quad \frac{2b^2\tan}{}$$

Mathematica [A]
time = 2.02, size = 568, normalized size = 0.51

$$\frac{-2a^2-\frac{4ab\left(\text{ArcTan}\left(\sqrt{\frac{c}{x^2}}\right)-\tanh^{-1}\left(\sqrt{\frac{c}{x^2}}\right)\right)}{\sqrt{\frac{c}{x^2}}}-4ab\tanh^{-1}\left(\frac{c}{x^2}\right)+\frac{b^2\left(2i\text{ArcTan}\left(\sqrt{\frac{c}{x^2}}\right)^2-4\text{ArcTan}\left(\sqrt{\frac{c}{x^2}}\right)\tanh^{-1}\left(\frac{c}{x^2}\right)-2\sqrt{\frac{c}{x^2}}\tanh^{-1}\left(\frac{c}{x^2}\right)^2-2\text{ArcTan}\left(\sqrt{\frac{c}{x^2}}\right)\log\left(1+e^{4i\text{ArcTan}\left(\sqrt{\frac{c}{x^2}}\right)}\right)-2\tanh^{-1}\left(\frac{c}{x^2}\right)\log\left(1-\sqrt{\frac{c}{x^2}}\right)+\log(2)\log\left(1-\sqrt{\frac{c}{x^2}}\right)-\frac{1}{2}\log^2\left(1-\sqrt{\frac{c}{x^2}}\right)+\log\left(1-\sqrt{\frac{c}{x^2}}\right)\log\left(\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i+\sqrt{\frac{c}{x^2}}\right)\right)+2\tanh^{-1}\left(\frac{c}{x^2}\right)\log\left(1+\sqrt{\frac{c}{x^2}}\right)-\log(2)\log\left(1+\sqrt{\frac{c}{x^2}}\right)-\log\left(\frac{1}{2}\left((1+i)-(1-i)\sqrt{\frac{c}{x^2}}\right)\right)\log\left(1+\sqrt{\frac{c}{x^2}}\right)-\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)\left(i+\sqrt{\frac{c}{x^2}}\right)\right)\log\left(1+\sqrt{\frac{c}{x^2}}\right)+\frac{1}{2}\log^2\left(1+\sqrt{\frac{c}{x^2}}\right)+\log\left(1-\sqrt{\frac{c}{x^2}}\right)\log\left(\frac{1}{2}\left((1+i)+(1-i)\sqrt{\frac{c}{x^2}}\right)\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{4i\text{ArcTan}\left(\sqrt{\frac{c}{x^2}}\right)}\right)-\text{PolyLog}\left(2,\frac{1}{2}\left(1-\sqrt{\frac{c}{x^2}}\right)\right)+\text{PolyLog}\left(2,\left(-\frac{1}{2}-\frac{i}{2}\right)\left(-1+\sqrt{\frac{c}{x^2}}\right)\right)+\text{PolyLog}\left(2,\left(-\frac{1}{2}+\frac{i}{2}\right)\left(-1+\sqrt{\frac{c}{x^2}}\right)\right)+\text{PolyLog}\left(2,\frac{1}{2}\left(1+\sqrt{\frac{c}{x^2}}\right)\right)-\text{PolyLog}\left(2,\left(\frac{1}{2}-\frac{i}{2}\right)\left(1+\sqrt{\frac{c}{x^2}}\right)\right)-\text{PolyLog}\left(2,\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+\sqrt{\frac{c}{x^2}}\right)\right)\right)}{\sqrt{\frac{c}{x^2}}}}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2/x^2,x]
```

```
[Out] (-2*a^2 - (4*a*b*(ArcTan[Sqrt[c/x^2]] - ArcTanh[Sqrt[c/x^2]]))/Sqrt[c/x^2]
- 4*a*b*ArcTanh[c/x^2] + (b^2*((2*I)*ArcTan[Sqrt[c/x^2]]^2 - 4*ArcTan[Sqrt[
c/x^2]]*ArcTanh[c/x^2] - 2*Sqrt[c/x^2]*ArcTanh[c/x^2]^2 - 2*ArcTan[Sqrt[c/x
^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c/x^2]])] - 2*ArcTanh[c/x^2]*Log[1 - Sqrt
[c/x^2]] + Log[2]*Log[1 - Sqrt[c/x^2]] - Log[1 - Sqrt[c/x^2]]^2/2 + Log[1 -
 Sqrt[c/x^2]]*Log[(1/2 + I/2)*(-I + Sqrt[c/x^2])] + 2*ArcTanh[c/x^2]*Log[1
+ Sqrt[c/x^2]] - Log[2]*Log[1 + Sqrt[c/x^2]] - Log[((1 + I) - (1 - I)*Sqrt[
c/x^2])/2]*Log[1 + Sqrt[c/x^2]] - Log[(-1/2 - I/2)*(I + Sqrt[c/x^2])]*Log[1
 + Sqrt[c/x^2]] + Log[1 + Sqrt[c/x^2]]^2/2 + Log[1 - Sqrt[c/x^2]]*Log[((1 +
 I) + (1 - I)*Sqrt[c/x^2])/2] + (I/2)*PolyLog[2, -E^((4*I)*ArcTan[Sqrt[c/x^
2]])] - PolyLog[2, (1 - Sqrt[c/x^2])/2] + PolyLog[2, (-1/2 - I/2)*(-1 + Sqr
t[c/x^2])] + PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[c/x^2])] + PolyLog[2, (1 +
Sqrt[c/x^2])/2] - PolyLog[2, (1/2 - I/2)*(1 + Sqrt[c/x^2])] - PolyLog[2, (1
/2 + I/2)*(1 + Sqrt[c/x^2])]))/Sqrt[c/x^2])/(2*x)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2/x^2,x)
```

```
[Out] int((a+b*arctanh(c/x^2))^2/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^2,x, algorithm="maxima")
```

```
[Out] (c*(2*arctan(x/sqrt(c))/c^(3/2) - log((x - sqrt(c))/(x + sqrt(c)))/c^(3/2))
 - 2*arctanh(c/x^2)/x)*a*b - 1/4*b^2*(log(x^2 - c)^2/x + integrate(-((x^2 -
 c)*log(x^2 + c)^2 + 2*(2*x^2 - (x^2 - c)*log(x^2 + c))*log(x^2 - c))/(x^4
- c*x^2), x)) - a^2/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x**2,x)
[Out] Integral((a + b*atanh(c/x**2))**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c/x^2) + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)}^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))^2/x^2,x)
[Out] int((a + b*atanh(c/x^2))^2/x^2, x)
```

3.180 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=1263

$$\frac{2ab}{9x^3}-\frac{2ab}{3cx}-\frac{2ab\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}+\frac{4b^2\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}+\frac{ib^2\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}}+\frac{4b^2\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}-\frac{b^2\tanh^{-1}\left(\right.}{3c^{3/2}}$$

```
[Out] -1/3*b^2*arctanh(x/c^(1/2))*ln(2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(
1/2)+c^(1/2)))/c^(3/2)-2/3*b^2*arctan(x/c^(1/2))*ln(2-2*c^(1/2)/(-I*x+c^(1/
2)))/c^(3/2)-2/3*b^2*arctanh(x/c^(1/2))*ln(2-2*c^(1/2)/(x+c^(1/2)))/c^(3/2)
-1/3*I*b^2*polylog(2,I*x/c^(1/2))/c^(3/2)-1/3*I*b^2*polylog(2,1-2*c^(1/2)/(
-I*x+c^(1/2)))/c^(3/2)-2/3*a*b*arctan(x/c^(1/2))/c^(3/2)+1/3*b^2*ln(1-c/x^2
)/c/x+1/3*b^2*arctan(x/c^(1/2))*ln(1-c/x^2)/c^(3/2)-1/3*b*(2*a-b*ln(1-c/x^2
))/c/x+1/3*b*arctanh(x/c^(1/2))*(2*a-b*ln(1-c/x^2))/c^(3/2)-1/3*a*b*ln(1+c/
x^2)/x^3-2/3*b^2*ln(1+c/x^2)/c/x-1/3*b^2*arctan(x/c^(1/2))*ln(1+c/x^2)/c^(3
/2)+1/3*b^2*arctanh(x/c^(1/2))*ln(1+c/x^2)/c^(3/2)+1/6*b^2*ln(1-c/x^2)*ln(1
+c/x^2)/x^3+2/3*b^2*arctan(x/c^(1/2))*ln(2*c^(1/2)/(-I*x+c^(1/2)))/c^(3/2)-
1/3*b^2*arctan(x/c^(1/2))*ln((1+I)*(-x+c^(1/2))/(-I*x+c^(1/2)))/c^(3/2)+2/3
*b^2*arctanh(x/c^(1/2))*ln(2*c^(1/2)/(x+c^(1/2)))/c^(3/2)-1/3*b^2*arctanh(x
/c^(1/2))*ln(2*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/2)))/c^
(3/2)-1/3*b^2*arctan(x/c^(1/2))*ln((1-I)*(x+c^(1/2))/(-I*x+c^(1/2)))/c^(3/2
)+1/3*I*b^2*polylog(2,-I*x/c^(1/2))/c^(3/2)+1/3*I*b^2*polylog(2,-1+2*c^(1/2
)/(-I*x+c^(1/2)))/c^(3/2)+1/6*I*b^2*polylog(2,1-(1+I)*(-x+c^(1/2))/(-I*x+c^
(1/2)))/c^(3/2)+1/6*I*b^2*polylog(2,1+(-1+I)*(x+c^(1/2))/(-I*x+c^(1/2)))/c^
(3/2)+1/3*I*b^2*arctan(x/c^(1/2))^2/c^(3/2)-2/3*a*b/c/x+4/3*b^2*arctan(x/c^
(1/2))/c^(3/2)+4/3*b^2*arctanh(x/c^(1/2))/c^(3/2)-1/3*b^2*arctanh(x/c^(1/2)
)^2/c^(3/2)-1/9*b^2*ln(1-c/x^2)/x^3-1/9*b*(2*a-b*ln(1-c/x^2))/x^3-1/12*b^2*
ln(1+c/x^2)^2/x^3-1/3*b^2*polylog(2,-x/c^(1/2))/c^(3/2)+1/3*b^2*polylog(2,x
/c^(1/2))/c^(3/2)-1/3*b^2*polylog(2,1-2*c^(1/2)/(x+c^(1/2)))/c^(3/2)+1/3*b^
2*polylog(2,-1+2*c^(1/2)/(x+c^(1/2)))/c^(3/2)+1/6*b^2*polylog(2,1-2*(-x+(-c
)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/2)))/c^(3/2)+1/6*b^2*polylog(
2,1-2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(1/2)+c^(1/2)))/c^(3/2)+2/9*
a*b/x^3-1/12*(2*a-b*ln(1-c/x^2))^2/x^3
```

Rubi [A]

time = 2.05, antiderivative size = 1263, normalized size of antiderivative = 1.00, number of steps used = 105, number of rules used = 31, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.938, Rules used = {6045, 6042, 2507, 2526, 2505, 269, 331, 213, 212, 2520, 12, 266, 6820, 6135, 6079, 2497, 6874, 209, 30, 2637, 6139, 6031, 6057, 2449, 2352, 5048, 4940, 2438, 4966, 5044, 4988}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2/x^4,x]
```

```
[Out] (2*a*b)/(9*x^3) - (2*a*b)/(3*c*x) - (2*a*b*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) +
 (4*b^2*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) + ((I/3)*b^2*ArcTan[x/Sqrt[c]]^2)/c^
(3/2) + (4*b^2*ArcTanh[x/Sqrt[c]])/(3*c^(3/2)) - (b^2*ArcTanh[x/Sqrt[c]]^2)
/(3*c^(3/2)) - (2*b^2*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)
])/(3*c^(3/2)) - (b^2*Log[1 - c/x^2])/(9*x^3) + (b^2*Log[1 - c/x^2])/(3*c*x
) + (b^2*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/(3*c^(3/2)) - (b*(2*a - b*Log[1
- c/x^2]))/(9*x^3) - (b*(2*a - b*Log[1 - c/x^2]))/(3*c*x) + (b*ArcTanh[x/Sq
rt[c]]*(2*a - b*Log[1 - c/x^2]))/(3*c^(3/2)) - (2*a - b*Log[1 - c/x^2])^2/(
12*x^3) - (a*b*Log[1 + c/x^2])/(3*x^3) - (2*b^2*Log[1 + c/x^2])/(3*c*x) - (
b^2*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/(3*c^(3/2)) + (b^2*ArcTanh[x/Sqrt[c]]
*Log[1 + c/x^2])/(3*c^(3/2)) + (b^2*Log[1 - c/x^2]*Log[1 + c/x^2])/(6*x^3)
- (b^2*Log[1 + c/x^2]^2)/(12*x^3) + (2*b^2*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c]
)/(Sqrt[c] - I*x)])/(3*c^(3/2)) - (b^2*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt
[c] - x))/(Sqrt[c] - I*x)])/(3*c^(3/2)) + (2*b^2*ArcTanh[x/Sqrt[c]]*Log[(2*
Sqrt[c])/(Sqrt[c] + x)])/(3*c^(3/2)) - (b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[
c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(3*c^(3/2)) - (b^
2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(
Sqrt[c] + x))])/(3*c^(3/2)) - (b^2*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c]
+ x))/(Sqrt[c] - I*x)])/(3*c^(3/2)) - (2*b^2*ArcTanh[x/Sqrt[c]]*Log[2 - (2*
Sqrt[c])/(Sqrt[c] + x)])/(3*c^(3/2)) - ((I/3)*b^2*PolyLog[2, 1 - (2*Sqrt[c]
)/(Sqrt[c] - I*x)])/c^(3/2) + ((I/3)*b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[
c] - I*x)])/c^(3/2) + ((I/6)*b^2*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sq
rt[c] - I*x)])/c^(3/2) - (b^2*PolyLog[2, -(x/Sqrt[c])])/(3*c^(3/2)) + ((I/3
)*b^2*PolyLog[2, ((-I)*x)/Sqrt[c]])/c^(3/2) - ((I/3)*b^2*PolyLog[2, (I*x)/S
qrt[c]])/c^(3/2) + (b^2*PolyLog[2, x/Sqrt[c]])/(3*c^(3/2)) - (b^2*PolyLog[2
, 1 - (2*Sqrt[c])/(Sqrt[c] + x)])/(3*c^(3/2)) + (b^2*PolyLog[2, -1 + (2*Sqr
t[c])/(Sqrt[c] + x)])/(3*c^(3/2)) + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c
] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(6*c^(3/2)) + (b^2*PolyLog[2
, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/(6*
c^(3/2)) + ((I/6)*b^2*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x
)])/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
```

```
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6042

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + 1/(x^n*c)]/2) - b*(Log[1 - 1/(x^n*c
)]/2))^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rule 6045

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[x^m*(a + b*ArcCoth[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c, m}, x] && IGtQ[
p, 1] && ILtQ[n, 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x^4} - \frac{b\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{2x^4} + \frac{b^2\log^2\left(\cdots\right.}{4\cdots}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{x^4}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{x^4}\,dx\cdots$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{12x^3} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{12x^3} - \frac{1}{2}b\int\left(-\frac{2a\log\left(1+\frac{c}{x^2}\right)}{x^4} + \frac{b\log\cdots}{\cdots}\right.$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{12x^3} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{12x^3} + (ab)\int \frac{\log\left(1+\frac{c}{x^2}\right)}{x^4}\,dx - \frac{1}{2}b^2\int\cdots$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{12x^3} - \frac{ab\log\left(1+\frac{c}{x^2}\right)}{3x^3} + \frac{b^2\log\left(1-\frac{c}{x^2}\right)\log\left(1+\frac{c}{x^2}\right)}{6x^3} - \frac{b^2\cdots}{\cdots}$$

$$= -\frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{9x^3} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{3cx} + \frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\cdots\right.}{3c^{3/2}}$$

$$= \frac{2ab}{9x^3} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{9x^3} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{3cx} + \frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2\cdots\right.}{3c^{3\cdots}}$$

$$= \frac{2ab}{9x^3} - \frac{4b^2}{27x^3} - \frac{2ab}{3cx} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{9x^3} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{3cx} + \frac{b\tan\cdots}{\cdots}$$

$$= \frac{2ab}{9x^3} - \frac{4b^2}{27x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{ib^2\tan^{-1}\left(\cdots\right.}{3c^{3/2}}$$

$$= \frac{2ab}{9x^3} - \frac{4b^2}{27x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{8b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{9c^{3/2}} + \frac{ib^2\tan^{-1}\left(\cdots\right.}{3c^{3/2}}$$

$$= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{8b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{9c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \cdots$$

$$= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{14b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{9c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \cdots$$

$$= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{4b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \cdots$$

Mathematica [F]
time = 1.93, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2/x^4,x]
```

```
[Out] Integrate[(a + b*ArcTanh[c/x^2])^2/x^4, x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2/x^4,x)
```

```
[Out] int((a+b*arctanh(c/x^2))^2/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(c*(2*arctan(x/sqrt(c))/c^(5/2) + log((x - sqrt(c))/(x + sqrt(c)))/c^(
5/2) + 4/(c^2*x)) + 2*arctanh(c/x^2)/x^3)*a*b - 1/12*b^2*(log(x^2 - c)^2/x^
3 + 3*integrate(-1/3*(3*(x^2 - c)*log(x^2 + c)^2 + 2*(2*x^2 - 3*(x^2 - c)*l
og(x^2 + c))*log(x^2 - c))/(x^6 - c*x^4), x)) - 1/3*a^2/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x**4,x)
[Out] Integral((a + b*atanh(c/x**2))**2/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^4,x, algorithm="giac")
[Out] integrate((b*arctanh(c/x^2) + a)^2/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))^2/x^4,x)
[Out] int((a + b*atanh(c/x^2))^2/x^4, x)
```

3.181 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx$

Optimal. Leaf size=1337

$$\frac{2ab}{25x^5}-\frac{2ab}{15cx^3}+\frac{2ab}{5c^2x}-\frac{8b^2}{15c^2x}+\frac{2ab\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}-\frac{4b^2\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)}{15c^{5/2}}-\frac{ib^2\text{ArcTan}\left(\frac{x}{\sqrt{c}}\right)^2}{5c^{5/2}}+\frac{4b^2\tanh^{-1}\left(\frac{\ldots}{\sqrt{\ldots}}\right.}{15c^{5/2}}$$

```
[Out] 2/25*a*b/x^5-8/15*b^2/c^2/x+2/5*a*b*arctan(x/c^(1/2))/c^(5/2)+1/15*b^2*ln(1
-c/x^2)/c/x^3-1/5*b^2*ln(1-c/x^2)/c^2/x-1/5*b^2*arctan(x/c^(1/2))*ln(1-c/x^
2)/c^(5/2)-1/15*b*(2*a-b*ln(1-c/x^2))/c/x^3-1/5*b*(2*a-b*ln(1-c/x^2))/c^2/x
+1/5*b*arctanh(x/c^(1/2))*(2*a-b*ln(1-c/x^2))/c^(5/2)-1/5*a*b*ln(1+c/x^2)/x
^5-2/15*b^2*ln(1+c/x^2)/c/x^3+1/5*b^2*arctan(x/c^(1/2))*ln(1+c/x^2)/c^(5/2)
+1/5*b^2*arctanh(x/c^(1/2))*ln(1+c/x^2)/c^(5/2)+1/10*b^2*ln(1-c/x^2)*ln(1+c
/x^2)/x^5-2/5*b^2*arctan(x/c^(1/2))*ln(2*c^(1/2)/(-I*x+c^(1/2)))/c^(5/2)+1/
5*b^2*arctan(x/c^(1/2))*ln((1+I)*(-x+c^(1/2))/(-I*x+c^(1/2)))/c^(5/2)+2/5*b
^2*arctanh(x/c^(1/2))*ln(2*c^(1/2)/(x+c^(1/2)))/c^(5/2)-1/5*b^2*arctanh(x/c
^(1/2))*ln(2*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/2)))/c^(5
/2)+1/5*b^2*arctan(x/c^(1/2))*ln((1-I)*(x+c^(1/2))/(-I*x+c^(1/2)))/c^(5/2)-
1/5*b^2*arctanh(x/c^(1/2))*ln(2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(1
/2)+c^(1/2)))/c^(5/2)+2/5*b^2*arctan(x/c^(1/2))*ln(2-2*c^(1/2)/(-I*x+c^(1/2
)))/c^(5/2)-2/5*b^2*arctanh(x/c^(1/2))*ln(2-2*c^(1/2)/(x+c^(1/2)))/c^(5/2)-
1/5*I*b^2*arctan(x/c^(1/2))^2/c^(5/2)-1/5*I*b^2*polylog(2,-I*x/c^(1/2))/c^(
5/2)-1/5*I*b^2*polylog(2,-1+2*c^(1/2)/(-I*x+c^(1/2)))/c^(5/2)-1/10*I*b^2*po
lylog(2,1-(1+I)*(-x+c^(1/2))/(-I*x+c^(1/2)))/c^(5/2)-1/10*I*b^2*polylog(2,1
+(-1+I)*(x+c^(1/2))/(-I*x+c^(1/2)))/c^(5/2)+1/5*I*b^2*polylog(2,I*x/c^(1/2)
)/c^(5/2)+1/5*I*b^2*polylog(2,1-2*c^(1/2)/(-I*x+c^(1/2)))/c^(5/2)-4/15*b^2*
arctan(x/c^(1/2))/c^(5/2)+4/15*b^2*arctanh(x/c^(1/2))/c^(5/2)-1/5*b^2*arcta
nh(x/c^(1/2))^2/c^(5/2)-1/25*b^2*ln(1-c/x^2)/x^5-1/25*b*(2*a-b*ln(1-c/x^2))
/x^5-1/20*b^2*ln(1+c/x^2)^2/x^5-1/5*b^2*polylog(2,-x/c^(1/2))/c^(5/2)+1/5*b
^2*polylog(2,x/c^(1/2))/c^(5/2)-1/5*b^2*polylog(2,1-2*c^(1/2)/(x+c^(1/2)))/
c^(5/2)+1/5*b^2*polylog(2,-1+2*c^(1/2)/(x+c^(1/2)))/c^(5/2)+1/10*b^2*polylo
g(2,1-2*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/2)))/c^(5/2)+1
/10*b^2*polylog(2,1-2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(1/2)+c^(1/2
)))/c^(5/2)-2/15*a*b/c/x^3+2/5*a*b/c^2/x-1/20*(2*a-b*ln(1-c/x^2))^2/x^5
```

Rubi [A]
time = 2.04, antiderivative size = 1337, normalized size of antiderivative = 1.00, number of steps used = 130, number of rules used = 31, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.938, Rules used = {6045, 6042, 2507, 2526, 2505, 269, 331, 213, 212, 2520, 12, 266, 6820, 6135, 6079, 2497, 6874, 209, 30, 2637, 6139, 6031, 6057, 2449, 2352, 5048, 4940, 2438, 4966, 5044, 4988}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2/x^6,x]
```

```
[Out] (2*a*b)/(25*x^5) - (2*a*b)/(15*c*x^3) + (2*a*b)/(5*c^2*x) - (8*b^2)/(15*c^2
*x) + (2*a*b*ArcTan[x/Sqrt[c]])/(5*c^(5/2)) - (4*b^2*ArcTan[x/Sqrt[c]])/(15
*c^(5/2)) - ((I/5)*b^2*ArcTan[x/Sqrt[c]]^2)/c^(5/2) + (4*b^2*ArcTanh[x/Sqrt
[c]])/(15*c^(5/2)) - (b^2*ArcTanh[x/Sqrt[c]]^2)/(5*c^(5/2)) + (2*b^2*ArcTan
[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/(5*c^(5/2)) - (b^2*Log[1
- c/x^2])/(25*x^5) + (b^2*Log[1 - c/x^2])/(15*c*x^3) - (b^2*Log[1 - c/x^2])
/(5*c^2*x) - (b^2*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/(5*c^(5/2)) - (b*(2*a -
 b*Log[1 - c/x^2]))/(25*x^5) - (b*(2*a - b*Log[1 - c/x^2]))/(15*c*x^3) - (b
*(2*a - b*Log[1 - c/x^2]))/(5*c^2*x) + (b*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1
 - c/x^2]))/(5*c^(5/2)) - (2*a - b*Log[1 - c/x^2])^2/(20*x^5) - (a*b*Log[1
+ c/x^2])/(5*x^5) - (2*b^2*Log[1 + c/x^2])/(15*c*x^3) + (b^2*ArcTan[x/Sqrt[
c]]*Log[1 + c/x^2])/(5*c^(5/2)) + (b^2*ArcTanh[x/Sqrt[c]]*Log[1 + c/x^2])/(
5*c^(5/2)) + (b^2*Log[1 - c/x^2]*Log[1 + c/x^2])/(10*x^5) - (b^2*Log[1 + c/
x^2]^2)/(20*x^5) - (2*b^2*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] - I*x)
])/(5*c^(5/2)) + (b^2*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[c] - x))/(Sqrt[c
] - I*x)])/(5*c^(5/2)) + (2*b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c]
 + x)])/(5*c^(5/2)) - (b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] - x)
)/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(5*c^(5/2)) - (b^2*ArcTanh[x/Sqrt[
c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/(
5*c^(5/2)) + (b^2*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c] -
I*x)])/(5*c^(5/2)) - (2*b^2*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c]
 + x)])/(5*c^(5/2)) + ((I/5)*b^2*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)
])/c^(5/2) - ((I/5)*b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)])/c^(5/
2) - ((I/10)*b^2*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)])/c
^(5/2) - (b^2*PolyLog[2, -(x/Sqrt[c])])/(5*c^(5/2)) - ((I/5)*b^2*PolyLog[2,
 ((-I)*x)/Sqrt[c]])/c^(5/2) + ((I/5)*b^2*PolyLog[2, (I*x)/Sqrt[c]])/c^(5/2)
 + (b^2*PolyLog[2, x/Sqrt[c]])/(5*c^(5/2)) - (b^2*PolyLog[2, 1 - (2*Sqrt[c]
)/(Sqrt[c] + x)])/(5*c^(5/2)) + (b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] +
 x)])/(5*c^(5/2)) + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-
c] - Sqrt[c])*(Sqrt[c] + x))])/(10*c^(5/2)) + (b^2*PolyLog[2, 1 - (2*Sqrt[c
]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/(10*c^(5/2)) - ((I
/10)*b^2*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)])/c^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
```

```
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6042

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + 1/(x^n*c)]/2) - b*(Log[1 - 1/(x^n*c
)]/2))^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rule 6045

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[x^m*(a + b*ArcCoth[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c, m}, x] && IGtQ[
p, 1] && ILtQ[n, 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x^6} - \frac{b\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{2x^6} + \frac{b^2\log^2\left(\right.}{4\ldots}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{x^6}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{x^6}\,dx$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{20x^5} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{20x^5} - \frac{1}{2}b\int\left(-\frac{2a\log\left(1+\frac{c}{x^2}\right)}{x^6} + \frac{b\log\ldots}{}\right.$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{20x^5} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{20x^5} + (ab)\int \frac{\log\left(1+\frac{c}{x^2}\right)}{x^6}\,dx - \frac{1}{2}b^2\int$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{20x^5} - \frac{ab\log\left(1+\frac{c}{x^2}\right)}{5x^5} + \frac{b^2\log\left(1-\frac{c}{x^2}\right)\log\left(1+\frac{c}{x^2}\right)}{10x^5} - \frac{b^2}{}$$

$$= -\frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{25x^5} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{15cx^3} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{5c^2x} +$$

$$= \frac{2ab}{25x^5} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{25x^5} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{15cx^3} - \frac{b\left(2a-b\log\left(1-\ldots\right)\right)}{5c^2x}$$

$$= \frac{2ab}{25x^5} - \frac{4b^2}{125x^5} - \frac{2ab}{15cx^3} - \frac{4b^2}{5c^2x} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{25x^5} - \frac{b\left(2a-b\log\left(1-\ldots\right)\right)}{15cx^3}$$

$$= \frac{2ab}{25x^5} - \frac{4b^2}{125x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{16b^2}{15c^2x} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{ib^2\tan^{-1}\left(\ldots\right)}{5c^{5/2}}$$

$$= \frac{2ab}{25x^5} - \frac{4b^2}{125x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{92b^2}{75c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{8b^2\tan^{-1}\left(\ldots\right)}{15c^{5/2}}$$

$$= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{32b^2}{75c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{46b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{75c^{5/2}} -$$

$$= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{52b^2}{75c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{16b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{75c^{5/2}} -$$

$$= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{8b^2}{15c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{26b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{75c^{5/2}} -$$

Mathematica [F]
time = 1.95, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2/x^6,x]
```

```
[Out] Integrate[(a + b*ArcTanh[c/x^2])^2/x^6, x]
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2/x^6,x)
```

```
[Out] int((a+b*arctanh(c/x^2))^2/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/15*(c*(6*arctan(x/sqrt(c))/c^(7/2) - 3*log((x - sqrt(c))/(x + sqrt(c)))/c
^(7/2) - 4/(c^2*x^3)) - 6*arctanh(c/x^2)/x^5)*a*b - 1/20*b^2*(log(x^2 - c)^
2/x^5 + 5*integrate(-1/5*(5*(x^2 - c)*log(x^2 + c)^2 + 2*(2*x^2 - 5*(x^2 -
c)*log(x^2 + c))*log(x^2 - c))/(x^8 - c*x^6), x)) - 1/5*a^2/x^5
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^6,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x^6, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x**6,x)
[Out] Integral((a + b*atanh(c/x**2))**2/x**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^6,x, algorithm="giac")
[Out] integrate((b*arctanh(c/x^2) + a)^2/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)}^2}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))^2/x^6,x)
[Out] int((a + b*atanh(c/x^2))^2/x^6, x)
```

3.182 $\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^3 dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^3, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c/x^2))^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^3 dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c/x^2])^3,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c/x^2])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^3 dx = \int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^3 dx$$

Mathematica [A]
time = 1.90, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^3 dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2])^3, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c/x^2))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c/x^2))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*d^m*x*x^m*log(x^2 - c)^3/(m + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1)) +
 integrate(1/8*((b^3*d^m*(m + 1)*x^2 - b^3*c*d^m*(m + 1))*x^m*log(x^2 + c)^
3 + 6*(a*b^2*d^m*(m + 1)*x^2 - a*b^2*c*d^m*(m + 1))*x^m*log(x^2 + c)^2 + 12
*(a^2*b*d^m*(m + 1)*x^2 - a^2*b*c*d^m*(m + 1))*x^m*log(x^2 + c) + 3*((b^3*d
^m*(m + 1)*x^2 - b^3*c*d^m*(m + 1))*x^m*log(x^2 + c) - 2*(a*b^2*c*d^m*(m +
1) - (a*b^2*d^m*(m + 1) + b^3*d^m)*x^2)*x^m)*log(x^2 - c)^2 - 3*((b^3*d^m*(
m + 1)*x^2 - b^3*c*d^m*(m + 1))*x^m*log(x^2 + c)^2 + 4*(a*b^2*d^m*(m + 1)*x
^2 - a*b^2*c*d^m*(m + 1))*x^m*log(x^2 + c) + 4*(a^2*b*d^m*(m + 1)*x^2 - a^2
*b*c*d^m*(m + 1))*x^m)*log(x^2 - c))/((m + 1)*x^2 - c*(m + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c/x^2)^3 + 3*a*b^2*arctanh(c/x^2)^2 + 3*a^2*b*arctanh
(c/x^2) + a^3)*(d*x)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c/x**2))**3,x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c/x**2))**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^3*(d*x)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c/x^2))^3,x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c/x^2))^3, x)
```

3.183 $\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c/x^2))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c/x^2])^2,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c/x^2])^2, x]
```

Rubi steps

$$\int (dx)^m\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx = \int (dx)^m\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Mathematica [A]
time = 1.20, size = 0, normalized size = 0.00

$$\int (dx)^m\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2])^2, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dx)^m\left(a+b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c/x^2))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c/x^2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*d^m*x*x^m*log(x^2 - c)^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1)) -
integrate(-1/4*((b^2*d^m*(m + 1)*x^2 - b^2*c*d^m*(m + 1))*x^m*log(x^2 + c)^
2 + 4*(a*b*d^m*(m + 1)*x^2 - a*b*c*d^m*(m + 1))*x^m*log(x^2 + c) - 2*((b^2*
d^m*(m + 1)*x^2 - b^2*c*d^m*(m + 1))*x^m*log(x^2 + c) - 2*(a*b*c*d^m*(m + 1
) - (a*b*d^m*(m + 1) + b^2*d^m)*x^2)*x^m)*log(x^2 - c))/((m + 1)*x^2 - c*(m
 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)*(d*x)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c/x**2))**2,x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c/x**2))**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2*(d*x)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c/x^2))^2,x)
[Out] int((d*x)^m*(a + b*atanh(c/x^2))^2, x)
```

3.184 $\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx$

Optimal. Leaf size=75

$$\frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{d(1+m)}-\frac{2bcd(dx)^{-1+m}\,{}_2F_1\left(1,\frac{1-m}{4};\frac{5-m}{4};\frac{c^2}{x^4}\right)}{1-m^2}$$

```
[Out] (d*x)^(1+m)*(a+b*arctanh(c/x^2))/d/(1+m)-2*b*c*d*(d*x)^(-1+m)*hypergeom([1,
 1/4-1/4*m],[5/4-1/4*m],c^2/x^4)/(-m^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6049, 346, 371}

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{d(m+1)}-\frac{2bcd(dx)^{m-1}\,{}_2F_1\left(1,\frac{1-m}{4};\frac{5-m}{4};\frac{c^2}{x^4}\right)}{1-m^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTanh[c/x^2]))/(d*(1 + m)) - (2*b*c*d*(d*x)^(-1 + m
)*Hypergeometric2F1[1, (1 - m)/4, (5 - m)/4, c^2/x^4])/(1 - m^2)
```

Rule 346

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1), Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x
, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6049

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n
/(d^n*(m + 1))), Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx = \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{d(1+m)} + \frac{(2bc)\int \frac{(dx)^{1+m}}{\left(1-\frac{c^2}{x^4}\right)x^3}dx}{d(1+m)}$$

$$= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{d(1+m)} + \frac{(2bcd^2)\int \frac{(dx)^{-2+m}}{1-\frac{c^2}{x^4}}dx}{1+m}$$

$$= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{d(1+m)} - \frac{\left(2bcd\left(\frac{1}{x}\right)^{-1+m}(dx)^{-1+m}\right)\text{Subst}\left(\int \frac{x^{-}}{1-c}\right.}{1+m}$$

$$= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{d(1+m)} - \frac{2bcd(dx)^{-1+m}\,{}_2F_1\left(1,\frac{1-m}{4};\frac{5-m}{4};\frac{c^2}{x^4}\right)}{1-m^2}$$

Mathematica [A]
time = 0.05, size = 68, normalized size = 0.91

$$\frac{(dx)^m\left((-1+m)x^2\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+2bc\,{}_2F_1\left(1,\frac{1}{4}-\frac{m}{4};\frac{5}{4}-\frac{m}{4};\frac{c^2}{x^4}\right)\right)}{(-1+m)(1+m)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2]),x]
[Out] ((d*x)^m*((-1 + m)*x^2*(a + b*ArcTanh[c/x^2]) + 2*b*c*Hypergeometric2F1[1,
1/4 - m/4, 5/4 - m/4, c^2/x^4]))/((-1 + m)*(1 + m)*x)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c/x^2)),x)
[Out] int((d*x)^m*(a+b*arctanh(c/x^2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] 1/2*(4*c*d^m*integrate(x^2*x^m/((m + 1)*x^4 - c^2*(m + 1)), x) + (d^m*x*x^m
*log(x^2 + c) - d^m*x*x^m*log(x^2 - c))/(m + 1))*b + (d*x)^(m + 1)*a/(d*(m
+ 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
[Out] integral((b*arctanh(c/x^2) + a)*(d*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c/x**2)),x)
[Out] Integral((d*x)**m*(a + b*atanh(c/x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2)),x, algorithm="giac")
[Out] integrate((b*arctanh(c/x^2) + a)*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,x\right)}^m\,\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c/x^2)),x)
[Out] int((d*x)^m*(a + b*atanh(c/x^2)), x)
```

3.185 $\int \frac{(dx)^m}{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(dx)^m}{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctanh(c/x^2)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c/x^2]),x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c/x^2]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}\,dx=\int \frac{(dx)^m}{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}\,dx$$

Mathematica [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c/x^2]),x]
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c/x^2]), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c/x^2)),x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c/x^2)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c/x^2) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b*arctanh(c/x^2) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c/x**2)),x)
```

```
[Out] Integral((d*x)**m/(a + b*atanh(c/x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c/x^2) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c/x^2)),x)
[Out] int((d*x)^m/(a + b*atanh(c/x^2)), x)
```

3.186 $\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(dx)^m}{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2},x\right)$$

`[Out] Unintegrable((d*x)^m/(a+b*arctanh(c/x^2))^2,x)`

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

`[In] Int[(d*x)^m/(a + b*ArcTanh[c/x^2])^2,x]`

`[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c/x^2])^2, x]`

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[(d*x)^m/(a + b*ArcTanh[c/x^2])^2,x]`

`[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c/x^2])^2, x]`

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c/x^2))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c/x^2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] (d^m*x^4 - c^2*d^m)*x^m/(b^2*c*x*log(x^2 + c) - b^2*c*x*log(x^2 - c) + 2*a*
b*c*x) + integrate(-(d^m*(m + 3)*x^4 - c^2*d^m*(m - 1))*x^m/(b^2*c*x^2*log(
x^2 + c) - b^2*c*x^2*log(x^2 - c) + 2*a*b*c*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c/x**2))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c/x^2) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c/x^2))^2,x)
[Out] int((d*x)^m/(a + b*atanh(c/x^2))^2, x)
```

3.187 $\int x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\,dx$

Optimal. Leaf size=88

$$\frac{b\sqrt{x}}{4c^7}+\frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{4c^8}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)$$

```
[Out] 1/12*b*x^(3/2)/c^5+1/20*b*x^(5/2)/c^3+1/28*b*x^(7/2)/c-1/4*b*arctanh(c*x^(1
/2))/c^8+1/4*x^4*(a+b*arctanh(c*x^(1/2)))+1/4*b*x^(1/2)/c^7
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6037, 52, 65, 212}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{4c^8}+\frac{b\sqrt{x}}{4c^7}+\frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

```
[Out] (b*Sqrt[x])/(4*c^7) + (b*x^(3/2))/(12*c^5) + (b*x^(5/2))/(20*c^3) + (b*x^(7
/2))/(28*c) - (b*ArcTanh[c*Sqrt[x]])/(4*c^8) + (x^4*(a + b*ArcTanh[c*Sqrt[x
]]))/4
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\,dx &= \frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{1}{8}(bc)\int\frac{x^{7/2}}{1-c^2x}\,dx\\ &= \frac{bx^{7/2}}{28c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{x^{5/2}}{1-c^2x}\,dx}{8c}\\ &= \frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{x^{3/2}}{1-c^2x}\,dx}{8c^3}\\ &= \frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{\sqrt{x}}{1-c^2x}\,dx}{8c^5}\\ &= \frac{b\sqrt{x}}{4c^7}+\frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{1}{\sqrt{x}\,(1-}}{8c^7}\\ &= \frac{b\sqrt{x}}{4c^7}+\frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\text{Subst}\left(\int\right.}{}\\ &= \frac{b\sqrt{x}}{4c^7}+\frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{4c^8}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\right.\right.\end{aligned}$$

Mathematica [A]
time = 0.04, size = 114, normalized size = 1.30

$$\frac{b\sqrt{x}}{4c^7}+\frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}+\frac{ax^4}{4}+\frac{1}{4}bx^4\tanh^{-1}\left(c\sqrt{x}\right)+\frac{b\log\left(1-c\sqrt{x}\right)}{8c^8}-\frac{b\log\left(1+c\sqrt{x}\right)}{8c^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

```
[Out] (b*Sqrt[x])/(4*c^7) + (b*x^(3/2))/(12*c^5) + (b*x^(5/2))/(20*c^3) + (b*x^(7
/2))/(28*c) + (a*x^4)/4 + (b*x^4*ArcTanh[c*Sqrt[x]])/4 + (b*Log[1 - c*Sqrt[
x]])/(8*c^8) - (b*Log[1 + c*Sqrt[x]])/(8*c^8)
```

Maple [A]
time = 0.06, size = 87, normalized size = 0.99

method	result	size
derivativedivides	$\frac{\frac{c^{8}x^{4}a}{4}+\frac{b\,c^{8}x^{4}\operatorname{arctanh}\left(c\sqrt{x}\right)}{4}+\frac{c^{7}x^{\frac{7}{2}}b}{28}+\frac{b\,c^{5}x^{\frac{5}{2}}}{20}+\frac{b\,c^{3}x^{\frac{3}{2}}}{12}+\frac{bc\sqrt{x}}{4}+\frac{b\ln\left(c\sqrt{x}-1\right)}{8}-\frac{b\ln\left(1+c\sqrt{x}\right)}{8}}{c^{8}}$	87
default	$\frac{\frac{c^{8}x^{4}a}{4}+\frac{b\,c^{8}x^{4}\operatorname{arctanh}\left(c\sqrt{x}\right)}{4}+\frac{c^{7}x^{\frac{7}{2}}b}{28}+\frac{b\,c^{5}x^{\frac{5}{2}}}{20}+\frac{b\,c^{3}x^{\frac{3}{2}}}{12}+\frac{bc\sqrt{x}}{4}+\frac{b\ln\left(c\sqrt{x}-1\right)}{8}-\frac{b\ln\left(1+c\sqrt{x}\right)}{8}}{c^{8}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^(1/2))),x,method=_RETURNVERBOSE)
[Out] 2/c^8*(1/8*c^8*x^4*a+1/8*b*c^8*x^4*arctanh(c*x^(1/2))+1/56*c^7*x^(7/2)*b+1/
40*b*c^5*x^(5/2)+1/24*b*c^3*x^(3/2)+1/8*b*c*x^(1/2)+1/16*b*ln(c*x^(1/2)-1)-
1/16*b*ln(1+c*x^(1/2)))
```

Maxima [A]
time = 0.26, size = 86, normalized size = 0.98

$$\frac{1}{4}\,ax^{4}+\frac{1}{840}\left(210\,x^{4}\operatorname{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(15\,c^{6}x^{\frac{7}{2}}+21\,c^{4}x^{\frac{5}{2}}+35\,c^{2}x^{\frac{3}{2}}+105\,\sqrt{x}\right)}{c^{8}}-\frac{105\,\log\left(c\sqrt{x}+1\right)}{c^{9}}+\frac{105\,\log\left(c\sqrt{x}-1\right)}{c^{9}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2))),x, algorithm="maxima")
[Out] 1/4*a*x^4 + 1/840*(210*x^4*arctanh(c*sqrt(x)) + c*(2*(15*c^6*x^(7/2) + 21*c
^4*x^(5/2) + 35*c^2*x^(3/2) + 105*sqrt(x))/c^8 - 105*log(c*sqrt(x) + 1)/c^9
 + 105*log(c*sqrt(x) - 1)/c^9))*b
```

Fricas [A]
time = 0.39, size = 89, normalized size = 1.01

$$\frac{210\,ac^{8}x^{4}+105\,(bc^{8}x^{4}-b)\log\left(-\frac{c^{2}x+2\,c\sqrt{x}+1}{c^{2}x-1}\right)+2\,(15\,bc^{7}x^{3}+21\,bc^{5}x^{2}+35\,bc^{3}x+105\,bc)\sqrt{x}}{840\,c^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2))),x, algorithm="fricas")
[Out] 1/840*(210*a*c^8*x^4 + 105*(b*c^8*x^4 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(
c^2*x - 1)) + 2*(15*b*c^7*x^3 + 21*b*c^5*x^2 + 35*b*c^3*x + 105*b*c)*sqrt(x
))/c^8
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(a+b\operatorname{atanh}\left(c\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(a+b*atanh(c*x**(1/2))),x)

[Out] Integral(x**3*(a + b*atanh(c*sqrt(x))), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(64) = 128.
time = 0.44, size = 359, normalized size = 4.08

$$\frac{1}{4}ax^4+\frac{2}{105}bc\left(\frac{\frac{105\left(c\sqrt{x}+1\right)^6}{\left(c\sqrt{x}-1\right)^6}-\frac{315\left(c\sqrt{x}+1\right)^5}{\left(c\sqrt{x}-1\right)^5}+\frac{770\left(c\sqrt{x}+1\right)^4}{\left(c\sqrt{x}-1\right)^4}-\frac{770\left(c\sqrt{x}+1\right)^3}{\left(c\sqrt{x}-1\right)^3}+\frac{609\left(c\sqrt{x}+1\right)^2}{\left(c\sqrt{x}-1\right)^2}-\frac{203\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1}+44}{c^9\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)^7}+\frac{105\left(\frac{\left(c\sqrt{x}+1\right)^7}{\left(c\sqrt{x}-1\right)^7}+\frac{7\left(c\sqrt{x}+1\right)^5}{\left(c\sqrt{x}-1\right)^5}+\frac{7\left(c\sqrt{x}+1\right)^3}{\left(c\sqrt{x}-1\right)^3}+\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)\log\left(-\frac{\frac{c\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}+1\right)}{\frac{\left(c\sqrt{x}+1\right)c}{c\sqrt{x}-1}-c}+1}{\frac{c\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}+1\right)}{\frac{\left(c\sqrt{x}+1\right)c}{c\sqrt{x}-1}-c}-1}\right)}{c^9\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)^8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arctanh(c*x^(1/2))),x, algorithm="giac")

[Out] 1/4*a*x^4 + 2/105*b*c*((105*(c*sqrt(x) + 1)^6/(c*sqrt(x) - 1)^6 - 315*(c*sqrt(x) + 1)^5/(c*sqrt(x) - 1)^5 + 770*(c*sqrt(x) + 1)^4/(c*sqrt(x) - 1)^4 - 770*(c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3 + 609*(c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 - 203*(c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 44)/(c^9*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) - 1)^7) + 105*((c*sqrt(x) + 1)^7/(c*sqrt(x) - 1)^7 + 7*(c*sqrt(x) + 1)^5/(c*sqrt(x) - 1)^5 + 7*(c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3 + (c*sqrt(x) + 1)/(c*sqrt(x) - 1))*log(-(c*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(c*sqrt(x) - 1) - c) + 1)/(c*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(c*sqrt(x) - 1) - c) - 1))/(c^9*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) - 1)^8))

Mupad [B]
time = 1.43, size = 86, normalized size = 0.98

$$\frac{\frac{b\,c^3\,x^{3/2}}{12}-\frac{b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{4}+\frac{b\,c^5\,x^{5/2}}{20}+\frac{b\,c^7\,x^{7/2}}{28}+\frac{b\,c\,\sqrt{x}}{4}}{c^8}+\frac{b\,\left(105\,x^4\,\ln\left(c\,\sqrt{x}+1\right)-105\,x^4\,\ln\left(1-c\,\sqrt{x}\right)\right)}{840}+\frac{a\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a + b*atanh(c*x^(1/2))),x)

[Out] ((b*c^3*x^(3/2))/12 - (b*atanh(c*x^(1/2)))/4 + (b*c^5*x^(5/2))/20 + (b*c^7*x^(7/2))/28 + (b*c*x^(1/2))/4)/c^8 + (b*(105*x^4*log(c*x^(1/2) + 1) - 105*x^4*log(1 - c*x^(1/2))))/840 + (a*x^4)/4

3.188 $\int x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\,dx$

Optimal. Leaf size=75

$$\frac{b\sqrt{x}}{3c^5}+\frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{3c^6}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)$$

```
[Out] 1/9*b*x^(3/2)/c^3+1/15*b*x^(5/2)/c-1/3*b*arctanh(c*x^(1/2))/c^6+1/3*x^3*(a+
b*arctanh(c*x^(1/2)))+1/3*b*x^(1/2)/c^5
```

Rubi [A]
time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6037, 52, 65, 212}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{3c^6}+\frac{b\sqrt{x}}{3c^5}+\frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

```
[Out] (b*Sqrt[x])/(3*c^5) + (b*x^(3/2))/(9*c^3) + (b*x^(5/2))/(15*c) - (b*ArcTanh
[c*Sqrt[x]])/(3*c^6) + (x^3*(a + b*ArcTanh[c*Sqrt[x]]))/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\,dx &= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{1}{6}(bc)\int\frac{x^{5/2}}{1-c^2x}\,dx\\
&= \frac{bx^{5/2}}{15c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{x^{3/2}}{1-c^2x}\,dx}{6c}\\
&= \frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{\sqrt{x}}{1-c^2x}\,dx}{6c^3}\\
&= \frac{b\sqrt{x}}{3c^5}+\frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{1}{\sqrt{x}\,(1-c^2x)}\,dx}{6c^5}\\
&= \frac{b\sqrt{x}}{3c^5}+\frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\text{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx\right.}{3c^5}\\
&= \frac{b\sqrt{x}}{3c^5}+\frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{3c^6}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 101, normalized size = 1.35

$$\frac{b\sqrt{x}}{3c^5}+\frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}+\frac{ax^3}{3}+\frac{1}{3}bx^3\tanh^{-1}\left(c\sqrt{x}\right)+\frac{b\log\left(1-c\sqrt{x}\right)}{6c^6}-\frac{b\log\left(1+c\sqrt{x}\right)}{6c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

```
[Out] (b*Sqrt[x])/(3*c^5) + (b*x^(3/2))/(9*c^3) + (b*x^(5/2))/(15*c) + (a*x^3)/3
+ (b*x^3*ArcTanh[c*Sqrt[x]])/3 + (b*Log[1 - c*Sqrt[x]])/(6*c^6) - (b*Log[1
+ c*Sqrt[x]])/(6*c^6)
```

Maple [A]
time = 0.07, size = 78, normalized size = 1.04

method	result	size

derivativedivides	$\frac{\frac{c^6x^3a}{3}+\frac{b\,c^6x^3\,\mathrm{arctanh}\left(c\sqrt{x}\right)}{3}+\frac{b\,c^5x^{\frac{5}{2}}}{15}+\frac{b\,c^3x^{\frac{3}{2}}}{9}+\frac{bc\sqrt{x}}{3}+\frac{b\ln\left(c\sqrt{x}-1\right)}{6}-\frac{b\ln\left(1+c\sqrt{x}\right)}{6}}{c^6}$	78
default	$\frac{\frac{c^6x^3a}{3}+\frac{b\,c^6x^3\,\mathrm{arctanh}\left(c\sqrt{x}\right)}{3}+\frac{b\,c^5x^{\frac{5}{2}}}{15}+\frac{b\,c^3x^{\frac{3}{2}}}{9}+\frac{bc\sqrt{x}}{3}+\frac{b\ln\left(c\sqrt{x}-1\right)}{6}-\frac{b\ln\left(1+c\sqrt{x}\right)}{6}}{c^6}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^(1/2))),x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^6*(1/6*c^6*x^3*a+1/6*b*c^6*x^3*arctanh(c*x^(1/2))+1/30*b*c^5*x^(5/2)+1/
18*b*c^3*x^(3/2)+1/6*b*c*x^(1/2)+1/12*b*ln(c*x^(1/2)-1)-1/12*b*ln(1+c*x^(1/
2)))
```

Maxima [A]
time = 0.27, size = 78, normalized size = 1.04

$$\frac{1}{3}\,ax^3+\frac{1}{90}\left(30\,x^3\,\mathrm{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(3\,c^4x^{\frac{5}{2}}+5\,c^2x^{\frac{3}{2}}+15\,\sqrt{x}\right)}{c^6}-\frac{15\,\log\left(c\sqrt{x}+1\right)}{c^7}+\frac{15\,\log\left(c\sqrt{x}-1\right)}{c^7}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2))),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/90*(30*x^3*arctanh(c*sqrt(x)) + c*(2*(3*c^4*x^(5/2) + 5*c^2*x
^(3/2) + 15*sqrt(x))/c^6 - 15*log(c*sqrt(x) + 1)/c^7 + 15*log(c*sqrt(x) - 1
)/c^7))*b
```

Fricas [A]
time = 0.35, size = 80, normalized size = 1.07

$$\frac{30\,ac^6x^3+15\,(bc^6x^3-b)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)+2\,(3\,bc^5x^2+5\,bc^3x+15\,bc)\sqrt{x}}{90\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2))),x, algorithm="fricas")
```

```
[Out] 1/90*(30*a*c^6*x^3 + 15*(b*c^6*x^3 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2
*x - 1)) + 2*(3*b*c^5*x^2 + 5*b*c^3*x + 15*b*c)*sqrt(x))/c^6
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\,\mathrm{atanh}\left(c\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a+b*atanh(c*x**(1/2))),x)

[Out] Integral(x**2*(a + b*atanh(c*sqrt(x))), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(55) = 110.
time = 0.44, size = 301, normalized size = 4.01

$$\frac{1}{3}ax^3+\frac{2}{45}bc\left(\frac{\frac{45\left(c\sqrt{x}+1\right)^4}{\left(c\sqrt{x}-1\right)^4}-\frac{90\left(c\sqrt{x}+1\right)^3}{\left(c\sqrt{x}-1\right)^3}+\frac{140\left(c\sqrt{x}+1\right)^2}{\left(c\sqrt{x}-1\right)^2}-\frac{70\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1}+23}{c^7\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)^5}+\frac{15\left(\frac{3\left(c\sqrt{x}+1\right)^5}{\left(c\sqrt{x}-1\right)^5}+\frac{10\left(c\sqrt{x}+1\right)^3}{\left(c\sqrt{x}-1\right)^3}+\frac{3\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1}\right)\log\left(-\frac{\frac{c\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}+1\right)}{\frac{\left(c\sqrt{x}+1\right)c}{c\sqrt{x}-1}-c}+1}{\frac{c\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}+1\right)}{\frac{\left(c\sqrt{x}+1\right)c}{c\sqrt{x}-1}-c}-1}\right)}{c^7\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arctanh(c*x^(1/2))),x, algorithm="giac")

[Out] 1/3*a*x^3 + 2/45*b*c*((45*(c*sqrt(x) + 1)^4/(c*sqrt(x) - 1)^4 - 90*(c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3 + 140*(c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 - 70*(c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 23)/(c^7*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) - 1)^5) + 15*(3*(c*sqrt(x) + 1)^5/(c*sqrt(x) - 1)^5 + 10*(c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3 + 3*(c*sqrt(x) + 1)/(c*sqrt(x) - 1))*log(-(c*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(c*sqrt(x) - 1) - c) + 1)/(c*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(c*sqrt(x) - 1) - c) - 1))/(c^7*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) - 1)^6))

Mupad [B]
time = 1.22, size = 58, normalized size = 0.77

$$\frac{a\,x^3}{3}+\frac{\frac{b\,c^3\,x^{3/2}}{9}-\frac{b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{3}+\frac{b\,c^5\,x^{5/2}}{15}+\frac{b\,c\,\sqrt{x}}{3}}{c^6}+\frac{b\,x^3\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + b*atanh(c*x^(1/2))),x)

[Out] (a*x^3)/3 + ((b*c^3*x^(3/2))/9 - (b*atanh(c*x^(1/2)))/3 + (b*c^5*x^(5/2))/15 + (b*c*x^(1/2))/3)/c^6 + (b*x^3*atanh(c*x^(1/2)))/3

3.189 $\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\,dx$

Optimal. Leaf size=62

$$\frac{b\sqrt{x}}{2c^3}+\frac{bx^{3/2}}{6c}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{2c^4}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)$$

```
[Out] 1/6*b*x^(3/2)/c-1/2*b*arctanh(c*x^(1/2))/c^4+1/2*x^2*(a+b*arctanh(c*x^(1/2)
))+1/2*b*x^(1/2)/c^3
```

Rubi [A]

time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6037, 52, 65, 212}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{2c^4}+\frac{b\sqrt{x}}{2c^3}+\frac{bx^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

```
[Out] (b*Sqrt[x])/(2*c^3) + (b*x^(3/2))/(6*c) - (b*ArcTanh[c*Sqrt[x]])/(2*c^4) +
(x^2*(a + b*ArcTanh[c*Sqrt[x]]))/2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\,dx&=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{1}{4}(bc)\int\frac{x^{3/2}}{1-c^2x}\,dx\\&=\frac{bx^{3/2}}{6c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{\sqrt{x}}{1-c^2x}\,dx}{4c}\\&=\frac{b\sqrt{x}}{2c^3}+\frac{bx^{3/2}}{6c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{1}{\sqrt{x}\,(1-c^2x)}\,dx}{4c^3}\\&=\frac{b\sqrt{x}}{2c^3}+\frac{bx^{3/2}}{6c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\text{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,\sqrt{x}\right)}{2c^3}\\&=\frac{b\sqrt{x}}{2c^3}+\frac{bx^{3/2}}{6c}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{2c^4}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 1.42

$$\frac{b\sqrt{x}}{2c^3}+\frac{bx^{3/2}}{6c}+\frac{ax^2}{2}+\frac{1}{2}bx^2\tanh^{-1}\left(c\sqrt{x}\right)+\frac{b\log\left(1-c\sqrt{x}\right)}{4c^4}-\frac{b\log\left(1+c\sqrt{x}\right)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

```
[Out] (b*Sqrt[x])/(2*c^3) + (b*x^(3/2))/(6*c) + (a*x^2)/2 + (b*x^2*ArcTanh[c*Sqrt
[x]])/2 + (b*Log[1 - c*Sqrt[x]])/(4*c^4) - (b*Log[1 + c*Sqrt[x]])/(4*c^4)
```

Maple [A]
time = 0.07, size = 69, normalized size = 1.11

method	result	size
derivativedivides	$\frac{\frac{c^4x^2a}{2}+\frac{b\,c^4x^2\,\mathrm{arctanh}\left(c\sqrt{x}\right)}{2}+\frac{b\,c^3x^{\frac{3}{2}}}{6}+\frac{bc\sqrt{x}}{2}+\frac{b\ln\left(c\sqrt{x}-1\right)}{4}-\frac{b\ln\left(1+c\sqrt{x}\right)}{4}}{c^4}$	69

default	$\frac{\frac{c^4x^2a}{2}+\frac{b\,c^4x^2\,\mathrm{arctanh}\left(c\sqrt{x}\right)}{2}+\frac{b\,c^3x^{\frac{3}{2}}}{6}+\frac{bc\sqrt{x}}{2}+\frac{b\ln\left(c\sqrt{x}-1\right)}{4}-\frac{b\ln\left(1+c\sqrt{x}\right)}{4}}{c^4}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^(1/2))),x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^4*(1/4*c^4*x^2*a+1/4*b*c^4*x^2*arctanh(c*x^(1/2))+1/12*b*c^3*x^(3/2)+1/
4*b*c*x^(1/2)+1/8*b*ln(c*x^(1/2)-1)-1/8*b*ln(1+c*x^(1/2)))
```

Maxima [A]
time = 0.26, size = 69, normalized size = 1.11

$$\frac{1}{2}\,ax^2+\frac{1}{12}\left(6\,x^2\,\mathrm{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(c^2x^{\frac{3}{2}}+3\sqrt{x}\right)}{c^4}-\frac{3\,\log\left(c\sqrt{x}+1\right)}{c^5}+\frac{3\,\log\left(c\sqrt{x}-1\right)}{c^5}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2))),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/12*(6*x^2*arctanh(c*sqrt(x)) + c*(2*(c^2*x^(3/2) + 3*sqrt(x))
/c^4 - 3*log(c*sqrt(x) + 1)/c^5 + 3*log(c*sqrt(x) - 1)/c^5))*b
```

Fricas [A]
time = 0.36, size = 70, normalized size = 1.13

$$\frac{6\,ac^4x^2+3\left(bc^4x^2-b\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)+2\left(bc^3x+3\,bc\right)\sqrt{x}}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2))),x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*c^4*x^2 + 3*(b*c^4*x^2 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x
 - 1)) + 2*(b*c^3*x + 3*b*c)*sqrt(x))/c^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\,\mathrm{atanh}\left(c\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**(1/2))),x)
```

```
[Out] Integral(x*(a + b*atanh(c*sqrt(x))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(46) = 92.

time = 0.45, size = 239, normalized size = 3.85

$$\frac{1}{2}ax^2+\frac{2}{3}bc\left(\frac{\frac{3\left(c\sqrt{x}+1\right)^2}{\left(c\sqrt{x}-1\right)^2}-\frac{3\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1}+2}{c^5\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)^3}+\frac{3\left(\frac{\left(c\sqrt{x}+1\right)^3}{\left(c\sqrt{x}-1\right)^3}+\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)\log\left(-\frac{\frac{c\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}+1\right)}{\frac{\left(c\sqrt{x}+1\right)c}{c\sqrt{x}-1}-c}+1}{\frac{c\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}+1\right)}{\frac{\left(c\sqrt{x}+1\right)c}{c\sqrt{x}-1}-c}-1}\right)}{c^5\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2))),x, algorithm="giac")
```

```
[Out] 1/2*a*x^2 + 2/3*b*c*((3*(c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 - 3*(c*sqrt(x)
+ 1)/(c*sqrt(x) - 1) + 2)/(c^5*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) - 1)^3) + 3
*((c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3 + (c*sqrt(x) + 1)/(c*sqrt(x) - 1))*lo
g(-(c*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(c*sqrt(x) -
 1) - c) + 1)/(c*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(
c*sqrt(x) - 1) - c) - 1))/(c^5*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) - 1)^4))
```

Mupad [B]

time = 1.17, size = 49, normalized size = 0.79

$$\frac{\frac{b\,c^3\,x^{3/2}}{6}-\frac{b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{2}+\frac{b\,c\,\sqrt{x}}{2}}{c^4}+\frac{a\,x^2}{2}+\frac{b\,x^2\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^(1/2))),x)
```

```
[Out] ((b*c^3*x^(3/2))/6 - (b*atanh(c*x^(1/2)))/2 + (b*c*x^(1/2))/2)/c^4 + (a*x^2
)/2 + (b*x^2*atanh(c*x^(1/2)))/2
```

3.190 $\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\,dx$

Optimal. Leaf size=39

$$\frac{b\sqrt{x}}{c}+ax-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{c^2}+bx\tanh^{-1}\left(c\sqrt{x}\right)$$

```
[Out] a*x-b*arctanh(c*x^(1/2))/c^2+b*x*arctanh(c*x^(1/2))+b*x^(1/2)/c
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {6021, 52, 65, 212}

$$ax-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{c^2}+\frac{b\sqrt{x}}{c}+bx\tanh^{-1}\left(c\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c*Sqrt[x]],x]
```

```
[Out] (b*Sqrt[x])/c + a*x - (b*ArcTanh[c*Sqrt[x]])/c^2 + b*x*ArcTanh[c*Sqrt[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\,dx &= ax+b\int \tanh^{-1}\left(c\sqrt{x}\right)\,dx\\
&= ax+bx\tanh^{-1}\left(c\sqrt{x}\right)-\frac{1}{2}(bc)\int \frac{\sqrt{x}}{1-c^2x}\,dx\\
&= \frac{b\sqrt{x}}{c}+ax+bx\tanh^{-1}\left(c\sqrt{x}\right)-\frac{b\int \frac{1}{\sqrt{x}\,(1-c^2x)}\,dx}{2c}\\
&= \frac{b\sqrt{x}}{c}+ax+bx\tanh^{-1}\left(c\sqrt{x}\right)-\frac{b\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx,x,\sqrt{x}\right)}{c}\\
&= \frac{b\sqrt{x}}{c}+ax-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{c^2}+bx\tanh^{-1}\left(c\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 42, normalized size = 1.08

$$ax+bx\tanh^{-1}\left(c\sqrt{x}\right)-bc\left(-\frac{\sqrt{x}}{c^2}+\frac{\tanh^{-1}\left(c\sqrt{x}\right)}{c^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c*Sqrt[x]],x]
[Out] a*x + b*x*ArcTanh[c*Sqrt[x]] - b*c*(-(Sqrt[x]/c^2) + ArcTanh[c*Sqrt[x]]/c^3
)
```

Maple [A]
time = 0.07, size = 50, normalized size = 1.28

method	result	size
default	$ax+bx\,\mathrm{arctanh}\left(c\sqrt{x}\right)+\frac{b\sqrt{x}}{c}+\frac{b\ln\left(c\sqrt{x}-1\right)}{2c^2}-\frac{b\ln\left(1+c\sqrt{x}\right)}{2c^2}$	50
derivativedivides	$\frac{a\,c^2x+b\,c^2x\,\mathrm{arctanh}\left(c\sqrt{x}\right)+bc\sqrt{x}+\frac{b\ln\left(c\sqrt{x}-1\right)}{2}-\frac{b\ln\left(1+c\sqrt{x}\right)}{2}}{c^2}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*x*arctanh(c*x^(1/2))+b*x^(1/2)/c+1/2*b/c^2*ln(c*x^(1/2)-1)-1/2*b/c^2*
ln(1+c*x^(1/2))
```

Maxima [A]
time = 0.26, size = 53, normalized size = 1.36

$$\frac{1}{2}\left(c\left(\frac{2\sqrt{x}}{c^2}-\frac{\log\left(c\sqrt{x}+1\right)}{c^3}+\frac{\log\left(c\sqrt{x}-1\right)}{c^3}\right)+2\,x\,\mathrm{artanh}\left(c\sqrt{x}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*(c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3 + log(c*sqrt(x) - 1)/c^3) +
2*x*arctanh(c*sqrt(x)))*b + a*x
```

Fricas [A]
time = 0.47, size = 56, normalized size = 1.44

$$\frac{2\,ac^2x+2\,bc\sqrt{x}+(bc^2x-b)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*c^2*x + 2*b*c*sqrt(x) + (b*c^2*x - b)*log(-(c^2*x + 2*c*sqrt(x) +
1)/(c^2*x - 1)))/c^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\,\mathrm{atanh}\left(c\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x**(1/2)),x)
```

```
[Out] Integral(a + b*atanh(c*sqrt(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(33) = 66.

time = 0.48, size = 174, normalized size = 4.46

$$2\,bc\left(\frac{1}{c^{3}\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)}+\frac{(c\sqrt{x}+1)\log\left(-\frac{\frac{c\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}+1\right)}{\frac{(c\sqrt{x}+1)c}{c\sqrt{x}-1}-c}+1}{\frac{c\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}+1\right)}{\frac{(c\sqrt{x}+1)c}{c\sqrt{x}-1}-c}-1}\right)}{(c\sqrt{x}-1)c^{3}\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)^{2}}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*b*c*(1/(c^3*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) - 1)) + (c*sqrt(x) + 1)*log(
-(c*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(c*sqrt(x) - 1
) - c) + 1)/(c*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(c*
sqrt(x) - 1) - c) - 1))/((c*sqrt(x) - 1)*c^3*((c*sqrt(x) + 1)/(c*sqrt(x) -
1) - 1)^2)) + a*x
```

Mupad [B]
time = 0.90, size = 32, normalized size = 0.82

$$a\,x+b\,x\,\mathrm{atanh}\left(c\,\sqrt{x}\right)-\frac{b\,\left(\mathrm{atanh}\left(c\,\sqrt{x}\right)-c\,\sqrt{x}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atanh(c*x^(1/2)),x)
```

```
[Out] a*x + b*x*atanh(c*x^(1/2)) - (b*(atanh(c*x^(1/2)) - c*x^(1/2)))/c^2
```

3.191 $\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}\,dx$

Optimal. Leaf size=29

$$a\log(x) - b\text{PolyLog}\left(2, -c\sqrt{x}\right) + b\text{PolyLog}\left(2, c\sqrt{x}\right)$$

```
[Out] a*ln(x)-b*polylog(2,-c*x^(1/2))+b*polylog(2,c*x^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6035, 6031}

$$a\log(x) - b\text{Li}_2\left(-c\sqrt{x}\right) + b\text{Li}_2\left(c\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/x,x]
```

```
[Out] a*Log[x] - b*PolyLog[2, -(c*Sqrt[x])] + b*PolyLog[2, c*Sqrt[x]]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}\,dx &= 2\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx, x, \sqrt{x}\right)\\ &= a\log(x) - b\text{Li}_2\left(-c\sqrt{x}\right) + b\text{Li}_2\left(c\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 1.00

$$a\log(x) - b\text{PolyLog}\left(2, -c\sqrt{x}\right) + b\text{PolyLog}\left(2, c\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/x,x]
```

```
[Out] a*Log[x] - b*PolyLog[2, -(c*Sqrt[x])] + b*PolyLog[2, c*Sqrt[x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(25) = 50$.
time = 0.08, size = 63, normalized size = 2.17

method	result
derivativedivides	$2a\ln\left(c\sqrt{x}\right)+2b\ln\left(c\sqrt{x}\right)\operatorname{arctanh}\left(c\sqrt{x}\right)-b\operatorname{dilog}\left(c\sqrt{x}\right)-b\operatorname{dilog}\left(1+c\sqrt{x}\right)-b\ln$
default	$2a\ln\left(c\sqrt{x}\right)+2b\ln\left(c\sqrt{x}\right)\operatorname{arctanh}\left(c\sqrt{x}\right)-b\operatorname{dilog}\left(c\sqrt{x}\right)-b\operatorname{dilog}\left(1+c\sqrt{x}\right)-b\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*ln(c*x^(1/2))+2*b*ln(c*x^(1/2))*arctanh(c*x^(1/2))-b*dilog(c*x^(1/2))-b
*dilog(1+c*x^(1/2))-b*ln(c*x^(1/2))*ln(1+c*x^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(23) = 46$.
time = 0.36, size = 61, normalized size = 2.10

$$-\left(\log\left(c\sqrt{x}\right)\log\left(-c\sqrt{x}+1\right)+\operatorname{Li}_2\left(-c\sqrt{x}+1\right)\right)b+\left(\log\left(c\sqrt{x}+1\right)\log\left(-c\sqrt{x}\right)+\operatorname{Li}_2\left(c\sqrt{x}+1\right)\right)b+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x,x, algorithm="maxima")
```

```
[Out] -(log(c*sqrt(x))*log(-c*sqrt(x) + 1) + dilog(-c*sqrt(x) + 1))*b + (log(c*sq
rt(x) + 1)*log(-c*sqrt(x)) + dilog(c*sqrt(x) + 1))*b + a*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*sqrt(x)) + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{atanh}\left(c\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/x,x)
```

```
[Out] Integral((a + b*atanh(c*sqrt(x)))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a+b\,\mathrm{atanh}\left(c\,\sqrt{x}\,\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))/x,x)
```

```
[Out] int((a + b*atanh(c*x^(1/2)))/x, x)
```

3.192 $\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^2}\,dx$

Optimal. Leaf size=40

$$-\frac{bc}{\sqrt{x}}+bc^2\tanh^{-1}\left(c\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}$$

```
[Out] b*c^2*arctanh(c*x^(1/2))+(-a-b*arctanh(c*x^(1/2)))/x-b*c/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6037, 53, 65, 212}

$$-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}+bc^2\tanh^{-1}\left(c\sqrt{x}\right)-\frac{bc}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/x^2,x]
```

```
[Out] -((b*c)/Sqrt[x]) + b*c^2*ArcTanh[c*Sqrt[x]] - (a + b*ArcTanh[c*Sqrt[x]])/x
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^2}\,dx &= -\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}+\frac{1}{2}(bc)\int \frac{1}{x^{3/2}\left(1-c^2x\right)}\,dx\\ &= -\frac{bc}{\sqrt{x}}-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}+\frac{1}{2}\left(bc^3\right)\int \frac{1}{\sqrt{x}\,\left(1-c^2x\right)}\,dx\\ &= -\frac{bc}{\sqrt{x}}-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}+\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx,x,\sqrt{x}\right)\\ &= -\frac{bc}{\sqrt{x}}+bc^2\tanh^{-1}\left(c\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 67, normalized size = 1.68

$$-\frac{a}{x}-\frac{bc}{\sqrt{x}}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{x}-\frac{1}{2}bc^2\log\left(1-c\sqrt{x}\right)+\frac{1}{2}bc^2\log\left(1+c\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/x^2,x]
```

```
[Out] -(a/x) - (b*c)/Sqrt[x] - (b*ArcTanh[c*Sqrt[x]])/x - (b*c^2*Log[1 - c*Sqrt[x
]])/2 + (b*c^2*Log[1 + c*Sqrt[x]])/2
```

Maple [A]
time = 0.07, size = 62, normalized size = 1.55

method	result	size
derivativedivides	$2c^2\left(-\frac{a}{2c^2x}-\frac{b\,\mathrm{arctanh}\left(c\sqrt{x}\right)}{2c^2x}-\frac{b\ln\left(c\sqrt{x}-1\right)}{4}+\frac{b\ln\left(1+c\sqrt{x}\right)}{4}-\frac{b}{2c\sqrt{x}}\right)$	62
default	$2c^2\left(-\frac{a}{2c^2x}-\frac{b\,\mathrm{arctanh}\left(c\sqrt{x}\right)}{2c^2x}-\frac{b\ln\left(c\sqrt{x}-1\right)}{4}+\frac{b\ln\left(1+c\sqrt{x}\right)}{4}-\frac{b}{2c\sqrt{x}}\right)$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*c^2*(-1/2*a/c^2/x-1/2*b/c^2/x*arctanh(c*x^(1/2))-1/4*b*ln(c*x^(1/2)-1)+1/
4*b*ln(1+c*x^(1/2))-1/2*b/c/x^(1/2))
```

Maxima [A]
time = 0.26, size = 51, normalized size = 1.28

$$\frac{1}{2}\left(\left(c\log\left(c\sqrt{x}+1\right)-c\log\left(c\sqrt{x}-1\right)-\frac{2}{\sqrt{x}}\right)c-\frac{2\,\operatorname{artanh}\left(c\sqrt{x}\right)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*((c*log(c*sqrt(x) + 1) - c*log(c*sqrt(x) - 1) - 2/sqrt(x))*c - 2*arctan
h(c*sqrt(x))/x)*b - a/x
```

Fricas [A]
time = 0.36, size = 53, normalized size = 1.32

$$-\frac{2\,bc\sqrt{x}-\left(bc^2x-b\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)+2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c*sqrt(x) - (b*c^2*x - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x -
 1)) + 2*a)/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(36) = 72.
time = 3.39, size = 231, normalized size = 5.78

$$\begin{cases} -\frac{a}{x}+\frac{b\operatorname{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} & \text{for}\: c=-\sqrt{\frac{1}{x}} \\ -\frac{a}{x}-\frac{b\operatorname{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} & \text{for}\: c=\sqrt{\frac{1}{x}} \\ -\frac{ac^2x^{\frac{3}{2}}}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{a\sqrt{x}}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{bc^4x^{\frac{5}{2}}\operatorname{atanh}\left(c\sqrt{x}\right)}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}}-\frac{bc^3x^2}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}}-\frac{2bc^2x^{\frac{3}{2}}\operatorname{atanh}\left(c\sqrt{x}\right)}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{bcx}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{b\sqrt{x}\operatorname{atanh}\left(c\sqrt{x}\right)}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/x**2,x)
```

```
[Out] Piecewise((-a/x + b*atanh(sqrt(x)*sqrt(1/x))/x, Eq(c, -sqrt(1/x))), (-a/x -
 b*atanh(sqrt(x)*sqrt(1/x))/x, Eq(c, sqrt(1/x))), (-a*c**2*x**(3/2)/(c**2*x
**(5/2) - x**(3/2)) + a*sqrt(x)/(c**2*x**(5/2) - x**(3/2)) + b*c**4*x**(5/2
)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) - b*c**3*x**2/(c**2*x**(5/2)
- x**(3/2)) - 2*b*c**2*x**(3/2)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2))
 + b*c*x/(c**2*x**(5/2) - x**(3/2)) + b*sqrt(x)*atanh(c*sqrt(x))/(c**2*x**(
5/2) - x**(3/2)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(34) = 68$.
time = 0.43, size = 168, normalized size = 4.20

$$2\left(\frac{\left(c\sqrt{x}+1\right)bc\log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)}{\left(c\sqrt{x}-1\right)\left(\frac{\left(c\sqrt{x}+1\right)^2}{\left(c\sqrt{x}-1\right)^2}+\frac{2\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1}+1\right)}+\frac{\frac{2\left(c\sqrt{x}+1\right)ac}{c\sqrt{x}-1}+\frac{\left(c\sqrt{x}+1\right)bc}{c\sqrt{x}-1}+bc}{\frac{\left(c\sqrt{x}+1\right)^2}{\left(c\sqrt{x}-1\right)^2}+\frac{2\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1}+1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2,x, algorithm="giac")
```

```
[Out] 2*((c*sqrt(x) + 1)*b*c*log(-(c*sqrt(x) + 1)/(c*sqrt(x) - 1))/((c*sqrt(x) -
1)*((c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 + 2*(c*sqrt(x) + 1)/(c*sqrt(x) - 1)
 + 1)) + (2*(c*sqrt(x) + 1)*a*c/(c*sqrt(x) - 1) + (c*sqrt(x) + 1)*b*c/(c*sq
rt(x) - 1) + b*c)/((c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 + 2*(c*sqrt(x) + 1)/
(c*sqrt(x) - 1) + 1))*c
```

Mupad [B]
time = 1.12, size = 52, normalized size = 1.30

$$b\,c\,\mathrm{atan}\left(\frac{c^2\,\sqrt{x}}{\sqrt{-c^2}}\right)\,\sqrt{-c^2}-\frac{a}{x}-\frac{b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)+b\,c\,\sqrt{x}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))/x^2,x)
```

```
[Out] b*c*atan((c^2*x^(1/2))/(-c^2)^(1/2))*(-c^2)^(1/2) - a/x - (b*atanh(c*x^(1/2
)) + b*c*x^(1/2))/x
```

3.193 $\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^3}\,dx$

Optimal. Leaf size=60

$$-\frac{bc}{6x^{3/2}}-\frac{bc^3}{2\sqrt{x}}+\frac{1}{2}bc^4\tanh^{-1}\left(c\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}$$

```
[Out] -1/6*b*c/x^(3/2)+1/2*b*c^4*arctanh(c*x^(1/2))+1/2*(-a-b*arctanh(c*x^(1/2)))
/x^2-1/2*b*c^3/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6037, 53, 65, 212}

$$-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}+\frac{1}{2}bc^4\tanh^{-1}\left(c\sqrt{x}\right)-\frac{bc^3}{2\sqrt{x}}-\frac{bc}{6x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/x^3,x]
```

```
[Out] -1/6*(b*c)/x^(3/2) - (b*c^3)/(2*Sqrt[x]) + (b*c^4*ArcTanh[c*Sqrt[x]])/2 - (
a + b*ArcTanh[c*Sqrt[x]])/(2*x^2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^3}\,dx &= -\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}+\frac{1}{4}(bc)\int \frac{1}{x^{5/2}\left(1-c^2x\right)}\,dx\\ &= -\frac{bc}{6x^{3/2}}-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}+\frac{1}{4}\left(bc^3\right)\int \frac{1}{x^{3/2}\left(1-c^2x\right)}\,dx\\ &= -\frac{bc}{6x^{3/2}}-\frac{bc^3}{2\sqrt{x}}-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}+\frac{1}{4}\left(bc^5\right)\int \frac{1}{\sqrt{x}\left(1-c^2x\right)}\,dx\\ &= -\frac{bc}{6x^{3/2}}-\frac{bc^3}{2\sqrt{x}}-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}+\frac{1}{2}\left(bc^5\right)\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx,x,\sqrt{}\right.\\ &= -\frac{bc}{6x^{3/2}}-\frac{bc^3}{2\sqrt{x}}+\frac{1}{2}bc^4\tanh^{-1}\left(c\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 86, normalized size = 1.43

$$-\frac{a}{2x^2}-\frac{bc}{6x^{3/2}}-\frac{bc^3}{2\sqrt{x}}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}-\frac{1}{4}bc^4\log\left(1-c\sqrt{x}\right)+\frac{1}{4}bc^4\log\left(1+c\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*c)/(6*x^(3/2)) - (b*c^3)/(2*Sqrt[x]) - (b*ArcTanh[c*Sqrt[x]
])/(2*x^2) - (b*c^4*Log[1 - c*Sqrt[x]])/4 + (b*c^4*Log[1 + c*Sqrt[x]])/4
```

Maple [A]
time = 0.07, size = 71, normalized size = 1.18

method	result	size

derivativedivides	$2c^4\left(-\frac{a}{4c^4x^2}-\frac{b\,\mathrm{arctanh}\left(c\sqrt{x}\right)}{4c^4x^2}-\frac{b\ln\left(c\sqrt{x}-1\right)}{8}-\frac{b}{12c^3x^{\frac{3}{2}}}-\frac{b}{4c\sqrt{x}}+\frac{b\ln\left(1+c\sqrt{x}\right)}{8}\right)$	71
default	$2c^4\left(-\frac{a}{4c^4x^2}-\frac{b\,\mathrm{arctanh}\left(c\sqrt{x}\right)}{4c^4x^2}-\frac{b\ln\left(c\sqrt{x}-1\right)}{8}-\frac{b}{12c^3x^{\frac{3}{2}}}-\frac{b}{4c\sqrt{x}}+\frac{b\ln\left(1+c\sqrt{x}\right)}{8}\right)$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*c^4*(-1/4*a/c^4/x^2-1/4*b/c^4/x^2*arctanh(c*x^(1/2))-1/8*b*ln(c*x^(1/2)-1
)-1/12*b/c^3/x^(3/2)-1/4*b/c/x^(1/2)+1/8*b*ln(1+c*x^(1/2)))
```

Maxima [A]
time = 0.27, size = 64, normalized size = 1.07

$$\frac{1}{12}\left(\left(3\,c^3\log\left(c\sqrt{x}+1\right)-3\,c^3\log\left(c\sqrt{x}-1\right)-\frac{2\,(3\,c^2x+1)}{x^{\frac{3}{2}}}\right)c-\frac{6\,\mathrm{artanh}\left(c\sqrt{x}\right)}{x^2}\right)b-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*((3*c^3*log(c*sqrt(x) + 1) - 3*c^3*log(c*sqrt(x) - 1) - 2*(3*c^2*x + 1
)/x^(3/2))*c - 6*arctanh(c*sqrt(x))/x^2)*b - 1/2*a/x^2
```

Fricas [A]
time = 0.37, size = 64, normalized size = 1.07

$$\frac{3\,(bc^4x^2-b)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)-2\,(3\,bc^3x+bc)\sqrt{x}-6\,a}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*(b*c^4*x^2 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)) - 2*(3*
b*c^3*x + b*c)*sqrt(x) - 6*a)/x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(56) = 112$.
time = 9.70, size = 342, normalized size = 5.70

$$\begin{cases} -\frac{a}{2x^2}+\frac{b\,\mathrm{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{2x^2} & \text{for}\ c=-\sqrt{\frac{1}{x}} \\ -\frac{a}{2x^2}-\frac{b\,\mathrm{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{2x^2} & \text{for}\ c=\sqrt{\frac{1}{x}} \\ -\frac{3ac^2x^{\frac{3}{2}}}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{3a\sqrt{x}}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{3bc^6x^{\frac{7}{2}}\,\mathrm{atanh}\left(c\sqrt{x}\right)}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3bc^5x^3}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3bc^4x^{\frac{5}{2}}\,\mathrm{atanh}\left(c\sqrt{x}\right)}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{2bc^3x^2}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3bc^2x^{\frac{3}{2}}\,\mathrm{atanh}\left(c\sqrt{x}\right)}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{bcx}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{3b\sqrt{x}\,\mathrm{atanh}\left(c\sqrt{x}\right)}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) + b*atanh(sqrt(x)*sqrt(1/x))/(2*x**2), Eq(c, -sqrt(1
/x))), (-a/(2*x**2) - b*atanh(sqrt(x)*sqrt(1/x))/(2*x**2), Eq(c, sqrt(1/x))
), (-3*a*c**2*x**(3/2)/(6*c**2*x**(7/2) - 6*x**(5/2)) + 3*a*sqrt(x)/(6*c**2
*x**(7/2) - 6*x**(5/2)) + 3*b*c**6*x**(7/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/
2) - 6*x**(5/2)) - 3*b*c**5*x**3/(6*c**2*x**(7/2) - 6*x**(5/2)) - 3*b*c**4*
x**(5/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 2*b*c**3*x**2/(6
*c**2*x**(7/2) - 6*x**(5/2)) - 3*b*c**2*x**(3/2)*atanh(c*sqrt(x))/(6*c**2*x
**(7/2) - 6*x**(5/2)) + b*c*x/(6*c**2*x**(7/2) - 6*x**(5/2)) + 3*b*sqrt(x)*
atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(44) = 88$.

time = 0.44, size = 356, normalized size = 5.93

$$\frac{2}{3}c\left(\frac{3\left(\frac{\left(c\sqrt{x}+1\right)^{3}bc^{3}}{\left(c\sqrt{x}-1\right)^{3}}+\frac{\left(c\sqrt{x}+1\right)bc^{3}}{c\sqrt{x}-1}\right)\log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)}{\frac{\left(c\sqrt{x}+1\right)^{4}}{\left(c\sqrt{x}-1\right)^{4}}+\frac{4\left(c\sqrt{x}+1\right)^{3}}{\left(c\sqrt{x}-1\right)^{3}}+\frac{6\left(c\sqrt{x}+1\right)^{2}}{\left(c\sqrt{x}-1\right)^{2}}+\frac{4\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1}+1}+\frac{\frac{6\left(c\sqrt{x}+1\right)^{3}ac^{3}}{\left(c\sqrt{x}-1\right)^{3}}+\frac{6\left(c\sqrt{x}+1\right)ac^{3}}{c\sqrt{x}-1}+\frac{3\left(c\sqrt{x}+1\right)^{3}bc^{3}}{\left(c\sqrt{x}-1\right)^{3}}+\frac{6\left(c\sqrt{x}+1\right)^{2}bc^{3}}{\left(c\sqrt{x}-1\right)^{2}}+\frac{5\left(c\sqrt{x}+1\right)bc^{3}}{c\sqrt{x}-1}+2\,bc^{3}}{\frac{\left(c\sqrt{x}+1\right)^{4}}{\left(c\sqrt{x}-1\right)^{4}}+\frac{4\left(c\sqrt{x}+1\right)^{3}}{\left(c\sqrt{x}-1\right)^{3}}+\frac{6\left(c\sqrt{x}+1\right)^{2}}{\left(c\sqrt{x}-1\right)^{2}}+\frac{4\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^3,x, algorithm="giac")
```

```
[Out] 2/3*c*(3*((c*sqrt(x) + 1)^3*b*c^3/(c*sqrt(x) - 1)^3 + (c*sqrt(x) + 1)*b*c^3
/(c*sqrt(x) - 1))*log(-(c*sqrt(x) + 1)/(c*sqrt(x) - 1))/((c*sqrt(x) + 1)^4/
(c*sqrt(x) - 1)^4 + 4*(c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3 + 6*(c*sqrt(x) +
1)^2/(c*sqrt(x) - 1)^2 + 4*(c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1) + (6*(c*sqr
t(x) + 1)^3*a*c^3/(c*sqrt(x) - 1)^3 + 6*(c*sqrt(x) + 1)*a*c^3/(c*sqrt(x) -
1) + 3*(c*sqrt(x) + 1)^3*b*c^3/(c*sqrt(x) - 1)^3 + 6*(c*sqrt(x) + 1)^2*b*c^
3/(c*sqrt(x) - 1)^2 + 5*(c*sqrt(x) + 1)*b*c^3/(c*sqrt(x) - 1) + 2*b*c^3)/((
c*sqrt(x) + 1)^4/(c*sqrt(x) - 1)^4 + 4*(c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3
+ 6*(c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 + 4*(c*sqrt(x) + 1)/(c*sqrt(x) - 1)
 + 1))
```

Mupad [B]

time = 1.36, size = 61, normalized size = 1.02

$$\frac{b\,c^4\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{2}-\frac{b\,\left(3\,\ln\left(c\,\sqrt{x}+1\right)-3\,\ln\left(1-c\,\sqrt{x}\right)+2\,c\,\sqrt{x}+6\,c^3\,x^{3/2}\right)}{12\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))/x^3,x)
```

```
[Out] (b*c^4*atanh(c*x^(1/2)))/2 - (b*(3*log(c*x^(1/2) + 1) - 3*log(1 - c*x^(1/2)
) + 2*c*x^(1/2) + 6*c^3*x^(3/2)))/(12*x^2) - a/(2*x^2)
```

3.194 $\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^4} dx$

Optimal. Leaf size=73

$$-\frac{bc}{15x^{5/2}}-\frac{bc^3}{9x^{3/2}}-\frac{bc^5}{3\sqrt{x}}+\frac{1}{3}bc^6\tanh^{-1}\left(c\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}$$

```
[Out] -1/15*b*c/x^(5/2)-1/9*b*c^3/x^(3/2)+1/3*b*c^6*arctanh(c*x^(1/2))+1/3*(-a-b*
arctanh(c*x^(1/2)))/x^3-1/3*b*c^5/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6037, 53, 65, 212}

$$-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}+\frac{1}{3}bc^6\tanh^{-1}\left(c\sqrt{x}\right)-\frac{bc^5}{3\sqrt{x}}-\frac{bc^3}{9x^{3/2}}-\frac{bc}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/x^4,x]
```

```
[Out] -1/15*(b*c)/x^(5/2) - (b*c^3)/(9*x^(3/2)) - (b*c^5)/(3*Sqrt[x]) + (b*c^6*Ar
cTanh[c*Sqrt[x]])/3 - (a + b*ArcTanh[c*Sqrt[x]])/(3*x^3)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^4}\,dx &= -\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}+\frac{1}{6}(bc)\int \frac{1}{x^{7/2}\left(1-c^2x\right)}\,dx\\
&= -\frac{bc}{15x^{5/2}}-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}+\frac{1}{6}\left(bc^3\right)\int \frac{1}{x^{5/2}\left(1-c^2x\right)}\,dx\\
&= -\frac{bc}{15x^{5/2}}-\frac{bc^3}{9x^{3/2}}-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}+\frac{1}{6}\left(bc^5\right)\int \frac{1}{x^{3/2}\left(1-c^2x\right)}\,dx\\
&= -\frac{bc}{15x^{5/2}}-\frac{bc^3}{9x^{3/2}}-\frac{bc^5}{3\sqrt{x}}-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}+\frac{1}{6}\left(bc^7\right)\int \frac{1}{\sqrt{x}\left(1-c^2x\right)}\\
&= -\frac{bc}{15x^{5/2}}-\frac{bc^3}{9x^{3/2}}-\frac{bc^5}{3\sqrt{x}}-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}+\frac{1}{3}\left(bc^7\right)\text{Subst}\left(\int \frac{1}{1-c^2}\right.\\
&= -\frac{bc}{15x^{5/2}}-\frac{bc^3}{9x^{3/2}}-\frac{bc^5}{3\sqrt{x}}+\frac{1}{3}bc^6\tanh^{-1}\left(c\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 99, normalized size = 1.36

$$-\frac{a}{3x^3}-\frac{bc}{15x^{5/2}}-\frac{bc^3}{9x^{3/2}}-\frac{bc^5}{3\sqrt{x}}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}-\frac{1}{6}bc^6\log\left(1-c\sqrt{x}\right)+\frac{1}{6}bc^6\log\left(1+c\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*c)/(15*x^(5/2)) - (b*c^3)/(9*x^(3/2)) - (b*c^5)/(3*Sqrt[x])
 - (b*ArcTanh[c*Sqrt[x]])/(3*x^3) - (b*c^6*Log[1 - c*Sqrt[x]])/6 + (b*c^6*L
og[1 + c*Sqrt[x]])/6
```

Maple [A]
time = 0.07, size = 80, normalized size = 1.10

method	result
derivativedivides	$2c^6\left(-\frac{a}{6c^6x^3}-\frac{b\,\mathrm{arctanh}\left(c\sqrt{x}\right)}{6c^6x^3}-\frac{b}{30c^5x^{\frac{5}{2}}}-\frac{b}{18c^3x^{\frac{3}{2}}}-\frac{b}{6c\sqrt{x}}-\frac{b\ln\left(c\sqrt{x}-1\right)}{12}+\frac{b\ln\left(1+c\sqrt{x}\right)}{12}\right)$
default	$2c^6\left(-\frac{a}{6c^6x^3}-\frac{b\,\mathrm{arctanh}\left(c\sqrt{x}\right)}{6c^6x^3}-\frac{b}{30c^5x^{\frac{5}{2}}}-\frac{b}{18c^3x^{\frac{3}{2}}}-\frac{b}{6c\sqrt{x}}-\frac{b\ln\left(c\sqrt{x}-1\right)}{12}+\frac{b\ln\left(1+c\sqrt{x}\right)}{12}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2*c^6*(-1/6*a/c^6/x^3-1/6*b/c^6/x^3*arctanh(c*x^(1/2))-1/30*b/c^5/x^(5/2)-1
/18*b/c^3/x^(3/2)-1/6*b/c/x^(1/2)-1/12*b*ln(c*x^(1/2)-1)+1/12*b*ln(1+c*x^(1
/2)))
```

Maxima [A]
time = 0.26, size = 72, normalized size = 0.99

$$\frac{1}{90}\left(\left(15\,c^5\log\left(c\sqrt{x}+1\right)-15\,c^5\log\left(c\sqrt{x}-1\right)-\frac{2\,(15\,c^4x^2+5\,c^2x+3)}{x^{\frac{5}{2}}}\right)c-\frac{30\,\mathrm{artanh}\left(c\sqrt{x}\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^4,x, algorithm="maxima")
```

```
[Out] 1/90*((15*c^5*log(c*sqrt(x) + 1) - 15*c^5*log(c*sqrt(x) - 1) - 2*(15*c^4*x^
2 + 5*c^2*x + 3)/x^(5/2))*c - 30*arctanh(c*sqrt(x))/x^3)*b - 1/3*a/x^3
```

Fricas [A]
time = 0.35, size = 74, normalized size = 1.01

$$\frac{15\,(bc^6x^3-b)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)-2\,(15\,bc^5x^2+5\,bc^3x+3\,bc)\sqrt{x}-30\,a}{90\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^4,x, algorithm="fricas")
```

```
[Out] 1/90*(15*(b*c^6*x^3 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)) - 2*(1
5*b*c^5*x^2 + 5*b*c^3*x + 3*b*c)*sqrt(x) - 30*a)/x^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(68) = 136.
time = 25.83, size = 371, normalized size = 5.08

$$\begin{cases} -\frac{a}{3x^3}+\frac{b\,\mathrm{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{3x^3} & \text{for}\ c=-\sqrt{\frac{1}{x}} \\ -\frac{a}{3x^3}-\frac{b\,\mathrm{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{3x^3} & \text{for}\ c=\sqrt{\frac{1}{x}} \\ -\frac{15ac^2x^{\frac{3}{2}}}{45c^2x^{\frac{9}{2}}-45x^{\frac{7}{2}}}+\frac{15a\sqrt{x}}{45c^2x^{\frac{9}{2}}-45x^{\frac{7}{2}}}+\frac{15bc^8x^{\frac{9}{2}}\,\mathrm{atanh}\left(c\sqrt{x}\right)}{45c^2x^{\frac{9}{2}}-45x^{\frac{7}{2}}}-\frac{15bc^7x^4}{45c^2x^{\frac{9}{2}}-45x^{\frac{7}{2}}}-\frac{15bc^6x^{\frac{7}{2}}\,\mathrm{atanh}\left(c\sqrt{x}\right)}{45c^2x^{\frac{9}{2}}-45x^{\frac{7}{2}}}+\frac{10bc^5x^3}{45c^2x^{\frac{9}{2}}-45x^{\frac{7}{2}}}+\frac{2bc^3x^2}{45c^2x^{\frac{9}{2}}-45x^{\frac{7}{2}}}-\frac{15bc^2x^{\frac{3}{2}}\,\mathrm{atanh}\left(c\sqrt{x}\right)}{45c^2x^{\frac{9}{2}}-45x^{\frac{7}{2}}}+\frac{3bcx}{45c^2x^{\frac{9}{2}}-45x^{\frac{7}{2}}}+\frac{15b\sqrt{x}\,\mathrm{atanh}\left(c\sqrt{x}\right)}{45c^2x^{\frac{9}{2}}-45x^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3) + b*atanh(sqrt(x)*sqrt(1/x))/(3*x**3), Eq(c, -sqrt(1
/x))), (-a/(3*x**3) - b*atanh(sqrt(x)*sqrt(1/x))/(3*x**3), Eq(c, sqrt(1/x))
), (-15*a*c**2*x**(3/2)/(45*c**2*x**(9/2) - 45*x**(7/2)) + 15*a*sqrt(x)/(45
*c**2*x**(9/2) - 45*x**(7/2)) + 15*b*c**8*x**(9/2)*atanh(c*sqrt(x))/(45*c**
2*x**(9/2) - 45*x**(7/2)) - 15*b*c**7*x**4/(45*c**2*x**(9/2) - 45*x**(7/2))
 - 15*b*c**6*x**(7/2)*atanh(c*sqrt(x))/(45*c**2*x**(9/2) - 45*x**(7/2)) + 1
0*b*c**5*x**3/(45*c**2*x**(9/2) - 45*x**(7/2)) + 2*b*c**3*x**2/(45*c**2*x**
(9/2) - 45*x**(7/2)) - 15*b*c**2*x**(3/2)*atanh(c*sqrt(x))/(45*c**2*x**(9/2
) - 45*x**(7/2)) + 3*b*c*x/(45*c**2*x**(9/2) - 45*x**(7/2)) + 15*b*sqrt(x)*
atanh(c*sqrt(x))/(45*c**2*x**(9/2) - 45*x**(7/2)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. 2(53) = 106.
time = 0.44, size = 534, normalized size = 7.32

$$\frac{2}{45}\,c\left(\frac{15\left(\frac{3\left(c\sqrt{x}+1\right)^{5}bc^{5}}{\left(c\sqrt{x}-1\right)^{5}}+\frac{10\left(c\sqrt{x}+1\right)^{3}bc^{5}}{\left(c\sqrt{x}-1\right)^{3}}+\frac{3\left(c\sqrt{x}+1\right)bc^{5}}{c\sqrt{x}-1}\right)\log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)}{\frac{\left(c\sqrt{x}+1\right)^{6}}{\left(c\sqrt{x}-1\right)^{6}}+\frac{6\left(c\sqrt{x}+1\right)^{5}}{\left(c\sqrt{x}-1\right)^{5}}+\frac{15\left(c\sqrt{x}+1\right)^{4}}{\left(c\sqrt{x}-1\right)^{4}}+\frac{20\left(c\sqrt{x}+1\right)^{3}}{\left(c\sqrt{x}-1\right)^{3}}+\frac{15\left(c\sqrt{x}+1\right)^{2}}{\left(c\sqrt{x}-1\right)^{2}}+\frac{6\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1}+1}+\frac{\frac{90\left(c\sqrt{x}+1\right)^{5}ac^{5}}{\left(c\sqrt{x}-1\right)^{5}}+\frac{300\left(c\sqrt{x}+1\right)^{3}ac^{5}}{\left(c\sqrt{x}-1\right)^{3}}+\frac{90\left(c\sqrt{x}+1\right)ac^{5}}{c\sqrt{x}-1}+\frac{45\left(c\sqrt{x}+1\right)^{5}bc^{5}}{\left(c\sqrt{x}-1\right)^{5}}+\frac{135\left(c\sqrt{x}+1\right)^{4}bc^{5}}{\left(c\sqrt{x}-1\right)^{4}}+\frac{230\left(c\sqrt{x}+1\right)^{3}bc^{5}}{\left(c\sqrt{x}-1\right)^{3}}+\frac{210\left(c\sqrt{x}+1\right)^{2}bc^{5}}{\left(c\sqrt{x}-1\right)^{2}}+\frac{93\left(c\sqrt{x}+1\right)bc^{5}}{c\sqrt{x}-1}+23\,bc^{5}}{\frac{\left(c\sqrt{x}+1\right)^{6}}{\left(c\sqrt{x}-1\right)^{6}}+\frac{6\left(c\sqrt{x}+1\right)^{5}}{\left(c\sqrt{x}-1\right)^{5}}+\frac{15\left(c\sqrt{x}+1\right)^{4}}{\left(c\sqrt{x}-1\right)^{4}}+\frac{20\left(c\sqrt{x}+1\right)^{3}}{\left(c\sqrt{x}-1\right)^{3}}+\frac{15\left(c\sqrt{x}+1\right)^{2}}{\left(c\sqrt{x}-1\right)^{2}}+\frac{6\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^4,x, algorithm="giac")
```

```
[Out] 2/45*c*(15*(3*(c*sqrt(x) + 1)^5*b*c^5/(c*sqrt(x) - 1)^5 + 10*(c*sqrt(x) + 1
)^3*b*c^5/(c*sqrt(x) - 1)^3 + 3*(c*sqrt(x) + 1)*b*c^5/(c*sqrt(x) - 1))*log(
-(c*sqrt(x) + 1)/(c*sqrt(x) - 1))/((c*sqrt(x) + 1)^6/(c*sqrt(x) - 1)^6 + 6*
(c*sqrt(x) + 1)^5/(c*sqrt(x) - 1)^5 + 15*(c*sqrt(x) + 1)^4/(c*sqrt(x) - 1)^
4 + 20*(c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3 + 15*(c*sqrt(x) + 1)^2/(c*sqrt(x
) - 1)^2 + 6*(c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1) + (90*(c*sqrt(x) + 1)^5*a
*c^5/(c*sqrt(x) - 1)^5 + 300*(c*sqrt(x) + 1)^3*a*c^5/(c*sqrt(x) - 1)^3 + 90
*(c*sqrt(x) + 1)*a*c^5/(c*sqrt(x) - 1) + 45*(c*sqrt(x) + 1)^5*b*c^5/(c*sqrt
(x) - 1)^5 + 135*(c*sqrt(x) + 1)^4*b*c^5/(c*sqrt(x) - 1)^4 + 230*(c*sqrt(x)
 + 1)^3*b*c^5/(c*sqrt(x) - 1)^3 + 210*(c*sqrt(x) + 1)^2*b*c^5/(c*sqrt(x) -
1)^2 + 93*(c*sqrt(x) + 1)*b*c^5/(c*sqrt(x) - 1) + 23*b*c^5)/((c*sqrt(x) + 1
)^6/(c*sqrt(x) - 1)^6 + 6*(c*sqrt(x) + 1)^5/(c*sqrt(x) - 1)^5 + 15*(c*sqrt(
x) + 1)^4/(c*sqrt(x) - 1)^4 + 20*(c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3 + 15*(
c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 + 6*(c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)
)
```

Mupad [B]
time = 1.39, size = 69, normalized size = 0.95

$$\frac{b\,c^6\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{3}-\frac{b\,\left(15\,\ln\left(c\,\sqrt{x}+1\right)-15\,\ln\left(1-c\,\sqrt{x}\right)+6\,c\,\sqrt{x}+10\,c^3\,x^{3/2}+30\,c^5\,x^{5/2}\right)}{90\,x^3}-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))/x^4,x)
```

```
[Out] (b*c^6*atanh(c*x^(1/2)))/3 - (b*(15*log(c*x^(1/2) + 1) - 15*log(1 - c*x^(1/
2)) + 6*c*x^(1/2) + 10*c^3*x^(3/2) + 30*c^5*x^(5/2)))/(90*x^3) - a/(3*x^3)
```

3.195 $\int x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx$

Optimal. Leaf size=211

$$\frac{ab\sqrt{x}}{2c^7}+\frac{71b^2x}{420c^6}+\frac{3b^2x^2}{70c^4}+\frac{b^2x^3}{84c^2}+\frac{b^2\sqrt{x}\,\tanh^{-1}\left(c\sqrt{x}\right)}{2c^7}+\frac{bx^{3/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{6c^5}+\frac{bx^{5/2}\left(a+b\tanh^{-1}\left(c\right.\right.}{10c^3}$$

```
[Out] 71/420*b^2*x/c^6+3/70*b^2*x^2/c^4+1/84*b^2*x^3/c^2+1/6*b*x^(3/2)*(a+b*arcta
nh(c*x^(1/2)))/c^5+1/10*b*x^(5/2)*(a+b*arctanh(c*x^(1/2)))/c^3+1/14*b*x^(7/
2)*(a+b*arctanh(c*x^(1/2)))/c-1/4*(a+b*arctanh(c*x^(1/2)))^2/c^8+1/4*x^4*(a
+b*arctanh(c*x^(1/2)))^2+44/105*b^2*ln(-c^2*x+1)/c^8+1/2*a*b*x^(1/2)/c^7+1/
2*b^2*arctanh(c*x^(1/2))*x^(1/2)/c^7
```

Rubi [A]
time = 0.37, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.444$, Rules used = {6039, 6037, 6127, 272, 45, 6021, 266, 6095}

$$-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{4c^8}+\frac{ab\sqrt{x}}{2c^7}+\frac{bx^{3/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{6c^5}+\frac{bx^{5/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{10c^3}+\frac{bx^{7/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{14c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+\frac{b^2\sqrt{x}\,\tanh^{-1}\left(c\sqrt{x}\right)}{2c^7}+\frac{71b^2x}{420c^6}+\frac{3b^2x^2}{70c^4}+\frac{b^2x^3}{84c^2}+\frac{44b^2\log\left(1-c^2x\right)}{105c^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] (a*b*Sqrt[x])/(2*c^7) + (71*b^2*x)/(420*c^6) + (3*b^2*x^2)/(70*c^4) + (b^2*
x^3)/(84*c^2) + (b^2*Sqrt[x]*ArcTanh[c*Sqrt[x]])/(2*c^7) + (b*x^(3/2)*(a +
b*ArcTanh[c*Sqrt[x]]))/(6*c^5) + (b*x^(5/2)*(a + b*ArcTanh[c*Sqrt[x]]))/(10
*c^3) + (b*x^(7/2)*(a + b*ArcTanh[c*Sqrt[x]]))/(14*c) - (a + b*ArcTanh[c*Sq
rt[x]])^2/(4*c^8) + (x^4*(a + b*ArcTanh[c*Sqrt[x]])^2)/4 + (44*b^2*Log[1 -
c^2*x])/(105*c^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$\int x^3 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 \, dx = \int x^3 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 \, dx$$

Mathematica [A]
time = 0.07, size = 224, normalized size = 1.06

$$\frac{210abc\sqrt{x}+71b^2c^2x+70abc^3x^{3/2}+18b^2c^4x^2+42abc^5x^{5/2}+5b^2c^6x^3+30abc^7x^{7/2}+105a^2c^8x^4+2bc\sqrt{x}\left(105ac^7x^{7/2}+b(105+35c^2x+21c^4x^2+15c^6x^3)\right)\tanh^{-1}\left(c\sqrt{x}\right)+105b^2(-1+c^8x^4)\tanh^{-1}\left(c\sqrt{x}\right)^2+b(105a+176b)\log\left(1-c\sqrt{x}\right)-105ab\log\left(1+c\sqrt{x}\right)+176b^2\log\left(1+c\sqrt{x}\right)}{420c^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] (210*a*b*c*Sqrt[x] + 71*b^2*c^2*x + 70*a*b*c^3*x^(3/2) + 18*b^2*c^4*x^2 + 4
2*a*b*c^5*x^(5/2) + 5*b^2*c^6*x^3 + 30*a*b*c^7*x^(7/2) + 105*a^2*c^8*x^4 +
2*b*c*Sqrt[x]*(105*a*c^7*x^(7/2) + b*(105 + 35*c^2*x + 21*c^4*x^2 + 15*c^6*
x^3))*ArcTanh[c*Sqrt[x]] + 105*b^2*(-1 + c^8*x^4)*ArcTanh[c*Sqrt[x]]^2 + b*
(105*a + 176*b)*Log[1 - c*Sqrt[x]] - 105*a*b*Log[1 + c*Sqrt[x]] + 176*b^2*L
og[1 + c*Sqrt[x]])/(420*c^8)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(167) = 334.
time = 0.18, size = 373, normalized size = 1.77

method	result
derivativedivides	$\frac{71b^2c^2x}{420}+\frac{c^8x^4a^2}{4}+\frac{ab\,c^8x^4\operatorname{arctanh}\left(c\sqrt{x}\right)}{2}-\frac{b^2\ln\left(c\sqrt{x}-1\right)\ln\left(\frac{c\sqrt{x}}{2}+\frac{1}{2}\right)}{8}+\frac{ab\ln\left(c\sqrt{x}-1\right)}{4}-\frac{ab\ln\left(1+c\sqrt{x}\right)}{4}+$
default	$\frac{71b^2c^2x}{420}+\frac{c^8x^4a^2}{4}+\frac{ab\,c^8x^4\operatorname{arctanh}\left(c\sqrt{x}\right)}{2}-\frac{b^2\ln\left(c\sqrt{x}-1\right)\ln\left(\frac{c\sqrt{x}}{2}+\frac{1}{2}\right)}{8}+\frac{ab\ln\left(c\sqrt{x}-1\right)}{4}-\frac{ab\ln\left(1+c\sqrt{x}\right)}{4}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^(1/2)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^8*(71/840*b^2*c^2*x+1/8*c^8*x^4*a^2-1/8*b^2*arctanh(c*x^(1/2))*ln(1+c*x
^(1/2))-1/16*b^2*ln(c*x^(1/2)-1)*ln(1/2*c*x^(1/2)+1/2)+1/8*b^2*arctanh(c*x^
(1/2))*ln(c*x^(1/2)-1)+1/8*a*b*ln(c*x^(1/2)-1)-1/8*a*b*ln(1+c*x^(1/2))+1/16
8*b^2*c^6*x^3+3/140*b^2*c^4*x^2+1/16*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2*c*x^
(1/2)+1/2)-1/16*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))+1/32*b^2*ln(1+c*
x^(1/2))^2+22/105*b^2*ln(c*x^(1/2)-1)+22/105*b^2*ln(1+c*x^(1/2))+1/32*b^2*l
n(c*x^(1/2)-1)^2+1/8*b^2*c^8*x^4*arctanh(c*x^(1/2))^2+1/28*b^2*arctanh(c*x^
(1/2))*c^7*x^(7/2)+1/20*b^2*arctanh(c*x^(1/2))*c^5*x^(5/2)+1/12*b^2*arctanh
(c*x^(1/2))*c^3*x^(3/2)+1/4*b^2*arctanh(c*x^(1/2))*c*x^(1/2)+1/28*c^7*x^(7/
2)*a*b+1/12*a*b*c^3*x^(3/2)+1/4*a*b*c*x^(1/2)+1/20*a*b*c^5*x^(5/2)+1/4*a*b*
c^8*x^4*arctanh(c*x^(1/2)))
```

Maxima [A]
time = 0.27, size = 265, normalized size = 1.26

$$\frac{1}{4}b^2x^4\operatorname{artanh}\left(c\sqrt{x}\right)^2+\frac{1}{4}a^2x^4+\frac{1}{420}\left(210x^4\operatorname{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(15c^6x^{\frac{7}{2}}+21c^4x^{\frac{5}{2}}+35c^2x^{\frac{3}{2}}+105\sqrt{x}\right)}{c^8}-\frac{105\log\left(c\sqrt{x}+1\right)}{c^9}+\frac{105\log\left(c\sqrt{x}-1\right)}{c^9}\right)\right)ab+\frac{1}{1680}\left(4c\left(\frac{2\left(15c^6x^{\frac{7}{2}}+21c^4x^{\frac{5}{2}}+35c^2x^{\frac{3}{2}}+105\sqrt{x}\right)}{c^8}-\frac{105\log\left(c\sqrt{x}+1\right)}{c^9}+\frac{105\log\left(c\sqrt{x}-1\right)}{c^9}\right)\operatorname{artanh}\left(c\sqrt{x}\right)+\frac{20c^6x^3+72c^4x^2+284c^2x-2\left(105\log\left(c\sqrt{x}-1\right)-352\right)\log\left(c\sqrt{x}+1\right)+105\log\left(c\sqrt{x}+1\right)^2+105\log\left(c\sqrt{x}-1\right)^2+704\log\left(c\sqrt{x}-1\right)}{c^8}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctanh(c*sqrt(x))^2 + 1/4*a^2*x^4 + 1/420*(210*x^4*arctanh(c*s
qrt(x)) + c*(2*(15*c^6*x^(7/2) + 21*c^4*x^(5/2) + 35*c^2*x^(3/2) + 105*sqrt
(x))/c^8 - 105*log(c*sqrt(x) + 1)/c^9 + 105*log(c*sqrt(x) - 1)/c^9))*a*b +
1/1680*(4*c*(2*(15*c^6*x^(7/2) + 21*c^4*x^(5/2) + 35*c^2*x^(3/2) + 105*sqrt
(x))/c^8 - 105*log(c*sqrt(x) + 1)/c^9 + 105*log(c*sqrt(x) - 1)/c^9)*arctanh
(c*sqrt(x)) + (20*c^6*x^3 + 72*c^4*x^2 + 284*c^2*x - 2*(105*log(c*sqrt(x) -
 1) - 352)*log(c*sqrt(x) + 1) + 105*log(c*sqrt(x) + 1)^2 + 105*log(c*sqrt(x
) - 1)^2 + 704*log(c*sqrt(x) - 1))/c^8)*b^2
```

Fricas [A]
time = 0.39, size = 273, normalized size = 1.29

$$\frac{420\,a^2c^8x^4+20\,b^2c^6x^3+72\,b^2c^4x^2+284\,b^2c^2x+105\,(b^2c^8x^4-b^2)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)^2+4\,(105\,abc^8-105\,ab+176\,b^2)\log\left(c\sqrt{x}+1\right)-4\,(105\,abc^8-105\,ab-176\,b^2)\log\left(c\sqrt{x}-1\right)+4\,(105\,abc^8x^4-105\,abc^8+(15\,b^2c^7x^3+21\,b^2c^5x^2+35\,b^2c^3x+105\,b^2c)\sqrt{x})\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)+8\,(15\,abc^7x^3+21\,abc^5x^2+35\,abc^3x+105\,abc)\sqrt{x}}{1680\,c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] 1/1680*(420*a^2*c^8*x^4 + 20*b^2*c^6*x^3 + 72*b^2*c^4*x^2 + 284*b^2*c^2*x +
 105*(b^2*c^8*x^4 - b^2)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1))^2 + 4*
(105*a*b*c^8 - 105*a*b + 176*b^2)*log(c*sqrt(x) + 1) - 4*(105*a*b*c^8 - 105
*a*b - 176*b^2)*log(c*sqrt(x) - 1) + 4*(105*a*b*c^8*x^4 - 105*a*b*c^8 + (15
*b^2*c^7*x^3 + 21*b^2*c^5*x^2 + 35*b^2*c^3*x + 105*b^2*c)*sqrt(x))*log(-(c^
2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)) + 8*(15*a*b*c^7*x^3 + 21*a*b*c^5*x^2 +
35*a*b*c^3*x + 105*a*b*c)*sqrt(x))/c^8
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\,\mathrm{atanh}\left(c\sqrt{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**(1/2)))**2,x)
```

```
[Out] Integral(x**3*(a + b*atanh(c*sqrt(x)))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2*x^3, x)
```

Mupad [B]
time = 4.87, size = 453, normalized size = 2.15

$$\frac{a^2 x^4}{4}+\frac{44 b^2 \ln\left(c \sqrt{x}-1\right)}{105 c^8}+\frac{44 b^2 \ln\left(c \sqrt{x}+1\right)}{105 c^8}+\frac{71 b^2 x}{420 c^6}-\frac{b^2 \ln\left(c \sqrt{x}+1\right)^2}{16 c^8}-\frac{b^2 \ln\left(1-c \sqrt{x}\right)^2}{16 c^8}+\frac{b^2 x^3}{84 c^2}+\frac{3 b^2 x^2}{70 c^4}+\frac{b^2 x^4 \ln\left(c \sqrt{x}+1\right)^2}{16}+\frac{b^2 x^4 \ln\left(1-c \sqrt{x}\right)^2}{16}+\frac{b^2 x^{7/2} \ln\left(c \sqrt{x}+1\right)}{28 c}+\frac{b^2 x^{5/2} \ln\left(c \sqrt{x}+1\right)}{20 c^3}+\frac{b^2 x^{3/2} \ln\left(c \sqrt{x}+1\right)}{12 c^5}+\frac{b^2 \sqrt{x} \ln\left(c \sqrt{x}+1\right)}{4 c^7}-\frac{b^2 x^{7/2} \ln\left(1-c \sqrt{x}\right)}{28 c}-\frac{b^2 x^{5/2} \ln\left(1-c \sqrt{x}\right)}{20 c^3}-\frac{b^2 x^{3/2} \ln\left(1-c \sqrt{x}\right)}{12 c^5}-\frac{b^2 \sqrt{x} \ln\left(1-c \sqrt{x}\right)}{4 c^7}+\frac{a b \ln\left(c \sqrt{x}-1\right)}{4 c^8}-\frac{a b \ln\left(c \sqrt{x}+1\right)}{4 c^8}+\frac{a b x^4 \ln\left(c \sqrt{x}+1\right)}{4}-\frac{a b x^4 \ln\left(1-c \sqrt{x}\right)}{4}+\frac{b^2 \ln\left(c \sqrt{x}+1\right) \ln\left(1-c \sqrt{x}\right)}{8 c^8}+\frac{a b x^{7/2}}{14 c}+\frac{a b x^{5/2}}{10 c^3}+\frac{a b x^{3/2}}{6 c^5}+\frac{a b \sqrt{x}}{2 c^7}-\frac{b^2 x^4 \ln\left(c \sqrt{x}+1\right) \ln\left(1-c \sqrt{x}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x^(1/2)))^2,x)
```

```
[Out] (a^2*x^4)/4 + (44*b^2*log(c*x^(1/2) - 1))/(105*c^8) + (44*b^2*log(c*x^(1/2)
 + 1))/(105*c^8) + (71*b^2*x)/(420*c^6) - (b^2*log(c*x^(1/2) + 1)^2)/(16*c^
8) - (b^2*log(1 - c*x^(1/2))^2)/(16*c^8) + (b^2*x^3)/(84*c^2) + (3*b^2*x^2)
/(70*c^4) + (b^2*x^4*log(c*x^(1/2) + 1)^2)/16 + (b^2*x^4*log(1 - c*x^(1/2))
^2)/16 + (b^2*x^(7/2)*log(c*x^(1/2) + 1))/(28*c) + (b^2*x^(5/2)*log(c*x^(1/
2) + 1))/(20*c^3) + (b^2*x^(3/2)*log(c*x^(1/2) + 1))/(12*c^5) + (b^2*x^(1/2
)*log(c*x^(1/2) + 1))/(4*c^7) - (b^2*x^(7/2)*log(1 - c*x^(1/2)))/(28*c) - (
b^2*x^(5/2)*log(1 - c*x^(1/2)))/(20*c^3) - (b^2*x^(3/2)*log(1 - c*x^(1/2)))
/(12*c^5) - (b^2*x^(1/2)*log(1 - c*x^(1/2)))/(4*c^7) + (a*b*log(c*x^(1/2) -
 1))/(4*c^8) - (a*b*log(c*x^(1/2) + 1))/(4*c^8) + (a*b*x^4*log(c*x^(1/2) +
1))/4 - (a*b*x^4*log(1 - c*x^(1/2)))/4 + (b^2*log(c*x^(1/2) + 1)*log(1 - c*
x^(1/2)))/(8*c^8) + (a*b*x^(7/2))/(14*c) + (a*b*x^(5/2))/(10*c^3) + (a*b*x^
(3/2))/(6*c^5) + (a*b*x^(1/2))/(2*c^7) - (b^2*x^4*log(c*x^(1/2) + 1)*log(1
- c*x^(1/2)))/8
```

3.196 $\int x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx$

Optimal. Leaf size=173

$$\frac{2ab\sqrt{x}}{3c^5}+\frac{8b^2x}{45c^4}+\frac{b^2x^2}{30c^2}+\frac{2b^2\sqrt{x}\,\tanh^{-1}\left(c\sqrt{x}\right)}{3c^5}+\frac{2bx^{3/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{9c^3}+\frac{2bx^{5/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{15c}$$

```
[Out] 8/45*b^2*x/c^4+1/30*b^2*x^2/c^2+2/9*b*x^(3/2)*(a+b*arctanh(c*x^(1/2)))/c^3+
2/15*b*x^(5/2)*(a+b*arctanh(c*x^(1/2)))/c-1/3*(a+b*arctanh(c*x^(1/2)))^2/c^
6+1/3*x^3*(a+b*arctanh(c*x^(1/2)))^2+23/45*b^2*ln(-c^2*x+1)/c^6+2/3*a*b*x^(
1/2)/c^5+2/3*b^2*arctanh(c*x^(1/2))*x^(1/2)/c^5
```

Rubi [A]
time = 0.27, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6039, 6037, 6127, 272, 45, 6021, 266, 6095}

$$-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{3c^6}+\frac{2ab\sqrt{x}}{3c^5}+\frac{2bx^{3/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{9c^3}+\frac{2bx^{5/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{15c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+\frac{2b^2\sqrt{x}\,\tanh^{-1}\left(c\sqrt{x}\right)}{3c^5}+\frac{8b^2x}{45c^4}+\frac{b^2x^2}{30c^2}+\frac{23b^2\log\left(1-c^2x\right)}{45c^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] (2*a*b*Sqrt[x])/(3*c^5) + (8*b^2*x)/(45*c^4) + (b^2*x^2)/(30*c^2) + (2*b^2*
Sqrt[x]*ArcTanh[c*Sqrt[x]])/(3*c^5) + (2*b*x^(3/2)*(a + b*ArcTanh[c*Sqrt[x]
]))/(9*c^3) + (2*b*x^(5/2)*(a + b*ArcTanh[c*Sqrt[x]]))/(15*c) - (a + b*ArcT
anh[c*Sqrt[x]])^2/(3*c^6) + (x^3*(a + b*ArcTanh[c*Sqrt[x]])^2)/3 + (23*b^2*
Log[1 - c^2*x])/(45*c^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx=\int x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx$$

Mathematica [A]
time = 0.07, size = 194, normalized size = 1.12

$$\frac{60abc\sqrt{x}+16b^2c^2x+20abc^3x^{3/2}+3b^2c^4x^2+12abc^5x^{5/2}+30a^2c^6x^3+4bc\sqrt{x}\left(15ac^5x^{5/2}+b(15+5c^2x+3c^4x^2)\right)\tanh^{-1}\left(c\sqrt{x}\right)+30b^2(-1+c^6x^3)\tanh^{-1}\left(c\sqrt{x}\right)^2+2b(15a+23b)\log\left(1-c\sqrt{x}\right)-30ab\log\left(1+c\sqrt{x}\right)+46b^2\log\left(1+c\sqrt{x}\right)}{90c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] (60*a*b*c*Sqrt[x] + 16*b^2*c^2*x + 20*a*b*c^3*x^(3/2) + 3*b^2*c^4*x^2 + 12*
a*b*c^5*x^(5/2) + 30*a^2*c^6*x^3 + 4*b*c*Sqrt[x]*(15*a*c^5*x^(5/2) + b*(15
+ 5*c^2*x + 3*c^4*x^2))*ArcTanh[c*Sqrt[x]] + 30*b^2*(-1 + c^6*x^3)*ArcTanh[
c*Sqrt[x]]^2 + 2*b*(15*a + 23*b)*Log[1 - c*Sqrt[x]] - 30*a*b*Log[1 + c*Sqrt
[x]] + 46*b^2*Log[1 + c*Sqrt[x]])/(90*c^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(137) = 274$.
time = 0.18, size = 335, normalized size = 1.94

method	result
derivativedivides	$\frac{c^6 x^3 a^2}{3}+\frac{b^2 c^6 x^3 \operatorname{arctanh}\left(c\sqrt{x}\right)^2}{3}+\frac{2 b^2 \operatorname{arctanh}\left(c\sqrt{x}\right) c^5 x^{\frac{5}{2}}}{15}+\frac{2 b^2 \operatorname{arctanh}\left(c\sqrt{x}\right) c^3 x^{\frac{3}{2}}}{9}+\frac{2 b^2 \operatorname{arctanh}\left(c\sqrt{x}\right) c\sqrt{x}}{3}+$
default	$\frac{c^6 x^3 a^2}{3}+\frac{b^2 c^6 x^3 \operatorname{arctanh}\left(c\sqrt{x}\right)^2}{3}+\frac{2 b^2 \operatorname{arctanh}\left(c\sqrt{x}\right) c^5 x^{\frac{5}{2}}}{15}+\frac{2 b^2 \operatorname{arctanh}\left(c\sqrt{x}\right) c^3 x^{\frac{3}{2}}}{9}+\frac{2 b^2 \operatorname{arctanh}\left(c\sqrt{x}\right) c\sqrt{x}}{3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^(1/2)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^6*(1/6*c^6*x^3*a^2+1/6*b^2*c^6*x^3*arctanh(c*x^(1/2))^2+1/15*b^2*arctan
h(c*x^(1/2))*c^5*x^(5/2)+1/9*b^2*arctanh(c*x^(1/2))*c^3*x^(3/2)+1/3*b^2*arc
tanh(c*x^(1/2))*c*x^(1/2)+1/6*b^2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-1/6*b^
2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-1/12*b^2*ln(c*x^(1/2)-1)*ln(1/2*c*x^(1
/2)+1/2)+1/24*b^2*ln(c*x^(1/2)-1)^2+1/24*b^2*ln(1+c*x^(1/2))^2-1/12*b^2*ln(
-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))+1/12*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2*
c*x^(1/2)+1/2)+1/60*b^2*c^4*x^2+4/45*b^2*c^2*x+23/90*b^2*ln(c*x^(1/2)-1)+23
/90*b^2*ln(1+c*x^(1/2))+1/3*a*b*c^6*x^3*arctanh(c*x^(1/2))+1/15*a*b*c^5*x^(
5/2)+1/9*a*b*c^3*x^(3/2)+1/3*a*b*c*x^(1/2)+1/6*a*b*ln(c*x^(1/2)-1)-1/6*a*b*
ln(1+c*x^(1/2)))
```

Maxima [A]
time = 0.26, size = 241, normalized size = 1.39

$$\frac{1}{3}b^2x^3\operatorname{artanh}\left(c\sqrt{x}\right)^2+\frac{1}{3}a^2x^3+\frac{1}{45}\left(30x^3\operatorname{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(3c^4x^{\frac{5}{2}}+5c^2x^{\frac{3}{2}}+15\sqrt{x}\right)}{c^6}-\frac{15\log\left(c\sqrt{x}+1\right)}{c^7}+\frac{15\log\left(c\sqrt{x}-1\right)}{c^7}\right)\right)ab+\frac{1}{180}\left(4c\left(\frac{2\left(3c^4x^{\frac{5}{2}}+5c^2x^{\frac{3}{2}}+15\sqrt{x}\right)}{c^6}-\frac{15\log\left(c\sqrt{x}+1\right)}{c^7}+\frac{15\log\left(c\sqrt{x}-1\right)}{c^7}\right)\operatorname{artanh}\left(c\sqrt{x}\right)+\frac{6c^4x^2+32c^2x-2\left(15\log\left(c\sqrt{x}-1\right)-46\right)\log\left(c\sqrt{x}+1\right)+15\log\left(c\sqrt{x}+1\right)^2+15\log\left(c\sqrt{x}-1\right)^2+92\log\left(c\sqrt{x}-1\right)}{c^6}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*arctanh(c*sqrt(x))^2 + 1/3*a^2*x^3 + 1/45*(30*x^3*arctanh(c*sqr
t(x)) + c*(2*(3*c^4*x^(5/2) + 5*c^2*x^(3/2) + 15*sqrt(x))/c^6 - 15*log(c*sq
```

```
rt(x) + 1)/c^7 + 15*log(c*sqrt(x) - 1)/c^7))*a*b + 1/180*(4*c*(2*(3*c^4*x^(
5/2) + 5*c^2*x^(3/2) + 15*sqrt(x))/c^6 - 15*log(c*sqrt(x) + 1)/c^7 + 15*log
(c*sqrt(x) - 1)/c^7)*arctanh(c*sqrt(x)) + (6*c^4*x^2 + 32*c^2*x - 2*(15*log
(c*sqrt(x) - 1) - 46)*log(c*sqrt(x) + 1) + 15*log(c*sqrt(x) + 1)^2 + 15*log
(c*sqrt(x) - 1)^2 + 92*log(c*sqrt(x) - 1))/c^6)*b^2
```

Fricas [A]
time = 0.42, size = 241, normalized size = 1.39

$$\frac{60\,a^2c^6x^3+6\,b^2c^4x^2+32\,b^2c^2x+15\,(b^2c^6x^3-b^2)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)^2+4\,(15\,abc^6-15\,ab+23\,b^2)\log\left(c\sqrt{x}+1\right)-4\,(15\,abc^6-15\,ab-23\,b^2)\log\left(c\sqrt{x}-1\right)+4\,\left(15\,abc^6x^3-15\,abc^6+(3\,b^2c^5x^2+5\,b^2c^3x+15\,b^2c)\sqrt{x}\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)+8\,(3\,abc^5x^2+5\,abc^3x+15\,abc)\sqrt{x}}{180\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] 1/180*(60*a^2*c^6*x^3 + 6*b^2*c^4*x^2 + 32*b^2*c^2*x + 15*(b^2*c^6*x^3 - b^
2)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1))^2 + 4*(15*a*b*c^6 - 15*a*b +
 23*b^2)*log(c*sqrt(x) + 1) - 4*(15*a*b*c^6 - 15*a*b - 23*b^2)*log(c*sqrt(x
) - 1) + 4*(15*a*b*c^6*x^3 - 15*a*b*c^6 + (3*b^2*c^5*x^2 + 5*b^2*c^3*x + 15
*b^2*c)*sqrt(x))*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)) + 8*(3*a*b*c^5
*x^2 + 5*a*b*c^3*x + 15*a*b*c)*sqrt(x))/c^6
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\,\mathrm{atanh}\left(c\sqrt{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**(1/2)))**2,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*sqrt(x)))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2*x^2, x)
```

Mupad [B]
time = 1.57, size = 185, normalized size = 1.07

$$\frac{46\,b^2\,\ln\left(c^2\,x-1\right)-30\,b^2\,\mathrm{atanh}\left(c\,\sqrt{x}\right)^2-60\,a\,b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)+16\,b^2\,c^2\,x+30\,a^2\,c^6\,x^3+3\,b^2\,c^4\,x^2+30\,b^2\,c^6\,x^3\,\mathrm{atanh}\left(c\,\sqrt{x}\right)^2+60\,b^2\,c\,\sqrt{x}\,\mathrm{atanh}\left(c\,\sqrt{x}\right)+60\,a\,b\,c\,\sqrt{x}+20\,b^2\,c^3\,x^{3/2}\,\mathrm{atanh}\left(c\,\sqrt{x}\right)+12\,b^2\,c^5\,x^{5/2}\,\mathrm{atanh}\left(c\,\sqrt{x}\right)+20\,a\,b\,c^3\,x^{3/2}+12\,a\,b\,c^5\,x^{5/2}+60\,a\,b\,c^6\,x^3\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{90\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^(1/2)))^2,x)
```

```
[Out] (46*b^2*log(c^2*x - 1) - 30*b^2*atanh(c*x^(1/2))^2 - 60*a*b*atanh(c*x^(1/2)
) + 16*b^2*c^2*x + 30*a^2*c^6*x^3 + 3*b^2*c^4*x^2 + 30*b^2*c^6*x^3*atanh(c*
x^(1/2))^2 + 60*b^2*c*x^(1/2)*atanh(c*x^(1/2)) + 60*a*b*c*x^(1/2) + 20*b^2*
c^3*x^(3/2)*atanh(c*x^(1/2)) + 12*b^2*c^5*x^(5/2)*atanh(c*x^(1/2)) + 20*a*b
*c^3*x^(3/2) + 12*a*b*c^5*x^(5/2) + 60*a*b*c^6*x^3*atanh(c*x^(1/2)))/(90*c^
6)
```

3.197 $\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx$

Optimal. Leaf size=129

$$\frac{ab\sqrt{x}}{c^3}+\frac{b^2x}{6c^2}+\frac{b^2\sqrt{x}\tanh^{-1}\left(c\sqrt{x}\right)}{c^3}+\frac{bx^{3/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{3c}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{2c^4}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\ldots\right.$$

```
[Out] 1/6*b^2*x/c^2+1/3*b*x^(3/2)*(a+b*arctanh(c*x^(1/2)))/c-1/2*(a+b*arctanh(c*x
^(1/2)))^2/c^4+1/2*x^2*(a+b*arctanh(c*x^(1/2)))^2+2/3*b^2*ln(-c^2*x+1)/c^4+
a*b*x^(1/2)/c^3+b^2*arctanh(c*x^(1/2))*x^(1/2)/c^3
```

Rubi [A]
time = 0.18, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6039, 6037, 6127, 272, 45, 6021, 266, 6095}

$$-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{2c^4}+\frac{ab\sqrt{x}}{c^3}+\frac{bx^{3/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{3c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+\frac{b^2\sqrt{x}\tanh^{-1}\left(c\sqrt{x}\right)}{c^3}+\frac{b^2x}{6c^2}+\frac{2b^2\log\left(1-c^2x\right)}{3c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] (a*b*Sqrt[x])/c^3 + (b^2*x)/(6*c^2) + (b^2*Sqrt[x]*ArcTanh[c*Sqrt[x]])/c^3
+ (b*x^(3/2)*(a + b*ArcTanh[c*Sqrt[x]]))/(3*c) - (a + b*ArcTanh[c*Sqrt[x]])
^2/(2*c^4) + (x^2*(a + b*ArcTanh[c*Sqrt[x]])^2)/2 + (2*b^2*Log[1 - c^2*x])/
(3*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx=\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx$$

Mathematica [A]
time = 0.06, size = 160, normalized size = 1.24

$$\frac{6abc\sqrt{x}+b^2c^2x+2abc^3x^{3/2}+3a^2c^4x^2+2bc\sqrt{x}\left(3ac^3x^{3/2}+b(3+c^2x)\right)\tanh^{-1}\left(c\sqrt{x}\right)+3b^2(-1+c^4x^2)\tanh^{-1}\left(c\sqrt{x}\right)^2+b(3a+4b)\log\left(1-c\sqrt{x}\right)-3ab\log\left(1+c\sqrt{x}\right)+4b^2\log\left(1+c\sqrt{x}\right)}{6c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] (6*a*b*c*Sqrt[x] + b^2*c^2*x + 2*a*b*c^3*x^(3/2) + 3*a^2*c^4*x^2 + 2*b*c*Sq
rt[x]*(3*a*c^3*x^(3/2) + b*(3 + c^2*x))*ArcTanh[c*Sqrt[x]] + 3*b^2*(-1 + c^
4*x^2)*ArcTanh[c*Sqrt[x]]^2 + b*(3*a + 4*b)*Log[1 - c*Sqrt[x]] - 3*a*b*Log[
1 + c*Sqrt[x]] + 4*b^2*Log[1 + c*Sqrt[x]])/(6*c^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(105) = 210$.
time = 0.18, size = 297, normalized size = 2.30

method	result
derivativedivides	$\frac{\frac{c^4 x^2 a^2}{2}+\frac{b^2 c^4 x^2 \operatorname{arctanh}\left(c\sqrt{x}\right)^2}{2}+\frac{b^2 \operatorname{arctanh}\left(c\sqrt{x}\right) c^3 x^{\frac{3}{2}}}{3}+b^2 \operatorname{arctanh}\left(c\sqrt{x}\right) c\sqrt{x}+\frac{b^2 \operatorname{arctanh}\left(c\sqrt{x}\right) \ln\left(c\sqrt{x}\right.}{2}}{}$
default	$\frac{\frac{c^4 x^2 a^2}{2}+\frac{b^2 c^4 x^2 \operatorname{arctanh}\left(c\sqrt{x}\right)^2}{2}+\frac{b^2 \operatorname{arctanh}\left(c\sqrt{x}\right) c^3 x^{\frac{3}{2}}}{3}+b^2 \operatorname{arctanh}\left(c\sqrt{x}\right) c\sqrt{x}+\frac{b^2 \operatorname{arctanh}\left(c\sqrt{x}\right) \ln\left(c\sqrt{x}\right.}{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^(1/2)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^4*(1/4*c^4*x^2*a^2+1/4*b^2*c^4*x^2*arctanh(c*x^(1/2))^2+1/6*b^2*arctanh
(c*x^(1/2))*c^3*x^(3/2)+1/2*b^2*arctanh(c*x^(1/2))*c*x^(1/2)+1/4*b^2*arctan
h(c*x^(1/2))*ln(c*x^(1/2)-1)-1/4*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))+1/1
6*b^2*ln(c*x^(1/2)-1)^2-1/8*b^2*ln(c*x^(1/2)-1)*ln(1/2*c*x^(1/2)+1/2)-1/8*b
^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))+1/8*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln
(1/2*c*x^(1/2)+1/2)+1/16*b^2*ln(1+c*x^(1/2))^2+1/12*b^2*c^2*x+1/3*b^2*ln(c*
x^(1/2)-1)+1/3*b^2*ln(1+c*x^(1/2))+1/2*a*b*c^4*x^2*arctanh(c*x^(1/2))+1/6*a
*b*c^3*x^(3/2)+1/2*a*b*c*x^(1/2)+1/4*a*b*ln(c*x^(1/2)-1)-1/4*a*b*ln(1+c*x^(
1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(105) = 210$.
time = 0.27, size = 215, normalized size = 1.67

$$\frac{1}{2} b^2 x^2 \operatorname{artanh}\left(c\sqrt{x}\right)^2+\frac{1}{2} a^2 x^2+\frac{1}{6}\left(6 x^2 \operatorname{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(c^2 x^{\frac{3}{2}}+3\sqrt{x}\right)}{c^4}-\frac{3 \log\left(c\sqrt{x}+1\right)}{c^5}+\frac{3 \log\left(c\sqrt{x}-1\right)}{c^5}\right)\right) ab+\frac{1}{24}\left(4 c\left(\frac{2\left(c^2 x^{\frac{3}{2}}+3\sqrt{x}\right)}{c^4}-\frac{3 \log\left(c\sqrt{x}+1\right)}{c^5}+\frac{3 \log\left(c\sqrt{x}-1\right)}{c^5}\right) \operatorname{artanh}\left(c\sqrt{x}\right)+\frac{4 c^2 x-2\left(3 \log\left(c\sqrt{x}-1\right)-8\right) \log\left(c\sqrt{x}+1\right)+3 \log\left(c\sqrt{x}+1\right)^2+3 \log\left(c\sqrt{x}-1\right)^2+16 \log\left(c\sqrt{x}-1\right)}{c^4}\right) b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arctanh(c*sqrt(x))^2 + 1/2*a^2*x^2 + 1/6*(6*x^2*arctanh(c*sqrt(
x)) + c*(2*(c^2*x^(3/2) + 3*sqrt(x))/c^4 - 3*log(c*sqrt(x) + 1)/c^5 + 3*log
(c*sqrt(x) - 1)/c^5))*a*b + 1/24*(4*c*(2*(c^2*x^(3/2) + 3*sqrt(x))/c^4 - 3*
```

```
log(c*sqrt(x) + 1)/c^5 + 3*log(c*sqrt(x) - 1)/c^5)*arctanh(c*sqrt(x)) + (4*
c^2*x - 2*(3*log(c*sqrt(x) - 1) - 8)*log(c*sqrt(x) + 1) + 3*log(c*sqrt(x) +
 1)^2 + 3*log(c*sqrt(x) - 1)^2 + 16*log(c*sqrt(x) - 1))/c^4)*b^2
```

Fricas [A]
time = 0.36, size = 207, normalized size = 1.60

$$\frac{12\,a^2c^4x^2 + 4\,b^2c^2x + 3\,(b^2c^4x^2 - b^2)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)^2 + 4\,(3\,abc^4 - 3\,ab + 4\,b^2)\log\left(c\sqrt{x}+1\right) - 4\,(3\,abc^4 - 3\,ab - 4\,b^2)\log\left(c\sqrt{x}-1\right) + 4\,\left(3\,abc^4x^2 - 3\,abc^4 + (b^2c^3x + 3\,b^2c)\sqrt{x}\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right) + 8\,(abc^3x + 3\,abc)\sqrt{x}}{24\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(12*a^2*c^4*x^2 + 4*b^2*c^2*x + 3*(b^2*c^4*x^2 - b^2)*log(-(c^2*x + 2*
c*sqrt(x) + 1)/(c^2*x - 1))^2 + 4*(3*a*b*c^4 - 3*a*b + 4*b^2)*log(c*sqrt(x)
 + 1) - 4*(3*a*b*c^4 - 3*a*b - 4*b^2)*log(c*sqrt(x) - 1) + 4*(3*a*b*c^4*x^2
 - 3*a*b*c^4 + (b^2*c^3*x + 3*b^2*c)*sqrt(x))*log(-(c^2*x + 2*c*sqrt(x) + 1
)/(c^2*x - 1)) + 8*(a*b*c^3*x + 3*a*b*c)*sqrt(x))/c^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a + b\operatorname{atanh}\left(c\sqrt{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**(1/2)))**2,x)
```

```
[Out] Integral(x*(a + b*atanh(c*sqrt(x)))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2*x, x)
```

Mupad [B]
time = 1.28, size = 143, normalized size = 1.11

$$\frac{a^2\,x^2}{2} - \frac{b^2\,\mathrm{atanh}\left(c\,\sqrt{x}\right)^2}{2\,c^4} + \frac{2\,b^2\,\ln\left(c^2\,x-1\right)}{3\,c^4} + \frac{b^2\,x^2\,\mathrm{atanh}\left(c\,\sqrt{x}\right)^2}{2} + \frac{b^2\,x}{6\,c^2} + \frac{b^2\,x^{3/2}\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{3\,c} + \frac{b^2\,\sqrt{x}\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{c^3} + \frac{a\,b\,x^{3/2}}{3\,c} + \frac{a\,b\,\sqrt{x}}{c^3} - \frac{a\,b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{c^4} + a\,b\,x^2\,\mathrm{atanh}\left(c\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^(1/2)))^2,x)
```

```
[Out] (a^2*x^2)/2 - (b^2*atanh(c*x^(1/2))^2)/(2*c^4) + (2*b^2*log(c^2*x - 1))/(3*
c^4) + (b^2*x^2*atanh(c*x^(1/2))^2)/2 + (b^2*x)/(6*c^2) + (b^2*x^(3/2)*atan
h(c*x^(1/2)))/(3*c) + (b^2*x^(1/2)*atanh(c*x^(1/2)))/c^3 + (a*b*x^(3/2))/(3
*c) + (a*b*x^(1/2))/c^3 - (a*b*atanh(c*x^(1/2)))/c^4 + a*b*x^2*atanh(c*x^(1
/2))
```

3.198 $\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 dx$

Optimal. Leaf size=85

$$\frac{2ab\sqrt{x}}{c}+\frac{2b^2\sqrt{x}\tanh^{-1}\left(c\sqrt{x}\right)}{c}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{c^2}+x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+\frac{b^2\log\left(1-c^2x\right)}{c^2}$$

```
[Out] -(a+b*arctanh(c*x^(1/2)))^2/c^2+x*(a+b*arctanh(c*x^(1/2)))^2+b^2*ln(-c^2*x+
1)/c^2+2*a*b*x^(1/2)/c+2*b^2*arctanh(c*x^(1/2))*x^(1/2)/c
```

Rubi [A]
time = 0.10, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6027, 6037, 6127, 6021, 266, 6095}

$$-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{c^2}+\frac{2ab\sqrt{x}}{c}+x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+\frac{b^2\log\left(1-c^2x\right)}{c^2}+\frac{2b^2\sqrt{x}\tanh^{-1}\left(c\sqrt{x}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] (2*a*b*Sqrt[x])/c + (2*b^2*Sqrt[x]*ArcTanh[c*Sqrt[x]])/c - (a + b*ArcTanh[c
*Sqrt[x]])^2/c^2 + x*(a + b*ArcTanh[c*Sqrt[x]])^2 + (b^2*Log[1 - c^2*x])/c^
2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6027

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> With[{k = D
enominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*ArcTanh[c*x^(k*n)])^p, x
], x, x^(1/k)], x]] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && FractionQ[n]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx=\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx$$

Mathematica [A]
time = 0.04, size = 115, normalized size = 1.35

$$\frac{2abc\sqrt{x}+a^2c^2x+2bc\left(b+ac\sqrt{x}\right)\sqrt{x}\,\tanh^{-1}\left(c\sqrt{x}\right)+b^2(-1+c^2x)\tanh^{-1}\left(c\sqrt{x}\right)^2+b(a+b)\log\left(1-c\sqrt{x}\right)-ab\log\left(1+c\sqrt{x}\right)+b^2\log\left(1+c\sqrt{x}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] (2*a*b*c*Sqrt[x] + a^2*c^2*x + 2*b*c*(b + a*c*Sqrt[x])*Sqrt[x]*ArcTanh[c*Sq
rt[x]] + b^2*(-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^2 + b*(a + b)*Log[1 - c*Sqrt[x
]] - a*b*Log[1 + c*Sqrt[x]] + b^2*Log[1 + c*Sqrt[x]])/c^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(75) = 150$.
time = 0.17, size = 252, normalized size = 2.96

method	result

derivativedivides	$\frac{a^2 x c^2+b^2 c^2 x \operatorname{arctanh}\left(c\sqrt{x}\right)^2+2b^2 \operatorname{arctanh}\left(c\sqrt{x}\right) c\sqrt{x}+b^2 \operatorname{arctanh}\left(c\sqrt{x}\right)\ln\left(c\sqrt{x}-1\right)-b^2 \operatorname{arctanh}\left(c\sqrt{x}\right.}{}$
default	$\frac{a^2 x c^2+b^2 c^2 x \operatorname{arctanh}\left(c\sqrt{x}\right)^2+2b^2 \operatorname{arctanh}\left(c\sqrt{x}\right) c\sqrt{x}+b^2 \operatorname{arctanh}\left(c\sqrt{x}\right)\ln\left(c\sqrt{x}-1\right)-b^2 \operatorname{arctanh}\left(c\sqrt{x}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^2*(1/2*a^2*x*c^2+1/2*b^2*c^2*x*arctanh(c*x^(1/2))^2+b^2*arctanh(c*x^(1/
2))*c*x^(1/2)+1/2*b^2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-1/2*b^2*arctanh(c*
x^(1/2))*ln(1+c*x^(1/2))+1/8*b^2*ln(c*x^(1/2)-1)^2-1/4*b^2*ln(c*x^(1/2)-1)*
ln(1/2*c*x^(1/2)+1/2)+1/2*b^2*ln(c*x^(1/2)-1)+1/2*b^2*ln(1+c*x^(1/2))+1/8*b
^2*ln(1+c*x^(1/2))^2-1/4*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))+1/4*b^2
*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2*c*x^(1/2)+1/2)+a*b*c^2*x*arctanh(c*x^(1/2))+
a*b*c*x^(1/2)+1/2*a*b*ln(c*x^(1/2)-1)-1/2*a*b*ln(1+c*x^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(75) = 150$.
time = 0.26, size = 175, normalized size = 2.06

$$\left(c\left(\frac{2\sqrt{x}}{c^2}-\frac{\log\left(c\sqrt{x}+1\right)}{c^3}+\frac{\log\left(c\sqrt{x}-1\right)}{c^3}\right)+2x\operatorname{artanh}\left(c\sqrt{x}\right)\right)ab+\frac{1}{4}\left(4c\left(\frac{2\sqrt{x}}{c^2}-\frac{\log\left(c\sqrt{x}+1\right)}{c^3}+\frac{\log\left(c\sqrt{x}-1\right)}{c^3}\right)\operatorname{artanh}\left(c\sqrt{x}\right)+4x\operatorname{artanh}\left(c\sqrt{x}\right)^2-\frac{2\left(\log\left(c\sqrt{x}-1\right)-2\right)\log\left(c\sqrt{x}+1\right)-\log\left(c\sqrt{x}+1\right)^2-\log\left(c\sqrt{x}-1\right)^2-4\log\left(c\sqrt{x}-1\right)}{c^2}\right)b^2+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] (c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3 + log(c*sqrt(x) - 1)/c^3) + 2*x*
arctanh(c*sqrt(x)))*a*b + 1/4*(4*c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3
+ log(c*sqrt(x) - 1)/c^3)*arctanh(c*sqrt(x)) + 4*x*arctanh(c*sqrt(x))^2 - (
2*(log(c*sqrt(x) - 1) - 2)*log(c*sqrt(x) + 1) - log(c*sqrt(x) + 1)^2 - log(
c*sqrt(x) - 1)^2 - 4*log(c*sqrt(x) - 1))/c^2)*b^2 + a^2*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(75) = 150$.
time = 0.36, size = 165, normalized size = 1.94

$$\frac{4a^2c^2x+8abc\sqrt{x}+\left(b^2c^2x-b^2\right)\log\left(-\frac{c^2x+2c\sqrt{x}+1}{c^2x-1}\right)^2+4\left(abc^2-ab+b^2\right)\log\left(c\sqrt{x}+1\right)-4\left(abc^2-ab-b^2\right)\log\left(c\sqrt{x}-1\right)+4\left(abc^2x-abc^2+b^2c\sqrt{x}\right)\log\left(-\frac{c^2x+2c\sqrt{x}+1}{c^2x-1}\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^2*c^2*x + 8*a*b*c*sqrt(x) + (b^2*c^2*x - b^2)*log(-(c^2*x + 2*c*sq
rt(x) + 1)/(c^2*x - 1))^2 + 4*(a*b*c^2 - a*b + b^2)*log(c*sqrt(x) + 1) - 4*
```

```
(a*b*c^2 - a*b - b^2)*log(c*sqrt(x) - 1) + 4*(a*b*c^2*x - a*b*c^2 + b^2*c*s
qrt(x))*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)))/c^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{atanh}\left(c\sqrt{x}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**2,x)
```

```
[Out] Integral((a + b*atanh(c*sqrt(x)))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2, x)
```

Mupad [B]
time = 1.06, size = 94, normalized size = 1.11

$$a^2 x + \frac{c\left(2 b^2 \sqrt{x}\, \operatorname{atanh}\left(c\sqrt{x}\right) + 2 a b \sqrt{x}\right) - b^2 \operatorname{atanh}\left(c\sqrt{x}\right)^2 + b^2 \ln\left(c^2 x - 1\right) - 2 a b \operatorname{atanh}\left(c\sqrt{x}\right)}{c^2} + b^2 x \operatorname{atanh}\left(c\sqrt{x}\right)^2 + 2 a b x \operatorname{atanh}\left(c\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))^2,x)
```

```
[Out] a^2*x + (c*(2*b^2*x^(1/2)*atanh(c*x^(1/2)) + 2*a*b*x^(1/2)) - b^2*atanh(c*x
^(1/2))^2 + b^2*log(c^2*x - 1) - 2*a*b*atanh(c*x^(1/2)))/c^2 + b^2*x*atanh(
c*x^(1/2))^2 + 2*a*b*x*atanh(c*x^(1/2))
```

3.199 $\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x}\,dx$

Optimal. Leaf size=145

$$4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-2b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)+$$

```
[Out] -4*(a+b*arctanh(c*x^(1/2)))^2*arctanh(-1+2/(1-c*x^(1/2)))-2*b*(a+b*arctanh(
c*x^(1/2)))*polylog(2,1-2/(1-c*x^(1/2)))+2*b*(a+b*arctanh(c*x^(1/2)))*polyl
og(2,-1+2/(1-c*x^(1/2)))+b^2*polylog(3,1-2/(1-c*x^(1/2)))-b^2*polylog(3,-1+
2/(1-c*x^(1/2)))
```

Rubi [A]
time = 0.23, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6035, 6033, 6199, 6095, 6205, 6745}

$$-2b\text{Li}_2\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+2b\text{Li}_2\left(\frac{2}{1-c\sqrt{x}}-1\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+b^2\text{Li}_3\left(1-\frac{2}{1-c\sqrt{x}}\right)-b^2\text{Li}_3\left(\frac{2}{1-c\sqrt{x}}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^2/x,x]
```

```
[Out] 4*ArcTanh[1 - 2/(1 - c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^2 - 2*b*(a + b*
ArcTanh[c*Sqrt[x]])*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])] + 2*b*(a + b*ArcTanh[
c*Sqrt[x]])*PolyLog[2, -1 + 2/(1 - c*Sqrt[x])] + b^2*PolyLog[3, 1 - 2/(1 -
c*Sqrt[x])] - b^2*PolyLog[3, -1 + 2/(1 - c*Sqrt[x])]
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x}\,dx = 2\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx, x, \sqrt{x}\right)$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-(8bc)\text{Subst}\left(\int \frac{(a+b$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+(4bc)\text{Subst}\left(\int \frac{(a+b$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-2b\big(a+b\tanh^{-1}\left(c\sqrt{x}$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-2b\big(a+b\tanh^{-1}\left(c\sqrt{x}$$

Mathematica [A]
time = 0.06, size = 164, normalized size = 1.13

$$4\tanh^{-1}\left(1+\frac{2}{-1+c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-b\left(-2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\text{PolyLog}\left(2,\frac{1+c\sqrt{x}}{1-c\sqrt{x}}\right)+2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\text{PolyLog}\left(2,\frac{1+c\sqrt{x}}{-1+c\sqrt{x}}\right)+b\left(\text{PolyLog}\left(3,\frac{1+c\sqrt{x}}{1-c\sqrt{x}}\right)-\text{PolyLog}\left(3,\frac{1+c\sqrt{x}}{-1+c\sqrt{x}}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^2/x,x]
```

```
[Out] 4*ArcTanh[1 + 2/(-1 + c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^2 - b*(-2*(a +
 b*ArcTanh[c*Sqrt[x]])*PolyLog[2, (1 + c*Sqrt[x])/(1 - c*Sqrt[x])] + 2*(a +
 b*ArcTanh[c*Sqrt[x]])*PolyLog[2, (1 + c*Sqrt[x])/(-1 + c*Sqrt[x])] + b*(Po
lyLog[3, (1 + c*Sqrt[x])/(1 - c*Sqrt[x])] - PolyLog[3, (1 + c*Sqrt[x])/(-1
+ c*Sqrt[x])]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.59, size = 742, normalized size = 5.12

method	result
derivativedivides	$2a^2 \ln\left(c\sqrt{x}\right) + 2b^2 \ln\left(c\sqrt{x}\right) \operatorname{arctanh}\left(c\sqrt{x}\right)^2 - 2b^2 \operatorname{arctanh}\left(c\sqrt{x}\right) \operatorname{polylog}\left(2, -\frac{(1\cdots}{\cdots}\right.$
default	$2a^2 \ln\left(c\sqrt{x}\right) + 2b^2 \ln\left(c\sqrt{x}\right) \operatorname{arctanh}\left(c\sqrt{x}\right)^2 - 2b^2 \operatorname{arctanh}\left(c\sqrt{x}\right) \operatorname{polylog}\left(2, -\frac{(1\cdots}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^2*ln(c*x^(1/2))+2*b^2*ln(c*x^(1/2))*arctanh(c*x^(1/2))^2-2*b^2*arctanh(
c*x^(1/2))*polylog(2,-(1+c*x^(1/2))^2/(-c^2*x+1))+b^2*polylog(3,-(1+c*x^(1/
2))^2/(-c^2*x+1))-2*b^2*arctanh(c*x^(1/2))^2*ln((1+c*x^(1/2))^2/(-c^2*x+1)-
1)+2*b^2*arctanh(c*x^(1/2))^2*ln(1-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+4*b^2*ar
ctanh(c*x^(1/2))*polylog(2,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-4*b^2*polylog(3,
(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+2*b^2*arctanh(c*x^(1/2))^2*ln(1+(1+c*x^(1/2
))/(-c^2*x+1)^(1/2))+4*b^2*arctanh(c*x^(1/2))*polylog(2,-(1+c*x^(1/2))/(-c^
2*x+1)^(1/2))-4*b^2*polylog(3,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+I*b^2*Pi*csg
n(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^3*arctan
h(c*x^(1/2))^2-I*b^2*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1))*csgn(I*((1+c
*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*arctanh(c*x^(1/
2))^2-I*b^2*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*csgn(I*((1+c*x^(1/2))
^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*arctanh(c*x^(1/2))^2+I*b
^2*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1))*csgn(I/(1+(1+c*x^(1/2))^2/(-c^
2*x+1)))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1
)))*arctanh(c*x^(1/2))^2+4*a*b*ln(c*x^(1/2))*arctanh(c*x^(1/2))-2*a*b*ln(c*
x^(1/2))*ln(1+c*x^(1/2))-2*a*b*dilog(c*x^(1/2))-2*a*b*dilog(1+c*x^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x,x, algorithm="maxima")
[Out] 1/4*b^2*integrate(log(c*sqrt(x) + 1)^2/x, x) - 1/2*b^2*integrate(log(c*sqrt
(x) + 1)*log(-c*sqrt(x) + 1)/x, x) + 1/4*b^2*integrate(log(-c*sqrt(x) + 1)^
2/x, x) + a*b*integrate(log(c*sqrt(x) + 1)/x, x) - a*b*integrate(log(-c*sqr
t(x) + 1)/x, x) + a^2*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x,x, algorithm="fricas")
[Out] integral((b^2*arctanh(c*sqrt(x))^2 + 2*a*b*arctanh(c*sqrt(x)) + a^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(c \sqrt{x}\right)\right)^{2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**2/x,x)
[Out] Integral((a + b*atanh(c*sqrt(x)))**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x,x, algorithm="giac")
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)\right)}^2}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))^2/x,x)
[Out] int((a + b*atanh(c*x^(1/2)))^2/x, x)
```

3.200 $\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=85

$$-\frac{2bc\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{\sqrt{x}}+c^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x}+b^2c^2\log(x)-b^2c^2\log\left(1-\right.$$

```
[Out] c^2*(a+b*arctanh(c*x^(1/2)))^2-(a+b*arctanh(c*x^(1/2)))^2/x+b^2*c^2*ln(x)-b
^2*c^2*ln(-c^2*x+1)-2*b*c*(a+b*arctanh(c*x^(1/2)))/x^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6039, 6037, 6129, 272, 36, 29, 31, 6095}

$$c^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-\frac{2bc\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{\sqrt{x}}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x}+b^2c^2\log(x)-b^2c^2\log\left(1-c^2x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^2/x^2,x]
```

```
[Out] (-2*b*c*(a + b*ArcTanh[c*Sqrt[x]]))/Sqrt[x] + c^2*(a + b*ArcTanh[c*Sqrt[x]]
)^2 - (a + b*ArcTanh[c*Sqrt[x]])^2/x + b^2*c^2*Log[x] - b^2*c^2*Log[1 - c^2
*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx$$

Mathematica [A]
time = 0.07, size = 129, normalized size = 1.52

$$-\frac{a^2+2abc\sqrt{x}+2b\left(a+bc\sqrt{x}\right)\tanh^{-1}\left(c\sqrt{x}\right)-b^2(-1+c^2x)\tanh^{-1}\left(c\sqrt{x}\right)^2+b(a+b)c^2x\log\left(1-c\sqrt{x}\right)-abc^2x\log\left(1+c\sqrt{x}\right)+b^2c^2x\log\left(1+c\sqrt{x}\right)-b^2c^2x\log(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^2/x^2,x]
```

```
[Out] -((a^2 + 2*a*b*c*Sqrt[x] + 2*b*(a + b*c*Sqrt[x])*ArcTanh[c*Sqrt[x]] - b^2*(
-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^2 + b*(a + b)*c^2*x*Log[1 - c*Sqrt[x]] - a*b
```

```
*c^2*x*Log[1 + c*Sqrt[x]] + b^2*c^2*x*Log[1 + c*Sqrt[x]] - b^2*c^2*x*Log[x]
)/x)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(77) = 154$.
time = 0.11, size = 275, normalized size = 3.24

method	result
derivativedivides	$2c^2\left(-\frac{a^2}{2c^2x}-\frac{b^2\operatorname{arctanh}\left(c\sqrt{x}\right)^2}{2c^2x}+\frac{b^2\operatorname{arctanh}\left(c\sqrt{x}\right)\ln\left(1+c\sqrt{x}\right)}{2}-\frac{b^2\operatorname{arctanh}\left(c\sqrt{x}\right)\ln\left(c\sqrt{x}-1\right)}{2}\right.$
default	$2c^2\left(-\frac{a^2}{2c^2x}-\frac{b^2\operatorname{arctanh}\left(c\sqrt{x}\right)^2}{2c^2x}+\frac{b^2\operatorname{arctanh}\left(c\sqrt{x}\right)\ln\left(1+c\sqrt{x}\right)}{2}-\frac{b^2\operatorname{arctanh}\left(c\sqrt{x}\right)\ln\left(c\sqrt{x}-1\right)}{2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*c^2*(-1/2*a^2/c^2/x-1/2*b^2/c^2/x*arctanh(c*x^(1/2))^2+1/2*b^2*arctanh(c*
x^(1/2))*ln(1+c*x^(1/2))-1/2*b^2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-b^2*arc
tanh(c*x^(1/2))/c/x^(1/2)-1/8*b^2*ln(c*x^(1/2)-1)^2+1/4*b^2*ln(c*x^(1/2)-1)
*ln(1/2*c*x^(1/2)+1/2)-1/2*b^2*ln(1+c*x^(1/2))+b^2*ln(c*x^(1/2))-1/2*b^2*ln
(c*x^(1/2)-1)-1/8*b^2*ln(1+c*x^(1/2))^2-1/4*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1
/2*c*x^(1/2)+1/2)+1/4*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))-a*b/c^2/x*
arctanh(c*x^(1/2))+1/2*a*b*ln(1+c*x^(1/2))-1/2*a*b*ln(c*x^(1/2)-1)-a*b/c/x^
(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(77) = 154$.
time = 0.27, size = 174, normalized size = 2.05

$$\left(\left(c\log\left(c\sqrt{x}+1\right)-c\log\left(c\sqrt{x}-1\right)-\frac{2}{\sqrt{x}}\right)c-\frac{2\,\operatorname{artanh}\left(c\sqrt{x}\right)}{x}\right)ab+\frac{1}{4}\left(\left(2\left(\log\left(c\sqrt{x}-1\right)-2\right)\log\left(c\sqrt{x}+1\right)-\log\left(c\sqrt{x}+1\right)^2-\log\left(c\sqrt{x}-1\right)^2-4\log\left(c\sqrt{x}-1\right)+4\log\left(x\right)\right)c^2+4\left(c\log\left(c\sqrt{x}+1\right)-c\log\left(c\sqrt{x}-1\right)-\frac{2}{\sqrt{x}}\right)c\,\operatorname{artanh}\left(c\sqrt{x}\right)\right)b^2-\frac{b^2\operatorname{artanh}\left(c\sqrt{x}\right)^2}{x}-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^2,x, algorithm="maxima")
```

```
[Out] ((c*log(c*sqrt(x) + 1) - c*log(c*sqrt(x) - 1) - 2/sqrt(x))*c - 2*arctanh(c*
sqrt(x))/x)*a*b + 1/4*((2*(log(c*sqrt(x) - 1) - 2)*log(c*sqrt(x) + 1) - log
(c*sqrt(x) + 1)^2 - log(c*sqrt(x) - 1)^2 - 4*log(c*sqrt(x) - 1) + 4*log(x))
*c^2 + 4*(c*log(c*sqrt(x) + 1) - c*log(c*sqrt(x) - 1) - 2/sqrt(x))*c*arctan
h(c*sqrt(x)))*b^2 - b^2*arctanh(c*sqrt(x))^2/x - a^2/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(77) = 154$.

time = 0.37, size = 157, normalized size = 1.85

$$\frac{8\,b^{2}c^{2}x\log\left(\sqrt{x}\right)+4\,(ab-b^{2})c^{2}x\log\left(c\sqrt{x}+1\right)-4\,(ab+b^{2})c^{2}x\log\left(c\sqrt{x}-1\right)-8\,abc\sqrt{x}+(b^{2}c^{2}x-b^{2})\log\left(-\frac{c^{2}x+2\,c\sqrt{x}+1}{c^{2}x-1}\right)^{2}-4\,a^{2}-4\,(b^{2}c\sqrt{x}+ab)\log\left(-\frac{c^{2}x+2\,c\sqrt{x}+1}{c^{2}x-1}\right)}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(8*b^2*c^2*x*log(sqrt(x)) + 4*(a*b - b^2)*c^2*x*log(c*sqrt(x) + 1) - 4*
(a*b + b^2)*c^2*x*log(c*sqrt(x) - 1) - 8*a*b*c*sqrt(x) + (b^2*c^2*x - b^2)*
log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1))^2 - 4*a^2 - 4*(b^2*c*sqrt(x) +
a*b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)))/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(78) = 156$.
time = 3.47, size = 680, normalized size = 8.00

$$\begin{cases} -\frac{a^{2}}{x}+\frac{2ab\operatorname{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x}-\frac{b^{2}\operatorname{atanh}^{2}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} & \text{for}\ c=-\sqrt{\frac{1}{x}} \\ -\frac{a^{2}}{x}-\frac{2ab\operatorname{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x}-\frac{b^{2}\operatorname{atanh}^{2}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} & \text{for}\ c=\sqrt{\frac{1}{x}} \\ -\frac{a^{2}}{x} & \text{for}\ c=0 \\ -\frac{a^{2}c^{2}x^{\frac{3}{2}}}{c^{2}x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{a^{2}\sqrt{x}}{c^{2}x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{2abc^{4}x^{\frac{5}{2}}\operatorname{atanh}\left(c\sqrt{x}\right)}{c^{2}x^{\frac{5}{2}}-x^{\frac{3}{2}}}-\frac{2abc^{3}x^{2}}{c^{2}x^{\frac{5}{2}}-x^{\frac{3}{2}}}-\frac{4abc^{2}x^{\frac{3}{2}}\operatorname{atanh}\left(c\sqrt{x}\right)}{c^{2}x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{2abcx}{c^{2}x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{2ab\sqrt{x}\operatorname{atanh}\left(c\sqrt{x}\right)}{c^{2}x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{b^{2}c^{4}x^{\frac{5}{2}}\log\left(x\right)}{c^{2}x^{\frac{5}{2}}-x^{\frac{3}{2}}}-\frac{2b^{2}c^{4}x^{\frac{5}{2}}\log\left(\sqrt{x}-\frac{1}{c}\right)}{c^{2}x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{b^{2}c^{4}x^{\frac{5}{2}}\operatorname{atanh}^{2}\left(c\sqrt{x}\right)}{c^{2}x^{\frac{5}{2}}-x^{\frac{3}{2}}}-\frac{2b^{2}c^{4}x^{\frac{5}{2}}\operatorname{atanh}\left(c\sqrt{x}\right)}{c^{2}x^{\frac{5}{2}}-x^{\frac{3}{2}}}-\frac{2b^{2}c^{3}x^{2}\operatorname{atanh}\left(c\sqrt{x}\right)}{c^{2}x^{\frac{5}{2}}-x^{\frac{3}{2}}}-\frac{b^{2}c^{2}x^{\frac{3}{2}}\log\left(x\right)}{c^{2}x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{2b^{2}c^{2}x^{\frac{3}{2}}\log\left(\sqrt{x}-\frac{1}{c}\right)}{c^{2}x^{\frac{5}{2}}-x^{\frac{3}{2}}}-\frac{2b^{2}c^{2}x^{\frac{3}{2}}\operatorname{atanh}^{2}\left(c\sqrt{x}\right)}{c^{2}x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{2b^{2}c^{2}x^{\frac{3}{2}}\operatorname{atanh}\left(c\sqrt{x}\right)}{c^{2}x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{2b^{2}cx\operatorname{atanh}\left(c\sqrt{x}\right)}{c^{2}x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{b^{2}\sqrt{x}\operatorname{atanh}^{2}\left(c\sqrt{x}\right)}{c^{2}x^{\frac{5}{2}}-x^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**2/x**2,x)
```

```
[Out] Piecewise((-a**2/x + 2*a*b*atanh(sqrt(x)*sqrt(1/x))/x - b**2*atanh(sqrt(x)*
sqrt(1/x))**2/x, Eq(c, -sqrt(1/x))), (-a**2/x - 2*a*b*atanh(sqrt(x)*sqrt(1/
x))/x - b**2*atanh(sqrt(x)*sqrt(1/x))**2/x, Eq(c, sqrt(1/x))), (-a**2/x, Eq
(c, 0)), (-a**2*c**2*x**(3/2)/(c**2*x**(5/2) - x**(3/2)) + a**2*sqrt(x)/(c*
*2*x**(5/2) - x**(3/2)) + 2*a*b*c**4*x**(5/2)*atanh(c*sqrt(x))/(c**2*x**(5/
2) - x**(3/2)) - 2*a*b*c**3*x**2/(c**2*x**(5/2) - x**(3/2)) - 4*a*b*c**2*x*
*(3/2)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) + 2*a*b*c*x/(c**2*x**(5/
2) - x**(3/2)) + 2*a*b*sqrt(x)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2))
+ b**2*c**4*x**(5/2)*log(x)/(c**2*x**(5/2) - x**(3/2)) - 2*b**2*c**4*x**(5/
2)*log(sqrt(x) - 1/c)/(c**2*x**(5/2) - x**(3/2)) + b**2*c**4*x**(5/2)*atanh
(c*sqrt(x))**2/(c**2*x**(5/2) - x**(3/2)) - 2*b**2*c**4*x**(5/2)*atanh(c*sq
rt(x))/(c**2*x**(5/2) - x**(3/2)) - 2*b**2*c**3*x**2*atanh(c*sqrt(x))/(c**2
*x**(5/2) - x**(3/2)) - b**2*c**2*x**(3/2)*log(x)/(c**2*x**(5/2) - x**(3/2)
) + 2*b**2*c**2*x**(3/2)*log(sqrt(x) - 1/c)/(c**2*x**(5/2) - x**(3/2)) - 2*
b**2*c**2*x**(3/2)*atanh(c*sqrt(x))**2/(c**2*x**(5/2) - x**(3/2)) + 2*b**2*
c**2*x**(3/2)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) + 2*b**2*c*x*atan
h(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) + b**2*sqrt(x)*atanh(c*sqrt(x))**2/
(c**2*x**(5/2) - x**(3/2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2/x^2, x)
```

Mupad [B]
time = 1.79, size = 278, normalized size = 3.27

$$2\,b^2\,c^2\,\ln\left(\sqrt{x}\right)-\frac{a^2}{x}-b^2\,c^2\,\ln\left(c\,\sqrt{x}-1\right)-b^2\,c^2\,\ln\left(c\,\sqrt{x}+1\right)+\frac{b^2\,c^2\ln\left(c\,\sqrt{x}+1\right)^2}{4}+\frac{b^2\,c^2\ln\left(1-c\,\sqrt{x}\right)^2}{4}-\frac{b^2\ln\left(c\,\sqrt{x}+1\right)^2}{4\,x}-\frac{b^2\ln\left(1-c\,\sqrt{x}\right)^2}{4\,x}-a\,b\,c^2\,\ln\left(c\,\sqrt{x}-1\right)+a\,b\,c^2\,\ln\left(c\,\sqrt{x}+1\right)-\frac{2\,a\,b\,c}{\sqrt{x}}-\frac{a\,b\,\ln\left(c\,\sqrt{x}+1\right)}{x}+\frac{a\,b\,\ln\left(1-c\,\sqrt{x}\right)}{x}-\frac{b^2\,c^2\,\ln\left(c\,\sqrt{x}+1\right)\,\ln\left(1-c\,\sqrt{x}\right)}{2}-\frac{b^2\,c\,\ln\left(c\,\sqrt{x}+1\right)}{\sqrt{x}}+\frac{b^2\,c\,\ln\left(1-c\,\sqrt{x}\right)}{\sqrt{x}}+\frac{b^2\,\ln\left(c\,\sqrt{x}+1\right)\,\ln\left(1-c\,\sqrt{x}\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))^2/x^2,x)
```

```
[Out] 2*b^2*c^2*log(x^(1/2)) - a^2/x - b^2*c^2*log(c*x^(1/2) - 1) - b^2*c^2*log(c
*x^(1/2) + 1) + (b^2*c^2*log(c*x^(1/2) + 1)^2)/4 + (b^2*c^2*log(1 - c*x^(1/
2))^2)/4 - (b^2*log(c*x^(1/2) + 1)^2)/(4*x) - (b^2*log(1 - c*x^(1/2))^2)/(4
*x) - a*b*c^2*log(c*x^(1/2) - 1) + a*b*c^2*log(c*x^(1/2) + 1) - (2*a*b*c)/x
^(1/2) - (a*b*log(c*x^(1/2) + 1))/x + (a*b*log(1 - c*x^(1/2)))/x - (b^2*c^2
*log(c*x^(1/2) + 1)*log(1 - c*x^(1/2)))/2 - (b^2*c*log(c*x^(1/2) + 1))/x^(1
/2) + (b^2*c*log(1 - c*x^(1/2)))/x^(1/2) + (b^2*log(c*x^(1/2) + 1)*log(1 -
c*x^(1/2)))/(2*x)
```

3.201 $\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=133

$$-\frac{b^2c^2}{6x}-\frac{bc\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{3x^{3/2}}-\frac{bc^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{\sqrt{x}}+\frac{1}{2}c^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-\frac{\left(a+b\tanh^{-1}\left(\right.\right.}{2x^2}$$

```
[Out] -1/6*b^2*c^2/x-1/3*b*c*(a+b*arctanh(c*x^(1/2)))/x^(3/2)+1/2*c^4*(a+b*arctan
h(c*x^(1/2)))^2-1/2*(a+b*arctanh(c*x^(1/2)))^2/x^2+2/3*b^2*c^4*ln(x)-2/3*b^
2*c^4*ln(-c^2*x+1)-b*c^3*(a+b*arctanh(c*x^(1/2)))/x^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6039, 6037, 6129, 272, 46, 36, 29, 31, 6095}

$$\frac{1}{2}c^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-\frac{bc^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{\sqrt{x}}-\frac{bc\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{3x^{3/2}}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{2x^2}+\frac{2}{3}b^2c^4\log(x)-\frac{b^2c^2}{6x}-\frac{2}{3}b^2c^4\log\left(1-c^2x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^2/x^3,x]
```

```
[Out] -1/6*(b^2*c^2)/x - (b*c*(a + b*ArcTanh[c*Sqrt[x]]))/(3*x^(3/2)) - (b*c^3*(a
 + b*ArcTanh[c*Sqrt[x]]))/Sqrt[x] + (c^4*(a + b*ArcTanh[c*Sqrt[x]])^2)/2 -
(a + b*ArcTanh[c*Sqrt[x]])^2/(2*x^2) + (2*b^2*c^4*Log[x])/3 - (2*b^2*c^4*Lo
g[1 - c^2*x])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx$$

Mathematica [A]
time = 0.08, size = 178, normalized size = 1.34

$$-\frac{3a^2+2abc\sqrt{x}+b^2c^2x+6abc^3x^{3/2}+2b\left(3a+bc\sqrt{x}\left(1+3c^2x\right)\right)\tanh^{-1}\left(c\sqrt{x}\right)-3b^2\left(-1+c^4x^2\right)\tanh^{-1}\left(c\sqrt{x}\right)^2+b(3a+4b)c^4x^2\log\left(1-c\sqrt{x}\right)-3abc^4x^2\log\left(1+c\sqrt{x}\right)+4b^2c^4x^2\log\left(1+c\sqrt{x}\right)-4b^2c^4x^2\log(x)}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^2/x^3,x]
```

```
[Out] -1/6*(3*a^2 + 2*a*b*c*Sqrt[x] + b^2*c^2*x + 6*a*b*c^3*x^(3/2) + 2*b*(3*a +
b*c*Sqrt[x]*(1 + 3*c^2*x))*ArcTanh[c*Sqrt[x]] - 3*b^2*(-1 + c^4*x^2)*ArcTan
h[c*Sqrt[x]]^2 + b*(3*a + 4*b)*c^4*x^2*Log[1 - c*Sqrt[x]] - 3*a*b*c^4*x^2*L
og[1 + c*Sqrt[x]] + 4*b^2*c^4*x^2*Log[1 + c*Sqrt[x]] - 4*b^2*c^4*x^2*Log[x]
)/x^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(109) = 218.
time = 0.12, size = 314, normalized size = 2.36

method	result
derivativedivides	$2c^4\left(-\frac{a^2}{4c^4x^2}-\frac{b^2\operatorname{arctanh}\left(c\sqrt{x}\right)^2}{4c^4x^2}-\frac{b^2\operatorname{arctanh}\left(c\sqrt{x}\right)}{6c^3x^{\frac{3}{2}}}-\frac{b^2\operatorname{arctanh}\left(c\sqrt{x}\right)}{2c\sqrt{x}}-\frac{b^2\operatorname{arctanh}\left(c\sqrt{x}\right)\ln(}{4}\right.$
default	$2c^4\left(-\frac{a^2}{4c^4x^2}-\frac{b^2\operatorname{arctanh}\left(c\sqrt{x}\right)^2}{4c^4x^2}-\frac{b^2\operatorname{arctanh}\left(c\sqrt{x}\right)}{6c^3x^{\frac{3}{2}}}-\frac{b^2\operatorname{arctanh}\left(c\sqrt{x}\right)}{2c\sqrt{x}}-\frac{b^2\operatorname{arctanh}\left(c\sqrt{x}\right)\ln(}{4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*c^4*(-1/4*a^2/c^4/x^2-1/4*b^2/c^4/x^2*arctanh(c*x^(1/2))^2-1/6*b^2*arctan
h(c*x^(1/2))/c^3/x^(3/2)-1/2*b^2*arctanh(c*x^(1/2))/c/x^(1/2)-1/4*b^2*arcta
nh(c*x^(1/2))*ln(c*x^(1/2)-1)+1/4*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-1/
16*b^2*ln(c*x^(1/2)-1)^2+1/8*b^2*ln(c*x^(1/2)-1)*ln(1/2*c*x^(1/2)+1/2)-1/16
*b^2*ln(1+c*x^(1/2))^2-1/8*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2*c*x^(1/2)+1/2)
+1/8*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))-1/12*b^2/c^2/x+2/3*b^2*ln(c
*x^(1/2))-1/3*b^2*ln(c*x^(1/2)-1)-1/3*b^2*ln(1+c*x^(1/2))-1/2*a*b/c^4/x^2*a
rctanh(c*x^(1/2))-1/6*a*b/c^3/x^(3/2)-1/2*a*b/c/x^(1/2)-1/4*a*b*ln(c*x^(1/2
)-1)+1/4*a*b*ln(1+c*x^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(109) = 218.
time = 0.26, size = 234, normalized size = 1.76

$$\frac{1}{6}\left(\left(3\,c^3\log\left(c\sqrt{x}+1\right)-3\,c^3\log\left(c\sqrt{x}-1\right)-\frac{2\left(3\,c^2x+1\right)}{x^{\frac{3}{2}}}\right)c-\frac{6\operatorname{artanh}\left(c\sqrt{x}\right)}{x^2}\right)ab+\frac{1}{24}\left(\left(16\,c^2\log\left(x\right)-\frac{3\,c^2x\log\left(c\sqrt{x}+1\right)^2+3\,c^2x\log\left(c\sqrt{x}-1\right)^2+16\,c^2x\log\left(c\sqrt{x}-1\right)-2\left(3\,c^2x\log\left(c\sqrt{x}-1\right)-8\,c^2x\right)\log\left(c\sqrt{x}+1\right)+4}{x}\right)c^2+4\left(3\,c^3\log\left(c\sqrt{x}+1\right)-3\,c^3\log\left(c\sqrt{x}-1\right)-\frac{2\left(3\,c^2x+1\right)}{x^{\frac{3}{2}}}\right)c\operatorname{artanh}\left(c\sqrt{x}\right)\right)b^2-\frac{b^2\operatorname{artanh}\left(c\sqrt{x}\right)^2}{2\,x^2}-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*((3*c^3*log(c*sqrt(x) + 1) - 3*c^3*log(c*sqrt(x) - 1) - 2*(3*c^2*x + 1)
/x^(3/2))*c - 6*arctanh(c*sqrt(x))/x^2)*a*b + 1/24*((16*c^2*log(x) - (3*c^2
*x*log(c*sqrt(x) + 1)^2 + 3*c^2*x*log(c*sqrt(x) - 1)^2 + 16*c^2*x*log(c*sqr
t(x) - 1) - 2*(3*c^2*x*log(c*sqrt(x) - 1) - 8*c^2*x)*log(c*sqrt(x) + 1) + 4
)/x)*c^2 + 4*(3*c^3*log(c*sqrt(x) + 1) - 3*c^3*log(c*sqrt(x) - 1) - 2*(3*c^
2*x + 1)/x^(3/2))*c*arctanh(c*sqrt(x)))*b^2 - 1/2*b^2*arctanh(c*sqrt(x))^2/
x^2 - 1/2*a^2/x^2
```

Fricas [A]
time = 0.38, size = 201, normalized size = 1.51

$$\frac{32\,b^{2}c^{4}x^{2}\log\left(\sqrt{x}\right)+4\,(3\,ab-4\,b^{2})c^{4}x^{2}\log\left(c\sqrt{x}+1\right)-4\,(3\,ab+4\,b^{2})c^{4}x^{2}\log\left(c\sqrt{x}-1\right)-4\,b^{2}c^{2}x+3\,(b^{2}c^{4}x^{2}-b^{2})\log\left(-\frac{c^{2}x+2\,c\sqrt{x}+1}{c^{2}x-1}\right)^{2}-12\,a^{2}-4\,\left(3\,ab+(3\,b^{2}c^{3}x+b^{2}c)\sqrt{x}\right)\log\left(-\frac{c^{2}x+2\,c\sqrt{x}+1}{c^{2}x-1}\right)-8\,(3\,abc^{3}x+abc)\sqrt{x}}{24\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(32*b^2*c^4*x^2*log(sqrt(x)) + 4*(3*a*b - 4*b^2)*c^4*x^2*log(c*sqrt(x)
 + 1) - 4*(3*a*b + 4*b^2)*c^4*x^2*log(c*sqrt(x) - 1) - 4*b^2*c^2*x + 3*(b^2
*c^4*x^2 - b^2)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1))^2 - 12*a^2 - 4*
(3*a*b + (3*b^2*c^3*x + b^2*c)*sqrt(x))*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2
*x - 1)) - 8*(3*a*b*c^3*x + a*b*c)*sqrt(x))/x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(122) = 244$.
time = 9.71, size = 972, normalized size = 7.31

$$\begin{cases} -\frac{a^{2}}{2x^{2}} & \text{for}\: c = 0 \\ -\frac{a^{2}}{2x^{2}} + \frac{ab\,\text{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x^{2}} - \frac{b^{2}\,\text{atanh}^{2}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{2x^{2}} & \text{for}\: c = -\sqrt{\frac{1}{x}} \\ -\frac{a^{2}}{2x^{2}} - \frac{ab\,\text{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x^{2}} - \frac{b^{2}\,\text{atanh}^{2}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{2x^{2}} & \text{for}\: c = \sqrt{\frac{1}{x}} \\ -\frac{3a^{2}c^{2}x^{\frac{3}{2}}}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} + \frac{3a^{2}\sqrt{x}}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} + \frac{6abc^{6}x^{\frac{7}{2}}\,\text{atanh}\left(c\sqrt{x}\right)}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} - \frac{6abc^{5}x^{3}}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} - \frac{6abc^{4}x^{\frac{5}{2}}\,\text{atanh}\left(c\sqrt{x}\right)}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} + \frac{4abc^{3}x^{2}}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} - \frac{6abc^{2}x^{\frac{3}{2}}\,\text{atanh}\left(c\sqrt{x}\right)}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} + \frac{2abcx}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} + \frac{6ab\sqrt{x}\,\text{atanh}\left(c\sqrt{x}\right)}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} + \frac{4b^{2}c^{6}x^{\frac{7}{2}}\log\left(x\right)}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} - \frac{8b^{2}c^{6}x^{\frac{7}{2}}\log\left(\sqrt{x}-\frac{1}{c}\right)}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} + \frac{3b^{2}c^{6}x^{\frac{7}{2}}\,\text{atanh}^{2}\left(c\sqrt{x}\right)}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} - \frac{8b^{2}c^{6}x^{\frac{7}{2}}\,\text{atanh}\left(c\sqrt{x}\right)}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} - \frac{6b^{2}c^{5}x^{3}\,\text{atanh}\left(c\sqrt{x}\right)}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} - \frac{4b^{2}c^{4}x^{\frac{5}{2}}\log\left(x\right)}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} + \frac{8b^{2}c^{4}x^{\frac{5}{2}}\log\left(\sqrt{x}-\frac{1}{c}\right)}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} - \frac{3b^{2}c^{4}x^{\frac{5}{2}}\,\text{atanh}^{2}\left(c\sqrt{x}\right)}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} + \frac{8b^{2}c^{4}x^{\frac{5}{2}}\,\text{atanh}\left(c\sqrt{x}\right)}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} - \frac{b^{2}c^{4}x^{\frac{5}{2}}}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} + \frac{4b^{2}c^{3}x^{2}\,\text{atanh}\left(c\sqrt{x}\right)}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} - \frac{3b^{2}c^{2}x^{\frac{3}{2}}\,\text{atanh}^{2}\left(c\sqrt{x}\right)}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} + \frac{b^{2}c^{2}x^{\frac{3}{2}}}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} + \frac{2b^{2}cx\,\text{atanh}\left(c\sqrt{x}\right)}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} + \frac{3b^{2}\sqrt{x}\,\text{atanh}^{2}\left(c\sqrt{x}\right)}{6c^{2}x^{\frac{7}{2}}-6x^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**2/x**3,x)
```

```
[Out] Piecewise((-a**2/(2*x**2), Eq(c, 0)), (-a**2/(2*x**2) + a*b*atanh(sqrt(x)*s
qrt(1/x))/x**2 - b**2*atanh(sqrt(x)*sqrt(1/x))**2/(2*x**2), Eq(c, -sqrt(1/x
))), (-a**2/(2*x**2) - a*b*atanh(sqrt(x)*sqrt(1/x))/x**2 - b**2*atanh(sqrt(
x)*sqrt(1/x))**2/(2*x**2), Eq(c, sqrt(1/x))), (-3*a**2*c**2*x**(3/2)/(6*c**
2*x**(7/2) - 6*x**(5/2)) + 3*a**2*sqrt(x)/(6*c**2*x**(7/2) - 6*x**(5/2)) +
6*a*b*c**6*x**(7/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) - 6*a*b
*c**5*x**3/(6*c**2*x**(7/2) - 6*x**(5/2)) - 6*a*b*c**4*x**(5/2)*atanh(c*sqr
t(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 4*a*b*c**3*x**2/(6*c**2*x**(7/2) - 6
*x**(5/2)) - 6*a*b*c**2*x**(3/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(
5/2)) + 2*a*b*c*x/(6*c**2*x**(7/2) - 6*x**(5/2)) + 6*a*b*sqrt(x)*atanh(c*sq
```

```
rt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 4*b**2*c**6*x**(7/2)*log(x)/(6*c**2
*x**(7/2) - 6*x**(5/2)) - 8*b**2*c**6*x**(7/2)*log(sqrt(x) - 1/c)/(6*c**2*x
**(7/2) - 6*x**(5/2)) + 3*b**2*c**6*x**(7/2)*atanh(c*sqrt(x))**2/(6*c**2*x*
*(7/2) - 6*x**(5/2)) - 8*b**2*c**6*x**(7/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/
2) - 6*x**(5/2)) - 6*b**2*c**5*x**3*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x
**(5/2)) - 4*b**2*c**4*x**(5/2)*log(x)/(6*c**2*x**(7/2) - 6*x**(5/2)) + 8*b
**2*c**4*x**(5/2)*log(sqrt(x) - 1/c)/(6*c**2*x**(7/2) - 6*x**(5/2)) - 3*b**
2*c**4*x**(5/2)*atanh(c*sqrt(x))**2/(6*c**2*x**(7/2) - 6*x**(5/2)) + 8*b**2
*c**4*x**(5/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) - b**2*c**4*
x**(5/2)/(6*c**2*x**(7/2) - 6*x**(5/2)) + 4*b**2*c**3*x**2*atanh(c*sqrt(x))
/(6*c**2*x**(7/2) - 6*x**(5/2)) - 3*b**2*c**2*x**(3/2)*atanh(c*sqrt(x))**2/
(6*c**2*x**(7/2) - 6*x**(5/2)) + b**2*c**2*x**(3/2)/(6*c**2*x**(7/2) - 6*x*
*(5/2)) + 2*b**2*c*x*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 3*b*
*2*sqrt(x)*atanh(c*sqrt(x))**2/(6*c**2*x**(7/2) - 6*x**(5/2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2/x^3, x)
```

Mupad [B]
time = 2.71, size = 341, normalized size = 2.56

$$\frac{4\,b^2\,c^4\,\ln\left(\sqrt{x}\right)}{3}-\frac{a^2}{2\,x^2}-\frac{2\,b^2\,c^4\,\ln\left(c\,\sqrt{x}-1\right)}{3}-\frac{2\,b^2\,c^4\,\ln\left(c\,\sqrt{x}+1\right)}{3}+\frac{b^2\,c^4\,\ln{\left(c\,\sqrt{x}+1\right)}^2}{8}+\frac{b^2\,c^4\,\ln{\left(1-c\,\sqrt{x}\right)}^2}{8}-\frac{b^2\,c^2}{6\,x}-\frac{b^2\,\ln{\left(c\,\sqrt{x}+1\right)}^2}{8\,x^2}-\frac{b^2\,\ln{\left(1-c\,\sqrt{x}\right)}^2}{8\,x^2}-\frac{b^2\,c^3\,\ln\left(c\,\sqrt{x}+1\right)}{2\,\sqrt{x}}+\frac{b^2\,c^3\,\ln\left(1-c\,\sqrt{x}\right)}{2\,\sqrt{x}}-\frac{a\,b\,c^4\,\ln\left(c\,\sqrt{x}-1\right)}{2}+\frac{a\,b\,c^4\,\ln\left(c\,\sqrt{x}+1\right)}{2}-\frac{a\,b\,c}{3\,x^{3/2}}-\frac{a\,b\,\ln\left(c\,\sqrt{x}+1\right)}{2\,x^2}+\frac{a\,b\,\ln\left(1-c\,\sqrt{x}\right)}{2\,x^2}-\frac{b^2\,c^4\,\ln\left(c\,\sqrt{x}+1\right)\,\ln\left(1-c\,\sqrt{x}\right)}{4}-\frac{a\,b\,c^3}{\sqrt{x}}-\frac{b^2\,c\,\ln\left(c\,\sqrt{x}+1\right)}{6\,x^{3/2}}+\frac{b^2\,c\,\ln\left(1-c\,\sqrt{x}\right)}{6\,x^{3/2}}+\frac{b^2\,\ln\left(c\,\sqrt{x}+1\right)\,\ln\left(1-c\,\sqrt{x}\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))^2/x^3,x)
```

```
[Out] (4*b^2*c^4*log(x^(1/2)))/3 - a^2/(2*x^2) - (2*b^2*c^4*log(c*x^(1/2) - 1))/3
 - (2*b^2*c^4*log(c*x^(1/2) + 1))/3 + (b^2*c^4*log(c*x^(1/2) + 1)^2)/8 + (b
^2*c^4*log(1 - c*x^(1/2))^2)/8 - (b^2*c^2)/(6*x) - (b^2*log(c*x^(1/2) + 1)^
2)/(8*x^2) - (b^2*log(1 - c*x^(1/2))^2)/(8*x^2) - (b^2*c^3*log(c*x^(1/2) +
1))/(2*x^(1/2)) + (b^2*c^3*log(1 - c*x^(1/2)))/(2*x^(1/2)) - (a*b*c^4*log(c
*x^(1/2) - 1))/2 + (a*b*c^4*log(c*x^(1/2) + 1))/2 - (a*b*c)/(3*x^(3/2)) - (
a*b*log(c*x^(1/2) + 1))/(2*x^2) + (a*b*log(1 - c*x^(1/2)))/(2*x^2) - (b^2*c
^4*log(c*x^(1/2) + 1)*log(1 - c*x^(1/2)))/4 - (a*b*c^3)/x^(1/2) - (b^2*c*lo
g(c*x^(1/2) + 1))/(6*x^(3/2)) + (b^2*c*log(1 - c*x^(1/2)))/(6*x^(3/2)) + (b
^2*log(c*x^(1/2) + 1)*log(1 - c*x^(1/2)))/(4*x^2)
```

3.202 $\int x^3 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 \, dx$

Optimal. Leaf size=374

$$\frac{47b^3\sqrt{x}}{70c^7}+\frac{23b^3x^{3/2}}{420c^5}+\frac{b^3x^{5/2}}{140c^3}-\frac{47b^3\tanh^{-1}\left(c\sqrt{x}\right)}{70c^8}+\frac{71b^2x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{140c^6}+\frac{9b^2x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right.\right.}{70c^4}$$

```
[Out] 23/420*b^3*x^(3/2)/c^5+1/140*b^3*x^(5/2)/c^3-47/70*b^3*arctanh(c*x^(1/2))/c
^8+71/140*b^2*x*(a+b*arctanh(c*x^(1/2)))/c^6+9/70*b^2*x^2*(a+b*arctanh(c*x^
(1/2)))/c^4+1/28*b^2*x^3*(a+b*arctanh(c*x^(1/2)))/c^2+44/35*b*(a+b*arctanh(
c*x^(1/2)))^2/c^8+1/4*b*x^(3/2)*(a+b*arctanh(c*x^(1/2)))^2/c^5+3/20*b*x^(5/
2)*(a+b*arctanh(c*x^(1/2)))^2/c^3+3/28*b*x^(7/2)*(a+b*arctanh(c*x^(1/2)))^2
/c-1/4*(a+b*arctanh(c*x^(1/2)))^3/c^8+1/4*x^4*(a+b*arctanh(c*x^(1/2)))^3-88
/35*b^2*(a+b*arctanh(c*x^(1/2)))*ln(2/(1-c*x^(1/2)))/c^8-44/35*b^3*polylog(
2,1-2/(1-c*x^(1/2)))/c^8+47/70*b^3*x^(1/2)/c^7+3/4*b*(a+b*arctanh(c*x^(1/2)
))^2*x^(1/2)/c^7
```

Rubi [A]
time = 1.09, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 54, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6039, 6037, 6127, 308, 212, 327, 6131, 6055, 2449, 2352, 6021, 6095}

$$-\frac{88b^2\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{35c^8}+\frac{71b^2x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{140c^6}+\frac{9b^2x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{70c^4}+\frac{b^2x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{28c^2}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{4c^8}+\frac{44b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{35c^8}+\frac{3b\sqrt{x}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{4c^7}+\frac{bx^{3/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{4c^5}+\frac{3bx^{5/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{20c^3}+\frac{3bx^{7/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{28c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3-\frac{44b^3\text{Li}_2\left(1-\frac{2}{1-c\sqrt{x}}\right)}{35c^8}-\frac{47b^3\tanh^{-1}\left(c\sqrt{x}\right)}{70c^8}+\frac{47b^3\sqrt{x}}{70c^7}+\frac{23b^3x^{3/2}}{420c^5}+\frac{b^3x^{5/2}}{140c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] (47*b^3*Sqrt[x])/(70*c^7) + (23*b^3*x^(3/2))/(420*c^5) + (b^3*x^(5/2))/(140
*c^3) - (47*b^3*ArcTanh[c*Sqrt[x]])/(70*c^8) + (71*b^2*x*(a + b*ArcTanh[c*S
qrt[x]]))/(140*c^6) + (9*b^2*x^2*(a + b*ArcTanh[c*Sqrt[x]]))/(70*c^4) + (b^
2*x^3*(a + b*ArcTanh[c*Sqrt[x]]))/(28*c^2) + (44*b*(a + b*ArcTanh[c*Sqrt[x]
])^2)/(35*c^8) + (3*b*Sqrt[x]*(a + b*ArcTanh[c*Sqrt[x]])^2)/(4*c^7) + (b*x^
(3/2)*(a + b*ArcTanh[c*Sqrt[x]])^2)/(4*c^5) + (3*b*x^(5/2)*(a + b*ArcTanh[c
*Sqrt[x]])^2)/(20*c^3) + (3*b*x^(7/2)*(a + b*ArcTanh[c*Sqrt[x]])^2)/(28*c)
- (a + b*ArcTanh[c*Sqrt[x]])^3/(4*c^8) + (x^4*(a + b*ArcTanh[c*Sqrt[x]])^3)
/4 - (88*b^2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 - c*Sqrt[x])])/(35*c^8) -
(44*b^3*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/(35*c^8)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx=\int x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx$$

Mathematica [A]
time = 0.83, size = 418, normalized size = 1.12

$$\frac{-564ab^2+630a^2bc\sqrt{x}+564b^3c\sqrt{x}+426ab^2c^2x+210a^2bc^3x^{3/2}+46b^3c^3x^{3/2}+108ab^2c^4x^2+126a^2bc^5x^{5/2}+6b^3c^5x^{5/2}+30ab^2c^6x^3+90a^2bc^7x^{7/2}+210a^3c^8x^4+6b^2\left(b\left(-176+105c\sqrt{x}+35c^3x^{3/2}+21c^5x^{5/2}+15c^7x^{7/2}\right)+105a(-1+c^8x^4)\right)\tanh^{-1}\left(c\sqrt{x}\right)^2+210b^3(-1+c^8x^4)\tanh^{-1}\left(c\sqrt{x}\right)^3+6b\tanh^{-1}\left(c\sqrt{x}\right)\left(105a^2c^8x^4+b^2(-94+71c^2x+18c^4x^2+5c^6x^3)+2abc\sqrt{x}\,(105+35c^2x+21c^4x^2+15c^6x^3)-352b^2\log\left(1+e^{-2\tanh^{-1}(c\sqrt{x})}\right)\right)+315a^2b\log\left(1-c\sqrt{x}\right)-315a^2b\log\left(1+c\sqrt{x}\right)+1056ab^2\log(1-c^2x)+1056b^3\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(c\sqrt{x})}\right)}{840c^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] (-564*a*b^2 + 630*a^2*b*c*Sqrt[x] + 564*b^3*c*Sqrt[x] + 426*a*b^2*c^2*x + 2
10*a^2*b*c^3*x^(3/2) + 46*b^3*c^3*x^(3/2) + 108*a*b^2*c^4*x^2 + 126*a^2*b*c
^5*x^(5/2) + 6*b^3*c^5*x^(5/2) + 30*a*b^2*c^6*x^3 + 90*a^2*b*c^7*x^(7/2) +
```

```
210*a^3*c^8*x^4 + 6*b^2*(b*(-176 + 105*c*Sqrt[x] + 35*c^3*x^(3/2) + 21*c^5*
x^(5/2) + 15*c^7*x^(7/2)) + 105*a*(-1 + c^8*x^4))*ArcTanh[c*Sqrt[x]]^2 + 21
0*b^3*(-1 + c^8*x^4)*ArcTanh[c*Sqrt[x]]^3 + 6*b*ArcTanh[c*Sqrt[x]]*(105*a^2
*c^8*x^4 + b^2*(-94 + 71*c^2*x + 18*c^4*x^2 + 5*c^6*x^3) + 2*a*b*c*Sqrt[x]*
(105 + 35*c^2*x + 21*c^4*x^2 + 15*c^6*x^3) - 352*b^2*Log[1 + E^(-2*ArcTanh[
c*Sqrt[x]])]) + 315*a^2*b*Log[1 - c*Sqrt[x]] - 315*a^2*b*Log[1 + c*Sqrt[x]]
 + 1056*a*b^2*Log[1 - c^2*x] + 1056*b^3*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]
])])/(840*c^8)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 19.47, size = 1431, normalized size = 3.83

method	result	size
derivativedivides	Expression too large to display	1431
default	Expression too large to display	1431

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^(1/2)))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^8*(-3/32*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*
x+1)))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(
1+(1+c*x^(1/2))^2/(-c^2*x+1)))+3/16*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I*(1
+c*x^(1/2))/(-c^2*x+1)^(1/2))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^2-11/30*b^3
+3/8*a*b^2*c^8*x^4*arctanh(c*x^(1/2))^2+3/28*a*b^2*arctanh(c*x^(1/2))*c^7*x
^(7/2)+3/20*a*b^2*arctanh(c*x^(1/2))*c^5*x^(5/2)+9/140*a*b^2*c^4*x^2+3/56*a
^2*b*c^7*x^(7/2)+3/40*a^2*b*c^5*x^(5/2)+1/8*a^2*b*c^3*x^(3/2)+3/8*a^2*b*c*x
^(1/2)-3/16*I*b^3*arctanh(c*x^(1/2))^2*Pi+3/8*a*b^2*arctanh(c*x^(1/2))*ln(c
*x^(1/2)-1)-3/8*a*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-3/16*a*b^2*ln(c*x^
(1/2)-1)*ln(1/2*c*x^(1/2)+1/2)-3/16*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(
1/2))+3/16*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2*c*x^(1/2)+1/2)+1/56*a*b^2*c^
6*x^3+9/140*b^3*arctanh(c*x^(1/2))*c^4*x^2+71/280*b^3*arctanh(c*x^(1/2))*c^
2*x+1/8*b^3*c^8*x^4*arctanh(c*x^(1/2))^3+3/56*b^3*arctanh(c*x^(1/2))^2*c^7*
x^(7/2)+3/40*b^3*arctanh(c*x^(1/2))^2*c^5*x^(5/2)+1/8*b^3*arctanh(c*x^(1/2)
)^2*c^3*x^(3/2)+3/8*b^3*arctanh(c*x^(1/2))^2*c*x^(1/2)+1/56*b^3*arctanh(c*x
^(1/2))*c^6*x^3-3/32*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I*(1+c*x^(1/2))^2/(
c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^
2+3/32*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))
^2*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))+3/32*I*b^3*arctanh(c*x^(1/2))^2*Pi*csg
n(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+
c*x^(1/2))^2/(-c^2*x+1)))^2+1/280*b^3*c^5*x^(5/2)+23/840*b^3*c^3*x^(3/2)+47
/140*b^3*c*x^(1/2)+1/8*c^8*x^4*a^3+3/32*a*b^2*ln(c*x^(1/2)-1)^2+3/32*a*b^2*
ln(1+c*x^(1/2))^2+22/35*a*b^2*ln(c*x^(1/2)-1)+22/35*a*b^2*ln(1+c*x^(1/2))+3
/16*a^2*b*ln(c*x^(1/2)-1)-3/16*a^2*b*ln(1+c*x^(1/2))-44/35*b^3*arctanh(c*x^
(1/2))*ln(1+I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-44/35*b^3*arctanh(c*x^(1/2))*
ln(1-I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/16*b^3*arctanh(c*x^(1/2))^2*ln(c*x
```

```
^(1/2)-1)-3/16*b^3*arctanh(c*x^(1/2))^2*ln(1+c*x^(1/2))+3/8*b^3*arctanh(c*x
^(1/2))^2*ln((1+c*x^(1/2))/(-c^2*x+1)^(1/2))+71/280*a*b^2*x*c^2+3/32*I*b^3*
arctanh(c*x^(1/2))^2*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2
/(-c^2*x+1)))^3+3/32*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I*(1+c*x^(1/2))^2/(
c^2*x-1))^3-3/16*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-
c^2*x+1)))^3+3/16*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(
-c^2*x+1)))^2+3/8*a^2*b*c^8*x^4*arctanh(c*x^(1/2))+1/4*a*b^2*arctanh(c*x^(1
/2))*c^3*x^(3/2)+3/4*a*b^2*arctanh(c*x^(1/2))*c*x^(1/2)+22/35*b^3*arctanh(c
*x^(1/2))^2-1/8*b^3*arctanh(c*x^(1/2))^3-44/35*b^3*dilog(1-I*(1+c*x^(1/2))/
(-c^2*x+1)^(1/2))-44/35*b^3*dilog(1+I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-47/14
0*b^3*arctanh(c*x^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1972 vs. $2(297) = 594$.
time = 0.85, size = 1972, normalized size = 5.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*x^4 - 1/26880*a*b^2*c*((315*c^7*x^4 + 500*c^5*x^3 + 1002*c^3*x^2 +
3684*c*x - 12*(105*c^7*x^4 + 120*c^6*x^(7/2) + 140*c^5*x^3 + 168*c^4*x^(5/2
) + 210*c^3*x^2 + 280*c^2*x^(3/2) + 420*c*x + 840*sqrt(x))*log(c*sqrt(x) +
1))/c^8 - 6396*log(c*sqrt(x) + 1)/c^9 - 6396*log(c*sqrt(x) - 1)/c^9) - 1/22
40*(840*x^4*log(c*sqrt(x) + 1) - c*((105*c^7*x^4 - 120*c^6*x^(7/2) + 140*c^
5*x^3 - 168*c^4*x^(5/2) + 210*c^3*x^2 - 280*c^2*x^(3/2) + 420*c*x - 840*sqr
t(x))/c^8 + 840*log(c*sqrt(x) + 1)/c^9))*a*b^2*log(-c*sqrt(x) + 1) + 1/2240
*(840*x^4*log(c*sqrt(x) + 1) - c*((105*c^7*x^4 - 120*c^6*x^(7/2) + 140*c^5*
x^3 - 168*c^4*x^(5/2) + 210*c^3*x^2 - 280*c^2*x^(3/2) + 420*c*x - 840*sqrt(
x))/c^8 + 840*log(c*sqrt(x) + 1)/c^9))*a^2*b - 1/2240*(840*x^4*log(-c*sqrt(
x) + 1) - c*((105*c^7*x^4 + 120*c^6*x^(7/2) + 140*c^5*x^3 + 168*c^4*x^(5/2)
 + 210*c^3*x^2 + 280*c^2*x^(3/2) + 420*c*x + 840*sqrt(x))/c^8 + 840*log(c*s
qrt(x) - 1)/c^9))*a^2*b + 1/1881600*(11025*(32*log(-c*sqrt(x) + 1)^2 - 8*lo
g(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^8 + 57600*(49*log(-c*sqrt(x) + 1)^2
- 14*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x) - 1)^7 + 548800*(18*log(-c*sqrt(x)
 + 1)^2 - 6*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^6 + 790272*(25*log(-c*
sqrt(x) + 1)^2 - 10*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x) - 1)^5 + 3087000*(8
*log(-c*sqrt(x) + 1)^2 - 4*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^4 + 219
5200*(9*log(-c*sqrt(x) + 1)^2 - 6*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x) - 1)^
3 + 4939200*(2*log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x
) - 1)^2 + 2822400*(log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1) + 2)*(c*s
qrt(x) - 1))*a*b^2/c^8 - 1/3161088000*(385875*(256*log(-c*sqrt(x) + 1)^3 -
96*log(-c*sqrt(x) + 1)^2 + 24*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^8 +
2304000*(343*log(-c*sqrt(x) + 1)^3 - 147*log(-c*sqrt(x) + 1)^2 + 42*log(-c*
```

```
sqrt(x) + 1) - 6)*(c*sqrt(x) - 1)^7 + 76832000*(36*log(-c*sqrt(x) + 1)^3 -
18*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 1)*(c*sqrt(x) - 1)^6 + 4
4255232*(125*log(-c*sqrt(x) + 1)^3 - 75*log(-c*sqrt(x) + 1)^2 + 30*log(-c*s
qrt(x) + 1) - 6)*(c*sqrt(x) - 1)^5 + 216090000*(32*log(-c*sqrt(x) + 1)^3 -
24*log(-c*sqrt(x) + 1)^2 + 12*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^4 +
614656000*(9*log(-c*sqrt(x) + 1)^3 - 9*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqr
t(x) + 1) - 2)*(c*sqrt(x) - 1)^3 + 691488000*(4*log(-c*sqrt(x) + 1)^3 - 6*l
og(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^2 + 79027
2000*(log(-c*sqrt(x) + 1)^3 - 3*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) +
1) - 6)*(c*sqrt(x) - 1))*b^3/c^8 + 44/35*(log(c*sqrt(x) + 1)*log(-1/2*c*sqr
t(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/2))*b^3/c^8 - 1881559/3763200*b^3*log
(c*sqrt(x) - 1)/c^8 + 1/2240*(2283*a*b^2 - 752*b^3)*log(c*sqrt(x) + 1)/c^8
+ 1/3161088000*(1157625*(16*a*b^2*c^8 - b^3*c^8)*x^4 - 27000*(1680*a*b^2*c^
7 + 169*b^3*c^7)*x^(7/2) + 3500*(24528*a*b^2*c^6 - 3565*b^3*c^6)*x^3 + 9878
4000*(b^3*c^8*x^4 - b^3)*log(c*sqrt(x) + 1)^3 - 168*(895440*a*b^2*c^5 + 442
69*b^3*c^5)*x^(5/2) + 210*(1248240*a*b^2*c^4 - 334699*b^3*c^4)*x^2 + 564480
0*(105*a*b^2*c^8*x^4 + 15*b^3*c^7*x^(7/2) + 21*b^3*c^5*x^(5/2) + 35*b^3*c^3
*x^(3/2) + 105*b^3*c*sqrt(x) - 105*a*b^2 + 176*b^3)*log(c*sqrt(x) + 1)^2 -
352800*(105*b^3*c^8*x^4 - 120*b^3*c^7*x^(7/2) + 140*b^3*c^6*x^3 - 168*b^3*c
^5*x^(5/2) + 210*b^3*c^4*x^2 - 280*b^3*c^3*x^(3/2) + 420*b^3*c^2*x - 840*b^
3*c*sqrt(x) + 533*b^3 - 840*(b^3*c^8*x^4 - b^3)*log(c*sqrt(x) + 1))*log(-c*
sqrt(x) + 1)^2 - 280*(1718640*a*b^2*c^3 + 2899*b^3*c^3)*x^(3/2) + 420*(2424
240*a*b^2*c^2 - 1227199*b^3*c^2)*x - 1411200*(105*a*b^2*c^8*x^4 - 120*a*b^2
*c^7*x^(7/2) - 168*a*b^2*c^5*x^(5/2) - 280*a*b^2*c^3*x^(3/2) - 840*a*b^2*c*
sqrt(x) + 20*(7*a*b^2*c^6 - 2*b^3*c^6)*x^3 + 6*(35*a*b^2*c^4 - 24*b^3*c^4)*
x^2 + 4*(105*a*b^2*c^2 - 142*b^3*c^2)*x)*log(c*sqrt(x) + 1) + 840*(11025*b^
3*c^8*x^4 + 27000*b^3*c^7*x^(7/2) - 16100*b^3*c^6*x^3 + 89544*b^3*c^5*x^(5/
2) - 85890*b^3*c^4*x^2 + 286440*b^3*c^3*x^(3/2) - 348180*b^3*c^2*x + 191772
0*b^3*c*sqrt(x) - 352800*(b^3*c^8*x^4 - b^3)*log(c*sqrt(x) + 1)^2 - 13440*(
15*b^3*c^7*x^(7/2) + 21*b^3*c^5*x^(5/2) + 35*b^3*c^3*x^(3/2) + 105*b^3*c*sq
rt(x) + 176*b^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1) - 840*(3835440*a*b
^2*c + 618199*b^3*c)*sqrt(x))/c^8
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*x^3*arctanh(c*sqrt(x))^2 +
3*a^2*b*x^3*arctanh(c*sqrt(x)) + a^3*x^3, x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + b \operatorname{atanh}\left(c\sqrt{x}\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**(1/2)))**3,x)
```

```
[Out] Integral(x**3*(a + b*atanh(c*sqrt(x)))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(a + b \operatorname{atanh}\left(c\sqrt{x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x^(1/2)))^3,x)
```

```
[Out] int(x^3*(a + b*atanh(c*x^(1/2)))^3, x)
```

3.203 $\int x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx$

Optimal. Leaf size=304

$$\frac{19b^3\sqrt{x}}{30c^5}+\frac{b^3x^{3/2}}{30c^3}-\frac{19b^3\tanh^{-1}\left(c\sqrt{x}\right)}{30c^6}+\frac{8b^2x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{15c^4}+\frac{b^2x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{10c^2}+\frac{23b(a-$$

```
[Out] 1/30*b^3*x^(3/2)/c^3-19/30*b^3*arctanh(c*x^(1/2))/c^6+8/15*b^2*x*(a+b*arcta
nh(c*x^(1/2)))/c^4+1/10*b^2*x^2*(a+b*arctanh(c*x^(1/2)))/c^2+23/15*b*(a+b*a
rctanh(c*x^(1/2)))^2/c^6+1/3*b*x^(3/2)*(a+b*arctanh(c*x^(1/2)))^2/c^3+1/5*b
*x^(5/2)*(a+b*arctanh(c*x^(1/2)))^2/c-1/3*(a+b*arctanh(c*x^(1/2)))^3/c^6+1/
3*x^3*(a+b*arctanh(c*x^(1/2)))^3-46/15*b^2*(a+b*arctanh(c*x^(1/2)))*ln(2/(1
-c*x^(1/2)))/c^6-23/15*b^3*polylog(2,1-2/(1-c*x^(1/2)))/c^6+19/30*b^3*x^(1/
2)/c^5+b*(a+b*arctanh(c*x^(1/2)))^2*x^(1/2)/c^5
```

Rubi [A]
time = 0.73, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6039, 6037, 6127, 308, 212, 327, 6131, 6055, 2449, 2352, 6021, 6095}

$$-\frac{46b^2\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{15c^6}+\frac{8b^2x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{15c^4}+\frac{b^2x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{10c^2}+\frac{23b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{15c^6}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{3c^6}+\frac{b\sqrt{x}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{c^5}+\frac{bx^{3/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{3c^3}+\frac{bx^{5/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{5c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3-\frac{23b^3\text{Li}_2\left(1-\frac{2}{1-c\sqrt{x}}\right)}{15c^6}-\frac{19b^3\tanh^{-1}\left(c\sqrt{x}\right)}{30c^6}+\frac{19b^3\sqrt{x}}{30c^5}+\frac{b^3x^{3/2}}{30c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] (19*b^3*Sqrt[x])/(30*c^5) + (b^3*x^(3/2))/(30*c^3) - (19*b^3*ArcTanh[c*Sqrt
[x]])/(30*c^6) + (8*b^2*x*(a + b*ArcTanh[c*Sqrt[x]]))/(15*c^4) + (b^2*x^2*(
a + b*ArcTanh[c*Sqrt[x]]))/(10*c^2) + (23*b*(a + b*ArcTanh[c*Sqrt[x]])^2)/(
15*c^6) + (b*Sqrt[x]*(a + b*ArcTanh[c*Sqrt[x]])^2)/c^5 + (b*x^(3/2)*(a + b*
ArcTanh[c*Sqrt[x]])^2)/(3*c^3) + (b*x^(5/2)*(a + b*ArcTanh[c*Sqrt[x]])^2)/(
5*c) - (a + b*ArcTanh[c*Sqrt[x]])^3/(3*c^6) + (x^3*(a + b*ArcTanh[c*Sqrt[x]
])^3)/3 - (46*b^2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 - c*Sqrt[x])])/(15*c^
6) - (23*b^3*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/(15*c^6)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
```

```
0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx=\int x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx$$

Mathematica [A]
time = 0.58, size = 351, normalized size = 1.15

$$\frac{-19ab^2+30a^2bc\sqrt{x}+19b^3c\sqrt{x}+16ab^2c^2x+10a^2bc^3x^{3/2}+b^3c^3x^{3/2}+3ab^2c^4x^2+6a^2bc^5x^{5/2}+10a^3c^6x^3+2b^2\left(b\left(-23+15c\sqrt{x}+5c^3x^{3/2}+3c^5x^{5/2}\right)+15a(-1+c^6x^3)\right)\tanh^{-1}\left(c\sqrt{x}\right)^2+10b^3(-1+c^6x^3)\tanh^{-1}\left(c\sqrt{x}\right)^3+b\tanh^{-1}\left(c\sqrt{x}\right)\left(30a^2c^6x^3+4abc\sqrt{x}\left(15+5c^2x+3c^4x^2\right)+b^2(-19+16c^2x+3c^4x^2)-92b^2\log\left(1+e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right)\right)+15a^2b\log\left(1-c\sqrt{x}\right)-15a^2b\log\left(1+c\sqrt{x}\right)+46ab^2\log\left(1-c^2x\right)+46b^3\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right)}{30c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] (-19*a*b^2 + 30*a^2*b*c*Sqrt[x] + 19*b^3*c*Sqrt[x] + 16*a*b^2*c^2*x + 10*a^
2*b*c^3*x^(3/2) + b^3*c^3*x^(3/2) + 3*a*b^2*c^4*x^2 + 6*a^2*b*c^5*x^(5/2) +
 10*a^3*c^6*x^3 + 2*b^2*(b*(-23 + 15*c*Sqrt[x] + 5*c^3*x^(3/2) + 3*c^5*x^(5
/2)) + 15*a*(-1 + c^6*x^3))*ArcTanh[c*Sqrt[x]]^2 + 10*b^3*(-1 + c^6*x^3)*Ar
cTanh[c*Sqrt[x]]^3 + b*ArcTanh[c*Sqrt[x]]*(30*a^2*c^6*x^3 + 4*a*b*c*Sqrt[x]
*(15 + 5*c^2*x + 3*c^4*x^2) + b^2*(-19 + 16*c^2*x + 3*c^4*x^2) - 92*b^2*Log
[1 + E^(-2*ArcTanh[c*Sqrt[x]])]) + 15*a^2*b*Log[1 - c*Sqrt[x]] - 15*a^2*b*L
```

```
og[1 + c*Sqrt[x]] + 46*a*b^2*Log[1 - c^2*x] + 46*b^3*PolyLog[2, -E^(-2*ArcT
anh[c*Sqrt[x]])])/(30*c^6)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.97, size = 1341, normalized size = 4.41

method	result	size
derivativedivides	Expression too large to display	1341
default	Expression too large to display	1341

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^(1/2)))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^6*(-1/8*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))
*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2-1/3*b^3
+1/5*a*b^2*arctanh(c*x^(1/2))*c^5*x^(5/2)+1/20*a*b^2*c^4*x^2+1/10*a^2*b*c^5
*x^(5/2)+1/6*a^2*b*c^3*x^(3/2)+1/2*a^2*b*c*x^(1/2)+1/2*a*b^2*arctanh(c*x^(1
/2))*ln(c*x^(1/2)-1)-1/2*a*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-1/4*a*b^2
*ln(c*x^(1/2)-1)*ln(1/2*c*x^(1/2)+1/2)-1/4*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(
1+c*x^(1/2))+1/4*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2*c*x^(1/2)+1/2)+1/20*b^
3*arctanh(c*x^(1/2))*c^4*x^2+4/15*b^3*arctanh(c*x^(1/2))*c^2*x+1/10*b^3*arc
tanh(c*x^(1/2))^2*c^5*x^(5/2)+1/6*b^3*arctanh(c*x^(1/2))^2*c^3*x^(3/2)+1/2*
b^3*arctanh(c*x^(1/2))^2*c*x^(1/2)-1/4*I*b^3*arctanh(c*x^(1/2))^2*Pi+1/6*b^
3*c^6*x^3*arctanh(c*x^(1/2))^3+1/60*b^3*c^3*x^(3/2)+19/60*b^3*c*x^(1/2)+1/8
*a*b^2*ln(c*x^(1/2)-1)^2+1/8*a*b^2*ln(1+c*x^(1/2))^2+23/30*a*b^2*ln(c*x^(1/
2)-1)+23/30*a*b^2*ln(1+c*x^(1/2))+1/4*a^2*b*ln(c*x^(1/2)-1)-1/4*a^2*b*ln(1+
c*x^(1/2))-23/15*b^3*arctanh(c*x^(1/2))*ln(1+I*(1+c*x^(1/2))/(-c^2*x+1)^(1/
2))-23/15*b^3*arctanh(c*x^(1/2))*ln(1-I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+1/4
*b^3*arctanh(c*x^(1/2))^2*ln(c*x^(1/2)-1)-1/4*b^3*arctanh(c*x^(1/2))^2*ln(1
+c*x^(1/2))+1/2*b^3*arctanh(c*x^(1/2))^2*ln((1+c*x^(1/2))/(-c^2*x+1)^(1/2))
+4/15*a*b^2*x*c^2+1/2*a*b^2*c^6*x^3*arctanh(c*x^(1/2))^2+1/8*I*b^3*arctanh(
c*x^(1/2))^2*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x
+1)))^3+1/8*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))
^3+1/4*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))
^2-1/4*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))
^3+1/2*a^2*b*c^6*x^3*arctanh(c*x^(1/2))+1/3*a*b^2*arctanh(c*x^(1/2))*c^3*x^
(3/2)+a*b^2*arctanh(c*x^(1/2))*c*x^(1/2)+1/6*c^6*x^3*a^3-1/8*I*b^3*arctanh(
c*x^(1/2))^2*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*csgn(I*(1+c*x^(1/2))
^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1
)))+1/8*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1))
)*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2+1/4*I*
b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))*csgn(I*(
1+c*x^(1/2))^2/(c^2*x-1))^2+1/8*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I*(1+c*x
^(1/2))/(-c^2*x+1)^(1/2))^2*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))+23/30*b^3*arc
tanh(c*x^(1/2))^2-1/6*b^3*arctanh(c*x^(1/2))^3-23/15*b^3*dilog(1-I*(1+c*x^(
```

```
1/2))/(-c^2*x+1)^(1/2))-23/15*b^3*dilog(1+I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))
-19/60*b^3*arctanh(c*x^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1579 vs. $2(243) = 486$.
time = 0.77, size = 1579, normalized size = 5.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*x^3 - 1/720*a*b^2*c*((20*c^5*x^3 + 39*c^3*x^2 + 138*c*x - 6*(10*c^5
*x^3 + 12*c^4*x^(5/2) + 15*c^3*x^2 + 20*c^2*x^(3/2) + 30*c*x + 60*sqrt(x))*
log(c*sqrt(x) + 1))/c^6 - 222*log(c*sqrt(x) + 1)/c^7 - 222*log(c*sqrt(x) -
1)/c^7) - 1/120*(60*x^3*log(c*sqrt(x) + 1) - c*((10*c^5*x^3 - 12*c^4*x^(5/2
) + 15*c^3*x^2 - 20*c^2*x^(3/2) + 30*c*x - 60*sqrt(x))/c^6 + 60*log(c*sqrt(
x) + 1)/c^7))*a*b^2*log(-c*sqrt(x) + 1) + 1/120*(60*x^3*log(c*sqrt(x) + 1)
- c*((10*c^5*x^3 - 12*c^4*x^(5/2) + 15*c^3*x^2 - 20*c^2*x^(3/2) + 30*c*x -
60*sqrt(x))/c^6 + 60*log(c*sqrt(x) + 1)/c^7))*a^2*b - 1/120*(60*x^3*log(-c*
sqrt(x) + 1) - c*((10*c^5*x^3 + 12*c^4*x^(5/2) + 15*c^3*x^2 + 20*c^2*x^(3/2
) + 30*c*x + 60*sqrt(x))/c^6 + 60*log(c*sqrt(x) - 1)/c^7))*a^2*b + 1/7200*(
100*(18*log(-c*sqrt(x) + 1)^2 - 6*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^
6 + 432*(25*log(-c*sqrt(x) + 1)^2 - 10*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x)
- 1)^5 + 3375*(8*log(-c*sqrt(x) + 1)^2 - 4*log(-c*sqrt(x) + 1) + 1)*(c*sqrt
(x) - 1)^4 + 4000*(9*log(-c*sqrt(x) + 1)^2 - 6*log(-c*sqrt(x) + 1) + 2)*(c*
sqrt(x) - 1)^3 + 13500*(2*log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1) + 1
)*(c*sqrt(x) - 1)^2 + 10800*(log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1)
+ 2)*(c*sqrt(x) - 1))*a*b^2/c^6 - 1/864000*(1000*(36*log(-c*sqrt(x) + 1)^3
- 18*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 1)*(c*sqrt(x) - 1)^6 +
 1728*(125*log(-c*sqrt(x) + 1)^3 - 75*log(-c*sqrt(x) + 1)^2 + 30*log(-c*sqr
t(x) + 1) - 6)*(c*sqrt(x) - 1)^5 + 16875*(32*log(-c*sqrt(x) + 1)^3 - 24*log
(-c*sqrt(x) + 1)^2 + 12*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^4 + 80000*
(9*log(-c*sqrt(x) + 1)^3 - 9*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1)
- 2)*(c*sqrt(x) - 1)^3 + 135000*(4*log(-c*sqrt(x) + 1)^3 - 6*log(-c*sqrt(x)
 + 1)^2 + 6*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^2 + 216000*(log(-c*sqr
t(x) + 1)^3 - 3*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 6)*(c*sqrt(
x) - 1))*b^3/c^6 + 23/15*(log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + di
log(1/2*c*sqrt(x) + 1/2))*b^3/c^6 - 8929/14400*b^3*log(c*sqrt(x) - 1)/c^6 +
 1/120*(147*a*b^2 - 38*b^3)*log(c*sqrt(x) + 1)/c^6 + 1/864000*(1000*(12*a*b
^2*c^6 - b^3*c^6)*x^3 + 36000*(b^3*c^6*x^3 - b^3)*log(c*sqrt(x) + 1)^3 - 48
*(660*a*b^2*c^5 + 91*b^3*c^5)*x^(5/2) + 15*(4440*a*b^2*c^4 - 919*b^3*c^4)*x
^2 + 14400*(15*a*b^2*c^6*x^3 + 3*b^3*c^5*x^(5/2) + 5*b^3*c^3*x^(3/2) + 15*b
^3*c*sqrt(x) - 15*a*b^2 + 23*b^3)*log(c*sqrt(x) + 1)^2 - 1800*(10*b^3*c^6*x
^3 - 12*b^3*c^5*x^(5/2) + 15*b^3*c^4*x^2 - 20*b^3*c^3*x^(3/2) + 30*b^3*c^2*
```

```
x - 60*b^3*c*sqrt(x) + 37*b^3 - 60*(b^3*c^6*x^3 - b^3)*log(c*sqrt(x) + 1))*
log(-c*sqrt(x) + 1)^2 - 20*(6840*a*b^2*c^3 + 619*b^3*c^3)*x^(3/2) + 870*(36
0*a*b^2*c^2 - 161*b^3*c^2)*x - 7200*(10*a*b^2*c^6*x^3 - 12*a*b^2*c^5*x^(5/2
) - 20*a*b^2*c^3*x^(3/2) - 60*a*b^2*c*sqrt(x) + 3*(5*a*b^2*c^4 - 2*b^3*c^4)
*x^2 + 2*(15*a*b^2*c^2 - 16*b^3*c^2)*x)*log(c*sqrt(x) + 1) + 60*(100*b^3*c^
6*x^3 + 264*b^3*c^5*x^(5/2) - 165*b^3*c^4*x^2 + 1140*b^3*c^3*x^(3/2) - 1230
*b^3*c^2*x + 8820*b^3*c*sqrt(x) - 1800*(b^3*c^6*x^3 - b^3)*log(c*sqrt(x) +
1)^2 - 480*(3*b^3*c^5*x^(5/2) + 5*b^3*c^3*x^(3/2) + 15*b^3*c*sqrt(x) + 23*b
^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1) - 60*(17640*a*b^2*c + 4369*b^3*
c)*sqrt(x))/c^6
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctanh(c*sqrt(x))^3 + 3*a*b^2*x^2*arctanh(c*sqrt(x))^2 +
3*a^2*b*x^2*arctanh(c*sqrt(x)) + a^3*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(a+b\operatorname{atanh}\left(c\sqrt{x}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**(1/2)))**3,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*sqrt(x)))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^(1/2)))^3,x)
[Out] int(x^2*(a + b*atanh(c*x^(1/2)))^3, x)
```

3.204 $\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx$

Optimal. Leaf size=234

$$\frac{b^3\sqrt{x}}{2c^3}-\frac{b^3\tanh^{-1}\left(c\sqrt{x}\right)}{2c^4}+\frac{b^2x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{2c^2}+\frac{2b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{c^4}+\frac{3b\sqrt{x}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right.}{2c^3}$$

```
[Out] -1/2*b^3*arctanh(c*x^(1/2))/c^4+1/2*b^2*x*(a+b*arctanh(c*x^(1/2)))/c^2+2*b*
(a+b*arctanh(c*x^(1/2)))^2/c^4+1/2*b*x^(3/2)*(a+b*arctanh(c*x^(1/2)))^2/c-1
/2*(a+b*arctanh(c*x^(1/2)))^3/c^4+1/2*x^2*(a+b*arctanh(c*x^(1/2)))^3-4*b^2*
(a+b*arctanh(c*x^(1/2)))*ln(2/(1-c*x^(1/2)))/c^4-2*b^3*polylog(2,1-2/(1-c*x
^(1/2)))/c^4+1/2*b^3*x^(1/2)/c^3+3/2*b*(a+b*arctanh(c*x^(1/2)))^2*x^(1/2)/c
^3
```

Rubi [A]
time = 0.42, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {6039, 6037, 6127, 327, 212, 6131, 6055, 2449, 2352, 6021, 6095}

$$-\frac{4b^2\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^4}+\frac{b^2x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{2c^2}+\frac{2b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{c^4}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{2c^4}+\frac{3b\sqrt{x}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{2c^3}+\frac{bx^{3/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3-\frac{2b^3\text{Li}_2\left(1-\frac{2}{1-c\sqrt{x}}\right)}{c^4}-\frac{b^3\tanh^{-1}\left(c\sqrt{x}\right)}{2c^4}+\frac{b^3\sqrt{x}}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] (b^3*Sqrt[x])/(2*c^3) - (b^3*ArcTanh[c*Sqrt[x]])/(2*c^4) + (b^2*x*(a + b*Ar
cTanh[c*Sqrt[x]]))/(2*c^2) + (2*b*(a + b*ArcTanh[c*Sqrt[x]])^2)/c^4 + (3*b*
Sqrt[x]*(a + b*ArcTanh[c*Sqrt[x]])^2)/(2*c^3) + (b*x^(3/2)*(a + b*ArcTanh[c
*Sqrt[x]])^2)/(2*c) - (a + b*ArcTanh[c*Sqrt[x]])^3/(2*c^4) + (x^2*(a + b*Ar
cTanh[c*Sqrt[x]])^3)/2 - (4*b^2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 - c*Sqr
t[x])])/c^4 - (2*b^3*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/c^4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx=\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx$$

Mathematica [A]
time = 0.38, size = 285, normalized size = 1.22

$$\frac{-2ab^2+6a^2bc\sqrt{x}+2b^3c\sqrt{x}+2ab^2c^2x+2a^2bc^3x^{3/2}+2a^3c^4x^2+2b^2\left(b\left(-4+3c\sqrt{x}+c^3x^{3/2}\right)+3a(-1+c^4x^2)\right)\tanh^{-1}\left(c\sqrt{x}\right)^2+2b^3(-1+c^4x^2)\tanh^{-1}\left(c\sqrt{x}\right)^3+2b\tanh^{-1}\left(c\sqrt{x}\right)\left(3a^2c^4x^2+b^2(-1+c^2x)+2abc\sqrt{x}\,(3+c^2x)-8b^2\log\left(1+e^{-2\tanh^{-1}(c\sqrt{x})}\right)\right)+3a^2b\log\left(1-c\sqrt{x}\right)-3a^2b\log\left(1+c\sqrt{x}\right)+8ab^2\log\left(1-c^2x\right)+8b^3\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(c\sqrt{x})}\right)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] (-2*a*b^2 + 6*a^2*b*c*Sqrt[x] + 2*b^3*c*Sqrt[x] + 2*a*b^2*c^2*x + 2*a^2*b*c
^3*x^(3/2) + 2*a^3*c^4*x^2 + 2*b^2*(b*(-4 + 3*c*Sqrt[x] + c^3*x^(3/2)) + 3*
a*(-1 + c^4*x^2))*ArcTanh[c*Sqrt[x]]^2 + 2*b^3*(-1 + c^4*x^2)*ArcTanh[c*Sqr
t[x]]^3 + 2*b*ArcTanh[c*Sqrt[x]]*(3*a^2*c^4*x^2 + b^2*(-1 + c^2*x) + 2*a*b*
c*Sqrt[x]*(3 + c^2*x) - 8*b^2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])]) + 3*a^2*b
*Log[1 - c*Sqrt[x]] - 3*a^2*b*Log[1 + c*Sqrt[x]] + 8*a*b^2*Log[1 - c^2*x] +
 8*b^3*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])])/(4*c^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.33, size = 1250, normalized size = 5.34

method	result	size
derivativedivides	Expression too large to display	1250
default	Expression too large to display	1250

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^(1/2)))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^4*(3/16*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(
1/2))^2*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))-1/4*b^3+1/4*a^2*b*c^3*x^(3/2)+3/4
*a^2*b*c*x^(1/2)+3/4*a*b^2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-3/4*a*b^2*arc
tanh(c*x^(1/2))*ln(1+c*x^(1/2))-3/8*a*b^2*ln(c*x^(1/2)-1)*ln(1/2*c*x^(1/2)+
1/2)-3/8*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))+3/8*a*b^2*ln(-1/2*c*x
^(1/2)+1/2)*ln(1/2*c*x^(1/2)+1/2)+1/4*b^3*arctanh(c*x^(1/2))*c^2*x+1/4*b^3*
arctanh(c*x^(1/2))^2*c^3*x^(3/2)+3/4*b^3*arctanh(c*x^(1/2))^2*c*x^(1/2)+1/4
*b^3*c*x^(1/2)+3/16*a*b^2*ln(c*x^(1/2)-1)^2+3/16*a*b^2*ln(1+c*x^(1/2))^2+a*
b^2*ln(c*x^(1/2)-1)+a*b^2*ln(1+c*x^(1/2))+3/8*a^2*b*ln(c*x^(1/2)-1)-3/8*a^2
*b*ln(1+c*x^(1/2))-2*b^3*arctanh(c*x^(1/2))*ln(1+I*(1+c*x^(1/2))/(-c^2*x+1)
^(1/2))-2*b^3*arctanh(c*x^(1/2))*ln(1-I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/8
*b^3*arctanh(c*x^(1/2))^2*ln(c*x^(1/2)-1)-3/8*b^3*arctanh(c*x^(1/2))^2*ln(1
+c*x^(1/2))+3/4*b^3*arctanh(c*x^(1/2))^2*ln((1+c*x^(1/2))/(-c^2*x+1)^(1/2))
+1/4*a*b^2*x*c^2+3/16*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I/(1+(1+c*x^(1/2))
^2/(-c^2*x+1)))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x
+1)))^2-3/16*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)
)*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2+3/8*I*
b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))*csgn(I*(
1+c*x^(1/2))^2/(c^2*x-1))^2+1/2*a*b^2*arctanh(c*x^(1/2))*c^3*x^(3/2)+3/2*a*
b^2*arctanh(c*x^(1/2))*c*x^(1/2)-3/16*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I/
(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*csgn(I*(1
+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))+3/8*I*b^3*arctanh(c
*x^(1/2))^2*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2-3/8*I*b^3*arctanh(c
*x^(1/2))^2*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^3+3/4*a^2*b*c^4*x^2*a
rctanh(c*x^(1/2))+3/4*a*b^2*c^4*x^2*arctanh(c*x^(1/2))^2+3/16*I*b^3*arctanh
(c*x^(1/2))^2*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^3+3/16*I*b^3*arctanh(c*x
^(1/2))^2*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)
))^3+1/4*b^3*c^4*x^2*arctanh(c*x^(1/2))^3-3/8*I*b^3*arctanh(c*x^(1/2))^2*Pi
+1/4*c^4*x^2*a^3+b^3*arctanh(c*x^(1/2))^2-1/4*b^3*arctanh(c*x^(1/2))^3-2*b^
3*dilog(1-I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-2*b^3*dilog(1+I*(1+c*x^(1/2))/(
-c^2*x+1)^(1/2))-1/4*b^3*arctanh(c*x^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1184 vs. $2(191) = 382$.
time = 0.69, size = 1184, normalized size = 5.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*x^2 - 1/32*a*b^2*c*((3*c^3*x^2 + 10*c*x - 2*(3*c^3*x^2 + 4*c^2*x^(3
/2) + 6*c*x + 12*sqrt(x))*log(c*sqrt(x) + 1))/c^4 - 14*log(c*sqrt(x) + 1)/c
^5 - 14*log(c*sqrt(x) - 1)/c^5) - 1/16*(12*x^2*log(c*sqrt(x) + 1) - c*((3*c
^3*x^2 - 4*c^2*x^(3/2) + 6*c*x - 12*sqrt(x))/c^4 + 12*log(c*sqrt(x) + 1)/c^
5))*a*b^2*log(-c*sqrt(x) + 1) + 1/16*(12*x^2*log(c*sqrt(x) + 1) - c*((3*c^3
*x^2 - 4*c^2*x^(3/2) + 6*c*x - 12*sqrt(x))/c^4 + 12*log(c*sqrt(x) + 1)/c^5)
)*a^2*b - 1/16*(12*x^2*log(-c*sqrt(x) + 1) - c*((3*c^3*x^2 + 4*c^2*x^(3/2)
+ 6*c*x + 12*sqrt(x))/c^4 + 12*log(c*sqrt(x) - 1)/c^5))*a^2*b + 1/192*(9*(8
*log(-c*sqrt(x) + 1)^2 - 4*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^4 + 32*
(9*log(-c*sqrt(x) + 1)^2 - 6*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x) - 1)^3 + 2
16*(2*log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^2
+ 288*(log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x) - 1))*
a*b^2/c^4 - 1/4608*(9*(32*log(-c*sqrt(x) + 1)^3 - 24*log(-c*sqrt(x) + 1)^2
+ 12*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^4 + 128*(9*log(-c*sqrt(x) + 1
)^3 - 9*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 2)*(c*sqrt(x) - 1)^
3 + 432*(4*log(-c*sqrt(x) + 1)^3 - 6*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(
x) + 1) - 3)*(c*sqrt(x) - 1)^2 + 1152*(log(-c*sqrt(x) + 1)^3 - 3*log(-c*sqr
t(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 6)*(c*sqrt(x) - 1))*b^3/c^4 + 2*(log(
c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/2))*b^3/
c^4 - 319/384*b^3*log(c*sqrt(x) - 1)/c^4 + 1/16*(25*a*b^2 - 4*b^3)*log(c*sq
rt(x) + 1)/c^4 + 1/4608*(288*(b^3*c^4*x^2 - b^3)*log(c*sqrt(x) + 1)^3 + 27*
(8*a*b^2*c^4 - b^3*c^4)*x^2 + 576*(3*a*b^2*c^4*x^2 + b^3*c^3*x^(3/2) + 3*b^
3*c*sqrt(x) - 3*a*b^2 + 4*b^3)*log(c*sqrt(x) + 1)^2 - 72*(3*b^3*c^4*x^2 - 4
*b^3*c^3*x^(3/2) + 6*b^3*c^2*x - 12*b^3*c*sqrt(x) + 7*b^3 - 12*(b^3*c^4*x^2
 - b^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1)^2 - 4*(168*a*b^2*c^3 + 37*b
^3*c^3)*x^(3/2) + 6*(312*a*b^2*c^2 - 115*b^3*c^2)*x - 288*(3*a*b^2*c^4*x^2
- 4*a*b^2*c^3*x^(3/2) - 12*a*b^2*c*sqrt(x) + 2*(3*a*b^2*c^2 - 2*b^3*c^2)*x)
*log(c*sqrt(x) + 1) + 12*(9*b^3*c^4*x^2 + 28*b^3*c^3*x^(3/2) - 18*b^3*c^2*x
 + 300*b^3*c*sqrt(x) - 72*(b^3*c^4*x^2 - b^3)*log(c*sqrt(x) + 1)^2 - 96*(b^
3*c^3*x^(3/2) + 3*b^3*c*sqrt(x) + 4*b^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x)
 + 1) - 12*(600*a*b^2*c + 223*b^3*c)*sqrt(x))/c^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctanh(c*sqrt(x))^3 + 3*a*b^2*x*arctanh(c*sqrt(x))^2 + 3*a^
2*b*x*arctanh(c*sqrt(x)) + a^3*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\operatorname{atanh}\left(c\sqrt{x}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**(1/2)))**3,x)
[Out] Integral(x*(a + b*atanh(c*sqrt(x)))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="giac")
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\,\mathrm{atanh}\left(c\,\sqrt{x}\,\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^(1/2)))^3,x)
[Out] int(x*(a + b*atanh(c*x^(1/2)))^3, x)
```

3.205 $\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx$

Optimal. Leaf size=142

$$\frac{3b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{c^2}+\frac{3b\sqrt{x}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{c}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{c^2}+x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right.\right.$$

```
[Out] 3*b*(a+b*arctanh(c*x^(1/2)))^2/c^2-(a+b*arctanh(c*x^(1/2)))^3/c^2+x*(a+b*ar
ctanh(c*x^(1/2)))^3-6*b^2*(a+b*arctanh(c*x^(1/2)))*ln(2/(1-c*x^(1/2)))/c^2-
3*b^3*polylog(2,1-2/(1-c*x^(1/2)))/c^2+3*b*(a+b*arctanh(c*x^(1/2)))^2*x^(1/
2)/c
```

Rubi [A]
time = 0.20, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6027, 6037, 6127, 6021, 6131, 6055, 2449, 2352, 6095}

$$-\frac{6b^2\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^2}+\frac{3b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{c^2}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{c^2}+\frac{3b\sqrt{x}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{c}+x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3-\frac{3b^3\text{Li}_2\left(1-\frac{2}{1-c\sqrt{x}}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] (3*b*(a + b*ArcTanh[c*Sqrt[x]])^2)/c^2 + (3*b*Sqrt[x]*(a + b*ArcTanh[c*Sqrt
[x]])^2)/c - (a + b*ArcTanh[c*Sqrt[x]])^3/c^2 + x*(a + b*ArcTanh[c*Sqrt[x]]
)^3 - (6*b^2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 - c*Sqrt[x])])/c^2 - (3*b^
3*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/c^2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6027

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> With[{k = D
enominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*ArcTanh[c*x^(k*n)])^p, x
], x, x^(1/k)], x]] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && FractionQ[n]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx=\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx$$

Mathematica [A]
time = 0.18, size = 201, normalized size = 1.42

$$\frac{6b^2(-1+c\sqrt{x})\left(a+b+ac\sqrt{x}\right)\tanh^{-1}\left(c\sqrt{x}\right)^2+2b^3(-1+c^2x)\tanh^{-1}\left(c\sqrt{x}\right)^3+6b\tanh^{-1}\left(c\sqrt{x}\right)\left(2abc\sqrt{x}+a^2c^2x-2b^2\log\left(1+e^{-2\tanh^{-1}(c\sqrt{x})}\right)\right)+a\left(6abc\sqrt{x}+2a^2c^2x+3ab\log\left(1-c\sqrt{x}\right)-3ab\log\left(1+c\sqrt{x}\right)+6b^2\log\left(1-c^2x\right)\right)+6b^3\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(c\sqrt{x})}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] (6*b^2*(-1 + c*Sqrt[x])*(a + b + a*c*Sqrt[x])*ArcTanh[c*Sqrt[x]]^2 + 2*b^3*
(-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^3 + 6*b*ArcTanh[c*Sqrt[x]]*(2*a*b*c*Sqrt[x]
 + a^2*c^2*x - 2*b^2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])]) + a*(6*a*b*c*Sqrt[
x] + 2*a^2*c^2*x + 3*a*b*Log[1 - c*Sqrt[x]] - 3*a*b*Log[1 + c*Sqrt[x]] + 6*
b^2*Log[1 - c^2*x]) + 6*b^3*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])])/(2*c^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.80, size = 5972, normalized size = 42.06

method	result	size
derivativedivides	Expression too large to display	5972
default	Expression too large to display	5972

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] 3/2*(c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3 + log(c*sqrt(x) - 1)/c^3) +
2*x*arctanh(c*sqrt(x)))*a^2*b + 3/4*(4*c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1
)/c^3 + log(c*sqrt(x) - 1)/c^3)*arctanh(c*sqrt(x)) + 4*x*arctanh(c*sqrt(x))
^2 - (2*(log(c*sqrt(x) - 1) - 2)*log(c*sqrt(x) + 1) - log(c*sqrt(x) + 1)^2
- log(c*sqrt(x) - 1)^2 - 4*log(c*sqrt(x) - 1))/c^2)*a*b^2 + a^3*x - 1/32*b^
```

```
3*(((4*log(-c*sqrt(x) + 1)^3 - 6*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) +
 1) - 3)*(c*sqrt(x) - 1)^2 + 8*(log(-c*sqrt(x) + 1)^3 - 3*log(-c*sqrt(x) +
1)^2 + 6*log(-c*sqrt(x) + 1) - 6)*(c*sqrt(x) - 1))/c^2 - 4*integrate(log(c*
sqrt(x) + 1)^3 - 3*log(c*sqrt(x) + 1)^2*log(-c*sqrt(x) + 1) + 3*log(c*sqrt(
x) + 1)*log(-c*sqrt(x) + 1)^2, x))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3,x, algorithm="fricas")
[Out] integral(b^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*arctanh(c*sqrt(x))^2 + 3*a^2*b*
arctanh(c*sqrt(x)) + a^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{atanh}\left(c\sqrt{x}\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**3,x)
[Out] Integral((a + b*atanh(c*sqrt(x)))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3,x, algorithm="giac")
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))^3,x)
[Out] int((a + b*atanh(c*x^(1/2)))^3, x)
```

3.206 $\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x}\,dx$

Optimal. Leaf size=224

$$4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3-3b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\text{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)+$$

```
[Out] -4*(a+b*arctanh(c*x^(1/2)))^3*arctanh(-1+2/(1-c*x^(1/2)))-3*b*(a+b*arctanh(
c*x^(1/2)))^2*polylog(2,1-2/(1-c*x^(1/2)))+3*b*(a+b*arctanh(c*x^(1/2)))^2*p
olylog(2,-1+2/(1-c*x^(1/2)))+3*b^2*(a+b*arctanh(c*x^(1/2)))*polylog(3,1-2/(
1-c*x^(1/2)))-3*b^2*(a+b*arctanh(c*x^(1/2)))*polylog(3,-1+2/(1-c*x^(1/2)))-
3/2*b^3*polylog(4,1-2/(1-c*x^(1/2)))+3/2*b^3*polylog(4,-1+2/(1-c*x^(1/2)))
```

Rubi [A]
time = 0.36, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6035, 6033, 6199, 6095, 6205, 6209, 6745}

$$3b^2\text{Li}_3\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-3b^2\text{Li}_3\left(\frac{2}{1-c\sqrt{x}}-1\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-3b\text{Li}_2\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+3b\text{Li}_2\left(\frac{2}{1-c\sqrt{x}}-1\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3-\frac{3}{2}b^3\text{Li}_4\left(1-\frac{2}{1-c\sqrt{x}}\right)+\frac{3}{2}b^3\text{Li}_4\left(\frac{2}{1-c\sqrt{x}}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^3/x,x]
```

```
[Out] 4*ArcTanh[1 - 2/(1 - c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^3 - 3*b*(a + b*
ArcTanh[c*Sqrt[x]])^2*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])] + 3*b*(a + b*ArcTan
h[c*Sqrt[x]])^2*PolyLog[2, -1 + 2/(1 - c*Sqrt[x])] + 3*b^2*(a + b*ArcTanh[c
*Sqrt[x]])*PolyLog[3, 1 - 2/(1 - c*Sqrt[x])] - 3*b^2*(a + b*ArcTanh[c*Sqrt[
x]])*PolyLog[3, -1 + 2/(1 - c*Sqrt[x])] - (3*b^3*PolyLog[4, 1 - 2/(1 - c*Sq
rt[x])])/2 + (3*b^3*PolyLog[4, -1 + 2/(1 - c*Sqrt[x])])/2
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6209

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x}\,dx = 2\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}\,dx, x, \sqrt{x}\right)$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 - (12bc)\text{Subst}\left(\int \frac{(a+}{}\right.$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 + (6bc)\text{Subst}\left(\int \frac{(a+b}{}\right.$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 - 3b\big(a+b\tanh^{-1}\left(c\sqrt{x}\right.$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 - 3b\big(a+b\tanh^{-1}\left(c\sqrt{x}\right.$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 - 3b\big(a+b\tanh^{-1}\left(c\sqrt{x}\right.$$

Mathematica [A]
time = 0.14, size = 248, normalized size = 1.11

$$4\tanh^{-1}\left(1+\frac{2}{-1+c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3+\frac{3}{2}b\left(2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\text{PolyLog}\left(2,\frac{1+c\sqrt{x}}{1-c\sqrt{x}}\right)-2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\text{PolyLog}\left(2,\frac{1+c\sqrt{x}}{-1+c\sqrt{x}}\right)+b\left(-2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\text{PolyLog}\left(3,\frac{1+c\sqrt{x}}{1-c\sqrt{x}}\right)+2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\text{PolyLog}\left(3,\frac{1+c\sqrt{x}}{-1+c\sqrt{x}}\right)+b\left(\text{PolyLog}\left(4,\frac{1+c\sqrt{x}}{1-c\sqrt{x}}\right)-\text{PolyLog}\left(4,\frac{1+c\sqrt{x}}{-1+c\sqrt{x}}\right)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3/x,x]
```

```
[Out] 4*ArcTanh[1 + 2/(-1 + c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^3 + (3*b*(2*(a
 + b*ArcTanh[c*Sqrt[x]])^2*PolyLog[2, (1 + c*Sqrt[x])/(1 - c*Sqrt[x])] - 2*
(a + b*ArcTanh[c*Sqrt[x]])^2*PolyLog[2, (1 + c*Sqrt[x])/(-1 + c*Sqrt[x])] +
 b*(-2*(a + b*ArcTanh[c*Sqrt[x]])*PolyLog[3, (1 + c*Sqrt[x])/(1 - c*Sqrt[x]
)] + 2*(a + b*ArcTanh[c*Sqrt[x]])*PolyLog[3, (1 + c*Sqrt[x])/(-1 + c*Sqrt[x
])] + b*(PolyLog[4, (1 + c*Sqrt[x])/(1 - c*Sqrt[x])] - PolyLog[4, (1 + c*Sq
rt[x])/(-1 + c*Sqrt[x])]))))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.51, size = 1542, normalized size = 6.88

method	result	size
derivativedivides	Expression too large to display	1542
default	Expression too large to display	1542

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(c*x^(1/2)))^3/x,x,method=_RETURNVERBOSE)

[Out] -3*I*a*b^2*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*arctanh(c*x^(1/2))^2+I*b^3*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1))*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*arctanh(c*x^(1/2))^3-3*I*a*b^2*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*arctanh(c*x^(1/2))^2-I*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*arctanh(c*x^(1/2))^3-I*b^3*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*arctanh(c*x^(1/2))^3+3*I*a*b^2*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^3*arctanh(c*x^(1/2))^2+3*I*a*b^2*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1))*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*arctanh(c*x^(1/2))^2+I*b^3*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^3*arctanh(c*x^(1/2))^3+2*b^3*ln(c*x^(1/2))*arctanh(c*x^(1/2))^3-2*b^3*arctanh(c*x^(1/2))^3*ln((1+c*x^(1/2))^2/(-c^2*x+1)-1)-3*b^3*arctanh(c*x^(1/2))^2*polylog(2,-(1+c*x^(1/2))^2/(-c^2*x+1))+3*b^3*arctanh(c*x^(1/2))*polylog(3,-(1+c*x^(1/2))^2/(-c^2*x+1))+2*b^3*arctanh(c*x^(1/2))^3*ln(1+(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+6*b^3*arctanh(c*x^(1/2))^2*polylog(2,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-12*b^3*arctanh(c*x^(1/2))*polylog(3,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+2*b^3*arctanh(c*x^(1/2))^3*ln(1-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+6*b^3*arctanh(c*x^(1/2))^2*polylog(2,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-12*b^3*arctanh(c*x^(1/2))*polylog(3,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-3*a^2*b*dilog(c*x^(1/2))-3*a^2*b*dilog(1+c*x^(1/2))-12*a*b^2*polylog(3,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3*a*b^2*polylog(3,-(1+c*x^(1/2))^2/(-c^2*x+1))-12*a*b^2*polylog(3,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-3/2*b^3*polylog(4,-(1+c*x^(1/2))^2/(-c^2*x+1))+12*b^3*polylog(4,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+12*b^3*polylog(4,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+2*a^3*ln(c*x^(1/2))-6*a*b^2*arctanh(c*x^(1/2))*polylog(2,-(1+c*x^(1/2))^2/(-c^2*x+1))-6*a*b^2*arctanh(c*x^(1/2))^2*ln((1+c*x^(1/2))^2/(-c^2*x+1)-1)+6*a*b^2*arctanh(c*x^(1/2))^2*ln(1+(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+12*a*b^2*arctanh(c*x^(1/2))*polylog(2,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+6*a*b^2*arctanh(c*x^(1/2))^2*ln(1-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+6*a^2*b*ln(c*x^(1/2))*arctanh(c*x^(1/2))-3*a^2*b*ln(c*x^(1/2))*ln(1+c*x^(1/2))+12*a*b^2*arctanh(c*x^(1/2))*polylog(2,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+6*a*b^2*ln(c*x^(1/2))*arctanh(c*x^(1/2))^2

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x,x, algorithm="maxima")

```
[Out] 1/8*b^3*integrate(log(c*sqrt(x) + 1)^3/x, x) - 3/8*b^3*integrate(log(c*sqrt
(x) + 1)^2*log(-c*sqrt(x) + 1)/x, x) + 3/8*b^3*integrate(log(c*sqrt(x) + 1)
*log(-c*sqrt(x) + 1)^2/x, x) - 1/8*b^3*integrate(log(-c*sqrt(x) + 1)^3/x, x
) + 3/4*a*b^2*integrate(log(c*sqrt(x) + 1)^2/x, x) - 3/2*a*b^2*integrate(lo
g(c*sqrt(x) + 1)*log(-c*sqrt(x) + 1)/x, x) + 3/4*a*b^2*integrate(log(-c*sqr
t(x) + 1)^2/x, x) + 3/2*a^2*b*integrate(log(c*sqrt(x) + 1)/x, x) - 3/2*a^2*
b*integrate(log(-c*sqrt(x) + 1)/x, x) + a^3*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x,x, algorithm="fricas")
[Out] integral((b^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*arctanh(c*sqrt(x))^2 + 3*a^2*b
*arctanh(c*sqrt(x)) + a^3)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}{\left(c \sqrt{x} \right)}\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**3/x,x)
[Out] Integral((a + b*atanh(c*sqrt(x)))**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x,x, algorithm="giac")
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)\right)}^3}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))^3/x,x)
[Out] int((a + b*atanh(c*x^(1/2)))^3/x, x)
```

3.207 $\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=142

$$3bc^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-\frac{3bc\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{\sqrt{x}}+c^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right.\right.}{x}$$

```
[Out] 3*b*c^2*(a+b*arctanh(c*x^(1/2)))^2+c^2*(a+b*arctanh(c*x^(1/2)))^3-(a+b*arct
anh(c*x^(1/2)))^3/x+6*b^2*c^2*(a+b*arctanh(c*x^(1/2)))*ln(2-2/(1+c*x^(1/2))
)-3*b^3*c^2*polylog(2,-1+2/(1+c*x^(1/2)))-3*b*c*(a+b*arctanh(c*x^(1/2)))^2/
x^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6039, 6037, 6129, 6135, 6079, 2497, 6095}

$$6b^2c^2\log\left(2-\frac{2}{c\sqrt{x}+1}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+3bc^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+c^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3-\frac{3bc\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{\sqrt{x}}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x}-3b^3c^2\text{Li}_2\left(\frac{2}{\sqrt{x}c+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^3/x^2,x]
```

```
[Out] 3*b*c^2*(a + b*ArcTanh[c*Sqrt[x]])^2 - (3*b*c*(a + b*ArcTanh[c*Sqrt[x]])^2)
/Sqrt[x] + c^2*(a + b*ArcTanh[c*Sqrt[x]])^3 - (a + b*ArcTanh[c*Sqrt[x]])^3/
x + 6*b^2*c^2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2 - 2/(1 + c*Sqrt[x])] - 3*b^3
*c^2*PolyLog[2, -1 + 2/(1 + c*Sqrt[x])]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
```

```
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx$$

Mathematica [A]
time = 0.21, size = 230, normalized size = 1.62

$$\frac{6b^2\left(-1+c\sqrt{x}\right)\left(a+ac\sqrt{x}+bc\sqrt{x}\right)\tanh^{-1}\left(c\sqrt{x}\right)^2+2b^3(-1+c^2x)\tanh^{-1}\left(c\sqrt{x}\right)^3-6b\tanh^{-1}\left(c\sqrt{x}\right)\left(a^2+2abc\sqrt{x}-2b^2c^2x\log\left(1-e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right)\right)+a\left(-2a^2-6abc\sqrt{x}-3abc^2x\log\left(1-c\sqrt{x}\right)+3abc^2x\log\left(1+c\sqrt{x}\right)+12b^2c^2x\log\left(\frac{c\sqrt{x}}{\sqrt{1-c^2x}}\right)\right)-6b^3c^2x\,\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3/x^2,x]
```

```
[Out] (6*b^2*(-1 + c*Sqrt[x])*(a + a*c*Sqrt[x] + b*c*Sqrt[x])*ArcTanh[c*Sqrt[x]]^
2 + 2*b^3*(-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^3 - 6*b*ArcTanh[c*Sqrt[x]]*(a^2 +
 2*a*b*c*Sqrt[x] - 2*b^2*c^2*x*Log[1 - E^(-2*ArcTanh[c*Sqrt[x]])]) + a*(-2*
a^2 - 6*a*b*c*Sqrt[x] - 3*a*b*c^2*x*Log[1 - c*Sqrt[x]] + 3*a*b*c^2*x*Log[1
+ c*Sqrt[x]] + 12*b^2*c^2*x*Log[(c*Sqrt[x])/Sqrt[1 - c^2*x]]) - 6*b^3*c^2*x
*PolyLog[2, E^(-2*ArcTanh[c*Sqrt[x]])])/(2*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.18, size = 4974, normalized size = 35.03

method	result	size
derivativedivides	Expression too large to display	4974
default	Expression too large to display	4974

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*c^2*(-3/8*I*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*csgn(I*(1+c*x^(
1/2))^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^
2*x+1)))*arctanh(c*x^(1/2))*ln(1-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/4*I*b^3*
Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))
^2*dilog((1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/8*I*b^3*Pi*csgn(I*(1+c*x^(1/2))^
2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^3*dilog((1+c*x^(1/2))/(-c^2*x+1
)^(1/2))-3/8*I*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(
-c^2*x+1)))^3*dilog(1+(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-3/8*I*b^3*Pi*csgn(I*(
1+c*x^(1/2))^2/(c^2*x-1))^3*dilog(1+(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/4*I*b
^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*polylog(2,(1+c*x^(1/2))/(-c^
2*x+1)^(1/2))-3/4*I*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*dilog(1
+(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/8*I*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x
-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^3*polylog(2,-(1+c*x^(1/2))/(-c^2*x+1)^(
1/2))-3/8*I*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^
2*x+1)))^3*arctanh(c*x^(1/2))^2+3/8*I*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-
1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^3*polylog(2,(1+c*x^(1/2))/(-c^2*x+1)^(1/
2))-3/4*I*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^3*polylog(2,(1+c*x^
(1/2))/(-c^2*x+1)^(1/2))-3/4*I*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1))
)^2*arctanh(c*x^(1/2))^2+3/4*I*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1))
)^3*arctanh(c*x^(1/2))^2+3/4*I*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1))
)^3*dilog(1+(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-3/4*I*b^3*Pi*csgn(I/(1+(1+c*x^(
1/2))^2/(-c^2*x+1)))^3*dilog((1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/4*I*b^3*Pi*c
sgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*dilog((1+c*x^(1/2))/(-c^2*x+1)^(1/2
))-3/4*I*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^3*polylog(2,-(1+c*x^
(1/2))/(-c^2*x+1)^(1/2))+3/4*I*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1))
)^2*polylog(2,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-3/2*a*b^2*arctanh(c*x^(1/2))
```

```
*ln(c*x^(1/2)-1)+3/2*a*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))+3/4*a*b^2*ln(
c*x^(1/2)-1)*ln(1/2*c*x^(1/2)+1/2)+3/4*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*
x^(1/2))-3/4*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2*c*x^(1/2)+1/2)+3/8*I*b^3*P
i*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(
1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*polylog(2,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-
3/8*I*b^3*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))^2*csgn(I*(1+c*x^(1/2))^
2/(c^2*x-1))*dilog(1+(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/8*I*b^3*Pi*csgn(I*(1
+c*x^(1/2))/(-c^2*x+1)^(1/2))^2*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*polylog(2
,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/4*I*b^3*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x
+1)^(1/2))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^2*polylog(2,(1+c*x^(1/2))/(-c^
2*x+1)^(1/2))+3/8*I*b^3*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))^2*csgn(I*
(1+c*x^(1/2))^2/(c^2*x-1))*dilog((1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/8*I*b^3*
Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/
(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*dilog((1+c*x^(1/2))/(-c^2*x+1)^(1/2))-3/8
*I*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*csgn(I*(1+c*x^(1/2))^2/(c^
2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*arctanh(c*x^(1/2))^2+3/8*I*b^3*Pi*
csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*
x^(1/2))^2/(-c^2*x+1)))^2*dilog(1+(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/4*I*b^3
*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*arctanh(c*x^(1/2))*ln(1-(1+c*x
^(1/2))/(-c^2*x+1)^(1/2))+3/8*I*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)
))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*polyl
og(2,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-3/4*I*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^
2/(-c^2*x+1)))^3*arctanh(c*x^(1/2))*ln(1-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-3/
8*I*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-
1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*dilog((1+c*x^(1/2))/(-c^2*x+1)^(1/2))-
3/8*a*b^2*ln(c*x^(1/2)-1)^2-3/8*a*b^2*ln(1+c*x^(1/2))^2-3/2*a*b^2*ln(c*x^(1
/2)-1)-3/2*a*b^2*ln(1+c*x^(1/2))-3/4*a^2*b*ln(c*x^(1/2)-1)+3/4*a^2*b*ln(1+c
*x^(1/2))-3/4*b^3*arctanh(c*x^(1/2))^2*ln(c*x^(1/2)-1)+3/4*b^3*arctanh(c*x^
(1/2))^2*ln(1+c*x^(1/2))-3/2*b^3*arctanh(c*x^(1/2))^2*ln((1+c*x^(1/2))/(-c^
2*x+1)^(1/2))+3/8*I*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^3*arctanh(c*x^
(1/2))*ln(1-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/8*I*b^3*Pi*csgn(I*(1+c*x^(1/2
))^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x
+1)))^2*arctanh(c*x^(1/2))^2-3/8*I*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))
*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*polylog
(2,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-3/8*I*b^3*Pi*csgn(I*(1+c*x^(1/2))/(-c^2
*x+1)^(1/2))^2*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*arctanh(c*x^(1/2))^2+3/4*I
*b^3*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))*csgn(I*(1+c*x^(1/2))^2/(c^2*
x-1))^2*polylog(2,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/8*I*b^3*Pi*csgn(I*(1+c
*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^3*arctanh(c*x^(1/2))*
ln(1-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-3/8*I*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c
^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1...
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(125) = 250$.

time = 1.10, size = 528, normalized size = 3.72

$$-3\left(\log\left(c\sqrt{x}+1\right)\log\left(-\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)\right)b^3c^2-3\left(\log\left(c\sqrt{x}\right)\log\left(-c\sqrt{x}+1\right)+\mathrm{Li}_2(-c\sqrt{x}+1)\right)b^3c^2+3\left(\log\left(c\sqrt{x}+1\right)\log\left(-c\sqrt{x}\right)+\mathrm{Li}_2(c\sqrt{x}+1)\right)b^3c^2-3ab^2c^2\log\left(c\sqrt{x}-1\right)-\frac{3}{4}\left(\left(2c\log\left(c\sqrt{x}-1\right)-c\log(x)+\frac{2}{\sqrt{x}}\right)c-\frac{2\log\left(-c\sqrt{x}+1\right)}{x}\right)a^2b-\frac{a^3}{x}+\frac{3}{2}\left(a^2bc^2-2ab^2c^2\right)\log\left(c\sqrt{x}+1\right)-\frac{3}{4}\left(a^2bc^2-4ab^2c^2\right)\log(x)-\frac{12a^2bc\sqrt{x}-(b^3c^2x-b^3)\log\left(c\sqrt{x}+1\right)^3+(b^3c^2x-b^3)\log\left(-c\sqrt{x}+1\right)^3+6\left(b^3c\sqrt{x}+ab^2-(ab^2c^2-b^3c^2)x\right)\log\left(c\sqrt{x}+1\right)^2+3\left(2b^3c\sqrt{x}+2ab^2-2(ab^2c^2+b^3c^2)x-(b^3c^2x-b^3)\log\left(c\sqrt{x}+1\right)\right)\log\left(-c\sqrt{x}+1\right)^2+12\left(2ab^2c\sqrt{x}+a^2b\right)\log\left(c\sqrt{x}+1\right)-3\left(8ab^2c\sqrt{x}-(b^3c^2x-b^3)\log\left(c\sqrt{x}+1\right)^2+4\left(b^3c\sqrt{x}+ab^2-(ab^2c^2-b^3c^2)x\right)\log\left(c\sqrt{x}+1\right)\right)\log\left(-c\sqrt{x}+1\right)}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^2,x, algorithm="maxima")
```

```
[Out] -3*(log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/
2))*b^3*c^2 - 3*(log(c*sqrt(x))*log(-c*sqrt(x) + 1) + dilog(-c*sqrt(x) + 1)
)*b^3*c^2 + 3*(log(c*sqrt(x) + 1)*log(-c*sqrt(x)) + dilog(c*sqrt(x) + 1))*b
^3*c^2 - 3*a*b^2*c^2*log(c*sqrt(x) - 1) - 3/4*((2*c*log(c*sqrt(x) - 1) - c*
log(x) + 2/sqrt(x))*c - 2*log(-c*sqrt(x) + 1)/x)*a^2*b - a^3/x + 3/2*(a^2*b
*c^2 - 2*a*b^2*c^2)*log(c*sqrt(x) + 1) - 3/4*(a^2*b*c^2 - 4*a*b^2*c^2)*log(
x) - 1/8*(12*a^2*b*c*sqrt(x) - (b^3*c^2*x - b^3)*log(c*sqrt(x) + 1)^3 + (b^
3*c^2*x - b^3)*log(-c*sqrt(x) + 1)^3 + 6*(b^3*c*sqrt(x) + a*b^2 - (a*b^2*c^
2 - b^3*c^2)*x)*log(c*sqrt(x) + 1)^2 + 3*(2*b^3*c*sqrt(x) + 2*a*b^2 - 2*(a*
b^2*c^2 + b^3*c^2)*x - (b^3*c^2*x - b^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x)
 + 1)^2 + 12*(2*a*b^2*c*sqrt(x) + a^2*b)*log(c*sqrt(x) + 1) - 3*(8*a*b^2*c*
sqrt(x) - (b^3*c^2*x - b^3)*log(c*sqrt(x) + 1)^2 + 4*(b^3*c*sqrt(x) + a*b^2
 - (a*b^2*c^2 - b^3*c^2)*x)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1))/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*arctanh(c*sqrt(x))^2 + 3*a^2*b
*arctanh(c*sqrt(x)) + a^3)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\operatorname{atanh}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**3/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*sqrt(x)))**3/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,\sqrt{x}\,\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))^3/x^2,x)
[Out] int((a + b*atanh(c*x^(1/2)))^3/x^2, x)
```

3.208 $\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=234

$$-\frac{b^3c^3}{2\sqrt{x}}+\frac{1}{2}b^3c^4\tanh^{-1}\left(c\sqrt{x}\right)-\frac{b^2c^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{2x}+2bc^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-\frac{bc\left(a+b\tanh^{-1}\right.}{2x^{3/2}}$$

```
[Out] 1/2*b^3*c^4*arctanh(c*x^(1/2))-1/2*b^2*c^2*(a+b*arctanh(c*x^(1/2)))/x+2*b*c
^4*(a+b*arctanh(c*x^(1/2)))^2-1/2*b*c*(a+b*arctanh(c*x^(1/2)))^2/x^(3/2)+1/
2*c^4*(a+b*arctanh(c*x^(1/2)))^3-1/2*(a+b*arctanh(c*x^(1/2)))^3/x^2+4*b^2*c
^4*(a+b*arctanh(c*x^(1/2)))*ln(2-2/(1+c*x^(1/2)))-2*b^3*c^4*polylog(2,-1+2/
(1+c*x^(1/2)))-1/2*b^3*c^3/x^(1/2)-3/2*b*c^3*(a+b*arctanh(c*x^(1/2)))^2/x^(
1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6039, 6037, 6129, 331, 212, 6135, 6079, 2497, 6095}

$$4b^2c^4\log\left(2-\frac{2}{c\sqrt{x}+1}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b^2c^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{2x}+\frac{1}{2}c^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3+2bc^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-\frac{3bc^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{2\sqrt{x}}-\frac{bc\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{2x^{3/2}}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{2x^2}-2b^3c^4\text{Li}_2\left(\frac{2}{\sqrt{x}c+1}-1\right)+\frac{1}{2}b^3c^4\tanh^{-1}\left(c\sqrt{x}\right)-\frac{b^3c^3}{2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^3/x^3,x]
```

```
[Out] -1/2*(b^3*c^3)/Sqrt[x] + (b^3*c^4*ArcTanh[c*Sqrt[x]])/2 - (b^2*c^2*(a + b*A
rcTanh[c*Sqrt[x]]))/(2*x) + 2*b*c^4*(a + b*ArcTanh[c*Sqrt[x]])^2 - (b*c*(a
+ b*ArcTanh[c*Sqrt[x]])^2)/(2*x^(3/2)) - (3*b*c^3*(a + b*ArcTanh[c*Sqrt[x]]
)^2)/(2*Sqrt[x]) + (c^4*(a + b*ArcTanh[c*Sqrt[x]])^3)/2 - (a + b*ArcTanh[c*
Sqrt[x]])^3/(2*x^2) + 4*b^2*c^4*(a + b*ArcTanh[c*Sqrt[x]])*Log[2 - 2/(1 + c
*Sqrt[x])] - 2*b^3*c^4*PolyLog[2, -1 + 2/(1 + c*Sqrt[x])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
```

```
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx$$

Mathematica [A]
time = 0.49, size = 333, normalized size = 1.42

$$-\frac{2a^3+2a^2bc\sqrt{x}+2ab^2c^2x+6a^2bc^3x^{3/2}+2b^3c^3x^{3/2}-2ab^2c^4x^2-2b^2\left(bc\sqrt{x}\left(-1-3c^2x+4c^3x^{3/2}\right)+3a(-1+c^4x^2)\right)\tanh^{-1}\left(c\sqrt{x}\right)^2-2b^3(-1+c^4x^2)\tanh^{-1}\left(c\sqrt{x}\right)^3+2b\tanh^{-1}\left(c\sqrt{x}\right)\left(3a^2+b^2c^2x(1-c^2x)+2abc\sqrt{x}(1+3c^2x)-8b^2c^4x^2\log\left(1-e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right)\right)+3a^2bc^4x^2\log\left(1-c\sqrt{x}\right)-3a^2bc^4x^2\log\left(1+c\sqrt{x}\right)-16ab^2c^4x^2\log\left(\frac{c\sqrt{x}}{\sqrt{1-c^2x}}\right)+8b^3c^4x^2\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3/x^3,x]
```

```
[Out] -1/4*(2*a^3 + 2*a^2*b*c*Sqrt[x] + 2*a*b^2*c^2*x + 6*a^2*b*c^3*x^(3/2) + 2*b
^3*c^3*x^(3/2) - 2*a*b^2*c^4*x^2 - 2*b^2*(b*c*Sqrt[x]*(-1 - 3*c^2*x + 4*c^3
*x^(3/2)) + 3*a*(-1 + c^4*x^2))*ArcTanh[c*Sqrt[x]]^2 - 2*b^3*(-1 + c^4*x^2)
*ArcTanh[c*Sqrt[x]]^3 + 2*b*ArcTanh[c*Sqrt[x]]*(3*a^2 + b^2*c^2*x*(1 - c^2*
x) + 2*a*b*c*Sqrt[x]*(1 + 3*c^2*x) - 8*b^2*c^4*x^2*Log[1 - E^(-2*ArcTanh[c*
Sqrt[x]])]) + 3*a^2*b*c^4*x^2*Log[1 - c*Sqrt[x]] - 3*a^2*b*c^4*x^2*Log[1 +
c*Sqrt[x]] - 16*a*b^2*c^4*x^2*Log[(c*Sqrt[x])/Sqrt[1 - c^2*x]] + 8*b^3*c^4*
x^2*PolyLog[2, E^(-2*ArcTanh[c*Sqrt[x]])])/x^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 8.17, size = 1289, normalized size = 5.51

method	result	size
derivativedivides	Expression too large to display	1289
default	Expression too large to display	1289

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*c^4*(-3/16*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*
x+1)))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2+1
/4*b^3/((-c^2*x+1)^(1/2)+c*x^(1/2)+1)*(-c^2*x+1)^(1/2)-1/4*b^3/(-(-c^2*x+1)
^(1/2)+c*x^(1/2)+1)*(-c^2*x+1)^(1/2)-1/4*a^3/c^4/x^2-3/4*a*b^2*arctanh(c*x^
(1/2))*ln(c*x^(1/2)-1)+3/4*a*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))+3/8*a*b
^2*ln(c*x^(1/2)-1)*ln(1/2*c*x^(1/2)+1/2)+3/8*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*l
```

```
n(1+c*x^(1/2))-3/8*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2*c*x^(1/2)+1/2)-3/16*
a*b^2*ln(c*x^(1/2)-1)^2-3/16*a*b^2*ln(1+c*x^(1/2))^2-a*b^2*ln(c*x^(1/2)-1)-
a*b^2*ln(1+c*x^(1/2))-3/8*a^2*b*ln(c*x^(1/2)-1)+3/8*a^2*b*ln(1+c*x^(1/2))-3
/8*b^3*arctanh(c*x^(1/2))^2*ln(c*x^(1/2)-1)+3/8*b^3*arctanh(c*x^(1/2))^2*ln
(1+c*x^(1/2))-3/4*b^3*arctanh(c*x^(1/2))^2*ln((1+c*x^(1/2))/(-c^2*x+1)^(1/2
))+3/8*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))
^3-3/16*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^3-3
/16*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*
x^(1/2))^2/(-c^2*x+1)))^3-3/4*a*b^2/c^4/x^2*arctanh(c*x^(1/2))^2-1/2*a*b^2*
arctanh(c*x^(1/2))/c^3/x^(3/2)-3/4*a^2*b/c^4/x^2*arctanh(c*x^(1/2))-3/8*I*b
^3*arctanh(c*x^(1/2))^2*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2-3/2*a*b
^2*arctanh(c*x^(1/2))/c/x^(1/2)+3/8*I*b^3*arctanh(c*x^(1/2))^2*Pi-1/4*a*b^2
/c^2/x-1/4*b^3*arctanh(c*x^(1/2))/c^2/x-1/4*b^3/c^4/x^2*arctanh(c*x^(1/2))^
3-1/4*b^3*arctanh(c*x^(1/2))^2/c^3/x^(3/2)-1/4*a^2*b/c^3/x^(3/2)+2*a*b^2*ln
(c*x^(1/2))+2*b^3*arctanh(c*x^(1/2))*ln(1+(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-2
*b^3*dilog((1+c*x^(1/2))/(-c^2*x+1)^(1/2))+2*b^3*dilog(1+(1+c*x^(1/2))/(-c^
2*x+1)^(1/2))+3/16*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I/(1+(1+c*x^(1/2))^2/
(-c^2*x+1)))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*
x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))-3/4*a^2*b/c/x^(1/2)-3/4*b^3*arctanh(c*
x^(1/2))^2/c/x^(1/2)-b^3*arctanh(c*x^(1/2))^2+1/4*b^3*arctanh(c*x^(1/2))^3+
1/4*b^3*arctanh(c*x^(1/2))-3/8*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I*(1+c*x^
(1/2))/(-c^2*x+1)^(1/2))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^2-3/16*I*b^3*arc
tanh(c*x^(1/2))^2*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))^2*csgn(I*(1+c*x
^(1/2))^2/(c^2*x-1))+3/16*I*b^3*arctanh(c*x^(1/2))^2*Pi*csgn(I*(1+c*x^(1/2)
)^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+
1)))^2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(193) = 386$.
time = 1.23, size = 703, normalized size = 3.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^3,x, algorithm="maxima")
```

```
[Out] -2*(log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/
2))*b^3*c^4 - 2*(log(c*sqrt(x))*log(-c*sqrt(x) + 1) + dilog(-c*sqrt(x) + 1)
)*b^3*c^4 + 2*(log(c*sqrt(x) + 1)*log(-c*sqrt(x)) + dilog(c*sqrt(x) + 1))*b
^3*c^4 - 1/8*((6*c^3*log(c*sqrt(x) - 1) - 3*c^3*log(x) + (6*c^2*x + 3*c*sqr
t(x) + 2)/x^(3/2))*c - 6*log(-c*sqrt(x) + 1)/x^2)*a^2*b + 1/4*(3*a^2*b*c^4
- 8*a*b^2*c^4 + b^3*c^4)*log(c*sqrt(x) + 1) - 1/4*(8*a*b^2*c^4 + b^3*c^4)*l
og(c*sqrt(x) - 1) - 1/8*(3*a^2*b*c^4 - 16*a*b^2*c^4)*log(x) - 1/2*a^3/x^2 -
 1/16*(4*a^2*b*c*sqrt(x) - (b^3*c^4*x^2 - b^3)*log(c*sqrt(x) + 1)^3 + (b^3*
c^4*x^2 - b^3)*log(-c*sqrt(x) + 1)^3 + 2*(3*b^3*c^3*x^(3/2) + b^3*c*sqrt(x)
```

```
 + 3*a*b^2 - (3*a*b^2*c^4 - 4*b^3*c^4)*x^2)*log(c*sqrt(x) + 1)^2 + (6*b^3*c
^3*x^(3/2) + 2*b^3*c*sqrt(x) + 6*a*b^2 - 2*(3*a*b^2*c^4 + 4*b^3*c^4)*x^2 -
3*(b^3*c^4*x^2 - b^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1)^2 + 4*(3*a^2*
b*c^3 + 2*b^3*c^3)*x^(3/2) - 2*(3*a^2*b*c^2 - 4*a*b^2*c^2)*x + 4*(6*a*b^2*c
^3*x^(3/2) + b^3*c^2*x + 2*a*b^2*c*sqrt(x) + 3*a^2*b)*log(c*sqrt(x) + 1) -
(24*a*b^2*c^3*x^(3/2) + 4*b^3*c^2*x + 8*a*b^2*c*sqrt(x) - 3*(b^3*c^4*x^2 -
b^3)*log(c*sqrt(x) + 1)^2 + 4*(3*b^3*c^3*x^(3/2) + b^3*c*sqrt(x) + 3*a*b^2
- (3*a*b^2*c^4 - 4*b^3*c^4)*x^2)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1))/x
^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^3,x, algorithm="fricas")
[Out] integral((b^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*arctanh(c*sqrt(x))^2 + 3*a^2*b
*arctanh(c*sqrt(x)) + a^3)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}{\left(c \sqrt{x} \right)}\right)^{3}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**3/x**3,x)
[Out] Integral((a + b*atanh(c*sqrt(x)))**3/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^3,x, algorithm="giac")
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)\right)}^3}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))^3/x^3,x)
```

```
[Out] int((a + b*atanh(c*x^(1/2)))^3/x^3, x)
```

3.209 $\int x^{3/2} \tanh^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=38

$$\frac{x}{5}+\frac{x^2}{10}+\frac{2}{5}x^{5/2}\tanh^{-1}\left(\sqrt{x}\right)+\frac{1}{5}\log(1-x)$$

```
[Out] 1/5*x+1/10*x^2+2/5*x^(5/2)*arctanh(x^(1/2))+1/5*ln(1-x)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6037, 45}

$$\frac{2}{5}x^{5/2}\tanh^{-1}\left(\sqrt{x}\right)+\frac{x^2}{10}+\frac{x}{5}+\frac{1}{5}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTanh[Sqrt[x]],x]
```

```
[Out] x/5 + x^2/10 + (2*x^(5/2)*ArcTanh[Sqrt[x]])/5 + Log[1 - x]/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)dx&=\frac{2}{5}x^{5/2}\tanh^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\int\frac{x^2}{1-x}\,dx\\&=\frac{2}{5}x^{5/2}\tanh^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\int\left(-1+\frac{1}{1-x}-x\right)dx\\&=\frac{x}{5}+\frac{x^2}{10}+\frac{2}{5}x^{5/2}\tanh^{-1}\left(\sqrt{x}\right)+\frac{1}{5}\log(1-x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.82

$$\frac{1}{10}\left(x(2+x)+4x^{5/2}\tanh^{-1}\left(\sqrt{x}\right)+2\log(1-x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTanh[Sqrt[x]],x]
```

```
[Out] (x*(2 + x) + 4*x^(5/2)*ArcTanh[Sqrt[x]] + 2*Log[1 - x])/10
```

Maple [A]
time = 0.07, size = 35, normalized size = 0.92

method	result	size
derivativedivides	$\frac{2x^{\frac{5}{2}}\operatorname{arctanh}\left(\sqrt{x}\right)}{5}+\frac{x^2}{10}+\frac{x}{5}+\frac{\ln\left(\sqrt{x}-1\right)}{5}+\frac{\ln\left(\sqrt{x}+1\right)}{5}$	35
default	$\frac{2x^{\frac{5}{2}}\operatorname{arctanh}\left(\sqrt{x}\right)}{5}+\frac{x^2}{10}+\frac{x}{5}+\frac{\ln\left(\sqrt{x}-1\right)}{5}+\frac{\ln\left(\sqrt{x}+1\right)}{5}$	35
meijerg	$\frac{x(3x+6)}{30}-\frac{x^{\frac{5}{2}}\left(\ln\left(1-\sqrt{x}\right)-\ln\left(\sqrt{x}+1\right)\right)}{5}+\frac{\ln(1-x)}{5}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x^(5/2)*arctanh(x^(1/2))+1/10*x^2+1/5*x+1/5*ln(x^(1/2)-1)+1/5*ln(x^(1/2
)+1)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.63

$$\frac{2}{5}\,x^{\frac{5}{2}}\operatorname{artanh}\left(\sqrt{x}\right)+\frac{1}{10}\,x^2+\frac{1}{5}\,x+\frac{1}{5}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2)*arctanh(sqrt(x)) + 1/10*x^2 + 1/5*x + 1/5*log(x - 1)
```

Fricas [A]
time = 0.38, size = 36, normalized size = 0.95

$$\frac{1}{5}\,x^{\frac{5}{2}}\log\left(-\frac{x+2\,\sqrt{x}+1}{x-1}\right)+\frac{1}{10}\,x^2+\frac{1}{5}\,x+\frac{1}{5}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/5*x^(5/2)*log(-(x + 2*sqrt(x) + 1)/(x - 1)) + 1/10*x^2 + 1/5*x + 1/5*log(
x - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(29) = 58$.
time = 1.04, size = 121, normalized size = 3.18

$$\frac{4x^{\frac{7}{2}}\operatorname{atanh}\left(\sqrt{x}\right)}{10x-10}-\frac{4x^{\frac{5}{2}}\operatorname{atanh}\left(\sqrt{x}\right)}{10x-10}+\frac{x^3}{10x-10}+\frac{x^2}{10x-10}+\frac{4x\log\left(\sqrt{x}+1\right)}{10x-10}-\frac{4x\operatorname{atanh}\left(\sqrt{x}\right)}{10x-10}-\frac{4\log\left(\sqrt{x}+1\right)}{10x-10}+\frac{4\operatorname{atanh}\left(\sqrt{x}\right)}{10x-10}-\frac{2}{10x-10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(x**(1/2)),x)
```

```
[Out] 4*x**(7/2)*atanh(sqrt(x))/(10*x - 10) - 4*x**(5/2)*atanh(sqrt(x))/(10*x - 1
0) + x**3/(10*x - 10) + x**2/(10*x - 10) + 4*x*log(sqrt(x) + 1)/(10*x - 10)
 - 4*x*atanh(sqrt(x))/(10*x - 10) - 4*log(sqrt(x) + 1)/(10*x - 10) + 4*atan
h(sqrt(x))/(10*x - 10) - 2/(10*x - 10)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(26) = 52$.
time = 0.41, size = 170, normalized size = 4.47

$$\frac{8\left(\frac{\left(\sqrt{x}+1\right)^3}{\left(\sqrt{x}-1\right)^3}-\frac{\left(\sqrt{x}+1\right)^2}{\left(\sqrt{x}-1\right)^2}+\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{5\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^4}+\frac{2\left(\frac{5\left(\sqrt{x}+1\right)^4}{\left(\sqrt{x}-1\right)^4}+\frac{10\left(\sqrt{x}+1\right)^2}{\left(\sqrt{x}-1\right)^2}+1\right)\log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{5\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^5}+\frac{2}{5}\log\left(\frac{\sqrt{x}+1}{\left|\sqrt{x}-1\right|}\right)-\frac{2}{5}\log\left(\left|-\frac{\sqrt{x}+1}{\sqrt{x}-1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x^(1/2)),x, algorithm="giac")
```

```
[Out] 8/5*((sqrt(x) + 1)^3/(sqrt(x) - 1)^3 - (sqrt(x) + 1)^2/(sqrt(x) - 1)^2 + (s
qrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^4 + 2/5*(5*(sq
rt(x) + 1)^4/(sqrt(x) - 1)^4 + 10*(sqrt(x) + 1)^2/(sqrt(x) - 1)^2 + 1)*log(
-(sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^5 + 2/5*log
((sqrt(x) + 1)/abs(sqrt(x) - 1)) - 2/5*log(abs(-(sqrt(x) + 1)/(sqrt(x) - 1)
 + 1))
```

Mupad [B]
time = 0.86, size = 24, normalized size = 0.63

$$\frac{x}{5}+\frac{\ln\left(x-1\right)}{5}+\frac{2\,x^{5/2}\,\mathrm{atanh}\left(\sqrt{x}\right)}{5}+\frac{x^2}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*atanh(x^(1/2)),x)
```

```
[Out] x/5 + log(x - 1)/5 + (2*x^(5/2)*atanh(x^(1/2)))/5 + x^2/10
```

3.210 $\int \sqrt{x} \tanh^{-1}\left(\sqrt{x}\right) \, dx$

Optimal. Leaf size=31

$$\frac{x}{3}+\frac{2}{3}x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)+\frac{1}{3}\log(1-x)$$

```
[Out] 1/3*x+2/3*x^(3/2)*arctanh(x^(1/2))+1/3*ln(1-x)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6037, 45}

$$\frac{2}{3}x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)+\frac{x}{3}+\frac{1}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[Sqrt[x]],x]
```

```
[Out] x/3 + (2*x^(3/2)*ArcTanh[Sqrt[x]])/3 + Log[1 - x]/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
 + 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\tanh^{-1}\left(\sqrt{x}\right)\,dx &= \frac{2}{3}x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int\frac{x}{1-x}\,dx\\ &= \frac{2}{3}x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int\left(-1+\frac{1}{1-x}\right)dx\\ &= \frac{x}{3}+\frac{2}{3}x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)+\frac{1}{3}\log(1-x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.81

$$\frac{1}{3}\left(x+2x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)+\log(1-x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[Sqrt[x]],x]
[Out] (x + 2*x^(3/2)*ArcTanh[Sqrt[x]] + Log[1 - x])/3
```

Maple [A]
time = 0.04, size = 30, normalized size = 0.97

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}\operatorname{arctanh}\left(\sqrt{x}\right)}{3}+\frac{x}{3}+\frac{\ln\left(\sqrt{x}-1\right)}{3}+\frac{\ln\left(\sqrt{x}+1\right)}{3}$	30
default	$\frac{2x^{\frac{3}{2}}\operatorname{arctanh}\left(\sqrt{x}\right)}{3}+\frac{x}{3}+\frac{\ln\left(\sqrt{x}-1\right)}{3}+\frac{\ln\left(\sqrt{x}+1\right)}{3}$	30
meijerg	$\frac{x}{3}-\frac{x^{\frac{3}{2}}\left(\ln\left(1-\sqrt{x}\right)-\ln\left(\sqrt{x}+1\right)\right)}{3}+\frac{\ln(1-x)}{3}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x^(1/2))*x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*x^(3/2)*arctanh(x^(1/2))+1/3*x+1/3*ln(x^(1/2)-1)+1/3*ln(x^(1/2)+1)
```

Maxima [A]
time = 0.25, size = 19, normalized size = 0.61

$$\frac{2}{3}\,x^{\frac{3}{2}}\operatorname{artanh}\left(\sqrt{x}\right)+\frac{1}{3}\,x+\frac{1}{3}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))*x^(1/2),x, algorithm="maxima")
[Out] 2/3*x^(3/2)*arctanh(sqrt(x)) + 1/3*x + 1/3*log(x - 1)
```

Fricas [A]
time = 0.39, size = 31, normalized size = 1.00

$$\frac{1}{3}\,x^{\frac{3}{2}}\log\left(-\frac{x+2\,\sqrt{x}+1}{x-1}\right)+\frac{1}{3}\,x+\frac{1}{3}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))*x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*x^(3/2)*log(-(x + 2*sqrt(x) + 1)/(x - 1)) + 1/3*x + 1/3*log(x - 1)
```

Sympy [A]
time = 0.55, size = 39, normalized size = 1.26

$$\frac{2x^{\frac{3}{2}}\operatorname{atanh}\left(\sqrt{x}\right)}{3}+\frac{x}{3}+\frac{2\log\left(\sqrt{x}+1\right)}{3}-\frac{2\operatorname{atanh}\left(\sqrt{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x**(1/2))*x**(1/2),x)
[Out] 2*x**(3/2)*atanh(sqrt(x))/3 + x/3 + 2*log(sqrt(x) + 1)/3 - 2*atanh(sqrt(x))
/3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(21) = 42.
time = 0.41, size = 121, normalized size = 3.90

$$\frac{2\left(\frac{3\left(\sqrt{x}+1\right)^{2}}{\left(\sqrt{x}-1\right)^{2}}+1\right)\log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{3\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^{3}}+\frac{4\left(\sqrt{x}+1\right)}{3\left(\sqrt{x}-1\right)\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^{2}}+\frac{2}{3}\log\left(\frac{\sqrt{x}+1}{\left|\sqrt{x}-1\right|}\right)-\frac{2}{3}\log\left(\left|-\frac{\sqrt{x}+1}{\sqrt{x}-1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))*x^(1/2),x, algorithm="giac")
[Out] 2/3*(3*(sqrt(x) + 1)^2/(sqrt(x) - 1)^2 + 1)*log(-(sqrt(x) + 1)/(sqrt(x) - 1
))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^3 + 4/3*(sqrt(x) + 1)/((sqrt(x) - 1)*(
(sqrt(x) + 1)/(sqrt(x) - 1) - 1)^2) + 2/3*log((sqrt(x) + 1)/abs(sqrt(x) - 1
)) - 2/3*log(abs(-(sqrt(x) + 1)/(sqrt(x) - 1) + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\sqrt{x}\,\operatorname{atanh}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atanh(x^(1/2)),x)
[Out] int(x^(1/2)*atanh(x^(1/2)), x)
```

3.211 $\int \frac{\tanh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=20

$$2\sqrt{x}\,\tanh^{-1}\left(\sqrt{x}\right)+\log(1-x)$$

```
[Out] ln(1-x)+2*arctanh(x^(1/2))*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6037, 31}

$$\log(1-x)+2\sqrt{x}\,\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTanh[Sqrt[x]] + Log[1 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\sqrt{x}\,\tanh^{-1}\left(\sqrt{x}\right)-\int \frac{1}{1-x}\,dx\\ &= 2\sqrt{x}\,\tanh^{-1}\left(\sqrt{x}\right)+\log(1-x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$2\sqrt{x}\,\tanh^{-1}\left(\sqrt{x}\right)+\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Sqrt[x]]/Sqrt[x],x]
[Out] 2*Sqrt[x]*ArcTanh[Sqrt[x]] + Log[1 - x]
```

Maple [A]
time = 0.05, size = 17, normalized size = 0.85

method	result	size
derivativedivides	$\ln(1-x)+2\,\mathrm{arctanh}\left(\sqrt{x}\right)\sqrt{x}$	17
default	$\ln(1-x)+2\,\mathrm{arctanh}\left(\sqrt{x}\right)\sqrt{x}$	17
meijerg	$-\sqrt{x}\,\left(\ln\left(1-\sqrt{x}\right)-\ln\left(\sqrt{x}+1\right)\right)+\ln(1-x)$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(1-x)+2*arctanh(x^(1/2))*x^(1/2)
```

Maxima [A]
time = 0.27, size = 16, normalized size = 0.80

$$2\sqrt{x}\,\mathrm{artanh}\left(\sqrt{x}\right)+\log\left(-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x)*arctanh(sqrt(x)) + log(-x + 1)
```

Fricas [A]
time = 0.36, size = 25, normalized size = 1.25

$$\sqrt{x}\,\log\left(-\frac{x+2\sqrt{x}+1}{x-1}\right)+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] sqrt(x)*log(-(x + 2*sqrt(x) + 1)/(x - 1)) + log(x - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(17) = 34$.
time = 0.21, size = 87, normalized size = 4.35

$$\frac{2x^{\frac{3}{2}}\,\mathrm{atanh}\left(\sqrt{x}\right)}{x-1}-\frac{2\sqrt{x}\,\mathrm{atanh}\left(\sqrt{x}\right)}{x-1}+\frac{2x\log\left(\sqrt{x}+1\right)}{x-1}-\frac{2x\,\mathrm{atanh}\left(\sqrt{x}\right)}{x-1}-\frac{2\log\left(\sqrt{x}+1\right)}{x-1}+\frac{2\,\mathrm{atanh}\left(\sqrt{x}\right)}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*x**(3/2)*atanh(sqrt(x))/(x - 1) - 2*sqrt(x)*atanh(sqrt(x))/(x - 1) + 2*x*
log(sqrt(x) + 1)/(x - 1) - 2*x*atanh(sqrt(x))/(x - 1) - 2*log(sqrt(x) + 1)/
(x - 1) + 2*atanh(sqrt(x))/(x - 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(16) = 32$.
time = 0.47, size = 72, normalized size = 3.60

$$\frac{2 \log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\frac{\sqrt{x}+1}{\sqrt{x}-1}-1} + 2 \log\left(\frac{\sqrt{x}+1}{|\sqrt{x}-1|}\right) - 2 \log\left(\left|-\frac{\sqrt{x}+1}{\sqrt{x}-1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(-(sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1) + 2*l
og((sqrt(x) + 1)/abs(sqrt(x) - 1)) - 2*log(abs(-(sqrt(x) + 1)/(sqrt(x) - 1)
 + 1))
```

Mupad [B]
time = 0.80, size = 14, normalized size = 0.70

$$\ln(x-1) + 2\sqrt{x}\ \mathrm{atanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x^(1/2))/x^(1/2),x)
```

```
[Out] log(x - 1) + 2*x^(1/2)*atanh(x^(1/2))
```

3.212 $\int \frac{\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=24

$$-\frac{2\tanh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}-\log(1-x)+\log(x)$$

```
[Out] -ln(1-x)+ln(x)-2*arctanh(x^(1/2))/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {6037, 36, 31, 29}

$$-\log(1-x)+\log(x)-\frac{2\tanh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Sqrt[x]]/x^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx &= -\frac{2\tanh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}+\int \frac{1}{(1-x)x}\,dx\\ &= -\frac{2\tanh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}+\int \frac{1}{1-x}\,dx+\int \frac{1}{x}\,dx\\ &= -\frac{2\tanh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}-\log(1-x)+\log(x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}-\log(1-x)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Sqrt[x]]/x^(3/2),x]
[Out] (-2*ArcTanh[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]
```

Maple [A]
time = 0.07, size = 29, normalized size = 1.21

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\sqrt{x}\right)}{\sqrt{x}}+\ln\left(x\right)-\ln\left(\sqrt{x}+1\right)-\ln\left(\sqrt{x}-1\right)$	29
default	$-\frac{2\,\mathrm{arctanh}\left(\sqrt{x}\right)}{\sqrt{x}}+\ln\left(x\right)-\ln\left(\sqrt{x}+1\right)-\ln\left(\sqrt{x}-1\right)$	29
meijerg	$\frac{\ln\left(1-\sqrt{x}\right)-\ln\left(\sqrt{x}+1\right)}{\sqrt{x}}-\ln\left(1-x\right)+\ln\left(x\right)+i\pi$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x^(1/2))/x^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh(x^(1/2))/x^(1/2)+ln(x)-ln(x^(1/2)+1)-ln(x^(1/2)-1)
```

Maxima [A]
time = 0.25, size = 18, normalized size = 0.75

$$-\frac{2\,\mathrm{artanh}\left(\sqrt{x}\right)}{\sqrt{x}}-\log\left(x-1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(3/2),x, algorithm="maxima")
```

```
[Out] -2*arctanh(sqrt(x))/sqrt(x) - log(x - 1) + log(x)
```

Fricas [A]
time = 0.38, size = 37, normalized size = 1.54

$$-\frac{x\log\left(x-1\right)-x\log\left(x\right)+\sqrt{x}\,\log\left(-\frac{x+2\sqrt{x}+1}{x-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(3/2),x, algorithm="fricas")
```

```
[Out] -(x*log(x - 1) - x*log(x) + sqrt(x)*log(-(x + 2*sqrt(x) + 1)/(x - 1)))/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(20) = 40$.
time = 0.42, size = 126, normalized size = 5.25

$$-\frac{2x^{\frac{3}{2}}\operatorname{atanh}\left(\sqrt{x}\right)}{x^2-x}+\frac{2\sqrt{x}\operatorname{atanh}\left(\sqrt{x}\right)}{x^2-x}+\frac{x^2\log\left(x\right)}{x^2-x}-\frac{2x^2\log\left(\sqrt{x}+1\right)}{x^2-x}+\frac{2x^2\operatorname{atanh}\left(\sqrt{x}\right)}{x^2-x}-\frac{x\log\left(x\right)}{x^2-x}+\frac{2x\log\left(\sqrt{x}+1\right)}{x^2-x}-\frac{2x\operatorname{atanh}\left(\sqrt{x}\right)}{x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x**(1/2))/x**(3/2),x)
```

```
[Out] -2*x**(3/2)*atanh(sqrt(x))/(x**2 - x) + 2*sqrt(x)*atanh(sqrt(x))/(x**2 - x)
 + x**2*log(x)/(x**2 - x) - 2*x**2*log(sqrt(x) + 1)/(x**2 - x) + 2*x**2*ata
nh(sqrt(x))/(x**2 - x) - x*log(x)/(x**2 - x) + 2*x*log(sqrt(x) + 1)/(x**2 -
 x) - 2*x*atanh(sqrt(x))/(x**2 - x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(20) = 40$.
time = 0.42, size = 72, normalized size = 3.00

$$\frac{2\log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\frac{\sqrt{x}+1}{\sqrt{x}-1}+1}-2\log\left(\frac{\sqrt{x}+1}{\left|\sqrt{x}-1\right|}\right)+2\log\left(\left|-\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] 2*log(-(sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) + 1) - 2*l
og((sqrt(x) + 1)/abs(sqrt(x) - 1)) + 2*log(abs(-(sqrt(x) + 1)/(sqrt(x) - 1)
 - 1))
```

Mupad [B]
time = 0.79, size = 22, normalized size = 0.92

$$2\,\ln\left(\sqrt{x}\right)-\ln\left(x-1\right)-\frac{2\,\mathrm{atanh}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x^(1/2))/x^(3/2),x)
[Out] 2*log(x^(1/2)) - log(x - 1) - (2*atanh(x^(1/2)))/x^(1/2)
```

3.213 $\int x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)\,dx$

Optimal. Leaf size=190

$$\frac{3bx^{5/2}}{20c}-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1-2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\,\sqrt{x}\right)}{4c^{8/3}}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\right.$$

```
[Out] 3/20*b*x^(5/2)/c+1/4*x^4*(a+b*arctanh(c*x^(3/2)))-1/4*b*arctanh(c^(1/3)*x^(
1/2))/c^(8/3)+1/16*b*ln(1+c^(2/3)*x-c^(1/3)*x^(1/2))/c^(8/3)-1/16*b*ln(1+c^
(2/3)*x+c^(1/3)*x^(1/2))/c^(8/3)-1/8*b*arctan(1/3*(1-2*c^(1/3)*x^(1/2))*3^(
1/2))*3^(1/2)/c^(8/3)+1/8*b*arctan(1/3*(1+2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/
2)/c^(8/3)
```

Rubi [A]
time = 0.25, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {6037, 327, 335, 302, 648, 632, 210, 642, 212}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1-2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{2\sqrt[3]{c}\,\sqrt{x}+1}{\sqrt{3}}\right)}{8c^{8/3}}+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\,\sqrt{x}+1\right)}{16c^{8/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\,\sqrt{x}+1\right)}{16c^{8/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\,\sqrt{x}\right)}{4c^{8/3}}+\frac{3bx^{5/2}}{20c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x^(3/2)]),x]
```

```
[Out] (3*b*x^(5/2))/(20*c) - (Sqrt[3]*b*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/
(8*c^(8/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(8*c^(8/3
)) - (b*ArcTanh[c^(1/3)*Sqrt[x]])/(4*c^(8/3)) + (x^4*(a + b*ArcTanh[c*x^(3/
2)]))/4 + (b*Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x])/(16*c^(8/3)) - (b*Log[1
+ c^(1/3)*Sqrt[x] + c^(2/3)*x])/(16*c^(8/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
```

```
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
```

```
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)\,dx=\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{1}{8}(3bc)\int\frac{x^{9/2}}{1-c^2x^3}\,dx$$
$$=\frac{3bx^{5/2}}{20c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{(3b)\int\frac{x^{3/2}}{1-c^2x^3}\,dx}{8c}$$
$$=\frac{3bx^{5/2}}{20c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{(3b)\text{Subst}\left(\int\frac{x^4}{1-c^2x^6}\,dx,x,\sqrt{x}\right)}{4c}$$
$$=\frac{3bx^{5/2}}{20c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{b\text{Subst}\left(\int\frac{1}{1-c^{2/3}x^2}\,dx,x,\sqrt{x}\right)}{4c^{7/3}}-\frac{b\text{S}}{}$$
$$=\frac{3bx^{5/2}}{20c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{4c^{8/3}}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\text{Subst}\left(\int\frac{-}{1-}\right.}{}$$
$$=\frac{3bx^{5/2}}{20c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{4c^{8/3}}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\log\left(1-\sqrt[3]{c}\right.}{16}$$
$$=\frac{3bx^{5/2}}{20c}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}}-\frac{b\tan}{}$$

Mathematica [A]
time = 0.04, size = 222, normalized size = 1.17

$$\frac{3bx^{5/2}}{20c}+\frac{ax^4}{4}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{-1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}}+\frac{1}{4}bx^4\tanh^{-1}\left(cx^{3/2}\right)+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}\right)}{8c^{8/3}}-\frac{b\log\left(1+\sqrt[3]{c}\sqrt{x}\right)}{8c^{8/3}}+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)}{16c^{8/3}}-\frac{b\log\left(1+\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)}{16c^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x^(3/2)]),x]
```

```
[Out] (3*b*x^(5/2))/(20*c) + (a*x^4)/4 + (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*Sqrt[x
])/Sqrt[3]])/(8*c^(8/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3
]])/(8*c^(8/3)) + (b*x^4*ArcTanh[c*x^(3/2)])/4 + (b*Log[1 - c^(1/3)*Sqrt[x]
])/(8*c^(8/3)) - (b*Log[1 + c^(1/3)*Sqrt[x]])/(8*c^(8/3)) + (b*Log[1 - c^(1
/3)*Sqrt[x] + c^(2/3)*x])/(16*c^(8/3)) - (b*Log[1 + c^(1/3)*Sqrt[x] + c^(2/
3)*x])/(16*c^(8/3))
```

Maple [A]
time = 0.05, size = 194, normalized size = 1.02

method	result
derivativedivides	$\frac{x^{4} a}{4}+\frac{b\,x^{4} \operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{4}+\frac{3 b\,x^{\frac{5}{2}}}{20 c}-\frac{b \ln\left(\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{8 c^{3} \left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b \ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}} \sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{16 c^{3} \left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\cdots\right.}{8c^{3}\cdots}$
default	$\frac{x^{4} a}{4}+\frac{b\,x^{4} \operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{4}+\frac{3 b\,x^{\frac{5}{2}}}{20 c}-\frac{b \ln\left(\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{8 c^{3} \left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b \ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}} \sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{16 c^{3} \left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\cdots\right.}{8c^{3}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^(3/2))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctanh(c*x^(3/2))+3/20*b*x^(5/2)/c-1/8*b/c^3/(1/c)^(1/
3)*ln(x^(1/2)+(1/c)^(1/3))+1/16*b/c^3/(1/c)^(1/3)*ln(x-(1/c)^(1/3)*x^(1/2)+
(1/c)^(2/3))+1/8*b/c^3*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3
)*x^(1/2)-1))+1/8*b/c^3/(1/c)^(1/3)*ln(x^(1/2)-(1/c)^(1/3))-1/16*b/c^3/(1/c
)^(1/3)*ln(x+(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))+1/8*b/c^3*3^(1/2)/(1/c)^(1/3)
*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)+1))
```

Maxima [A]
time = 0.47, size = 172, normalized size = 0.91

$$\frac{1}{4} a x^{4}+\frac{1}{80}\left(20 x^{4} \operatorname{artanh}\left(c x^{\frac{3}{2}}\right)+c\left(\frac{12 x^{\frac{5}{2}}}{c^{2}}+\frac{10 \sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2 c^{\frac{2}{3}} \sqrt{x}+c^{\frac{1}{3}}\right)}{3 c^{\frac{1}{3}}}\right)}{c^{\frac{11}{3}}}+\frac{10 \sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2 c^{\frac{2}{3}} \sqrt{x}-c^{\frac{1}{3}}\right)}{3 c^{\frac{1}{3}}}\right)}{c^{\frac{11}{3}}}-\frac{5 \log\left(c^{\frac{2}{3}} x+c^{\frac{1}{3}} \sqrt{x}+1\right)}{c^{\frac{11}{3}}}+\frac{5 \log\left(c^{\frac{2}{3}} x-c^{\frac{1}{3}} \sqrt{x}+1\right)}{c^{\frac{11}{3}}}-\frac{10 \log\left(\frac{c^{\frac{1}{3}} \sqrt{x}+1}{c^{\frac{1}{3}}}\right)}{c^{\frac{11}{3}}}+\frac{10 \log\left(\frac{c^{\frac{1}{3}} \sqrt{x}-1}{c^{\frac{1}{3}}}\right)}{c^{\frac{11}{3}}}\right)\right) b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(3/2))),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/80*(20*x^4*arctanh(c*x^(3/2)) + c*(12*x^(5/2)/c^2 + 10*sqrt(3
)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) + c^(1/3))/c^(1/3))/c^(11/3) + 10*s
qrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) - c^(1/3))/c^(1/3))/c^(11/3) -
 5*log(c^(2/3)*x + c^(1/3)*sqrt(x) + 1)/c^(11/3) + 5*log(c^(2/3)*x - c^(1/3
)*sqrt(x) + 1)/c^(11/3) - 10*log((c^(1/3)*sqrt(x) + 1)/c^(1/3))/c^(11/3) +
10*log((c^(1/3)*sqrt(x) - 1)/c^(1/3))/c^(11/3)))*b
```

Fricas [C] Result contains complex when optimal does not.
time = 1.18, size = 1803, normalized size = 9.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(3/2))),x, algorithm="fricas")

[Out] 1/160*(40*a*c*x^4 + 24*b*x^(5/2) - 20*sqrt(3)*sqrt(((1/2)^(1/3)*(b^3 - (c^8
 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b
^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2)*c
*arctan(1/24*(4*sqrt(3)*sqrt(((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^
8)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*b^2*c^5*sqrt(x) + 4*b^4*c^5*sqrt(x) + 4*b
^4*c^2 + 4*b^4*x - 2*(2*b^3*c^5*sqrt(x) + b^3*c^2)*((1/2)^(1/3)*(b^3 - (c^8
 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b))*sqrt(((1/2)^(1/3)*(b
^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)
^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b +
 4*b^2)*c^3 - sqrt(3)*(((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/
3)*(I*sqrt(3) + 1) + 2*b)^2*c^8 - 4*((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 +
 b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b*c^8 + 4*b^2*c^8 + 8*b^2*c^3*sqrt(x
))*sqrt(((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) +
 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*
sqrt(3) + 1) + 2*b)*b + 4*b^2))/b^3) - 10*((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3
/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c*log(-1/4*((1/2)^(1/3)*(b^3 -
 (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c^5 + ((1/2)^(
1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b*c^5
 - b^2*c^5 + b^2*sqrt(x)) - 20*(4*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 +
 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)*c*log((4*(-1/1024*b^3 + 1/1024*
(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^5 + 2*(4
*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3)
 + 1) - b)*b*c^5 + b^2*c^5 + b^2*sqrt(x)) - 40*sqrt(3*(4*(-1/1024*b^3 + 1/1
024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)^2 + 6*(4
*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3)
 + 1) - b)*b + 3*b^2)*c*arctan(1/3*((4*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/
c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^8 + 2*(4*(-1/1024*b^3
+ 1/1024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)*b*c
^8 + b^2*c^8 - 2*b^2*c^3*sqrt(x) + sqrt(-4*(4*(-1/1024*b^3 + 1/1024*(c^8 -
1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)^2*b^2*c^5*sqrt(x) -
 4*b^4*c^5*sqrt(x) + 4*b^4*c^2 + 4*b^4*x - 4*(2*b^3*c^5*sqrt(x) - b^3*c^2)*
(4*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(
3) + 1) - b))*c^3)*sqrt(3*(4*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 + 1/10
24*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)^2 + 6*(4*(-1/1024*b^3 + 1/1024*(c^8
- 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)*b + 3*b^2)/b^3) +
 5*(((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1)
+ 2*b)*c - 6*b*c)*log(((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3
)*(I*sqrt(3) + 1) + 2*b)^2*b^2*c^5*sqrt(x) + 4*b^4*c^5*sqrt(x) + 4*b^4*c^2
+ 4*b^4*x - 2*(2*b^3*c^5*sqrt(x) + b^3*c^2)*((1/2)^(1/3)*(b^3 - (c^8 - 1)*b
^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)) + 10*((4*(-1/1024*b^3 + 1/1
024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)*c + 3*b*
c)*log(-4*(4*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3
)*(I*sqrt(3) + 1) - b)^2*b^2*c^5*sqrt(x) - 4*b^4*c^5*sqrt(x) + 4*b^4*c^2 +
```

```
4*b^4*x - 4*(2*b^3*c^5*sqrt(x) - b^3*c^2)*(4*(-1/1024*b^3 + 1/1024*(c^8 - 1
)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)) + 20*(b*c*x^4 - b*c
)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)))/c
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**(3/2))),x)
```

```
[Out] Timed out
```

Giac [**C**] Result contains complex when optimal does not.
time = 0.49, size = 227, normalized size = 1.19

$$\frac{1}{4}\,ax^{4}+\frac{1}{320}\left(40\,x^{4}\log\left(-\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}\right)+c\left(\frac{48\,x^{\frac{5}{2}}}{c^{2}}-\frac{10\,\sqrt{3}\left(-i\,\sqrt{3}-1\right)^{2}|c|^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,\sqrt{x}+\left(-\frac{1}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{1}{c}\right)^{\frac{1}{3}}}\right)}{c^{5}}+\frac{5\left(-i\,\sqrt{3}-1\right)^{2}|c|^{\frac{4}{3}}\log\left(x+\sqrt{x}\left(-\frac{1}{c}\right)^{\frac{1}{3}}+\left(-\frac{1}{c}\right)^{\frac{2}{3}}\right)}{c^{5}}-\frac{40\left(-\frac{1}{c}\right)^{\frac{2}{3}}\log\left(\left|\sqrt{x}-\left(-\frac{1}{c}\right)^{\frac{1}{3}}\right|\right)}{c^{3}}+\frac{40\,\sqrt{3}\,|c|^{\frac{4}{3}}\arctan\left(\frac{1}{3}\,\sqrt{3}\,c^{\frac{1}{3}}\left(2\,\sqrt{x}+\frac{1}{c^{\frac{1}{3}}}\right)\right)}{c^{5}}-\frac{20\,|c|^{\frac{4}{3}}\log\left(x+\frac{\sqrt{x}}{c^{\frac{1}{3}}}+\frac{1}{c^{\frac{2}{3}}}\right)}{c^{5}}+\frac{40\log\left(\left|\sqrt{x}-\frac{1}{c^{\frac{1}{3}}}\right|\right)}{c^{\frac{11}{3}}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(3/2))),x, algorithm="giac")
```

```
[Out] 1/4*a*x^4 + 1/320*(40*x^4*log(-(c*x^(3/2) + 1)/(c*x^(3/2) - 1)) + c*(48*x^(
5/2)/c^2 - 10*sqrt(3)*(-I*sqrt(3) - 1)^2*abs(c)^(4/3)*arctan(1/3*sqrt(3)*(2
*sqrt(x) + (-1/c)^(1/3))/(-1/c)^(1/3))/c^5 + 5*(-I*sqrt(3) - 1)^2*abs(c)^(4
/3)*log(x + sqrt(x)*(-1/c)^(1/3) + (-1/c)^(2/3))/c^5 - 40*(-1/c)^(2/3)*log(
abs(sqrt(x) - (-1/c)^(1/3)))/c^3 + 40*sqrt(3)*abs(c)^(4/3)*arctan(1/3*sqrt(
3)*c^(1/3)*(2*sqrt(x) + 1/c^(1/3)))/c^5 - 20*abs(c)^(4/3)*log(x + sqrt(x)/c
^(1/3) + 1/c^(2/3))/c^5 + 40*log(abs(sqrt(x) - 1/c^(1/3)))/c^(11/3)))*b
```

Mupad [**B**]
time = 13.38, size = 231, normalized size = 1.22

$$\frac{a\,x^4}{4}+\frac{3\,b\,x^{5/2}}{20\,c}+\frac{b\,\ln\left(\frac{c^{1/3}\,\sqrt{x}-1}{c^{1/3}\,\sqrt{x}+1}\right)}{8\,c^{8/3}}+\frac{\ln\left(1-c\,x^{3/2}\right)\,\left(\frac{b\,x^4}{4}-\frac{b\,c^2\,x^7}{4}\right)}{2\,c^2\,x^3-2}+\frac{b\,x^4\,\ln\left(c\,x^{3/2}+1\right)}{8}+\frac{b\,\ln\left(\frac{\sqrt{3}+c^{2/3}\,x\,1{}\mathrm{i}-c^{1/3}\,\sqrt{x}\,4{}\mathrm{i}-\sqrt{3}\,c^{2/3}\,x+1{}\mathrm{i}}{2\,c^{2/3}\,x+1+\sqrt{3}\,1{}\mathrm{i}}\right)\,\sqrt{-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}{8\,c^{8/3}}+\frac{\sqrt{2}\,b\,\ln\left(\frac{\sqrt{3}\,c^{2/3}\,x+c^{2/3}\,x\,1{}\mathrm{i}+c^{1/3}\,\sqrt{x}\,4{}\mathrm{i}-\sqrt{3}+1{}\mathrm{i}}{2\,c^{2/3}\,x+1-\sqrt{3}\,1{}\mathrm{i}}\right)\,\sqrt{1+\sqrt{3}\,1{}\mathrm{i}}\,1{}\mathrm{i}}{16\,c^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x^(3/2))),x)
```

```
[Out] (a*x^4)/4 + (3*b*x^(5/2))/(20*c) + (b*log((c^(1/3)*x^(1/2) - 1)/(c^(1/3)*x^
(1/2) + 1)))/(8*c^(8/3)) + (log(1 - c*x^(3/2))*((b*x^4)/4 - (b*c^2*x^7)/4))
/(2*c^2*x^3 - 2) + (b*x^4*log(c*x^(3/2) + 1))/8 + (b*log((3^(1/2) + c^(2/3)
*x*1i - c^(1/3)*x^(1/2)*4i - 3^(1/2)*c^(2/3)*x + 1i)/(3^(1/2)*1i + 2*c^(2/3
)*x + 1))*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(8*c^(8/3)) + (2^(1/2)*b*log((c^(2/
3)*x*1i - 3^(1/2) + c^(1/3)*x^(1/2)*4i + 3^(1/2)*c^(2/3)*x + 1i)/(2*c^(2/3)
*x - 3^(1/2)*1i + 1))*(3^(1/2)*1i + 1)^(1/2)*1i)/(16*c^(8/3))
```

3.214 $\int x^2 \left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right) \, dx$

Optimal. Leaf size=49

$$\frac{bx^{3/2}}{3c}-\frac{b\tanh^{-1}\left(cx^{3/2}\right)}{3c^2}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)$$

```
[Out] 1/3*b*x^(3/2)/c-1/3*b*arctanh(c*x^(3/2))/c^2+1/3*x^3*(a+b*arctanh(c*x^(3/2)
))
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6037, 327, 335, 281, 212}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{b\tanh^{-1}\left(cx^{3/2}\right)}{3c^2}+\frac{bx^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^(3/2)]),x]
```

```
[Out] (b*x^(3/2))/(3*c) - (b*ArcTanh[c*x^(3/2)])/(3*c^2) + (x^3*(a + b*ArcTanh[c*
x^(3/2)]))/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)\,dx &= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{1}{2}(bc)\int\frac{x^{7/2}}{1-c^2x^3}\,dx\\
&= \frac{bx^{3/2}}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{b\int\frac{\sqrt{x}}{1-c^2x^3}\,dx}{2c}\\
&= \frac{bx^{3/2}}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{b\text{Subst}\left(\int\frac{x^2}{1-c^2x^6}\,dx,x,\sqrt{x}\right)}{c}\\
&= \frac{bx^{3/2}}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{b\text{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,x^{3/2}\right)}{3c}\\
&= \frac{bx^{3/2}}{3c}-\frac{b\tanh^{-1}\left(cx^{3/2}\right)}{3c^2}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 75, normalized size = 1.53

$$\frac{bx^{3/2}}{3c}+\frac{ax^3}{3}+\frac{1}{3}bx^3\tanh^{-1}\left(cx^{3/2}\right)+\frac{b\log\left(1-cx^{3/2}\right)}{6c^2}-\frac{b\log\left(1+cx^{3/2}\right)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^(3/2)]),x]
```

```
[Out] (b*x^(3/2))/(3*c) + (a*x^3)/3 + (b*x^3*ArcTanh[c*x^(3/2)])/3 + (b*Log[1 - c
*x^(3/2)])/(6*c^2) - (b*Log[1 + c*x^(3/2)])/(6*c^2)
```

Maple [A]
time = 0.10, size = 60, normalized size = 1.22

method	result	size

derivativedivides	$\frac{\frac{c^2x^3a}{3}+\frac{b\,c^2x^3\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{3}+\frac{c\,x^{\frac{3}{2}}b}{3}+\frac{b\ln\left(c\,x^{\frac{3}{2}}-1\right)}{6}-\frac{b\ln\left(c\,x^{\frac{3}{2}}+1\right)}{6}}{c^2}$	60
default	$\frac{\frac{c^2x^3a}{3}+\frac{b\,c^2x^3\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{3}+\frac{c\,x^{\frac{3}{2}}b}{3}+\frac{b\ln\left(c\,x^{\frac{3}{2}}-1\right)}{6}-\frac{b\ln\left(c\,x^{\frac{3}{2}}+1\right)}{6}}{c^2}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^(3/2))),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/c^2*(1/2*c^2*x^3*a+1/2*b*c^2*x^3*arctanh(c*x^(3/2))+1/2*c*x^(3/2)*b+1/4
*b*ln(c*x^(3/2)-1)-1/4*b*ln(c*x^(3/2)+1))
```

Maxima [A]
time = 0.25, size = 58, normalized size = 1.18

$$\frac{1}{3}\,ax^3+\frac{1}{6}\left(2\,x^3\,\mathrm{artanh}\left(cx^{\frac{3}{2}}\right)+c\left(\frac{2\,x^{\frac{3}{2}}}{c^2}-\frac{\log\left(cx^{\frac{3}{2}}+1\right)}{c^3}+\frac{\log\left(cx^{\frac{3}{2}}-1\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(3/2))),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctanh(c*x^(3/2)) + c*(2*x^(3/2)/c^2 - log(c*x^(3/2
) + 1)/c^3 + log(c*x^(3/2) - 1)/c^3))*b
```

Fricas [A]
time = 0.34, size = 64, normalized size = 1.31

$$\frac{2\,ac^2x^3+2\,bcx^{\frac{3}{2}}+\left(bc^2x^3-b\right)\log\left(-\frac{c^2x^3+2\,cx^{\frac{3}{2}}+1}{c^2x^3-1}\right)}{6\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(3/2))),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^2*x^3 + 2*b*c*x^(3/2) + (b*c^2*x^3 - b)*log(-(c^2*x^3 + 2*c*x^(3
/2) + 1)/(c^2*x^3 - 1)))/c^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**(3/2))),x)
```

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(37) = 74.
time = 0.42, size = 97, normalized size = 1.98

$$\frac{1}{3}ax^3+\frac{2}{3}bc\left(\frac{1}{c^3\left(\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}-1\right)}+\frac{\left(cx^{\frac{3}{2}}+1\right)\log\left(-\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}\right)}{\left(cx^{\frac{3}{2}}-1\right)c^3\left(\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(3/2))),x, algorithm="giac")
```

```
[Out] 1/3*a*x^3 + 2/3*b*c*(1/(c^3*((c*x^(3/2) + 1)/(c*x^(3/2) - 1) - 1)) + (c*x^(
3/2) + 1)*log(-(c*x^(3/2) + 1)/(c*x^(3/2) - 1))/((c*x^(3/2) - 1)*c^3*((c*x^
(3/2) + 1)/(c*x^(3/2) - 1) - 1)^2))
```

Mupad [B]
time = 1.76, size = 110, normalized size = 2.24

$$\frac{a\,x^3}{3}+\frac{b\,x^{3/2}}{3\,c}+\frac{b\,\ln\left(\frac{c\,x^{3/2}-1}{c\,x^{3/2}+1}\right)}{6\,c^2}+\frac{b\,x^3\,\ln\left(c\,x^{3/2}+1\right)}{6}+\frac{b\,x^3\,\ln\left(1-c\,x^{3/2}\right)}{3\,\left(2\,c^2\,x^3-2\right)}-\frac{b\,c^2\,x^6\,\ln\left(1-c\,x^{3/2}\right)}{3\,\left(2\,c^2\,x^3-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^(3/2))),x)
```

```
[Out] (a*x^3)/3 + (b*x^(3/2))/(3*c) + (b*log((c*x^(3/2) - 1)/(c*x^(3/2) + 1)))/(6
*c^2) + (b*x^3*log(c*x^(3/2) + 1))/6 + (b*x^3*log(1 - c*x^(3/2)))/(3*(2*c^2
*x^3 - 2)) - (b*c^2*x^6*log(1 - c*x^(3/2)))/(3*(2*c^2*x^3 - 2))
```

3.215 $\int x\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)\,dx$

Optimal. Leaf size=190

$$\frac{3b\sqrt{x}}{2c}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1-2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\,\sqrt{x}\right)}{2c^{4/3}}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\right.$$

```
[Out] 1/2*x^2*(a+b*arctanh(c*x^(3/2)))-1/2*b*arctanh(c^(1/3)*x^(1/2))/c^(4/3)+1/8
*b*ln(1+c^(2/3)*x-c^(1/3)*x^(1/2))/c^(4/3)-1/8*b*ln(1+c^(2/3)*x+c^(1/3)*x^(
1/2))/c^(4/3)+1/4*b*arctan(1/3*(1-2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)/c^(4/
3)-1/4*b*arctan(1/3*(1+2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)/c^(4/3)+3/2*b*x^
(1/2)/c
```

Rubi [A]
time = 0.25, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6037, 327, 335, 216, 648, 632, 210, 642, 212}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1-2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{2\sqrt[3]{c}\,\sqrt{x}+1}{\sqrt{3}}\right)}{4c^{4/3}}+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\,\sqrt{x}+1\right)}{8c^{4/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\,\sqrt{x}+1\right)}{8c^{4/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\,\sqrt{x}\right)}{2c^{4/3}}+\frac{3b\sqrt{x}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x^(3/2)]),x]
```

```
[Out] (3*b*Sqrt[x])/(2*c) + (Sqrt[3]*b*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(
4*c^(4/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(4*c^(4/3)
) - (b*ArcTanh[c^(1/3)*Sqrt[x]])/(2*c^(4/3)) + (x^2*(a + b*ArcTanh[c*x^(3/2
)]))/2 + (b*Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x])/(8*c^(4/3)) - (b*Log[1 +
c^(1/3)*Sqrt[x] + c^(2/3)*x])/(8*c^(4/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
```

```
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)\,dx &= \frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{1}{4}(3bc)\int \frac{x^{5/2}}{1-c^2x^3}\,dx\\
&= \frac{3b\sqrt{x}}{2c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{(3b)\int \frac{1}{\sqrt{x}\,(1-c^2x^3)}\,dx}{4c}\\
&= \frac{3b\sqrt{x}}{2c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{(3b)\text{Subst}\left(\int \frac{1}{1-c^2x^6}\,dx,x,\sqrt{x}\right)}{2c}\\
&= \frac{3b\sqrt{x}}{2c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{b\text{Subst}\left(\int \frac{1}{1-c^{2/3}x^2}\,dx,x,\sqrt{x}\right)}{2c}-\frac{b\text{Su}}{}\\
&= \frac{3b\sqrt{x}}{2c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{2c^{4/3}}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\text{Subst}\left(\int \frac{-1}{1-}\right.}{}\\
&= \frac{3b\sqrt{x}}{2c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{2c^{4/3}}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\log\left(1-\sqrt[3]{c}\right.}{8c}\\
&= \frac{3b\sqrt{x}}{2c}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}-\frac{b\tan}{}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 222, normalized size = 1.17

$$
\frac{3b\sqrt{x}}{2c}+\frac{ax^2}{2}-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{-1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}+\frac{1}{2}bx^2\tanh^{-1}\left(cx^{3/2}\right)+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}\right)}{4c^{4/3}}-\frac{b\log\left(1+\sqrt[3]{c}\sqrt{x}\right)}{4c^{4/3}}+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)}{8c^{4/3}}-\frac{b\log\left(1+\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)}{8c^{4/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^(3/2)]),x]
```

```
[Out] (3*b*Sqrt[x])/(2*c) + (a*x^2)/2 - (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*Sqrt[x]
)/Sqrt[3]])/(4*c^(4/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]
])/(4*c^(4/3)) + (b*x^2*ArcTanh[c*x^(3/2)])/2 + (b*Log[1 - c^(1/3)*Sqrt[x]]
)/(4*c^(4/3)) - (b*Log[1 + c^(1/3)*Sqrt[x]])/(4*c^(4/3)) + (b*Log[1 - c^(1/
3)*Sqrt[x] + c^(2/3)*x])/(8*c^(4/3)) - (b*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3)
*x])/(8*c^(4/3))
```

Maple [A]
time = 0.06, size = 194, normalized size = 1.02

method	result

derivativedivides	$\frac{a\,x^{2}}{2}+\frac{b\,x^{2}\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{2}+\frac{3b\sqrt{x}}{2c}+\frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4c^{2}\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8c^{2}\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\sqrt{3}\,\arctan}{}$
default	$\frac{a\,x^{2}}{2}+\frac{b\,x^{2}\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{2}+\frac{3b\sqrt{x}}{2c}+\frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4c^{2}\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8c^{2}\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\sqrt{3}\,\arctan}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^(3/2))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctanh(c*x^(3/2))+3/2*b*x^(1/2)/c+1/4*b/c^2/(1/c)^(2/3
)*ln(x^(1/2)-(1/c)^(1/3))-1/8*b/c^2/(1/c)^(2/3)*ln(x+(1/c)^(1/3)*x^(1/2)+(1
/c)^(2/3))-1/4*b/c^2/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*
x^(1/2)+1))-1/4*b/c^2/(1/c)^(2/3)*ln(x^(1/2)+(1/c)^(1/3))+1/8*b/c^2/(1/c)^(
2/3)*ln(x-(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))-1/4*b/c^2/(1/c)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)-1))
```

Maxima [A]
time = 0.47, size = 172, normalized size = 0.91

$$\frac{1}{2}\,ax^{2}+\frac{1}{8}\left(4\,x^{2}\,\mathrm{artanh}\left(cx^{\frac{3}{2}}\right)-c\left(\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}\sqrt{x}+c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)}{c^{\frac{7}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}\sqrt{x}-c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)}{c^{\frac{7}{3}}}+\frac{\log\left(c^{\frac{2}{3}}x+c^{\frac{1}{3}}\sqrt{x}+1\right)}{c^{\frac{7}{3}}}-\frac{\log\left(c^{\frac{2}{3}}x-c^{\frac{1}{3}}\sqrt{x}+1\right)}{c^{\frac{7}{3}}}+\frac{2\,\log\left(\frac{c^{\frac{1}{3}}\sqrt{x}+1}{c^{\frac{1}{3}}}\right)}{c^{\frac{7}{3}}}-\frac{2\,\log\left(\frac{c^{\frac{1}{3}}\sqrt{x}-1}{c^{\frac{1}{3}}}\right)}{c^{\frac{7}{3}}}-\frac{12\,\sqrt{x}}{c^{2}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(3/2))),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/8*(4*x^2*arctanh(c*x^(3/2)) - c*(2*sqrt(3)*arctan(1/3*sqrt(3)
*(2*c^(2/3)*sqrt(x) + c^(1/3))/c^(1/3))/c^(7/3) + 2*sqrt(3)*arctan(1/3*sqrt
(3)*(2*c^(2/3)*sqrt(x) - c^(1/3))/c^(1/3))/c^(7/3) + log(c^(2/3)*x + c^(1/3
)*sqrt(x) + 1)/c^(7/3) - log(c^(2/3)*x - c^(1/3)*sqrt(x) + 1)/c^(7/3) + 2*l
og((c^(1/3)*sqrt(x) + 1)/c^(1/3))/c^(7/3) - 2*log((c^(1/3)*sqrt(x) - 1)/c^(
1/3))/c^(7/3) - 12*sqrt(x)/c^2))*b
```

Fricas [C] Result contains complex when optimal does not.
time = 1.22, size = 1848, normalized size = 9.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(3/2))),x, algorithm="fricas")

[Out] 1/16*(8*a*c*x^2 + 4*sqrt(3)*sqrt(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^
3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3
/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2)*c*arctan(-1/24*(2*s
qrt(3)*sqrt(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(
3) + 1) + 2*b)^2*c^2 + 4*b^2*c^2 + 4*b^2*c*sqrt(x) + 4*b^2*x - 2*(2*b*c^2 +
 b*c*sqrt(x))*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqr
t(3) + 1) + 2*b))*(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(
I*sqrt(3) + 1) + 2*b)*c^3 - 2*b*c^3)*sqrt(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3
/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^4
 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2) + sqrt(3)*
(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2
*b)^2*c^4 + 4*b^2*c^4 + 8*b^2*c^3*sqrt(x) - 4*(b*c^4 + b*c^3*sqrt(x))*((1/2
)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b))*s
qrt(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1)
+ 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt
(3) + 1) + 2*b)*b + 4*b^2))/b^3) - 2*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4
+ b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c*log(1/2*((1/2)^(1/3)*(b^3 - (c^4
- 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c - b*c + b*sqrt(x)) -
 4*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(
3) + 1) - b)*c*log((2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4
)^(1/3)*(I*sqrt(3) + 1) - b)*c + b*c + b*sqrt(x)) - 8*sqrt(3*(2*(-1/128*b^3
 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)^2 +
6*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3
) + 1) - b)*b + 3*b^2)*c*arctan(-1/3*(2*sqrt((2*(-1/128*b^3 + 1/128*(c^4 -
1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^2 + b^2*c^2 - b^
2*c*sqrt(x) + b^2*x + (2*b*c^2 - b*c*sqrt(x))*(2*(-1/128*b^3 + 1/128*(c^4 -
 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b))*((2*(-1/128*b^3 +
1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)*c^3 + b
*c^3)*sqrt(3*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3
)*(I*sqrt(3) + 1) - b)^2 + 6*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/1
28*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)*b + 3*b^2) + ((2*(-1/128*b^3 + 1/128
*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^4 + b^2*
c^4 - 2*b^2*c^3*sqrt(x) + 2*(b*c^4 - b*c^3*sqrt(x))*(2*(-1/128*b^3 + 1/128*
(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b))*sqrt(3*(2*(-
1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1)
- b)^2 + 6*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*
(I*sqrt(3) + 1) - b)*b + 3*b^2))/b^3) + (((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/
c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c - 6*b*c)*log(((1/2)^(1/3)*(b^
3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c^2 + 4*b^2
*c^2 + 4*b^2*c*sqrt(x) + 4*b^2*x - 2*(2*b*c^2 + b*c*sqrt(x))*((1/2)^(1/3)*(
b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)) + 2*((2*(-
1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1)
- b)*c + 3*b*c)*log(4*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/
```

```
c^4)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^2 + 4*b^2*c^2 - 4*b^2*c*sqrt(x) + 4*b^2
*x + 4*(2*b*c^2 - b*c*sqrt(x))*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1
/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)) + 4*(b*c*x^2 - b*c)*log(-(c^2*x^3
 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)) + 24*b*sqrt(x))/c
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**(3/2))),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(3/2))),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^(3/2)) + a)*x, x)
```

Mupad [**B**]
time = 11.97, size = 247, normalized size = 1.30

$$\frac{a\,x^2}{2}+\frac{3\,b\,\sqrt{x}}{2\,c}+\frac{b\,\ln\left(\frac{c^{1/3}\,\sqrt{x}-1}{c^{1/3}\,\sqrt{x}+1}\right)}{4\,c^{4/3}}+\frac{\ln\left(1-c\,x^{3/2}\right)\,\left(\frac{b\,x^2}{2}-\frac{b\,c^2\,x^5}{2}\right)}{2\,c^2\,x^3-2}+\frac{b\,x^2\,\ln\left(c\,x^{3/2}+1\right)}{4}+\frac{b\,\ln\left(\frac{\sqrt{3}\,c^{2/3}\,x+c^{2/3}\,x\,1\mathrm{i}-c^{1/3}\,\sqrt{x}\,4\mathrm{i}-\sqrt{3}+1\mathrm{i}}{2\,c^{2/3}\,x+1-\sqrt{3}\,1\mathrm{i}}\right)\,\sqrt{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}{4\,c^{4/3}}+\frac{\sqrt{2}\,b\,\ln\left(\frac{2\,\sqrt{2}-c^{1/3}\,\sqrt{x}\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{5/2}\,1\mathrm{i}-\sqrt{2}\,c^{2/3}\,x+\sqrt{2}\,\sqrt{3}\,c^{2/3}\,x\,1\mathrm{i}}{2\,c^{2/3}\,x+1+\sqrt{3}\,1\mathrm{i}}\right)\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{8\,c^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^(3/2))),x)
```

```
[Out] (a*x^2)/2 + (3*b*x^(1/2))/(2*c) + (b*log((c^(1/3)*x^(1/2) - 1)/(c^(1/3)*x^(
1/2) + 1)))/(4*c^(4/3)) + (log(1 - c*x^(3/2))*((b*x^2)/2 - (b*c^2*x^5)/2))/
(2*c^2*x^3 - 2) + (b*x^2*log(c*x^(3/2) + 1))/4 + (b*log((c^(2/3)*x*1i - 3^(
1/2) - c^(1/3)*x^(1/2)*4i + 3^(1/2)*c^(2/3)*x + 1i)/(2*c^(2/3)*x - 3^(1/2)*
1i + 1))*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(4*c^(4/3)) + (2^(1/2)*b*log((2*2^(1
/2) - c^(1/3)*x^(1/2)*(3^(1/2)*1i + 1)^(5/2)*1i - 2^(1/2)*c^(2/3)*x + 2^(1/
2)*3^(1/2)*c^(2/3)*x*1i)/(3^(1/2)*1i + 2*c^(2/3)*x + 1))*(3^(1/2)*1i + 1)^(
1/2)*1i)/(8*c^(4/3))
```

3.216 $\int \left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)\,dx$

Optimal. Leaf size=170

$$ax-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1-2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right)}{2c^{2/3}}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right)}{2c^{2/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\,\sqrt{x}\right)}{c^{2/3}}+bx\tanh^{-1}\left(cx^{3/2}\right)+\frac{b\,\mathrm{lo}}{}$$

```
[Out] a*x+b*x*arctanh(c*x^(3/2))-b*arctanh(c^(1/3)*x^(1/2))/c^(2/3)+1/4*b*ln(1+c^
(2/3)*x-c^(1/3)*x^(1/2))/c^(2/3)-1/4*b*ln(1+c^(2/3)*x+c^(1/3)*x^(1/2))/c^(2
/3)-1/2*b*arctan(1/3*(1-2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)/c^(2/3)+1/2*b*a
rctan(1/3*(1+2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)/c^(2/3)
```

Rubi [A]
time = 0.21, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6021, 335, 302, 648, 632, 210, 642, 212}

$$ax-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1-2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right)}{2c^{2/3}}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{2\sqrt[3]{c}\,\sqrt{x}+1}{\sqrt{3}}\right)}{2c^{2/3}}+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\,\sqrt{x}+1\right)}{4c^{2/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\,\sqrt{x}+1\right)}{4c^{2/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\,\sqrt{x}\right)}{c^{2/3}}+bx\tanh^{-1}\left(cx^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c*x^(3/2)],x]
```

```
[Out] a*x - (Sqrt[3]*b*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(2*c^(2/3)) + (Sq
rt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(2*c^(2/3)) - (b*ArcTanh[c
^(1/3)*Sqrt[x]])/c^(2/3) + b*x*ArcTanh[c*x^(3/2)] + (b*Log[1 - c^(1/3)*Sqrt
[x] + c^(2/3)*x])/(4*c^(2/3)) - (b*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3)*x])/(4
*c^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
```

```
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)\,dx = ax+b\int \tanh^{-1}\left(cx^{3/2}\right)\,dx$$
$$= ax+bx\tanh^{-1}\left(cx^{3/2}\right)-\frac{1}{2}(3bc)\int \frac{x^{3/2}}{1-c^2x^3}\,dx$$
$$= ax+bx\tanh^{-1}\left(cx^{3/2}\right)-(3bc)\text{Subst}\left(\int \frac{x^4}{1-c^2x^6}\,dx,x,\sqrt{x}\right)$$
$$= ax+bx\tanh^{-1}\left(cx^{3/2}\right)-\frac{b\text{Subst}\left(\int \frac{1}{1-c^{2/3}x^2}\,dx,x,\sqrt{x}\right)}{\sqrt[3]{c}}-\frac{b\text{Subst}\left(\int \frac{-\frac{1}{2}-\ldots}{1-\sqrt[3]{c}\,x\ldots}\ldots\right)}{\sqrt[3]{\ldots}}$$
$$= ax-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{c^{2/3}}+bx\tanh^{-1}\left(cx^{3/2}\right)+\frac{b\text{Subst}\left(\int \frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx,\ldots\right)}{4c^{2/3}}$$
$$= ax-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{c^{2/3}}+bx\tanh^{-1}\left(cx^{3/2}\right)+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)}{4c^{2/3}}$$
$$= ax-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{2c^{2/3}}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{2c^{2/3}}-\frac{b\tanh^{-1}\left(\ldots\right.}{c^{2/\ldots}}$$

Mathematica [A]
time = 0.11, size = 114, normalized size = 0.67

$$ax+bx\tanh^{-1}\left(cx^{3/2}\right)-\frac{b\left(\sqrt{3}\left(\text{ArcTan}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)-\text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)\right)+2\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)+\tanh^{-1}\left(\frac{\sqrt[3]{c}\sqrt{x}}{1+c^{2/3}x}\right)\right)}{2c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c*x^(3/2)],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x^(3/2)] - (b*(Sqrt[3]*(ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/
Sqrt[3]] - ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]]) + 2*ArcTanh[c^(1/3)*Sqr
t[x]] + ArcTanh[(c^(1/3)*Sqrt[x])/(1 + c^(2/3)*x)]))/(2*c^(2/3))
```

Maple [A]
time = 0.06, size = 179, normalized size = 1.05

method	result

derivativedivides	$ax+bx\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)+\frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\cdots\right.}{\cdots}\right.}{2c\left(\frac{1}{c}\right)^{\frac{1}{3}}}$ …
default	$ax+bx\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)+\frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\cdots\right.}{\cdots}\right.}{2c\left(\frac{1}{c}\right)^{\frac{1}{3}}}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x^(3/2)),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*x*arctanh(c*x^(3/2))+1/2*b/c/(1/c)^(1/3)*ln(x^(1/2)-(1/c)^(1/3))-1/4*
b/c/(1/c)^(1/3)*ln(x+(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))+1/2*b*3^(1/2)/c/(1/c)
^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)+1))-1/2*b/c/(1/c)^(1/3)*ln
(x^(1/2)+(1/c)^(1/3))+1/4*b/c/(1/c)^(1/3)*ln(x-(1/c)^(1/3)*x^(1/2)+(1/c)^(2
/3))+1/2*b*3^(1/2)/c/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)-
1))
```

Maxima [A]
time = 0.48, size = 158, normalized size = 0.93

$$\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}-\frac{\log\left(c^{\frac{2}{3}}x+c^{\frac{1}{3}}\sqrt{x}+1\right)}{c^{\frac{5}{3}}}+\frac{\log\left(c^{\frac{2}{3}}x-c^{\frac{1}{3}}\sqrt{x}+1\right)}{c^{\frac{5}{3}}}-\frac{2\log\left(\frac{c^{\frac{1}{3}}\sqrt{x}+1}{c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}+\frac{2\log\left(\frac{c^{\frac{1}{3}}\sqrt{x}-1}{c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}\right)+4x\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(3/2)),x, algorithm="maxima")
```

```
[Out] 1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) + c^(1/3))/c^(1/3))
/c^(5/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) - c^(1/3))/c^(1/
3))/c^(5/3) - log(c^(2/3)*x + c^(1/3)*sqrt(x) + 1)/c^(5/3) + log(c^(2/3)*x
- c^(1/3)*sqrt(x) + 1)/c^(5/3) - 2*log((c^(1/3)*sqrt(x) + 1)/c^(1/3))/c^(5/
3) + 2*log((c^(1/3)*sqrt(x) - 1)/c^(1/3))/c^(5/3)) + 4*x*arctanh(c*x^(3/2))
)*b + a*x
```

Fricas [C] Result contains complex when optimal does not.
time = 1.22, size = 1682, normalized size = 9.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(3/2)),x, algorithm="fricas")

[Out] a*x - 1/2*sqrt(3)*sqrt(((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/
3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3
/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2)*arctan(1/24*(4*sqrt(3)*sqrt((
(1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b
)^2*b^2*c*sqrt(x) + 4*b^4*c*sqrt(x) + 4*b^4*x + 4*b^4 - 2*(2*b^3*c*sqrt(x)
+ b^3)*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) +
1) + 2*b))*sqrt(((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*s
qrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(
1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2)*c - sqrt(3)*(((1/2)^(1/3)*(b^3 - (c^
2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c^2 - 4*((1/2)^(1/
3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b*c^2 +
 4*b^2*c^2 + 8*b^2*c*sqrt(x))*sqrt(((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 +
b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b
^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2))/b^3) + 1/8*((1/2
)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - 4*b)*lo
g(((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) +
2*b)^2*b^2*c*sqrt(x) + 4*b^4*c*sqrt(x) + 4*b^4*x + 4*b^4 - 2*(2*b^3*c*sqrt(
x) + b^3)*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3)
 + 1) + 2*b)) + 1/4*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1
/3)*(I*sqrt(3) + 1) + 2*b)*log(-4*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/
16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)^2*b^2*c*sqrt(x) - 4*b^4*c*sqrt(x) +
4*b^4*x + 4*b^4 - 4*(2*b^3*c*sqrt(x) - b^3)*((-1/16*b^3 + 1/16*(c^2 - 1)*b^
3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)) - 1/4*((1/2)^(1/3)*(b^3 -
 (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)*log(-1/4*((1/2)^
(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c
+ ((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) +
2*b)*b*c - b^2*c + b^2*sqrt(x)) - 1/2*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3/c^2
+ 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)*log(((-1/16*b^3 + 1/16*(c^2 - 1)
*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)^2*c + 2*((-1/16*b^3 + 1
/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)*b*c + b^2*
c + b^2*sqrt(x)) + 1/2*(b*x - b)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3
- 1)) - sqrt(3*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(
I*sqrt(3) + 1) - b)^2 + 6*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c
^2)^(1/3)*(I*sqrt(3) + 1) - b)*b + 3*b^2)*arctan(1/3*(((-1/16*b^3 + 1/16*(c
^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^2 + 2*((-1/1
6*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)*b
*c^2 + b^2*c^2 - 2*b^2*c*sqrt(x) + sqrt(-4*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3
/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)^2*b^2*c*sqrt(x) - 4*b^4*c*s
qrt(x) + 4*b^4*x + 4*b^4 - 4*(2*b^3*c*sqrt(x) - b^3)*((-1/16*b^3 + 1/16*(c^
2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b))*c)*sqrt(3*((-1/1
6*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)^2
 + 6*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3)
+ 1) - b)*b + 3*b^2)/b^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x**(3/2)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.43, size = 186, normalized size = 1.09

$$\frac{1}{4}\left(c\left(\frac{2\sqrt{3}|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^2}+\frac{2\sqrt{3}|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^2}-\frac{|c|^{\frac{1}{3}}\log\left(x+\frac{\sqrt{x}}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2}+\frac{|c|^{\frac{1}{3}}\log\left(x-\frac{\sqrt{x}}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2}-\frac{2|c|^{\frac{1}{3}}\log\left(\sqrt{x}+\frac{1}{|c|^{\frac{1}{3}}}\right)}{c^2}+\frac{2|c|^{\frac{1}{3}}\log\left(\left|\sqrt{x}-\frac{1}{|c|^{\frac{1}{3}}}\right|\right)}{c^2}\right)+2x\log\left(-\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(3/2)),x, algorithm="giac")
```

```
[Out] 1/4*(c*(2*sqrt(3)*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1/abs(c)^(1/
3))*abs(c)^(1/3))/c^2 + 2*sqrt(3)*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*sqrt(x
) - 1/abs(c)^(1/3))*abs(c)^(1/3))/c^2 - abs(c)^(1/3)*log(x + sqrt(x)/abs(c)
^(1/3) + 1/abs(c)^(2/3))/c^2 + abs(c)^(1/3)*log(x - sqrt(x)/abs(c)^(1/3) +
1/abs(c)^(2/3))/c^2 - 2*abs(c)^(1/3)*log(sqrt(x) + 1/abs(c)^(1/3))/c^2 + 2*
abs(c)^(1/3)*log(abs(sqrt(x) - 1/abs(c)^(1/3)))/c^2) + 2*x*log(-(c*x^(3/2)
+ 1)/(c*x^(3/2) - 1)))*b + a*x
```

Mupad [**B**]
time = 5.10, size = 107, normalized size = 0.63

$$a\,x+b\,x\,\mathrm{atanh}\left(c\,x^{3/2}\right)-\frac{b\,\mathrm{atanh}\left(c^{1/3}\,\sqrt{x}\right)}{c^{2/3}}+\frac{b\,\mathrm{atanh}\left(\frac{486\,c^8\,\sqrt{x}}{-243\,c^{23/3}+\sqrt{3}\,c^{23/3}\,243\mathrm{i}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,c^{2/3}}+\frac{b\,\mathrm{atanh}\left(\frac{486\,c^8\,\sqrt{x}}{243\,c^{23/3}+\sqrt{3}\,c^{23/3}\,243\mathrm{i}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,c^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atanh(c*x^(3/2)),x)
```

```
[Out] a*x + b*x*atanh(c*x^(3/2)) - (b*atanh(c^(1/3)*x^(1/2)))/c^(2/3) + (b*atanh(
(486*c^8*x^(1/2))/(3^(1/2)*c^(23/3)*243i - 243*c^(23/3)))*(3^(1/2)*1i + 1))
/(2*c^(2/3)) + (b*atanh((486*c^8*x^(1/2))/(3^(1/2)*c^(23/3)*243i + 243*c^(2
3/3)))*(3^(1/2)*1i - 1))/(2*c^(2/3))
```

3.217 $\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}\,dx$

Optimal. Leaf size=34

$$a\log(x)-\frac{1}{3}b\text{PolyLog}\left(2,-cx^{3/2}\right)+\frac{1}{3}b\text{PolyLog}\left(2,cx^{3/2}\right)$$

```
[Out] a*ln(x)-1/3*b*polylog(2,-c*x^(3/2))+1/3*b*polylog(2,c*x^(3/2))
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6035, 6031}

$$a\log(x)-\frac{1}{3}b\text{Li}_2\left(-cx^{3/2}\right)+\frac{1}{3}b\text{Li}_2\left(cx^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])/x,x]
[Out] a*Log[x] - (b*PolyLog[2, -(c*x^(3/2))])/3 + (b*PolyLog[2, c*x^(3/2)])/3
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}\,dx &= \frac{2}{3}\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx,x,x^{3/2}\right)\\ &= a\log(x)-\frac{1}{3}b\text{Li}_2\left(-cx^{3/2}\right)+\frac{1}{3}b\text{Li}_2\left(cx^{3/2}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 0.94

$$a\log(x)+\frac{1}{3}b\left(-\text{PolyLog}\left(2,-cx^{3/2}\right)+\text{PolyLog}\left(2,cx^{3/2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])/x,x]
```

```
[Out] a*Log[x] + (b*(-PolyLog[2, -(c*x^(3/2))] + PolyLog[2, c*x^(3/2)]))/3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(26) = 52$.
time = 0.12, size = 63, normalized size = 1.85

method	result	si
derivativedivides	$\frac{2a\ln\left(c\,x^{\frac{3}{2}}\right)}{3}+\frac{2b\ln\left(c\,x^{\frac{3}{2}}\right)\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{3}-\frac{b\,\mathrm{dilog}\left(c\,x^{\frac{3}{2}}\right)}{3}-\frac{b\,\mathrm{dilog}\left(c\,x^{\frac{3}{2}}+1\right)}{3}-\frac{b\ln\left(c\,x^{\frac{3}{2}}\right)\ln\left(c\,x^{\frac{3}{2}}+1\right)}{3}$	63
default	$\frac{2a\ln\left(c\,x^{\frac{3}{2}}\right)}{3}+\frac{2b\ln\left(c\,x^{\frac{3}{2}}\right)\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{3}-\frac{b\,\mathrm{dilog}\left(c\,x^{\frac{3}{2}}\right)}{3}-\frac{b\,\mathrm{dilog}\left(c\,x^{\frac{3}{2}}+1\right)}{3}-\frac{b\ln\left(c\,x^{\frac{3}{2}}\right)\ln\left(c\,x^{\frac{3}{2}}+1\right)}{3}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(3/2)))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a*ln(c*x^(3/2))+2/3*b*ln(c*x^(3/2))*arctanh(c*x^(3/2))-1/3*b*dilog(c*x^
(3/2))-1/3*b*dilog(c*x^(3/2)+1)-1/3*b*ln(c*x^(3/2))*ln(c*x^(3/2)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(24) = 48$.
time = 0.37, size = 62, normalized size = 1.82

$$-\frac{1}{3}\left(\log\left(cx^{\frac{3}{2}}\right)\log\left(-cx^{\frac{3}{2}}+1\right)+\mathrm{Li}_2\left(-cx^{\frac{3}{2}}+1\right)\right)b+\frac{1}{3}\left(\log\left(cx^{\frac{3}{2}}+1\right)\log\left(-cx^{\frac{3}{2}}\right)+\mathrm{Li}_2\left(cx^{\frac{3}{2}}+1\right)\right)b+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x,x, algorithm="maxima")
```

```
[Out] -1/3*(log(c*x^(3/2))*log(-c*x^(3/2) + 1) + dilog(-c*x^(3/2) + 1))*b + 1/3*(
log(c*x^(3/2) + 1)*log(-c*x^(3/2)) + dilog(c*x^(3/2) + 1))*b + a*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^(3/2)) + a)/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))/x,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^(3/2)) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a+b\,\mathrm{atanh}\left(c\,x^{3/2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(3/2)))/x,x)
```

```
[Out] int((a + b*atanh(c*x^(3/2)))/x, x)
```

3.218 $\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x^2}\,dx$

Optimal. Leaf size=172

$$-\frac{1}{2}\sqrt{3}\,bc^{2/3}\text{ArcTan}\left(\frac{1-2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right)+\frac{1}{2}\sqrt{3}\,bc^{2/3}\text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right)+bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}\,\sqrt{x}\right)-\frac{a+b}{}$$

```
[Out] (-a-b*arctanh(c*x^(3/2)))/x+b*c^(2/3)*arctanh(c^(1/3)*x^(1/2))-1/4*b*c^(2/3
)*ln(1+c^(2/3)*x-c^(1/3)*x^(1/2))+1/4*b*c^(2/3)*ln(1+c^(2/3)*x+c^(1/3)*x^(1
/2))-1/2*b*c^(2/3)*arctan(1/3*(1-2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)+1/2*b*
c^(2/3)*arctan(1/3*(1+2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6037, 335, 216, 648, 632, 210, 642, 212}

$$-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}-\frac{1}{2}\sqrt{3}\,bc^{2/3}\text{ArcTan}\left(\frac{1-2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right)+\frac{1}{2}\sqrt{3}\,bc^{2/3}\text{ArcTan}\left(\frac{2\sqrt[3]{c}\,\sqrt{x}+1}{\sqrt{3}}\right)-\frac{1}{4}bc^{2/3}\log\left(c^{2/3}x-\sqrt[3]{c}\,\sqrt{x}+1\right)+\frac{1}{4}bc^{2/3}\log\left(c^{2/3}x+\sqrt[3]{c}\,\sqrt{x}+1\right)+bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])/x^2,x]
```

```
[Out] -1/2*(Sqrt[3]*b*c^(2/3)*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sqrt[3]]) + (Sqrt[3]
*b*c^(2/3)*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/2 + b*c^(2/3)*ArcTanh[c
^(1/3)*Sqrt[x]] - (a + b*ArcTanh[c*x^(3/2)])/x - (b*c^(2/3)*Log[1 - c^(1/3)
*Sqrt[x] + c^(2/3)*x])/4 + (b*c^(2/3)*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3)*x])
/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
```

```
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x^2}\,dx = -\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x} + \frac{1}{2}(3bc)\int \frac{1}{\sqrt{x}\,\left(1-c^2x^3\right)}\,dx$$

$$= -\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x} + (3bc)\text{Subst}\left(\int \frac{1}{1-c^2x^6}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x} + (bc)\text{Subst}\left(\int \frac{1}{1-c^{2/3}x^2}\,dx, x, \sqrt{x}\right) + (bc)\text{Subst}\Bigg($$

$$= bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}\,\sqrt{x}\right) - \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x} - \frac{1}{4}\left(bc^{2/3}\right)\text{Subst}\Bigg(\int \frac{-\sqrt[3]{c}\,}{1-\sqrt[3]{c}\,}$$

$$= bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}\,\sqrt{x}\right) - \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x} - \frac{1}{4}bc^{2/3}\log\left(1-\sqrt[3]{c}\,\sqrt{x}+c^{2}\right.$$

$$= -\frac{1}{2}\sqrt{3}\,bc^{2/3}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right) + \frac{1}{2}\sqrt{3}\,bc^{2/3}\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right) + b$$

Mathematica [A]
time = 0.03, size = 205, normalized size = 1.19

$$-\frac{a}{x}+\frac{1}{2}\sqrt{3}\,bc^{2/3}\text{ArcTan}\left(\frac{-1+2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right)+\frac{1}{2}\sqrt{3}\,bc^{2/3}\text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right)-\frac{b\tanh^{-1}\left(cx^{3/2}\right)}{x}-\frac{1}{2}bc^{2/3}\log\left(1-\sqrt[3]{c}\,\sqrt{x}\right)+\frac{1}{2}bc^{2/3}\log\left(1+\sqrt[3]{c}\,\sqrt{x}\right)-\frac{1}{4}bc^{2/3}\log\left(1-\sqrt[3]{c}\,\sqrt{x}+c^{2/3}x\right)+\frac{1}{4}bc^{2/3}\log\left(1+\sqrt[3]{c}\,\sqrt{x}+c^{2/3}x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])/x^2,x]
```

```
[Out] -(a/x) + (Sqrt[3]*b*c^(2/3)*ArcTan[(-1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/2 + (
Sqrt[3]*b*c^(2/3)*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/2 - (b*ArcTanh[c
*x^(3/2)])/x - (b*c^(2/3)*Log[1 - c^(1/3)*Sqrt[x]])/2 + (b*c^(2/3)*Log[1 +
c^(1/3)*Sqrt[x]])/2 - (b*c^(2/3)*Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x])/4 +
(b*c^(2/3)*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3)*x])/4
```

Maple [A]
time = 0.05, size = 167, normalized size = 0.97

method	result
derivativedivides	$-\frac{a}{x}-\frac{b\,\text{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{x}+\frac{b\ln\left(\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{x}}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}\right.}{3}\right.}{2\left(\frac{1}{c}\right)^{\frac{2}{3}}}$

default	$-\frac{a}{x}-\frac{b\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{x}+\frac{b\ln\left(\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2\sqrt{x}}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\right.}{3}\right.}{2\left(\frac{1}{c}\right)^{\frac{2}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(3/2)))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/x-b/x*arctanh(c*x^(3/2))+1/2*b/(1/c)^(2/3)*ln(x^(1/2)+(1/c)^(1/3))-1/4*b
/(1/c)^(2/3)*ln(x-(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))+1/2*b/(1/c)^(2/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)-1))-1/2*b/(1/c)^(2/3)*ln(x^(1/2
)-(1/c)^(1/3))+1/4*b/(1/c)^(2/3)*ln(x+(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))+1/2*
b/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)+1))
```

<u>Maxima</u> [A]
time = 0.48, size = 163, normalized size = 0.95

$$\frac{1}{4}\left(\left(\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}+\frac{\log\left(c^{\frac{2}{3}}x+c^{\frac{1}{3}}\sqrt{x}+1\right)}{c^{\frac{1}{3}}}-\frac{\log\left(c^{\frac{2}{3}}x-c^{\frac{1}{3}}\sqrt{x}+1\right)}{c^{\frac{1}{3}}}+\frac{2\log\left(\frac{c^{\frac{1}{3}}\sqrt{x}+1}{c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}-\frac{2\log\left(\frac{c^{\frac{1}{3}}\sqrt{x}-1}{c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}\right)c-\frac{4\,\mathrm{artanh}\left(cx^{\frac{3}{2}}\right)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) + c^(1/3))/c^(1/3))/c
^(1/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) - c^(1/3))/c^(1/3)
)/c^(1/3) + log(c^(2/3)*x + c^(1/3)*sqrt(x) + 1)/c^(1/3) - log(c^(2/3)*x -
c^(1/3)*sqrt(x) + 1)/c^(1/3) + 2*log((c^(1/3)*sqrt(x) + 1)/c^(1/3))/c^(1/3)
 - 2*log((c^(1/3)*sqrt(x) - 1)/c^(1/3))/c^(1/3))*c - 4*arctanh(c*x^(3/2))/x
)*b - a/x
```

<u>Fricas</u> [A]
time = 0.37, size = 234, normalized size = 1.36

$$-\frac{2\sqrt{3}\left(-c^2\right)^{\frac{1}{3}}bx\arctan\left(\frac{2\sqrt{3}\left(-c^2\right)^{\frac{2}{3}}\sqrt{x}+\sqrt{3}\,c}{3c}\right)-2\sqrt{3}\,b\left(c^2\right)^{\frac{1}{3}}x\arctan\left(\frac{2\sqrt{3}\left(c^2\right)^{\frac{2}{3}}\sqrt{x}-\sqrt{3}\,c}{3c}\right)+\left(-c^2\right)^{\frac{1}{3}}bx\log\left(c^2x-\left(-c^2\right)^{\frac{1}{3}}c\sqrt{x}+\left(-c^2\right)^{\frac{2}{3}}\right)+b\left(c^2\right)^{\frac{1}{3}}x\log\left(c^2x-\left(c^2\right)^{\frac{1}{3}}c\sqrt{x}+\left(c^2\right)^{\frac{2}{3}}\right)-2\left(-c^2\right)^{\frac{1}{3}}bx\log\left(c\sqrt{x}+\left(-c^2\right)^{\frac{1}{3}}\right)-2b\left(c^2\right)^{\frac{1}{3}}x\log\left(c\sqrt{x}+\left(c^2\right)^{\frac{1}{3}}\right)+2b\log\left(-\frac{c^2x^3+2cx^{\frac{3}{2}}+1}{c^2x^3-1}\right)+4a}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-c^2)^(1/3)*b*x*arctan(1/3*(2*sqrt(3)*(-c^2)^(2/3)*sqrt(x)
 + sqrt(3)*c)/c) - 2*sqrt(3)*b*(c^2)^(1/3)*x*arctan(1/3*(2*sqrt(3)*(c^2)^(2
/3)*sqrt(x) - sqrt(3)*c)/c) + (-c^2)^(1/3)*b*x*log(c^2*x - (-c^2)^(1/3)*c*s
qrt(x) + (-c^2)^(2/3)) + b*(c^2)^(1/3)*x*log(c^2*x - (c^2)^(1/3)*c*sqrt(x)
+ (c^2)^(2/3)) - 2*(-c^2)^(1/3)*b*x*log(c*sqrt(x) + (-c^2)^(1/3)) - 2*b*(c^
```

```
2)^(1/3)*x*log(c*sqrt(x) + (c^2)^(1/3)) + 2*b*log(-(c^2*x^3 + 2*c*x^(3/2) +
 1)/(c^2*x^3 - 1)) + 4*a)/x
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))/x**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 172, normalized size = 1.00

$$\frac{1}{4}\left(\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}}+\frac{\log\left(x+\frac{\sqrt{x}}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{1}{3}}}-\frac{\log\left(x-\frac{\sqrt{x}}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{1}{3}}}+\frac{2\log\left(\sqrt{x}+\frac{1}{|c|^{\frac{1}{3}}}\right)}{|c|^{\frac{1}{3}}}-\frac{2\log\left(\left|\sqrt{x}-\frac{1}{|c|^{\frac{1}{3}}}\right|\right)}{|c|^{\frac{1}{3}}}\right)bc-\frac{b\log\left(-\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}\right)}{2x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1/abs(c)^(1/3))*abs(c)^(1/3)
)/abs(c)^(1/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) - 1/abs(c)^(1/3))*
abs(c)^(1/3))/abs(c)^(1/3) + log(x + sqrt(x)/abs(c)^(1/3) + 1/abs(c)^(2/3))
/abs(c)^(1/3) - log(x - sqrt(x)/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3)
 + 2*log(sqrt(x) + 1/abs(c)^(1/3))/abs(c)^(1/3) - 2*log(abs(sqrt(x) - 1/abs
(c)^(1/3)))/abs(c)^(1/3))*b*c - 1/2*b*log(-(c*x^(3/2) + 1)/(c*x^(3/2) - 1))
/x - a/x
```

Mupad [**B**]
time = 7.46, size = 220, normalized size = 1.28

$$\frac{b\,c^{2/3}\,\ln\left(\frac{c^{1/3}\,\sqrt{x}+1}{c^{1/3}\,\sqrt{x}-1}\right)}{2}-\frac{a}{x}+\frac{\ln\left(1-c\,x^{3/2}\right)\,\left(b\,x-b\,c^2\,x^4\right)}{2\,x^2-2\,c^2\,x^5}-\frac{b\,\ln\left(c\,x^{3/2}+1\right)}{2\,x}+\frac{b\,c^{2/3}\,\ln\left(\frac{\sqrt{3}+c^{2/3}\,x\,1\mathrm{i}-c^{1/3}\,\sqrt{x}\,4\mathrm{i}-\sqrt{3}\,c^{2/3}\,x+1\mathrm{i}}{2\,c^{2/3}\,x+1+\sqrt{3}\,1\mathrm{i}}\right)\,\sqrt{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\,1\mathrm{i}}{2}+\frac{b\,c^{2/3}\,\ln\left(\frac{\sqrt{3}\,c^{2/3}\,x+c^{2/3}\,x\,1\mathrm{i}+c^{1/3}\,\sqrt{x}\,4\mathrm{i}-\sqrt{3}+1\mathrm{i}}{2\,c^{2/3}\,x+1-\sqrt{3}\,1\mathrm{i}}\right)\,\sqrt{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(3/2)))/x^2,x)
```

```
[Out] (b*c^(2/3)*log((c^(1/3)*x^(1/2) + 1)/(c^(1/3)*x^(1/2) - 1)))/2 - a/x + (log
(1 - c*x^(3/2))*(b*x - b*c^2*x^4))/(2*x^2 - 2*c^2*x^5) - (b*log(c*x^(3/2) +
 1))/(2*x) + (b*c^(2/3)*log((3^(1/2) + c^(2/3)*x*1i - c^(1/3)*x^(1/2)*4i -
3^(1/2)*c^(2/3)*x + 1i)/(3^(1/2)*1i + 2*c^(2/3)*x + 1))*((3^(1/2)*1i)/2 + 1
/2)^(1/2)*1i)/2 + (b*c^(2/3)*log((c^(2/3)*x*1i - 3^(1/2) + c^(1/3)*x^(1/2)*
4i + 3^(1/2)*c^(2/3)*x + 1i)/(2*c^(2/3)*x - 3^(1/2)*1i + 1))*((3^(1/2)*1i)/
2 - 1/2)^(1/2))/2
```

3.219 $\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x^3}\,dx$

Optimal. Leaf size=188

$$-\frac{3bc}{2\sqrt{x}}+\frac{1}{4}\sqrt{3}\,bc^{4/3}\text{ArcTan}\left(\frac{1-2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right)-\frac{1}{4}\sqrt{3}\,bc^{4/3}\text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right)+\frac{1}{2}bc^{4/3}\tanh^{-1}\left(\sqrt[3]{c}\,\sqrt{x}\right.$$

```
[Out] 1/2*(-a-b*arctanh(c*x^(3/2)))/x^2+1/2*b*c^(4/3)*arctanh(c^(1/3)*x^(1/2))-1/
8*b*c^(4/3)*ln(1+c^(2/3)*x-c^(1/3)*x^(1/2))+1/8*b*c^(4/3)*ln(1+c^(2/3)*x+c^
(1/3)*x^(1/2))+1/4*b*c^(4/3)*arctan(1/3*(1-2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1
/2)-1/4*b*c^(4/3)*arctan(1/3*(1+2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)-3/2*b*c
/x^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {6037, 331, 335, 302, 648, 632, 210, 642, 212}

$$-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}+\frac{1}{4}\sqrt{3}\,bc^{4/3}\text{ArcTan}\left(\frac{1-2\sqrt[3]{c}\,\sqrt{x}}{\sqrt{3}}\right)-\frac{1}{4}\sqrt{3}\,bc^{4/3}\text{ArcTan}\left(\frac{2\sqrt[3]{c}\,\sqrt{x}+1}{\sqrt{3}}\right)-\frac{1}{8}bc^{4/3}\log\left(c^{2/3}x-\sqrt[3]{c}\,\sqrt{x}+1\right)+\frac{1}{8}bc^{4/3}\log\left(c^{2/3}x+\sqrt[3]{c}\,\sqrt{x}+1\right)+\frac{1}{2}bc^{4/3}\tanh^{-1}\left(\sqrt[3]{c}\,\sqrt{x}\right)-\frac{3bc}{2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])/x^3,x]
```

```
[Out] (-3*b*c)/(2*Sqrt[x]) + (Sqrt[3]*b*c^(4/3)*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sq
rt[3]])/4 - (Sqrt[3]*b*c^(4/3)*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/4 +
 (b*c^(4/3)*ArcTanh[c^(1/3)*Sqrt[x]])/2 - (a + b*ArcTanh[c*x^(3/2)])/(2*x^2
) - (b*c^(4/3)*Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x])/8 + (b*c^(4/3)*Log[1 +
 c^(1/3)*Sqrt[x] + c^(2/3)*x])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
```

```
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
```

```
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x^3}\,dx = -\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}+\frac{1}{4}(3bc)\int \frac{1}{x^{3/2}\left(1-c^2x^3\right)}\,dx$$

$$= -\frac{3bc}{2\sqrt{x}}-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}+\frac{1}{4}\left(3bc^3\right)\int \frac{x^{3/2}}{1-c^2x^3}\,dx$$

$$= -\frac{3bc}{2\sqrt{x}}-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}+\frac{1}{2}\left(3bc^3\right)\text{Subst}\left(\int \frac{x^4}{1-c^2x^6}\,dx,x,\sqrt{x}\right)$$

$$= -\frac{3bc}{2\sqrt{x}}-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}+\frac{1}{2}\left(bc^{5/3}\right)\text{Subst}\left(\int \frac{1}{1-c^{2/3}x^2}\,dx,x,\sqrt{x}\right)+$$

$$= -\frac{3bc}{2\sqrt{x}}+\frac{1}{2}bc^{4/3}\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}-\frac{1}{8}\left(bc^{4/3}\right)\text{Subst}\Big($$

$$= -\frac{3bc}{2\sqrt{x}}+\frac{1}{2}bc^{4/3}\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}-\frac{1}{8}bc^{4/3}\log\left(1-\sqrt[3]{}\right.$$

$$= -\frac{3bc}{2\sqrt{x}}+\frac{1}{4}\sqrt{3}\,bc^{4/3}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)-\frac{1}{4}\sqrt{3}\,bc^{4/3}\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\sqrt{}}{\sqrt{3}}\right.$$

Mathematica [A]
time = 0.04, size = 220, normalized size = 1.17

$$-\frac{a}{2x^2}-\frac{3bc}{2\sqrt{x}}-\frac{1}{4}\sqrt{3}\,bc^{4/3}\text{ArcTan}\left(\frac{-1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)-\frac{1}{4}\sqrt{3}\,bc^{4/3}\text{ArcTan}\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)-\frac{b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}-\frac{1}{4}bc^{4/3}\log\left(1-\sqrt[3]{c}\sqrt{x}\right)+\frac{1}{4}bc^{4/3}\log\left(1+\sqrt[3]{c}\sqrt{x}\right)-\frac{1}{8}bc^{4/3}\log\left(1-\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)+\frac{1}{8}bc^{4/3}\log\left(1+\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (3*b*c)/(2*Sqrt[x]) - (Sqrt[3]*b*c^(4/3)*ArcTan[(-1 + 2*c^(1/3
)*Sqrt[x])/Sqrt[3]])/4 - (Sqrt[3]*b*c^(4/3)*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/
Sqrt[3]])/4 - (b*ArcTanh[c*x^(3/2)])/(2*x^2) - (b*c^(4/3)*Log[1 - c^(1/3)*S
qrt[x]])/4 + (b*c^(4/3)*Log[1 + c^(1/3)*Sqrt[x]])/4 - (b*c^(4/3)*Log[1 - c^
(1/3)*Sqrt[x] + c^(2/3)*x])/8 + (b*c^(4/3)*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3
)*x])/8
```

Maple [A]
time = 0.06, size = 180, normalized size = 0.96

method	result
derivativedivides	$-\frac{a}{2x^2}-\frac{b\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{2x^2}+\frac{bc\ln\left(\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{bc\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2 \ldots\right.}{\ldots}\right.}{4\left(\frac{1}{c}\right)^{\frac{1}{3}}}$
default	$-\frac{a}{2x^2}-\frac{b\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{2x^2}+\frac{bc\ln\left(\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{bc\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2 \ldots\right.}{\ldots}\right.}{4\left(\frac{1}{c}\right)^{\frac{1}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(3/2)))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctanh(c*x^(3/2))+1/4*b*c/(1/c)^(1/3)*ln(x^(1/2)+(1/c
)^(1/3))-1/8*b*c/(1/c)^(1/3)*ln(x-(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))-1/4*b*c*
3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)-1))-3/2*b*c/x
^(1/2)-1/4*b*c/(1/c)^(1/3)*ln(x^(1/2)-(1/c)^(1/3))+1/8*b*c/(1/c)^(1/3)*ln(x
+(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))-1/4*b*c*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^
(1/2)*(2/(1/c)^(1/3)*x^(1/2)+1))
```

Maxima [A]
time = 0.47, size = 168, normalized size = 0.89

$$-\frac{1}{8}\left(\left(2\sqrt{3}\,c^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}\sqrt{x}+c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)+2\sqrt{3}\,c^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}\sqrt{x}-c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)-c^{\frac{1}{3}}\log\left(c^{\frac{2}{3}}x+c^{\frac{1}{3}}\sqrt{x}+1\right)+c^{\frac{1}{3}}\log\left(c^{\frac{2}{3}}x-c^{\frac{1}{3}}\sqrt{x}+1\right)-2\,c^{\frac{1}{3}}\log\left(\frac{c^{\frac{1}{3}}\sqrt{x}+1}{c^{\frac{1}{3}}}\right)+2\,c^{\frac{1}{3}}\log\left(\frac{c^{\frac{1}{3}}\sqrt{x}-1}{c^{\frac{1}{3}}}\right)+\frac{12}{\sqrt{x}}\right)c+\frac{4\,\mathrm{artanh}\left(cx^{\frac{3}{2}}\right)}{x^2}\right)b-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*((2*sqrt(3)*c^(1/3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) + c^(1/3))/c
^(1/3)) + 2*sqrt(3)*c^(1/3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) - c^(1/3)
)/c^(1/3)) - c^(1/3)*log(c^(2/3)*x + c^(1/3)*sqrt(x) + 1) + c^(1/3)*log(c^(
2/3)*x - c^(1/3)*sqrt(x) + 1) - 2*c^(1/3)*log((c^(1/3)*sqrt(x) + 1)/c^(1/3)
) + 2*c^(1/3)*log((c^(1/3)*sqrt(x) - 1)/c^(1/3)) + 12/sqrt(x))*c + 4*arctan
h(c*x^(3/2))/x^2)*b - 1/2*a/x^2
```

Fricas [A]
time = 0.38, size = 214, normalized size = 1.14

$$-\frac{2\sqrt{3}\,b(-c)^{\frac{1}{3}}\,cx^2\arctan\left(\frac{2}{3}\sqrt{3}\,(-c)^{\frac{1}{3}}\sqrt{x}-\frac{1}{3}\sqrt{3}\right)+2\sqrt{3}\,bc^{\frac{4}{3}}x^2\arctan\left(\frac{2}{3}\sqrt{3}\,c^{\frac{1}{3}}\sqrt{x}-\frac{1}{3}\sqrt{3}\right)+b(-c)^{\frac{1}{3}}\,cx^2\log\left(cx+(-c)^{\frac{2}{3}}\sqrt{x}-(-c)^{\frac{1}{3}}\right)+bc^{\frac{4}{3}}x^2\log\left(cx-c^{\frac{2}{3}}\sqrt{x}+c^{\frac{1}{3}}\right)-2\,b(-c)^{\frac{1}{3}}\,cx^2\log\left(c\sqrt{x}-(-c)^{\frac{2}{3}}\right)-2\,bc^{\frac{4}{3}}x^2\log\left(c\sqrt{x}+c^{\frac{2}{3}}\right)+12\,bcx^{\frac{3}{2}}+2\,b\log\left(-\frac{c^2x^3+2\,cx^{\frac{3}{2}}+1}{c^2x^3-1}\right)+4\,a}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(3)*b*(-c)^(1/3)*c*x^2*arctan(2/3*sqrt(3)*(-c)^(1/3)*sqrt(x) -
1/3*sqrt(3)) + 2*sqrt(3)*b*c^(4/3)*x^2*arctan(2/3*sqrt(3)*c^(1/3)*sqrt(x) -
 1/3*sqrt(3)) + b*(-c)^(1/3)*c*x^2*log(c*x + (-c)^(2/3)*sqrt(x) - (-c)^(1/3
)) + b*c^(4/3)*x^2*log(c*x - c^(2/3)*sqrt(x) + c^(1/3)) - 2*b*(-c)^(1/3)*c*
x^2*log(c*sqrt(x) - (-c)^(2/3)) - 2*b*c^(4/3)*x^2*log(c*sqrt(x) + c^(2/3))
+ 12*b*c*x^(3/2) + 2*b*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)) + 4*
a)/x^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))/x**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 194, normalized size = 1.03

$$-\frac{1}{4}\sqrt{3}\,bc|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)-\frac{1}{4}\sqrt{3}\,bc|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)+\frac{bc^{3}\log\left(x+\frac{\sqrt{x}}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{8\,|c|^{\frac{5}{3}}}-\frac{bc^{3}\log\left(x-\frac{\sqrt{x}}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{8\,|c|^{\frac{5}{3}}}+\frac{1}{4}\,bc|c|^{\frac{1}{3}}\log\left(\sqrt{x}+\frac{1}{|c|^{\frac{1}{3}}}\right)-\frac{bc^{3}\log\left(\left|\sqrt{x}-\frac{1}{|c|^{\frac{1}{3}}}\right|\right)}{4\,|c|^{\frac{5}{3}}}-\frac{b\log\left(-\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}\right)}{4\,x^{2}}-\frac{3\,bcx^{\frac{3}{2}}+a}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(3)*b*c*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1/abs(c)^(1/3
))*abs(c)^(1/3)) - 1/4*sqrt(3)*b*c*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*sqrt(
x) - 1/abs(c)^(1/3))*abs(c)^(1/3)) + 1/8*b*c^3*log(x + sqrt(x)/abs(c)^(1/3)
 + 1/abs(c)^(2/3))/abs(c)^(5/3) - 1/8*b*c^3*log(x - sqrt(x)/abs(c)^(1/3) +
1/abs(c)^(2/3))/abs(c)^(5/3) + 1/4*b*c*abs(c)^(1/3)*log(sqrt(x) + 1/abs(c)^
(1/3)) - 1/4*b*c^3*log(abs(sqrt(x) - 1/abs(c)^(1/3)))/abs(c)^(5/3) - 1/4*b*
log(-(c*x^(3/2) + 1)/(c*x^(3/2) - 1))/x^2 - 1/2*(3*b*c*x^(3/2) + a)/x^2
```

Mupad [**B**]
time = 7.37, size = 228, normalized size = 1.21

$$\frac{b\,c^{4/3}\,\ln\left(\frac{c^{1/3}\,\sqrt{x}+1}{c^{1/3}\,\sqrt{x}-1}\right)}{4}-\frac{a}{2\,x^2}+\frac{\ln\left(1-c\,x^{3/2}\right)\,\left(\frac{b\,x}{2}-\frac{b\,c^2\,x^4}{2}\right)}{2\,x^3-2\,c^2\,x^6}-\frac{3\,b\,c}{2\,\sqrt{x}}-\frac{b\,\ln\left(c\,x^{3/2}+1\right)}{4\,x^2}+\frac{b\,c^{4/3}\,\ln\left(\frac{\sqrt{3}+c^{2/3}\,x\,1{}\mathrm{i}+c^{1/3}\,\sqrt{x}\,4{}\mathrm{i}-\sqrt{3}\,c^{2/3}\,x+1{}\mathrm{i}}{2\,c^{2/3}\,x+1+\sqrt{3}\,1{}\mathrm{i}}\right)\,\sqrt{-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}{4}+\frac{b\,c^{4/3}\,\ln\left(\frac{\sqrt{3}\,c^{2/3}\,x+c^{2/3}\,x\,1{}\mathrm{i}-c^{1/3}\,\sqrt{x}\,4{}\mathrm{i}-\sqrt{3}+1{}\mathrm{i}}{2\,c^{2/3}\,x+1-\sqrt{3}\,1{}\mathrm{i}}\right)\,\sqrt{\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(3/2)))/x^3,x)
```

```
[Out] (b*c^(4/3)*log((c^(1/3)*x^(1/2) + 1)/(c^(1/3)*x^(1/2) - 1)))/4 - a/(2*x^2)
+ (log(1 - c*x^(3/2))*((b*x)/2 - (b*c^2*x^4)/2))/(2*x^3 - 2*c^2*x^6) - (3*b
```

```
*c)/(2*x^(1/2)) - (b*log(c*x^(3/2) + 1))/(4*x^2) + (b*c^(4/3)*log((3^(1/2)
+ c^(2/3)*x*1i + c^(1/3)*x^(1/2)*4i - 3^(1/2)*c^(2/3)*x + 1i)/(3^(1/2)*1i +
 2*c^(2/3)*x + 1))*((3^(1/2)*1i)/2 - 1/2)^(1/2))/4 + (b*c^(4/3)*log((c^(2/3
)*x*1i - 3^(1/2) - c^(1/3)*x^(1/2)*4i + 3^(1/2)*c^(2/3)*x + 1i)/(2*c^(2/3)*
x - 3^(1/2)*1i + 1))*((3^(1/2)*1i)/2 + 1/2)^(1/2)*1i)/4
```

3.220 $\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x^4} dx$

Optimal. Leaf size=47

$$-\frac{bc}{3x^{3/2}}+\frac{1}{3}bc^2\tanh^{-1}\left(cx^{3/2}\right)-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}$$

```
[Out] -1/3*b*c/x^(3/2)+1/3*b*c^2*arctanh(c*x^(3/2))+1/3*(-a-b*arctanh(c*x^(3/2)))
/x^3
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6037, 331, 335, 281, 212}

$$-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}+\frac{1}{3}bc^2\tanh^{-1}\left(cx^{3/2}\right)-\frac{bc}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])/x^4,x]
```

```
[Out] -1/3*(b*c)/x^(3/2) + (b*c^2*ArcTanh[c*x^(3/2)])/3 - (a + b*ArcTanh[c*x^(3/2
)])/(3*x^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x^4}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}+\frac{1}{2}(bc)\int \frac{1}{x^{5/2}\left(1-c^2x^3\right)}\,dx\\ &= -\frac{bc}{3x^{3/2}}-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}+\frac{1}{2}\left(bc^3\right)\int \frac{\sqrt{x}}{1-c^2x^3}\,dx\\ &= -\frac{bc}{3x^{3/2}}-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}+\left(bc^3\right)\text{Subst}\left(\int \frac{x^2}{1-c^2x^6}\,dx,x,\sqrt{x}\right)\\ &= -\frac{bc}{3x^{3/2}}-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}+\frac{1}{3}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx,x,x^{3/2}\right)\\ &= -\frac{bc}{3x^{3/2}}+\frac{1}{3}bc^2\tanh^{-1}\left(cx^{3/2}\right)-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 73, normalized size = 1.55

$$-\frac{a}{3x^3}-\frac{bc}{3x^{3/2}}-\frac{b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}-\frac{1}{6}bc^2\log\left(1-cx^{3/2}\right)+\frac{1}{6}bc^2\log\left(1+cx^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*c)/(3*x^(3/2)) - (b*ArcTanh[c*x^(3/2)])/(3*x^3) - (b*c^2*Lo
g[1 - c*x^(3/2)])/6 + (b*c^2*Log[1 + c*x^(3/2)])/6
```

Maple [A]
time = 0.06, size = 55, normalized size = 1.17

method	result	size
derivativedivides	$-\frac{a}{3x^3}-\frac{b\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{3x^3}-\frac{b\,c^2\ln\left(c\,x^{\frac{3}{2}}-1\right)}{6}-\frac{bc}{3x^{\frac{3}{2}}}+\frac{b\,c^2\ln\left(c\,x^{\frac{3}{2}}+1\right)}{6}$	55
default	$-\frac{a}{3x^3}-\frac{b\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{3x^3}-\frac{b\,c^2\ln\left(c\,x^{\frac{3}{2}}-1\right)}{6}-\frac{bc}{3x^{\frac{3}{2}}}+\frac{b\,c^2\ln\left(c\,x^{\frac{3}{2}}+1\right)}{6}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(3/2)))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctanh(c*x^(3/2))-1/6*b*c^2*ln(c*x^(3/2)-1)-1/3*b*c/x
^(3/2)+1/6*b*c^2*ln(c*x^(3/2)+1)
```

Maxima [A]
time = 0.27, size = 51, normalized size = 1.09

$$\frac{1}{6}\left(\left(c\log\left(cx^{\frac{3}{2}}+1\right)-c\log\left(cx^{\frac{3}{2}}-1\right)-\frac{2}{x^{\frac{3}{2}}}\right)c-\frac{2\,\mathrm{artanh}\left(cx^{\frac{3}{2}}\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((c*log(c*x^(3/2) + 1) - c*log(c*x^(3/2) - 1) - 2/x^(3/2))*c - 2*arctan
h(c*x^(3/2))/x^3)*b - 1/3*a/x^3
```

Fricas [A]
time = 0.35, size = 59, normalized size = 1.26

$$-\frac{2\,bcx^{\frac{3}{2}}-(bc^2x^3-b)\log\left(-\frac{c^2x^3+2\,cx^{\frac{3}{2}}+1}{c^2x^3-1}\right)+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*c*x^(3/2) - (b*c^2*x^3 - b)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2
*x^3 - 1)) + 2*a)/x^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))/x**4,x)
```

[Out] Timed out

Giac [A]
time = 0.44, size = 67, normalized size = 1.43

$$\frac{1}{6}\,bc^{2}\log\left(cx^{\frac{3}{2}}+1\right)-\frac{1}{6}\,bc^{2}\log\left(cx^{\frac{3}{2}}-1\right)-\frac{b\log\left(-\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}\right)}{6\,x^{3}}-\frac{bcx^{\frac{3}{2}}+a}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^4,x, algorithm="giac")
```

```
[Out] 1/6*b*c^2*log(c*x^(3/2) + 1) - 1/6*b*c^2*log(c*x^(3/2) - 1) - 1/6*b*log(-(c
*x^(3/2) + 1)/(c*x^(3/2) - 1))/x^3 - 1/3*(b*c*x^(3/2) + a)/x^3
```

Mupad [B]
time = 1.36, size = 114, normalized size = 2.43

$$\frac{b\,c^{2}\,\ln\left(\frac{c\,x^{3/2}+1}{c\,x^{3/2}-1}\right)}{6}-\frac{a}{3\,x^{3}}-\frac{b\,c}{3\,x^{3/2}}-\frac{b\,\ln\left(c\,x^{3/2}+1\right)}{6\,x^{3}}+\frac{b\,x\,\ln\left(1-c\,x^{3/2}\right)}{3\,\left(2\,x^{4}-2\,c^{2}\,x^{7}\right)}-\frac{b\,c^{2}\,x^{4}\,\ln\left(1-c\,x^{3/2}\right)}{3\,\left(2\,x^{4}-2\,c^{2}\,x^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(3/2)))/x^4,x)
```

```
[Out] (b*c^2*log((c*x^(3/2) + 1)/(c*x^(3/2) - 1)))/6 - a/(3*x^3) - (b*c)/(3*x^(3/
2)) - (b*log(c*x^(3/2) + 1))/(6*x^3) + (b*x*log(1 - c*x^(3/2)))/(3*(2*x^4 -
 2*c^2*x^7)) - (b*c^2*x^4*log(1 - c*x^(3/2)))/(3*(2*x^4 - 2*c^2*x^7))
```

3.221 $\int x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2\,dx$

Optimal. Leaf size=101

$$\frac{2abx^{3/2}}{3c}+\frac{2b^2x^{3/2}\tanh^{-1}\left(cx^{3/2}\right)}{3c}-\frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{3c^2}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2+\frac{b^2\log\left(1-c^2x^3\right)}{3c^2}$$

```
[Out] 2/3*a*b*x^(3/2)/c+2/3*b^2*x^(3/2)*arctanh(c*x^(3/2))/c-1/3*(a+b*arctanh(c*x
^(3/2)))^2/c^2+1/3*x^3*(a+b*arctanh(c*x^(3/2)))^2+1/3*b^2*ln(-c^2*x^3+1)/c^
2
```

Rubi [A]
time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6039, 6037, 6127, 6021, 266, 6095}

$$-\frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{3c^2}+\frac{2abx^{3/2}}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2+\frac{b^2\log\left(1-c^2x^3\right)}{3c^2}+\frac{2b^2x^{3/2}\tanh^{-1}\left(cx^{3/2}\right)}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^(3/2)])^2,x]
```

```
[Out] (2*a*b*x^(3/2))/(3*c) + (2*b^2*x^(3/2)*ArcTanh[c*x^(3/2)])/(3*c) - (a + b*A
rcTanh[c*x^(3/2)])^2/(3*c^2) + (x^3*(a + b*ArcTanh[c*x^(3/2)])^2)/3 + (b^2*
Log[1 - c^2*x^3])/(3*c^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2\,dx=\int x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2\,dx$$

Mathematica [A]
time = 0.06, size = 122, normalized size = 1.21

$$\frac{2abcx^{3/2}+a^2c^2x^3+2bcx^{3/2}\left(b+acx^{3/2}\right)\tanh^{-1}\left(cx^{3/2}\right)+b^2\left(-1+c^2x^3\right)\tanh^{-1}\left(cx^{3/2}\right)^2+b(a+b)\log\left(1-cx^{3/2}\right)-ab\log\left(1+cx^{3/2}\right)+b^2\log\left(1+cx^{3/2}\right)}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^(3/2)])^2,x]
```

```
[Out] (2*a*b*c*x^(3/2) + a^2*c^2*x^3 + 2*b*c*x^(3/2)*(b + a*c*x^(3/2))*ArcTanh[c*
x^(3/2)] + b^2*(-1 + c^2*x^3)*ArcTanh[c*x^(3/2)]^2 + b*(a + b)*Log[1 - c*x^
(3/2)] - a*b*Log[1 + c*x^(3/2)] + b^2*Log[1 + c*x^(3/2)])/(3*c^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(81) = 162$.
time = 0.27, size = 258, normalized size = 2.55

method	result

derivativedivides	$\frac{c^2x^3a^2}{3}+\frac{b^2c^2x^3\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)^2}{3}+\frac{2b^2\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)c\,x^{\frac{3}{2}}}{3}+\frac{b^2\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)\ln\left(c\,x^{\frac{3}{2}}-1\right)}{3}-\frac{b^2\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)\ln\left(c\,x^{\frac{3}{2}}+1\right)}{3}-$
default	$\frac{c^2x^3a^2}{3}+\frac{b^2c^2x^3\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)^2}{3}+\frac{2b^2\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)c\,x^{\frac{3}{2}}}{3}+\frac{b^2\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)\ln\left(c\,x^{\frac{3}{2}}-1\right)}{3}-\frac{b^2\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)\ln\left(c\,x^{\frac{3}{2}}+1\right)}{3}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^(3/2)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/c^2*(1/2*c^2*x^3*a^2+1/2*b^2*c^2*x^3*arctanh(c*x^(3/2))^2+b^2*arctanh(c
*x^(3/2))*c*x^(3/2)+1/2*b^2*arctanh(c*x^(3/2))*ln(c*x^(3/2)-1)-1/2*b^2*arct
anh(c*x^(3/2))*ln(c*x^(3/2)+1)-1/4*b^2*ln(c*x^(3/2)-1)*ln(1/2*c*x^(3/2)+1/2
)+1/8*b^2*ln(c*x^(3/2)-1)^2+1/2*b^2*ln(c*x^(3/2)-1)+1/2*b^2*ln(c*x^(3/2)+1)
+1/8*b^2*ln(c*x^(3/2)+1)^2-1/4*b^2*ln(-1/2*c*x^(3/2)+1/2)*ln(c*x^(3/2)+1)+1
/4*b^2*ln(-1/2*c*x^(3/2)+1/2)*ln(1/2*c*x^(3/2)+1/2)+a*b*c^2*x^3*arctanh(c*x
^(3/2))+c*x^(3/2)*a*b+1/2*a*b*ln(c*x^(3/2)-1)-1/2*a*b*ln(c*x^(3/2)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(81) = 162$.
time $= 0.26$, size $= 186$, normalized size $= 1.84$

$$\frac{1}{3}b^2x^3\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)^2+\frac{1}{3}a^2x^3+\frac{1}{3}\left(2x^3\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)+c\left(\frac{2x^{\frac{3}{2}}}{c^2}-\frac{\log\left(cx^{\frac{3}{2}}+1\right)}{c^3}+\frac{\log\left(cx^{\frac{3}{2}}-1\right)}{c^3}\right)\right)ab+\frac{1}{12}\left(4c\left(\frac{2x^{\frac{3}{2}}}{c^2}-\frac{\log\left(cx^{\frac{3}{2}}+1\right)}{c^3}+\frac{\log\left(cx^{\frac{3}{2}}-1\right)}{c^3}\right)\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)-\frac{2\left(\log\left(cx^{\frac{3}{2}}-1\right)-2\right)\log\left(cx^{\frac{3}{2}}+1\right)-\log\left(cx^{\frac{3}{2}}+1\right)^2-\log\left(cx^{\frac{3}{2}}-1\right)^2-4\log\left(cx^{\frac{3}{2}}-1\right)}{c^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(3/2)))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*arctanh(c*x^(3/2))^2 + 1/3*a^2*x^3 + 1/3*(2*x^3*arctanh(c*x^(3/
2)) + c*(2*x^(3/2)/c^2 - log(c*x^(3/2) + 1)/c^3 + log(c*x^(3/2) - 1)/c^3))*
a*b + 1/12*(4*c*(2*x^(3/2)/c^2 - log(c*x^(3/2) + 1)/c^3 + log(c*x^(3/2) - 1
)/c^3)*arctanh(c*x^(3/2)) - (2*(log(c*x^(3/2) - 1) - 2)*log(c*x^(3/2) + 1)
- log(c*x^(3/2) + 1)^2 - log(c*x^(3/2) - 1)^2 - 4*log(c*x^(3/2) - 1))/c^2)*
b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(81) = 162$.
time $= 0.38$, size $= 179$, normalized size $= 1.77$

$$\frac{4a^2c^2x^3+8abcx^{\frac{3}{2}}+\left(b^2c^2x^3-b^2\right)\log\left(-\frac{c^2x^3+2cx^{\frac{3}{2}}+1}{c^2x^3-1}\right)^2+4\left(abc^2-ab+b^2\right)\log\left(cx^{\frac{3}{2}}+1\right)-4\left(abc^2-ab-b^2\right)\log\left(cx^{\frac{3}{2}}-1\right)+4\left(abc^2x^3+b^2cx^{\frac{3}{2}}-abc^2\right)\log\left(-\frac{c^2x^3+2cx^{\frac{3}{2}}+1}{c^2x^3-1}\right)}{12c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(3/2)))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^2*c^2*x^3 + 8*a*b*c*x^(3/2) + (b^2*c^2*x^3 - b^2)*log(-(c^2*x^3 +
 2*c*x^(3/2) + 1)/(c^2*x^3 - 1))^2 + 4*(a*b*c^2 - a*b + b^2)*log(c*x^(3/2)
```

```
+ 1) - 4*(a*b*c^2 - a*b - b^2)*log(c*x^(3/2) - 1) + 4*(a*b*c^2*x^3 + b^2*c*
x^(3/2) - a*b*c^2)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)))/c^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**(3/2)))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(3/2)))^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^(3/2)) + a)^2*x^2, x)
```

Mupad [**B**]
time = 1.31, size = 105, normalized size = 1.04

$$\frac{c\left(\frac{2\,b^2\,x^{3/2}\,\mathrm{atanh}\left(c\,x^{3/2}\right)}{3}+\frac{2\,a\,b\,x^{3/2}}{3}\right)-\frac{b^2\,\mathrm{atanh}\left(c\,x^{3/2}\right)^2}{3}+\frac{b^2\,\ln\left(c^2\,x^3-1\right)}{3}-\frac{2\,a\,b\,\mathrm{atanh}\left(c\,x^{3/2}\right)}{3}}{c^2}+\frac{a^2\,x^3}{3}+\frac{b^2\,x^3\,\mathrm{atanh}\left(c\,x^{3/2}\right)^2}{3}+\frac{2\,a\,b\,x^3\,\mathrm{atanh}\left(c\,x^{3/2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^(3/2)))^2,x)
[Out] (c*((2*b^2*x^(3/2)*atanh(c*x^(3/2)))/3 + (2*a*b*x^(3/2))/3) - (b^2*atanh(c*
x^(3/2))^2)/3 + (b^2*log(c^2*x^3 - 1))/3 - (2*a*b*atanh(c*x^(3/2)))/3)/c^2
+ (a^2*x^3)/3 + (b^2*x^3*atanh(c*x^(3/2))^2)/3 + (2*a*b*x^3*atanh(c*x^(3/2)
))/3
```

3.222 $\int \frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{x}\,dx$

Optimal. Leaf size=156

$$\frac{4}{3}\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^{3/2}}\right)-\frac{2}{3}b\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1-cx^{3/2}}\right)+\frac{2}{3}$$

```
[Out] -4/3*(a+b*arctanh(c*x^(3/2)))^2*arctanh(-1+2/(1-c*x^(3/2)))-2/3*b*(a+b*arct
anh(c*x^(3/2)))*polylog(2,1-2/(1-c*x^(3/2)))+2/3*b*(a+b*arctanh(c*x^(3/2)))
*polylog(2,-1+2/(1-c*x^(3/2)))+1/3*b^2*polylog(3,1-2/(1-c*x^(3/2)))-1/3*b^2
*polylog(3,-1+2/(1-c*x^(3/2)))
```

Rubi [A]
time = 0.22, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6035, 6033, 6199, 6095, 6205, 6745}

$$-\frac{2}{3}b\text{Li}_2\left(1-\frac{2}{1-cx^{3/2}}\right)\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{2}{3}b\text{Li}_2\left(\frac{2}{1-cx^{3/2}}-1\right)\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{4}{3}\tanh^{-1}\left(1-\frac{2}{1-cx^{3/2}}\right)\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2+\frac{1}{3}b^2\text{Li}_3\left(1-\frac{2}{1-cx^{3/2}}\right)-\frac{1}{3}b^2\text{Li}_3\left(\frac{2}{1-cx^{3/2}}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])^2/x,x]
```

```
[Out] (4*(a + b*ArcTanh[c*x^(3/2)])^2*ArcTanh[1 - 2/(1 - c*x^(3/2))])/3 - (2*b*(a
 + b*ArcTanh[c*x^(3/2)])*PolyLog[2, 1 - 2/(1 - c*x^(3/2))])/3 + (2*b*(a + b
*ArcTanh[c*x^(3/2)])*PolyLog[2, -1 + 2/(1 - c*x^(3/2))])/3 + (b^2*PolyLog[3
, 1 - 2/(1 - c*x^(3/2))])/3 - (b^2*PolyLog[3, -1 + 2/(1 - c*x^(3/2))])/3
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{x}\,dx = \frac{2}{3}\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx, x, x^{3/2}\right)$$

$$= \frac{4}{3}\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^{3/2}}\right) - \frac{1}{3}(8bc)\text{Subst}\left(\int \frac{(a+ \cdots}{\cdots}\right.$$

$$= \frac{4}{3}\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^{3/2}}\right) + \frac{1}{3}(4bc)\text{Subst}\left(\int \frac{(a+ \cdots}{\cdots}\right.$$

$$= \frac{4}{3}\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^{3/2}}\right) - \frac{2}{3}b\left(a+b\tanh^{-1}\left(cx^{3/\cdots}\right.\right.$$

$$= \frac{4}{3}\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^{3/2}}\right) - \frac{2}{3}b\left(a+b\tanh^{-1}\left(cx^{3/\cdots}\right.\right.$$

Mathematica [A]
time = 0.09, size = 167, normalized size = 1.07

$$\frac{1}{3}\left(4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2\tanh^{-1}\left(1+\frac{2}{-1+cx^{3/2}}\right)+b\left(2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)\text{PolyLog}\left(2,\frac{1+cx^{3/2}}{1-cx^{3/2}}\right)-2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)\text{PolyLog}\left(2,\frac{1+cx^{3/2}}{-1+cx^{3/2}}\right)+b\left(-\text{PolyLog}\left(3,\frac{1+cx^{3/2}}{1-cx^{3/2}}\right)+\text{PolyLog}\left(3,\frac{1+cx^{3/2}}{-1+cx^{3/2}}\right)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])^2/x,x]
```

```
[Out] (4*(a + b*ArcTanh[c*x^(3/2)])^2*ArcTanh[1 + 2/(-1 + c*x^(3/2))] + b*(2*(a +
 b*ArcTanh[c*x^(3/2)])*PolyLog[2, (1 + c*x^(3/2))/(1 - c*x^(3/2))] - 2*(a +
 b*ArcTanh[c*x^(3/2)])*PolyLog[2, (1 + c*x^(3/2))/(-1 + c*x^(3/2))] + b*(-P
olyLog[3, (1 + c*x^(3/2))/(1 - c*x^(3/2))] + PolyLog[3, (1 + c*x^(3/2))/(-1
 + c*x^(3/2))])))/3
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.80, size = 785, normalized size = 5.03 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(3/2)))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a^2*ln(c*x^(3/2))+2/3*b^2*ln(c*x^(3/2))*arctanh(c*x^(3/2))^2-2/3*b^2*ar
ctanh(c*x^(3/2))*polylog(2,-(c*x^(3/2)+1)^2/(-c^2*x^3+1))+1/3*b^2*polylog(3
,-(c*x^(3/2)+1)^2/(-c^2*x^3+1))-2/3*b^2*arctanh(c*x^(3/2))^2*ln((c*x^(3/2)+
1)^2/(-c^2*x^3+1)-1)+2/3*b^2*arctanh(c*x^(3/2))^2*ln(1-(c*x^(3/2)+1)/(-c^2*
x^3+1)^(1/2))+4/3*b^2*arctanh(c*x^(3/2))*polylog(2,(c*x^(3/2)+1)/(-c^2*x^3+
1)^(1/2))-4/3*b^2*polylog(3,(c*x^(3/2)+1)/(-c^2*x^3+1)^(1/2))+2/3*b^2*arcta
nh(c*x^(3/2))^2*ln(1+(c*x^(3/2)+1)/(-c^2*x^3+1)^(1/2))+4/3*b^2*arctanh(c*x^
(3/2))*polylog(2,-(c*x^(3/2)+1)/(-c^2*x^3+1)^(1/2))-4/3*b^2*polylog(3,-(c*x
^(3/2)+1)/(-c^2*x^3+1)^(1/2))-1/3*I*b^2*Pi*csgn(I/(1+(c*x^(3/2)+1)^2/(-c^2*
x^3+1)))*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x^3+1)-1)/(1+(c*x^(3/2)+1)^2/(-c^2*x
^3+1)))^2*arctanh(c*x^(3/2))^2+1/3*I*b^2*Pi*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x
^3+1)-1)/(1+(c*x^(3/2)+1)^2/(-c^2*x^3+1)))^3*arctanh(c*x^(3/2))^2-1/3*I*b^2
*Pi*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x^3+1)-1))*csgn(I*((c*x^(3/2)+1)^2/(-c^2*
x^3+1)-1)/(1+(c*x^(3/2)+1)^2/(-c^2*x^3+1)))^2*arctanh(c*x^(3/2))^2+1/3*I*b^
2*Pi*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x^3+1)-1))*csgn(I/(1+(c*x^(3/2)+1)^2/(-c
^2*x^3+1)))*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x^3+1)-1)/(1+(c*x^(3/2)+1)^2/(-c^
2*x^3+1)))*arctanh(c*x^(3/2))^2+4/3*a*b*ln(c*x^(3/2))*arctanh(c*x^(3/2))-2/
3*a*b*ln(c*x^(3/2))*ln(c*x^(3/2)+1)-2/3*a*b*dilog(c*x^(3/2)+1)-2/3*a*b*dilo
g(c*x^(3/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*integrate(log(c*x^(3/2) + 1)^2/x, x) - 1/2*b^2*integrate(log(c*x^(3
/2) + 1)*log(-c*x^(3/2) + 1)/x, x) + 1/4*b^2*integrate(log(-c*x^(3/2) + 1)^
2/x, x) + a*b*integrate(log(c*x^(3/2) + 1)/x, x) - a*b*integrate(log(-c*x^(
3/2) + 1)/x, x) + a^2*log(x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x,x, algorithm="fricas")
[Out] integral((b^2*arctanh(c*x^(3/2))^2 + 2*a*b*arctanh(c*x^(3/2)) + a^2)/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))**2/x,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^(3/2)) + a)^2/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,x^{3/2}\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(3/2)))^2/x,x)
[Out] int((a + b*atanh(c*x^(3/2)))^2/x, x)
```

3.223 $\int \frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=96

$$-\frac{2bc\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)}{3x^{3/2}}+\frac{1}{3}c^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2-\frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{3x^3}+b^2c^2\log(x)-\frac{1}{3}b^2c^2\log\left(1\right.$$

```
[Out] -2/3*b*c*(a+b*arctanh(c*x^(3/2)))/x^(3/2)+1/3*c^2*(a+b*arctanh(c*x^(3/2)))^
2-1/3*(a+b*arctanh(c*x^(3/2)))^2/x^3+b^2*c^2*ln(x)-1/3*b^2*c^2*ln(-c^2*x^3+
1)
```

Rubi [A]
time = 0.13, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6039, 6037, 6129, 272, 36, 29, 31, 6095}

$$\frac{1}{3}c^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2-\frac{2bc\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)}{3x^{3/2}}-\frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{3x^3}-\frac{1}{3}b^2c^2\log\left(1-c^2x^3\right)+b^2c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])^2/x^4,x]
```

```
[Out] (-2*b*c*(a + b*ArcTanh[c*x^(3/2)]))/(3*x^(3/2)) + (c^2*(a + b*ArcTanh[c*x^(
3/2)])^2)/3 - (a + b*ArcTanh[c*x^(3/2)])^2/(3*x^3) + b^2*c^2*Log[x] - (b^2*
c^2*Log[1 - c^2*x^3])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6039

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x]
, x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpli
fy[(m + 1)/n]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx$$

Mathematica [A]
time = 0.09, size = 123, normalized size = 1.28

$$\frac{1}{3}\left(-\frac{a^2}{x^3}-\frac{2abc}{x^{3/2}}-\frac{2b\left(a+bcx^{3/2}\right)\tanh^{-1}\left(cx^{3/2}\right)}{x^3}+\frac{b^2\left(-1+c^2x^3\right)\tanh^{-1}\left(cx^{3/2}\right)^2}{x^3}+3b^2c^2\log(x)-b(a+b)c^2\log\left(1-cx^{3/2}\right)+(a-b)bc^2\log\left(1+cx^{3/2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])^2/x^4,x]
```

```
[Out] (-(a^2/x^3) - (2*a*b*c)/x^(3/2) - (2*b*(a + b*c*x^(3/2))*ArcTanh[c*x^(3/2)]
)/x^3 + (b^2*(-1 + c^2*x^3)*ArcTanh[c*x^(3/2)]^2)/x^3 + 3*b^2*c^2*Log[x] -
b*(a + b)*c^2*Log[1 - c*x^(3/2)] + (a - b)*b*c^2*Log[1 + c*x^(3/2)])/3
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(3/2)))^2/x^4,x)
```

```
[Out] int((a+b*arctanh(c*x^(3/2)))^2/x^4,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(80) = 160.
time = 0.28, size = 175, normalized size = 1.82

$$\frac{1}{3}\left(\left(c\log\left(cx^{\frac{3}{2}}+1\right)-c\log\left(cx^{\frac{3}{2}}-1\right)-\frac{2}{x^{\frac{3}{2}}}\right)c-\frac{2\,\mathrm{artanh}\left(cx^{\frac{3}{2}}\right)}{x^{3}}\right)ab+\frac{1}{12}\left(\left(2\left(\log\left(cx^{\frac{3}{2}}-1\right)-2\right)\log\left(cx^{\frac{3}{2}}+1\right)-\log\left(cx^{\frac{3}{2}}+1\right)^{2}-\log\left(cx^{\frac{3}{2}}-1\right)^{2}-4\,\log\left(cx^{\frac{3}{2}}-1\right)+12\,\log\left(x\right)\right)c^{2}+4\left(c\log\left(cx^{\frac{3}{2}}+1\right)-c\log\left(cx^{\frac{3}{2}}-1\right)-\frac{2}{x^{\frac{3}{2}}}\right)c\,\mathrm{artanh}\left(cx^{\frac{3}{2}}\right)\right)b^{2}-\frac{b^{2}\,\mathrm{artanh}\left(cx^{\frac{3}{2}}\right)^{2}}{3\,x^{3}}-\frac{a^{2}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*((c*log(c*x^(3/2) + 1) - c*log(c*x^(3/2) - 1) - 2/x^(3/2))*c - 2*arctan
h(c*x^(3/2))/x^3)*a*b + 1/12*((2*(log(c*x^(3/2) - 1) - 2)*log(c*x^(3/2) + 1
) - log(c*x^(3/2) + 1)^2 - log(c*x^(3/2) - 1)^2 - 4*log(c*x^(3/2) - 1) + 12
*log(x))*c^2 + 4*(c*log(c*x^(3/2) + 1) - c*log(c*x^(3/2) - 1) - 2/x^(3/2))*
c*arctanh(c*x^(3/2)))*b^2 - 1/3*b^2*arctanh(c*x^(3/2))^2/x^3 - 1/3*a^2/x^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(80) = 160.
time = 0.37, size = 173, normalized size = 1.80

$$\frac{24\,b^{2}c^{2}x^{3}\log\left(\sqrt{x}\right)+4\,(ab-b^{2})c^{2}x^{3}\log\left(cx^{\frac{3}{2}}+1\right)-4\,(ab+b^{2})c^{2}x^{3}\log\left(cx^{\frac{3}{2}}-1\right)-8\,abcx^{\frac{3}{2}}+(b^{2}c^{2}x^{3}-b^{2})\log\left(-\frac{c^{2}x^{3}+2\,cx^{\frac{3}{2}}+1}{c^{2}x^{3}-1}\right)^{2}-4\,a^{2}-4\left(b^{2}cx^{\frac{3}{2}}+ab\right)\log\left(-\frac{c^{2}x^{3}+2\,cx^{\frac{3}{2}}+1}{c^{2}x^{3}-1}\right)}{12\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(24*b^2*c^2*x^3*log(sqrt(x)) + 4*(a*b - b^2)*c^2*x^3*log(c*x^(3/2) + 1
) - 4*(a*b + b^2)*c^2*x^3*log(c*x^(3/2) - 1) - 8*a*b*c*x^(3/2) + (b^2*c^2*x
^3 - b^2)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1))^2 - 4*a^2 - 4*(b^
2*c*x^(3/2) + a*b)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)))/x^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))**2/x**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^(3/2)) + a)^2/x^4, x)
```

Mupad [B]
time = 2.04, size = 281, normalized size = 2.93

$$\frac{2\,b^2\,c^2\,\ln\left(x^{3/2}\right)}{3}-\frac{a^2}{3\,x^3}-\frac{b^2\,c^2\,\ln\left(c\,x^{3/2}-1\right)}{3}-\frac{b^2\,c^2\,\ln\left(c\,x^{3/2}+1\right)}{3}+\frac{b^2\,c^2\,\ln\left(c\,x^{3/2}+1\right)^2}{12}+\frac{b^2\,c^2\,\ln\left(1-c\,x^{3/2}\right)^2}{12}-\frac{b^2\,\ln\left(c\,x^{3/2}+1\right)^2}{12\,x^3}-\frac{b^2\,\ln\left(1-c\,x^{3/2}\right)^2}{12\,x^3}-\frac{a\,b\,c^2\,\ln\left(c\,x^{3/2}-1\right)}{3}+\frac{a\,b\,c^2\,\ln\left(c\,x^{3/2}+1\right)}{3}-\frac{2\,a\,b\,c}{3\,x^{3/2}}-\frac{a\,b\,\ln\left(c\,x^{3/2}+1\right)}{3\,x^3}+\frac{a\,b\,\ln\left(1-c\,x^{3/2}\right)}{3\,x^3}-\frac{b^2\,c^2\,\ln\left(c\,x^{3/2}+1\right)\,\ln\left(1-c\,x^{3/2}\right)}{6}-\frac{b^2\,c\,\ln\left(c\,x^{3/2}+1\right)}{3\,x^{3/2}}+\frac{b^2\,c\,\ln\left(1-c\,x^{3/2}\right)}{3\,x^{3/2}}+\frac{b^2\,\ln\left(c\,x^{3/2}+1\right)\,\ln\left(1-c\,x^{3/2}\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(3/2)))^2/x^4,x)
```

```
[Out] (2*b^2*c^2*log(x^(3/2)))/3 - a^2/(3*x^3) - (b^2*c^2*log(c*x^(3/2) - 1))/3 -
 (b^2*c^2*log(c*x^(3/2) + 1))/3 + (b^2*c^2*log(c*x^(3/2) + 1)^2)/12 + (b^2*
c^2*log(1 - c*x^(3/2))^2)/12 - (b^2*log(c*x^(3/2) + 1)^2)/(12*x^3) - (b^2*l
og(1 - c*x^(3/2))^2)/(12*x^3) - (a*b*c^2*log(c*x^(3/2) - 1))/3 + (a*b*c^2*l
og(c*x^(3/2) + 1))/3 - (2*a*b*c)/(3*x^(3/2)) - (a*b*log(c*x^(3/2) + 1))/(3*
x^3) + (a*b*log(1 - c*x^(3/2)))/(3*x^3) - (b^2*c^2*log(c*x^(3/2) + 1)*log(1
 - c*x^(3/2)))/6 - (b^2*c*log(c*x^(3/2) + 1))/(3*x^(3/2)) + (b^2*c*log(1 -
c*x^(3/2)))/(3*x^(3/2)) + (b^2*log(c*x^(3/2) + 1)*log(1 - c*x^(3/2)))/(6*x^
3)
```

3.224 $\int x^2 (a + b \tanh^{-1}(cx^n)) \, dx$

Optimal. Leaf size=66

$$\frac{1}{3}x^3 (a + b \tanh^{-1}(cx^n)) - \frac{bcnx^{3+n} {}_2F_1\left(1, \frac{3+n}{2n}; \frac{3(1+n)}{2n}; c^2x^{2n}\right)}{3(3+n)}$$

```
[Out] 1/3*x^3*(a+b*arctanh(c*x^n))-1/3*b*c*n*x^(3+n)*hypergeom([1, 1/2*(3+n)/n],[
3/2*(1+n)/n],c^2*x^(2*n))/(3+n)
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6037, 371}

$$\frac{1}{3}x^3 (a + b \tanh^{-1}(cx^n)) - \frac{bcnx^{n+3} {}_2F_1\left(1, \frac{n+3}{2n}; \frac{3(n+1)}{2n}; c^2x^{2n}\right)}{3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] (x^3*(a + b*ArcTanh[c*x^n]))/3 - (b*c*n*x^(3 + n)*Hypergeometric2F1[1, (3 +
 n)/(2*n), (3*(1 + n))/(2*n), c^2*x^(2*n)])/(3*(3 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2 (a + b \tanh^{-1}(cx^n)) \, dx &= \frac{1}{3}x^3 (a + b \tanh^{-1}(cx^n)) - \frac{1}{3}(bcn) \int \frac{x^{2+n}}{1 - c^2x^{2n}} \, dx \\
&= \frac{1}{3}x^3 (a + b \tanh^{-1}(cx^n)) - \frac{bcnx^{3+n} {}_2F_1\left(1, \frac{3+n}{2n}; \frac{3(1+n)}{2n}; c^2x^{2n}\right)}{3(3+n)}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 73, normalized size = 1.11

$$\frac{ax^3}{3}+\frac{1}{3}bx^3\tanh^{-1}(cx^n)-\frac{bcnx^{3+n}\,{}_2F_1\left(1,\frac{3+n}{2n};1+\frac{3+n}{2n};c^2x^{2n}\right)}{3(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] (a*x^3)/3 + (b*x^3*ArcTanh[c*x^n])/3 - (b*c*n*x^(3 + n)*Hypergeometric2F1[1
, (3 + n)/(2*n), 1 + (3 + n)/(2*n), c^2*x^(2*n)])/(3*(3 + n))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^2(a+b\,\mathrm{arctanh}\,(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^n)),x)
```

```
[Out] int(x^2*(a+b*arctanh(c*x^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(x^3*log(c*x^n + 1) - x^3*log(-c*x^n + 1) + 3*n*integrate(1
/3*x^2/(c*x^n + 1), x) + 3*n*integrate(1/3*x^2/(c*x^n - 1), x))*b
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^n)),x, algorithm="fricas")
```

```
[Out] integral(b*x^2*arctanh(c*x^n) + a*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+b\,\mathrm{atanh}\,(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**n)),x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,(a+b\,\mathrm{atanh}(c\,x^n))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^n)),x)
```

```
[Out] int(x^2*(a + b*atanh(c*x^n)), x)
```

3.225 $\int x\left(a+b\tanh^{-1}(cx^n)\right)\,dx$

Optimal. Leaf size=67

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx^n)\right)-\frac{bcnx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{2n};\frac{1}{2}\left(3+\frac{2}{n}\right);c^2x^{2n}\right)}{2(2+n)}$$

```
[Out] 1/2*x^2*(a+b*arctanh(c*x^n))-1/2*b*c*n*x^(2+n)*hypergeom([1, 1/2*(2+n)/n],[
3/2+1/n],c^2*x^(2*n))/(2+n)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6037, 371}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx^n)\right)-\frac{bcnx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{2n};\frac{1}{2}\left(3+\frac{2}{n}\right);c^2x^{2n}\right)}{2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] (x^2*(a + b*ArcTanh[c*x^n]))/2 - (b*c*n*x^(2 + n)*Hypergeometric2F1[1, (2 +
 n)/(2*n), (3 + 2/n)/2, c^2*x^(2*n)])/(2*(2 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
 + 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\tanh^{-1}(cx^n)\right)\,dx&=\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx^n)\right)-\frac{1}{2}(bcn)\int\frac{x^{1+n}}{1-c^2x^{2n}}\,dx\\&=\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx^n)\right)-\frac{bcnx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{2n};\frac{1}{2}\left(3+\frac{2}{n}\right);c^2x^{2n}\right)}{2(2+n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 73, normalized size = 1.09

$$\frac{ax^2}{2}+\frac{1}{2}bx^2\tanh^{-1}(cx^n)-\frac{bcnx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{2n};1+\frac{2+n}{2n};c^2x^{2n}\right)}{2(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*ArcTanh[c*x^n])/2 - (b*c*n*x^(2 + n)*Hypergeometric2F1[1
, (2 + n)/(2*n), 1 + (2 + n)/(2*n), c^2*x^(2*n)])/(2*(2 + n))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x(a+b\,\mathrm{arctanh}\,(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^n)),x)
```

```
[Out] int(x*(a+b*arctanh(c*x^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(x^2*log(c*x^n + 1) - x^2*log(-c*x^n + 1) + 2*n*integrate(1
/2*x/(c*x^n + 1), x) + 2*n*integrate(1/2*x/(c*x^n - 1), x))*b
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n)),x, algorithm="fricas")
```

```
[Out] integral(b*x*arctanh(c*x^n) + a*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+b\,\mathrm{atanh}\,(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**n)),x)
```

```
[Out] Integral(x*(a + b*atanh(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\mathrm{atanh}\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^n)),x)
```

```
[Out] int(x*(a + b*atanh(c*x^n)), x)
```

3.226 $\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=58

$$ax+bx\tanh^{-1}\left(cx^n\right)-\frac{bcnx^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);c^2x^{2n}\right)}{1+n}$$

```
[Out] a*x+b*x*arctanh(c*x^n)-b*c*n*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n]
,c^2*x^(2*n))/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6021, 371}

$$ax-\frac{bcnx^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);c^2x^{2n}\right)}{n+1}+bx\tanh^{-1}\left(cx^n\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c*x^n],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x^n] - (b*c*n*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2
*n), (3 + n^(-1))/2, c^2*x^(2*n)])/(1 + n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)\,dx &= ax+b\int \tanh^{-1}\left(cx^n\right)\,dx\\ &= ax+bx\tanh^{-1}\left(cx^n\right)-(bcn)\int \frac{x^n}{1-c^2x^{2n}}\,dx\\ &= ax+bx\tanh^{-1}\left(cx^n\right)-\frac{bcnx^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);c^2x^{2n}\right)}{1+n}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 58, normalized size = 1.00

$$ax + bx\tanh^{-1}(cx^n) - \frac{bcnx^{1+n}\,{}_2F_1\left(1, \frac{1+n}{2n}; \frac{1}{2}\left(3+\frac{1}{n}\right); c^2x^{2n}\right)}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c*x^n],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x^n] - (b*c*n*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2
*n), (3 + n^(-1))/2, c^2*x^(2*n)])/(1 + n)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int a + b\,\mathrm{arctanh}(c\,x^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x^n),x)
```

```
[Out] int(a+b*arctanh(c*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*(n*integrate(1/(c*x^n + 1), x) + n*integrate(1/(c*x^n - 1), x) + x*log(
c*x^n + 1) - x*log(-c*x^n + 1))*b + a*x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^n),x, algorithm="fricas")
```

```
[Out] integral(b*arctanh(c*x^n) + a, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{atanh}(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x**n),x)
```

```
[Out] Integral(a + b*atanh(c*x**n), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^n),x, algorithm="giac")
```

```
[Out] integrate(b*arctanh(c*x^n) + a, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int a + b\,\mathrm{atanh}(c\,x^n)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atanh(c*x^n),x)
```

```
[Out] int(a + b*atanh(c*x^n), x)
```

3.227 $\int \frac{a+b\tanh^{-1}(cx^n)}{x}\,dx$

Optimal. Leaf size=36

$$a\log(x) - \frac{b\text{PolyLog}(2,-cx^n)}{2n} + \frac{b\text{PolyLog}(2,cx^n)}{2n}$$

```
[Out] a*ln(x)-1/2*b*polylog(2,-c*x^n)/n+1/2*b*polylog(2,c*x^n)/n
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6035, 6031}

$$a\log(x) - \frac{b\text{Li}_2(-cx^n)}{2n} + \frac{b\text{Li}_2(cx^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^n])/x,x]
[Out] a*Log[x] - (b*PolyLog[2, -(c*x^n)])/(2*n) + (b*PolyLog[2, c*x^n])/(2*n)
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx^n)}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx, x, x^n\right)}{n}\\ &= a\log(x) - \frac{b\text{Li}_2(-cx^n)}{2n} + \frac{b\text{Li}_2(cx^n)}{2n}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.06, size = 39, normalized size = 1.08

$$\frac{bcx^n\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2};c^2x^{2n}\right)}{n} + a\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c*x^n])/x,x]

[Out] (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, c^2*x^(2*n)])/n + a*Log[x]

Maple [A]
time = 0.14, size = 65, normalized size = 1.81

method	result	size
risch	$a\ln\left(x\right)+\frac{b\,\mathrm{dilog}\left(1-c\,x^{n}\right)}{2n}-\frac{b\,\mathrm{dilog}\left(c\,x^{n}+1\right)}{2n}$	35
derivativedivides	$\frac{a\ln\left(c\,x^{n}\right)+b\ln\left(c\,x^{n}\right)\mathrm{arctanh}\left(c\,x^{n}\right)-\frac{b\,\mathrm{dilog}\left(c\,x^{n}+1\right)}{2}-\frac{b\ln\left(c\,x^{n}\right)\ln\left(c\,x^{n}+1\right)}{2}-\frac{b\,\mathrm{dilog}\left(c\,x^{n}\right)}{2}}{n}$	65
default	$\frac{a\ln\left(c\,x^{n}\right)+b\ln\left(c\,x^{n}\right)\mathrm{arctanh}\left(c\,x^{n}\right)-\frac{b\,\mathrm{dilog}\left(c\,x^{n}+1\right)}{2}-\frac{b\ln\left(c\,x^{n}\right)\ln\left(c\,x^{n}+1\right)}{2}-\frac{b\,\mathrm{dilog}\left(c\,x^{n}\right)}{2}}{n}$	65

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(c*x^n))/x,x,method=_RETURNVERBOSE)

[Out] 1/n*(a*ln(c*x^n)+b*ln(c*x^n)*arctanh(c*x^n)-1/2*b*dilog(c*x^n+1)-1/2*b*ln(c*x^n)*ln(c*x^n+1)-1/2*b*dilog(c*x^n))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^n))/x,x, algorithm="maxima")

[Out] 1/2*(n*integrate(log(x)/(c*x*x^n + x), x) + n*integrate(log(x)/(c*x*x^n - x), x) + log(c*x^n + 1)*log(x) - log(-c*x^n + 1)*log(x))*b + a*log(x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(30) = 60$.
time = 0.38, size = 141, normalized size = 3.92

$$-\frac{bn\log\left(c\cosh\left(n\log\left(x\right)\right)+c\sinh\left(n\log\left(x\right)\right)+1\right)\log\left(x\right)-bn\log\left(-c\cosh\left(n\log\left(x\right)\right)-c\sinh\left(n\log\left(x\right)\right)+1\right)\log\left(x\right)-bn\log\left(x\right)\log\left(-\frac{c\cosh(n\log(x))+c\sinh(n\log(x))+1}{c\cosh(n\log(x))+c\sinh(n\log(x))-1}\right)-2\,an\log\left(x\right)-b\mathrm{Li}_2(c\cosh\left(n\log\left(x\right)\right)+c\sinh\left(n\log\left(x\right)\right))+b\mathrm{Li}_2(-c\cosh\left(n\log\left(x\right)\right)-c\sinh\left(n\log\left(x\right)\right))}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^n))/x,x, algorithm="fricas")

[Out] -1/2*(b*n*log(c*cosh(n*log(x)) + c*sinh(n*log(x)) + 1)*log(x) - b*n*log(-c*cosh(n*log(x)) - c*sinh(n*log(x)) + 1)*log(x) - b*n*log(x)*log(-(c*cosh(n*log(x)) + c*sinh(n*log(x)) + 1)/(c*cosh(n*log(x)) + c*sinh(n*log(x)) - 1)) -

```
 2*a*n*log(x) - b*dilog(c*cosh(n*log(x)) + c*sinh(n*log(x))) + b*dilog(-c*c
osh(n*log(x)) - c*sinh(n*log(x))))/n
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atanh}(cx^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))/x,x)
[Out] Integral((a + b*atanh(c*x**n))/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^n) + a)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b \operatorname{atanh}(c\,x^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^n))/x,x)
[Out] int((a + b*atanh(c*x^n))/x, x)
```

3.228 $\int \frac{a+b\tanh^{-1}(cx^n)}{x^2}\,dx$

Optimal. Leaf size=67

$$-\frac{a+b\tanh^{-1}(cx^n)}{x}-\frac{bcnx^{-1+n}\,{}_2F_1\left(1,-\frac{1-n}{2n};\frac{1}{2}\left(3-\frac{1}{n}\right);c^2x^{2n}\right)}{1-n}$$

```
[Out] (-a-b*arctanh(c*x^n))/x-b*c*n*x^(-1+n)*hypergeom([1, 1/2*(-1+n)/n],[3/2-1/2
/n],c^2*x^(2*n))/(1-n)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6037, 371}

$$-\frac{a+b\tanh^{-1}(cx^n)}{x}-\frac{bcnx^{n-1}\,{}_2F_1\left(1,-\frac{1-n}{2n};\frac{1}{2}\left(3-\frac{1}{n}\right);c^2x^{2n}\right)}{1-n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^n])/x^2,x]
```

```
[Out] -((a + b*ArcTanh[c*x^n])/x) - (b*c*n*x^(-1 + n)*Hypergeometric2F1[1, -1/2*(
1 - n)/n, (3 - n^(-1))/2, c^2*x^(2*n)])/(1 - n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx^n)}{x^2}\,dx &= -\frac{a+b\tanh^{-1}(cx^n)}{x}+(bcn)\int \frac{x^{-2+n}}{1-c^2x^{2n}}\,dx\\ &= -\frac{a+b\tanh^{-1}(cx^n)}{x}-\frac{bcnx^{-1+n}\,{}_2F_1\left(1,-\frac{1-n}{2n};\frac{1}{2}\left(3-\frac{1}{n}\right);c^2x^{2n}\right)}{1-n}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 66, normalized size = 0.99

$$-\frac{a}{x}-\frac{b\tanh^{-1}(cx^n)}{x}+\frac{bcnx^{-1+n}\,{}_2F_1\left(1,\frac{-1+n}{2n};1+\frac{-1+n}{2n};c^2x^{2n}\right)}{-1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcTanh[c*x^n])/x + (b*c*n*x^(-1 + n)*Hypergeometric2F1[1, (-1
 + n)/(2*n), 1 + (-1 + n)/(2*n), c^2*x^(2*n)])/(-1 + n)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{arctanh}\left(c\,x^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))/x^2,x)
```

```
[Out] int((a+b*arctanh(c*x^n))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(n*integrate(1/(c*x^2*x^n + x^2), x) + n*integrate(1/(c*x^2*x^n - x^2)
, x) + (log(c*x^n + 1) - log(-c*x^n + 1))/x)*b - a/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^n) + a)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{atanh}\left(cx^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atanh}(c\,x^n)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^n))/x^2,x)
```

```
[Out] int((a + b*atanh(c*x^n))/x^2, x)
```

3.229 $\int \frac{a+b\tanh^{-1}(cx^n)}{x^3}\,dx$

Optimal. Leaf size=70

$$-\frac{a+b\tanh^{-1}(cx^n)}{2x^2}-\frac{bcnx^{-2+n}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{2}{n}\right);\frac{1}{2}\left(3-\frac{2}{n}\right);c^2x^{2n}\right)}{2(2-n)}$$

```
[Out] 1/2*(-a-b*arctanh(c*x^n))/x^2-1/2*b*c*n*x^(-2+n)*hypergeom([1, 1/2-1/n],[3/
2-1/n],c^2*x^(2*n))/(2-n)
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6037, 371}

$$-\frac{a+b\tanh^{-1}(cx^n)}{2x^2}-\frac{bcnx^{n-2}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{2}{n}\right);\frac{1}{2}\left(3-\frac{2}{n}\right);c^2x^{2n}\right)}{2(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^n])/x^3,x]
```

```
[Out] -1/2*(a + b*ArcTanh[c*x^n])/x^2 - (b*c*n*x^(-2 + n)*Hypergeometric2F1[1, (1
 - 2/n)/2, (3 - 2/n)/2, c^2*x^(2*n)])/(2*(2 - n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx^n)}{x^3}\,dx &= -\frac{a+b\tanh^{-1}(cx^n)}{2x^2}+\frac{1}{2}(bcn)\int \frac{x^{-3+n}}{1-c^2x^{2n}}\,dx\\ &= -\frac{a+b\tanh^{-1}(cx^n)}{2x^2}-\frac{bcnx^{-2+n}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{2}{n}\right);\frac{1}{2}\left(3-\frac{2}{n}\right);c^2x^{2n}\right)}{2(2-n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 73, normalized size = 1.04

$$-\frac{a}{2x^2}-\frac{b\tanh^{-1}(cx^n)}{2x^2}+\frac{bcnx^{-2+n}\,{}_2F_1\left(1,\frac{-2+n}{2n};1+\frac{-2+n}{2n};c^2x^{2n}\right)}{2(-2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*ArcTanh[c*x^n])/(2*x^2) + (b*c*n*x^(-2 + n)*Hypergeometric2
F1[1, (-2 + n)/(2*n), 1 + (-2 + n)/(2*n), c^2*x^(2*n)])/(2*(-2 + n))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{arctanh}\left(c\,x^{n}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))/x^3,x)
```

```
[Out] int((a+b*arctanh(c*x^n))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*n*integrate(1/2/(c*x^3*x^n + x^3), x) + 2*n*integrate(1/2/(c*x^3*x^
n - x^3), x) + (log(c*x^n + 1) - log(-c*x^n + 1))/x^2)*b - 1/2*a/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^n) + a)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{atanh}\left(cx^{n}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))/x**3,x)
[Out] Integral((a + b*atanh(c*x**n))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^3,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^n) + a)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atanh}(c\,x^n)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^n))/x^3,x)
[Out] int((a + b*atanh(c*x^n))/x^3, x)
```

3.230 $\int \frac{a+b\tanh^{-1}(cx^n)}{x^4}\,dx$

Optimal. Leaf size=72

$$-\frac{a+b\tanh^{-1}(cx^n)}{3x^3}-\frac{bcnx^{-3+n}\,{}_2F_1\left(1,-\frac{3-n}{2n};-\frac{3(1-n)}{2n};c^2x^{2n}\right)}{3(3-n)}$$

```
[Out] 1/3*(-a-b*arctanh(c*x^n))/x^3-1/3*b*c*n*x^(-3+n)*hypergeom([1, 1/2*(-3+n)/n
],[-3/2*(1-n)/n],c^2*x^(2*n))/(3-n)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {6037, 371}

$$-\frac{a+b\tanh^{-1}(cx^n)}{3x^3}-\frac{bcnx^{n-3}\,{}_2F_1\left(1,-\frac{3-n}{2n};-\frac{3(1-n)}{2n};c^2x^{2n}\right)}{3(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^n])/x^4,x]
```

```
[Out] -1/3*(a + b*ArcTanh[c*x^n])/x^3 - (b*c*n*x^(-3 + n)*Hypergeometric2F1[1, -1
/2*(3 - n)/n, (-3*(1 - n))/(2*n), c^2*x^(2*n)])/(3*(3 - n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx^n)}{x^4}\,dx = -\frac{a+b\tanh^{-1}(cx^n)}{3x^3} + \frac{1}{3}(bcn)\int \frac{x^{-4+n}}{1-c^2x^{2n}}\,dx$$
$$= -\frac{a+b\tanh^{-1}(cx^n)}{3x^3} - \frac{bcnx^{-3+n}\,{}_2F_1\left(1,-\frac{3-n}{2n};-\frac{3(1-n)}{2n};c^2x^{2n}\right)}{3(3-n)}$$

Mathematica [**A**]
time = 0.03, size = 73, normalized size = 1.01

$$-\frac{a}{3x^3} - \frac{b\tanh^{-1}(cx^n)}{3x^3} + \frac{bcnx^{-3+n}\,{}_2F_1\left(1,\frac{-3+n}{2n};1+\frac{-3+n}{2n};c^2x^{2n}\right)}{3(-3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*ArcTanh[c*x^n])/(3*x^3) + (b*c*n*x^(-3 + n)*Hypergeometric2
F1[1, (-3 + n)/(2*n), 1 + (-3 + n)/(2*n), c^2*x^(2*n)])/(3*(-3 + n))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arctanh}(c\,x^n)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))/x^4,x)
```

```
[Out] int((a+b*arctanh(c*x^n))/x^4,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*n*integrate(1/3/(c*x^4*x^n + x^4), x) + 3*n*integrate(1/3/(c*x^4*x^
n - x^4), x) + (log(c*x^n + 1) - log(-c*x^n + 1))/x^3)*b - 1/3*a/x^3
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^n) + a)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{atanh}(cx^n)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))/x**4,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{atanh}(c\,x^n)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^n))/x^4,x)
```

```
[Out] int((a + b*atanh(c*x^n))/x^4, x)
```

3.231 $\int x\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left(x\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2,x\right)$$

```
[Out] Unintegrable(x*(a+b*arctanh(c*x^n))^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*(a + b*ArcTanh[c*x^n])^2,x]
```

```
[Out] Defer[Int][x*(a + b*ArcTanh[c*x^n])^2, x]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2\,dx=\int x\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2\,dx$$

Mathematica [A]
time = 7.78, size = 0, normalized size = 0.00

$$\int x\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^n])^2,x]
```

```
[Out] Integrate[x*(a + b*ArcTanh[c*x^n])^2, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int x(a+b\,\mathrm{arctanh}\,(c\,x^{n}))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^n))^2,x)
```

```
[Out] int(x*(a+b*arctanh(c*x^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^2*log(-c*x^n + 1)^2 + 1/2*a^2*x^2 - integrate(-1/4*((b^2*c*x*x^n
- b^2*x)*log(c*x^n + 1)^2 + 4*(a*b*c*x*x^n - a*b*x)*log(c*x^n + 1) + (4*a*b
*x - (b^2*c*n + 4*a*b*c)*x*x^n - 2*(b^2*c*x*x^n - b^2*x)*log(c*x^n + 1))*lo
g(-c*x^n + 1))/(c*x^n - 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x*arctanh(c*x^n)^2 + 2*a*b*x*arctanh(c*x^n) + a^2*x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b\operatorname{atanh}(cx^n))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**n))**2,x)
```

```
[Out] Integral(x*(a + b*atanh(c*x**n))**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2*x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int x\,(a+b\,\mathrm{atanh}(c\,x^n))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^n))^2,x)
```

```
[Out] int(x*(a + b*atanh(c*x^n))^2, x)
```

3.232 $\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 dx$

Optimal. Leaf size=15

$$\mathrm{Int}\left(\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2,x\right)$$

```
[Out] Unintegrable((a+b*arctanh(c*x^n))^2,x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^n])^2,x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*x^n])^2, x]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 dx=\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 dx$$

Mathematica [A]
time = 1.72, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])^2,x]
```

```
[Out] Integrate[(a + b*ArcTanh[c*x^n])^2, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arctanh}\left(c\,x^{n}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))^2,x)
```

```
[Out] int((a+b*arctanh(c*x^n))^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x*log(-c*x^n + 1)^2 + a^2*x - integrate(-1/4*((b^2*c*x^n - b^2)*log
(c*x^n + 1)^2 + 4*(a*b*c*x^n - a*b)*log(c*x^n + 1) + 2*(2*a*b - (b^2*c*n +
2*a*b*c)*x^n - (b^2*c*x^n - b^2)*log(c*x^n + 1))*log(-c*x^n + 1))/(c*x^n -
1), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2, x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{atanh}(cx^n))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))**2,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))**2, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int (a + b \operatorname{atanh}(c x^n))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^n))^2,x)
[Out] int((a + b*atanh(c*x^n))^2, x)
```

3.233 $\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)^2}{x}\,dx$

Optimal. Leaf size=148

$$\frac{2\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^n}\right)}{n}-\frac{b\left(a+b\tanh^{-1}\left(cx^n\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1-cx^n}\right)}{n}+\frac{b\left(a+b\tanh^{-1}\left(\right.\right.}{}$$

```
[Out] -2*(a+b*arctanh(c*x^n))^2*arctanh(-1+2/(1-c*x^n))/n-b*(a+b*arctanh(c*x^n))*
polylog(2,1-2/(1-c*x^n))/n+b*(a+b*arctanh(c*x^n))*polylog(2,-1+2/(1-c*x^n))
/n+1/2*b^2*polylog(3,1-2/(1-c*x^n))/n-1/2*b^2*polylog(3,-1+2/(1-c*x^n))/n
```

Rubi [A]
time = 0.23, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6035, 6033, 6199, 6095, 6205, 6745}

$$-\frac{b\text{Li}_2\left(1-\frac{2}{1-cx^n}\right)\left(a+b\tanh^{-1}\left(cx^n\right)\right)}{n}+\frac{b\text{Li}_2\left(\frac{2}{1-cx^n}-1\right)\left(a+b\tanh^{-1}\left(cx^n\right)\right)}{n}+\frac{2\tanh^{-1}\left(1-\frac{2}{1-cx^n}\right)\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{n}+\frac{b^2\text{Li}_3\left(1-\frac{2}{1-cx^n}\right)}{2n}-\frac{b^2\text{Li}_3\left(\frac{2}{1-cx^n}-1\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^n])^2/x,x]
```

```
[Out] (2*(a + b*ArcTanh[c*x^n])^2*ArcTanh[1 - 2/(1 - c*x^n)])/n - (b*(a + b*ArcTa
nh[c*x^n])*PolyLog[2, 1 - 2/(1 - c*x^n)])/n + (b*(a + b*ArcTanh[c*x^n])*Pol
yLog[2, -1 + 2/(1 - c*x^n)])/n + (b^2*PolyLog[3, 1 - 2/(1 - c*x^n)])/(2*n)
- (b^2*PolyLog[3, -1 + 2/(1 - c*x^n)])/(2*n)
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)^2}{x}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\tanh^{-1}(cx))^2}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx^n)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^n}\right)}{n} - \frac{(4bc)\text{Subst}\left(\int \frac{(a+b\tanh^{-1}(cx))\tanh}{1-c^2x^2}\right.}{n}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx^n)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^n}\right)}{n} + \frac{(2bc)\text{Subst}\left(\int \frac{(a+b\tanh^{-1}(cx))\log\left(\right.}{1-c^2x^2}\right.}{n}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx^n)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^n}\right)}{n} - \frac{b\left(a+b\tanh^{-1}(cx^n)\right)\text{Li}_2\left(1-\frac{}{1-}\right.}{n}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx^n)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^n}\right)}{n} - \frac{b\left(a+b\tanh^{-1}(cx^n)\right)\text{Li}_2\left(1-\frac{}{1-}\right.}{n}$$

Mathematica [A]

time = 0.14, size = 183, normalized size = 1.24

$$\frac{2\left(a+b\tanh^{-1}(cx^n)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^n}\right)-4bc\left(\frac{1}{2}\left(\frac{\left(-a-b\tanh^{-1}(cx^n)\right)\text{PolyLog}\left(2,\frac{-1-cx^n}{-1+cx^n}\right)}{2c}+\frac{b\text{PolyLog}\left(3,\frac{-1-cx^n}{-1+cx^n}\right)}{4c}\right)+\frac{1}{2}\left(-\frac{\left(-a-b\tanh^{-1}(cx^n)\right)\text{PolyLog}\left(2,\frac{1+cx^n}{-1+cx^n}\right)}{2c}-\frac{b\text{PolyLog}\left(3,\frac{1+cx^n}{-1+cx^n}\right)}{4c}\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])^2/x,x]
```

```
[Out] (2*(a + b*ArcTanh[c*x^n])^2*ArcTanh[1 - 2/(1 - c*x^n)] - 4*b*c*((((-a - b*A
rcTanh[c*x^n])*PolyLog[2, (-1 - c*x^n)/(-1 + c*x^n)])/(2*c) + (b*PolyLog[3,
 (-1 - c*x^n)/(-1 + c*x^n)])/(4*c))/2 + (-1/2*((-a - b*ArcTanh[c*x^n])*Poly
Log[2, (1 + c*x^n)/(-1 + c*x^n)])/c - (b*PolyLog[3, (1 + c*x^n)/(-1 + c*x^n
)])/(4*c))/2))/n
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.90, size = 827, normalized size = 5.59 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(a^2*ln(c*x^n)+b^2*ln(c*x^n)*arctanh(c*x^n)^2-b^2*arctanh(c*x^n)*polylo
g(2,-(c*x^n+1)^2/(-c^2*(x^n)^2+1))+1/2*b^2*polylog(3,-(c*x^n+1)^2/(-c^2*(x^
n)^2+1))-b^2*arctanh(c*x^n)^2*ln((c*x^n+1)^2/(-c^2*(x^n)^2+1)-1)+b^2*arctan
h(c*x^n)^2*ln(1+(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))+2*b^2*arctanh(c*x^n)*poly
log(2,-(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))-2*b^2*polylog(3,-(c*x^n+1)/(-c^2*(
x^n)^2+1)^(1/2))+b^2*arctanh(c*x^n)^2*ln(1-(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2)
)+2*b^2*arctanh(c*x^n)*polylog(2,(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))-2*b^2*po
lylog(3,(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))+1/2*I*b^2*Pi*csgn(I*((c*x^n+1)^2/
(-c^2*(x^n)^2+1)-1)/(1+(c*x^n+1)^2/(-c^2*(x^n)^2+1)))^3*arctanh(c*x^n)^2-1/
2*I*b^2*Pi*csgn(I*((c*x^n+1)^2/(-c^2*(x^n)^2+1)-1))*csgn(I*((c*x^n+1)^2/(-c
^2*(x^n)^2+1)-1)/(1+(c*x^n+1)^2/(-c^2*(x^n)^2+1)))^2*arctanh(c*x^n)^2+1/2*I
*b^2*Pi*csgn(I*((c*x^n+1)^2/(-c^2*(x^n)^2+1)-1))*csgn(I/(1+(c*x^n+1)^2/(-c^
2*(x^n)^2+1)))*csgn(I*((c*x^n+1)^2/(-c^2*(x^n)^2+1)-1)/(1+(c*x^n+1)^2/(-c^2
*(x^n)^2+1)))*arctanh(c*x^n)^2-1/2*I*b^2*Pi*csgn(I/(1+(c*x^n+1)^2/(-c^2*(x^
n)^2+1)))*csgn(I*((c*x^n+1)^2/(-c^2*(x^n)^2+1)-1)/(1+(c*x^n+1)^2/(-c^2*(x^n
)^2+1)))^2*arctanh(c*x^n)^2+2*a*b*ln(c*x^n)*arctanh(c*x^n)-a*b*ln(c*x^n)*ln
(c*x^n+1)-a*b*dilog(c*x^n+1)-a*b*dilog(c*x^n))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*log(-c*x^n + 1)^2*log(x) + a^2*log(x) - integrate(-1/4*((b^2*c*x^n
- b^2)*log(c*x^n + 1)^2 + 4*(a*b*c*x^n - a*b)*log(c*x^n + 1) + 2*(2*a*b - (
b^2*c*n*log(x) + 2*a*b*c)*x^n - (b^2*c*x^n - b^2)*log(c*x^n + 1))*log(-c*x^
n + 1))/(c*x*x^n - x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x,x, algorithm="fricas")
[Out] integral((b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}{\left(c x^{n} \right)}\right)^{2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))**2/x,x)
[Out] Integral((a + b*atanh(c*x**n))**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^n) + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x^n\right)\right)}^2}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^n))^2/x,x)
[Out] int((a + b*atanh(c*x^n))^2/x, x)
```

3.234 $\int \frac{(a+b\tanh^{-1}(cx^n))^2}{x^2} \, dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*arctanh(c*x^n))^2/x^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^n])^2/x^2,x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*x^n])^2/x^2, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^2} \, dx = \int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^2} \, dx$$

Mathematica [A]
time = 10.44, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])^2/x^2,x]
```

```
[Out] Integrate[(a + b*ArcTanh[c*x^n])^2/x^2, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\,x^{n}\right)\right)^{2}}{x^{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))^2/x^2,x)
```

```
[Out] int((a+b*arctanh(c*x^n))^2/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*log(-c*x^n + 1)^2/x - a^2/x - integrate(-1/4*((b^2*c*x^n - b^2)*lo
g(c*x^n + 1)^2 + 4*(a*b*c*x^n - a*b)*log(c*x^n + 1) + 2*(2*a*b + (b^2*c*n -
 2*a*b*c)*x^n - (b^2*c*x^n - b^2)*log(c*x^n + 1))*log(-c*x^n + 1))/(c*x^2*x
^n - x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(cx^{n}\right)\right)^{2}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))**2/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))**2/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x^n))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^n))^2/x^2,x)
[Out] int((a + b*atanh(c*x^n))^2/x^2, x)
```

3.235 $\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)^2}{x^3}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^3},x\right)$$

```
[Out] Unintegrable((a+b*arctanh(c*x^n))^2/x^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^n])^2/x^3,x]
[Out] Defer[Int][(a + b*ArcTanh[c*x^n])^2/x^3, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^3}\,dx = \int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^3}\,dx$$

Mathematica [A]
time = 10.39, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])^2/x^3,x]
[Out] Integrate[(a + b*ArcTanh[c*x^n])^2/x^3, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\,x^n\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))^2/x^3,x)
```

```
[Out] int((a+b*arctanh(c*x^n))^2/x^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*log(-c*x^n + 1)^2/x^2 - 1/2*a^2/x^2 - integrate(-1/4*((b^2*c*x^n -
 b^2)*log(c*x^n + 1)^2 + 4*(a*b*c*x^n - a*b)*log(c*x^n + 1) + (4*a*b + (b^2
*c*n - 4*a*b*c)*x^n - 2*(b^2*c*x^n - b^2)*log(c*x^n + 1))*log(-c*x^n + 1))/
(c*x^3*x^n - x^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2)/x^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(c x^{n}\right)\right)^{2}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))**2/x**3,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))**2/x**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x^3,x, algorithm="giac")
```

[Out] integrate((b*arctanh(c*x^n) + a)^2/x^3, x)

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,x^n\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x^n))^2/x^3,x)

[Out] int((a + b*atanh(c*x^n))^2/x^3, x)

3.236 $\int \frac{\tanh^{-1}(ax^n)}{x} dx$

Optimal. Leaf size=30

$$-\frac{\text{PolyLog}(2,-ax^n)}{2n}+\frac{\text{PolyLog}(2,ax^n)}{2n}$$

```
[Out] -1/2*polylog(2,-a*x^n)/n+1/2*polylog(2,a*x^n)/n
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6035, 6031}

$$\frac{\text{Li}_2(ax^n)}{2n}-\frac{\text{Li}_2(-ax^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x^n]/x,x]
```

```
[Out] -1/2*PolyLog[2, -(a*x^n)]/n + PolyLog[2, a*x^n]/(2*n)
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax^n)}{x} dx &= \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(ax)}{x} dx, x, x^n\right)}{n}\\ &= -\frac{\text{Li}_2(-ax^n)}{2n}+\frac{\text{Li}_2(ax^n)}{2n}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.03, size = 33, normalized size = 1.10

$$\frac{ax^n\, {}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2};a^2x^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x^n]/x,x]
```

```
[Out] (a*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, a^2*x^(2*n)])/n
```

Maple [A]
time = 0.13, size = 53, normalized size = 1.77

method	result	size
risch	$\frac{\operatorname{dilog}(1-a\,x^{n})}{2n}-\frac{\operatorname{dilog}(a\,x^{n}+1)}{2n}$	29
derivativedivides	$\frac{\ln(a\,x^{n})\operatorname{arctanh}(a\,x^{n})-\frac{\operatorname{dilog}(a\,x^{n})}{2}-\frac{\operatorname{dilog}(a\,x^{n}+1)}{2}-\frac{\ln(a\,x^{n})\ln(a\,x^{n}+1)}{2}}{n}$	53
default	$\frac{\ln(a\,x^{n})\operatorname{arctanh}(a\,x^{n})-\frac{\operatorname{dilog}(a\,x^{n})}{2}-\frac{\operatorname{dilog}(a\,x^{n}+1)}{2}-\frac{\ln(a\,x^{n})\ln(a\,x^{n}+1)}{2}}{n}$	53
meijerg	$-\frac{i\left(\frac{2ia\,x^{n}\operatorname{polylog}\left(2,\sqrt{x^{2n}a^{2}}\right)}{\sqrt{x^{2n}a^{2}}}-\frac{2ia\,x^{n}\operatorname{polylog}\left(2,-\sqrt{x^{2n}a^{2}}\right)}{\sqrt{x^{2n}a^{2}}}\right)}{4n}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x^n)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(ln(a*x^n)*arctanh(a*x^n)-1/2*dilog(a*x^n)-1/2*dilog(a*x^n+1)-1/2*ln(a*
x^n)*ln(a*x^n+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(24) = 48$.
time = 0.32, size = 147, normalized size = 4.90

$$-\frac{1}{2}\,an\left(\frac{\log\left(\frac{ax^{n}+1}{a}\right)}{an}-\frac{\log\left(\frac{ax^{n}-1}{a}\right)}{an}\right)\log\left(x\right)+\frac{1}{2}\,an\left(\frac{\log\left(ax^{n}+1\right)\log\left(x\right)-\log\left(ax^{n}-1\right)\log\left(x\right)}{an}-\frac{n\log\left(ax^{n}+1\right)\log\left(x\right)+\operatorname{Li}_2(-ax^{n})}{an^{2}}+\frac{n\log\left(-ax^{n}+1\right)\log\left(x\right)+\operatorname{Li}_2(ax^{n})}{an^{2}}\right)+\operatorname{artanh}\left(ax^{n}\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*n*(log((a*x^n + 1)/a)/(a*n) - log((a*x^n - 1)/a)/(a*n))*log(x) + 1/2
*a*n*((log(a*x^n + 1)*log(x) - log(a*x^n - 1)*log(x))/(a*n) - (n*log(a*x^n
+ 1)*log(x) + dilog(-a*x^n))/(a*n^2) + (n*log(-a*x^n + 1)*log(x) + dilog(a*
x^n))/(a*n^2)) + arctanh(a*x^n)*log(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(24) = 48$.
time = 0.36, size = 129, normalized size = 4.30

$$-\frac{n\log\left(a\cosh\left(n\log\left(x\right)\right)+a\sinh\left(n\log\left(x\right)\right)+1\right)\log\left(x\right)-n\log\left(-a\cosh\left(n\log\left(x\right)\right)-a\sinh\left(n\log\left(x\right)\right)+1\right)\log\left(x\right)-n\log\left(x\right)\log\left(-\frac{a\cosh(n\log(x))+a\sinh(n\log(x))+1}{a\cosh(n\log(x))+a\sinh(n\log(x))-1}\right)-\operatorname{Li}_2(a\cosh\left(n\log\left(x\right)\right)+a\sinh\left(n\log\left(x\right)\right))+\operatorname{Li}_2(-a\cosh\left(n\log\left(x\right)\right)-a\sinh\left(n\log\left(x\right)\right))}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(n*log(a*cosh(n*log(x)) + a*sinh(n*log(x)) + 1)*log(x) - n*log(-a*cosh
(n*log(x)) - a*sinh(n*log(x)) + 1)*log(x) - n*log(x)*log(-(a*cosh(n*log(x))
 + a*sinh(n*log(x)) + 1)/(a*cosh(n*log(x)) + a*sinh(n*log(x)) - 1)) - dilog
(a*cosh(n*log(x)) + a*sinh(n*log(x))) + dilog(-a*cosh(n*log(x)) - a*sinh(n*
log(x))))/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x**n)/x,x)
```

```
[Out] Integral(atanh(a*x**n)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x^n)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{atanh}(a\,x^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x^n)/x,x)
```

```
[Out] int(atanh(a*x^n)/x, x)
```

3.237 $\int \frac{\tanh^{-1}(ax^5)}{x} \, dx$

Optimal. Leaf size=24

$$-\frac{1}{10}\text{PolyLog}(2,-ax^5)+\frac{1}{10}\text{PolyLog}(2,ax^5)$$

```
[Out] -1/10*polylog(2,-a*x^5)+1/10*polylog(2,a*x^5)
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6035, 6031}

$$\frac{\text{Li}_2(ax^5)}{10}-\frac{1}{10}\text{Li}_2(-ax^5)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x^5]/x,x]
```

```
[Out] -1/10*PolyLog[2, -(a*x^5)] + PolyLog[2, a*x^5]/10
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax^5)}{x}\,dx &= \frac{1}{5}\text{Subst}\left(\int \frac{\tanh^{-1}(ax)}{x}\,dx,x,x^5\right)\\ &= -\frac{1}{10}\text{Li}_2(-ax^5)+\frac{\text{Li}_2(ax^5)}{10}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.92

$$\frac{1}{10}\left(-\text{PolyLog}(2,-ax^5)+\text{PolyLog}(2,ax^5)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x^5]/x,x]
```

```
[Out] (-PolyLog[2, -(a*x^5)] + PolyLog[2, a*x^5])/10
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 95, normalized size = 3.96

method	result
meijerg	$-\frac{i\left(\frac{2ia\,x^5\,\mathrm{polylog}\left(2,\sqrt{a^2x^{10}}\right)}{\sqrt{a^2x^{10}}}-\frac{2ia\,x^5\,\mathrm{polylog}\left(2,-\sqrt{a^2x^{10}}\right)}{\sqrt{a^2x^{10}}}\right)}{20}$
default	$\ln\left(x\right)\mathrm{arctanh}\left(a\,x^5\right)-5a\left(-\frac{\sum_{\_R1=\mathrm{RootOf}\left(a\_Z^5-1\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)}{10a}+\frac{\sum_{\_R1=\mathrm{RootOf}\left(a\_Z\right.}}{}\right.$
risch	$\frac{\ln(x)\ln\left(a\,x^5+1\right)}{2}-\frac{\left(\sum_{\_R1=\mathrm{RootOf}\left(a\_Z^5+1\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{2}-\frac{\ln(x)\ln\left(-a\,x^5+1\right)}{2}+\frac{\left(\sum_{\_R1=\mathrm{Ro}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x^5)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*arctanh(a*x^5)-5*a*(-1/10/a*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_
R1),_R1=RootOf(_Z^5*a-1))+1/10/a*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R
1),_R1=RootOf(_Z^5*a+1)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(18) = 36.
time = 0.26, size = 104, normalized size = 4.33

$$-\frac{1}{2}a\left(\frac{\log\left(ax^5+1\right)}{a}-\frac{\log\left(ax^5-1\right)}{a}\right)\log\left(x\right)-\frac{1}{10}a\left(\frac{\log\left(ax^5-1\right)\log\left(ax^5\right)+\mathrm{Li}_2(-ax^5+1)}{a}-\frac{\log\left(ax^5+1\right)\log\left(-ax^5\right)+\mathrm{Li}_2(ax^5+1)}{a}\right)+\mathrm{artanh}\left(ax^5\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x^5)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(a*x^5 + 1)/a - log(a*x^5 - 1)/a)*log(x) - 1/10*a*((log(a*x^5 -
1)*log(a*x^5) + dilog(-a*x^5 + 1))/a - (log(a*x^5 + 1)*log(-a*x^5) + dilog(
a*x^5 + 1))/a) + arctanh(a*x^5)*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x^5)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax^5)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x**5)/x,x)
```

```
[Out] Integral(atanh(a*x**5)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x^5)/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x^5)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atanh}(a\,x^5)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x^5)/x,x)
```

```
[Out] int(atanh(a*x^5)/x, x)
```

3.238 $\int \tanh^{-1}\left(\frac{1}{x}\right)\,dx$

Optimal. Leaf size=19

$$x\tanh^{-1}\left(\frac{1}{x}\right)+\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] x*arctanh(1/x)+1/2*ln(-x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6021, 269, 266}

$$\frac{1}{2}\log\left(1-x^2\right)+x\tanh^{-1}\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x^(-1)],x]
```

```
[Out] x*ArcTanh[x^(-1)] + Log[1 - x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}\left(\frac{1}{x}\right)\,dx &= x\tanh^{-1}\left(\frac{1}{x}\right)+\int \frac{1}{\left(1-\frac{1}{x^2}\right)x}\,dx\\ &= x\tanh^{-1}\left(\frac{1}{x}\right)+\int \frac{x}{-1+x^2}\,dx\\ &= x\tanh^{-1}\left(\frac{1}{x}\right)+\frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 0.89

$$x \tanh^{-1}\left(\frac{1}{x}\right)+\frac{1}{2}\log\left(-1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[x^(-1)],x]
[Out] x*ArcTanh[x^(-1)] + Log[-1 + x^2]/2
```

Maple [A]
time = 0.14, size = 30, normalized size = 1.58

method	result
derivativedivides	$x \,\mathrm{arctanh}\left(\frac{1}{x}\right)+\frac{\ln\left(\frac{1}{x}+1\right)}{2}+\frac{\ln\left(\frac{1}{x}-1\right)}{2}-\ln\left(\frac{1}{x}\right)$
default	$x \,\mathrm{arctanh}\left(\frac{1}{x}\right)+\frac{\ln\left(\frac{1}{x}+1\right)}{2}+\frac{\ln\left(\frac{1}{x}-1\right)}{2}-\ln\left(\frac{1}{x}\right)$
meijerg	$-\frac{\ln\left(1-\sqrt{\frac{1}{x^2}}\right)-\ln\left(1+\sqrt{\frac{1}{x^2}}\right)}{2\sqrt{\frac{1}{x^2}}}+\frac{\ln\left(-\frac{1}{x^2}+1\right)}{2}+\ln\left(x\right)-\frac{i\pi}{2}$
risch	$-\frac{x\ln(x-1)}{2}+\frac{\ln(1+x)x}{2}+\frac{i\pi\,\mathrm{csgn}(i(1+x))\mathrm{csgn}\left(\frac{i(1+x)}{x}\right)^2 x}{4}-\frac{i\pi\,\mathrm{csgn}(i(1+x))\mathrm{csgn}\left(\frac{i(1+x)}{x}\right)\mathrm{csgn}\left(\frac{i}{x}\right)x}{4}-$ iπ csgn(

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1/x),x,method=_RETURNVERBOSE)
[Out] x*arctanh(1/x)+1/2*ln(1/x+1)+1/2*ln(1/x-1)-ln(1/x)
```

Maxima [A]
time = 0.25, size = 15, normalized size = 0.79

$$x \,\mathrm{artanh}\left(\frac{1}{x}\right)+\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1/x),x, algorithm="maxima")
[Out] x*arctanh(1/x) + 1/2*log(x^2 - 1)
```

Fricas [A]
time = 0.35, size = 22, normalized size = 1.16

$$\frac{1}{2}\,x\log\left(\frac{x+1}{x-1}\right)+\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1/x),x, algorithm="fricas")
```

```
[Out] 1/2*x*log((x + 1)/(x - 1)) + 1/2*log(x^2 - 1)
```

Sympy [A]
time = 0.08, size = 15, normalized size = 0.79

$$x\operatorname{atanh}\left(\frac{1}{x}\right)+\log\left(x+1\right)-\operatorname{atanh}\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1/x),x)
```

```
[Out] x*atanh(1/x) + log(x + 1) - atanh(1/x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(17) = 34.
time = 0.41, size = 101, normalized size = 5.32

$$\frac{\log\left(-\frac{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}+1}{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}-1}\right)}{\frac{x+1}{x-1}-1}+\log\left(\frac{|x+1|}{|x-1|}\right)-\log\left(\left|\frac{x+1}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1/x),x, algorithm="giac")
```

```
[Out] log(-(((x + 1)/(x - 1) - 1)/((x + 1)/(x - 1) + 1) + 1)/(((x + 1)/(x - 1) -
1)/((x + 1)/(x - 1) + 1) - 1))/((x + 1)/(x - 1) - 1) + log(abs(x + 1)/abs(x
 - 1)) - log(abs((x + 1)/(x - 1) - 1))
```

Mupad [B]
time = 0.07, size = 15, normalized size = 0.79

$$\frac{\ln\left(x^2-1\right)}{2}+x\operatorname{atanh}\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(1/x),x)
```

```
[Out] log(x^2 - 1)/2 + x*atanh(1/x)
```

3.239 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^3\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left((dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^3,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c*x^n))^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^n])^3,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x^n])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^3\,dx = \int (dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^3\,dx$$

Mathematica [A]
time = 6.30, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n])^3, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\operatorname{arctanh}\left(c\,x^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^n))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^n))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*d^m*x*x^m*log(-c*x^n + 1)^3/(m + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1)
) + integrate(1/8*((b^3*c*d^m*(m + 1)*e^(m*log(x) + n*log(x)) - b^3*d^m*(m
+ 1)*x^m)*log(c*x^n + 1)^3 + 6*(a*b^2*c*d^m*(m + 1)*e^(m*log(x) + n*log(x))
 - a*b^2*d^m*(m + 1)*x^m)*log(c*x^n + 1)^2 - 3*(2*a*b^2*d^m*(m + 1)*x^m - (
2*a*b^2*c*d^m*(m + 1) + b^3*c*d^m*n)*e^(m*log(x) + n*log(x)) - (b^3*c*d^m*(
m + 1)*e^(m*log(x) + n*log(x)) - b^3*d^m*(m + 1)*x^m)*log(c*x^n + 1))*log(-
c*x^n + 1)^2 + 12*(a^2*b*c*d^m*(m + 1)*e^(m*log(x) + n*log(x)) - a^2*b*d^m*
(m + 1)*x^m)*log(c*x^n + 1) - 3*(4*a^2*b*c*d^m*(m + 1)*e^(m*log(x) + n*log(
x)) - 4*a^2*b*d^m*(m + 1)*x^m + (b^3*c*d^m*(m + 1)*e^(m*log(x) + n*log(x))
- b^3*d^m*(m + 1)*x^m)*log(c*x^n + 1)^2 + 4*(a*b^2*c*d^m*(m + 1)*e^(m*log(x
) + n*log(x)) - a*b^2*d^m*(m + 1)*x^m)*log(c*x^n + 1))*log(-c*x^n + 1))/(c*
(m + 1)*x^n - m - 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*b^3*arctanh(c*x^n)^3 + 3*(d*x)^m*a*b^2*arctanh(c*x^n)^2 +
3*(d*x)^m*a^2*b*arctanh(c*x^n) + (d*x)^m*a^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**n))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^3*(d*x)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atanh}\left(c\,x^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c*x^n))^3,x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c*x^n))^3, x)
```

3.240 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 \, dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c*x^n))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^n])^2,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x^n])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 \, dx = \int (dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 \, dx$$

Mathematica [A]
time = 5.10, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n])^2,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n])^2, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx)^m (a+b\,\mathrm{arctanh}\,(c\,x^n))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^n))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*d^m*x*x^m*log(-c*x^n + 1)^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1))
 - integrate(-1/4*((b^2*c*d^m*(m + 1)*e^(m*log(x) + n*log(x)) - b^2*d^m*(m
+ 1)*x^m)*log(c*x^n + 1)^2 + 4*(a*b*c*d^m*(m + 1)*e^(m*log(x) + n*log(x)) -
 a*b*d^m*(m + 1)*x^m)*log(c*x^n + 1) + 2*(2*a*b*d^m*(m + 1)*x^m - (2*a*b*c*
d^m*(m + 1) + b^2*c*d^m*n)*e^(m*log(x) + n*log(x)) - (b^2*c*d^m*(m + 1)*e^(
m*log(x) + n*log(x)) - b^2*d^m*(m + 1)*x^m)*log(c*x^n + 1))*log(-c*x^n + 1)
)/(c*(m + 1)*x^n - m - 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*b^2*arctanh(c*x^n)^2 + 2*(d*x)^m*a*b*arctanh(c*x^n) + (d*x
)^m*a^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**n))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2*(d*x)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atanh}\left(c\,x^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c*x^n))^2,x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c*x^n))^2, x)
```

3.241 $\int (dx)^m \left(a+b\tanh^{-1}(cx^n)\right) dx$

Optimal. Leaf size=84

$$\frac{x(dx)^m\left(a+b\tanh^{-1}(cx^n)\right)}{1+m}-\frac{bcnx^{1+n}(dx)^m\,{}_2F_1\left(1,\frac{1+m+n}{2n};\frac{1+m+3n}{2n};c^2x^{2n}\right)}{(1+m)(1+m+n)}$$

```
[Out] x*(d*x)^m*(a+b*arctanh(c*x^n))/(1+m)-b*c*n*x^(1+n)*(d*x)^m*hypergeom([1, 1/
2*(1+m+n)/n],[1/2*(1+m+3*n)/n],c^2*x^(2*n))/(1+m)/(1+m+n)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {6051, 6037, 371}

$$\frac{x(dx)^m\left(a+b\tanh^{-1}(cx^n)\right)}{m+1}-\frac{bcnx^{n+1}(dx)^m\,{}_2F_1\left(1,\frac{m+n+1}{2n};\frac{m+3n+1}{2n};c^2x^{2n}\right)}{(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] (x*(d*x)^m*(a + b*ArcTanh[c*x^n]))/(1 + m) - (b*c*n*x^(1 + n)*(d*x)^m*Hyper
geometric2F1[1, (1 + m + n)/(2*n), (1 + m + 3*n)/(2*n), c^2*x^(2*n)])/((1 +
 m)*(1 + m + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6051

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_)*(x_))^(m_), x_Sym
bol] :> Dist[d^IntPart[m]*((d*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a + b
*ArcTanh[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] &&
 (EqQ[p, 1] || RationalQ[m, n])
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}(cx^n)\right) dx = \frac{(dx)^{1+m}\left(a+b\tanh^{-1}(cx^n)\right)}{d(1+m)} - \frac{(bcn)\int \frac{x^{-1+n}(dx)^{1+m}}{1-c^2x^{2n}}\,dx}{d(1+m)}$$
$$= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}(cx^n)\right)}{d(1+m)} - \frac{(bcnx^{-m}(dx)^m)\int \frac{x^{m+n}}{1-c^2x^{2n}}\,dx}{1+m}$$
$$= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}(cx^n)\right)}{d(1+m)} - \frac{bcnx^{1+n}(dx)^m\,{}_2F_1\left(1,\frac{1+m+n}{2n};\frac{1+m+3n}{2n};c^2\right.}{(1+m)(1+m+n)}$$

Mathematica [A]
time = 0.06, size = 77, normalized size = 0.92

$$\frac{x(dx)^m\left((1+m+n)\left(a+b\tanh^{-1}(cx^n)\right)-bcnx^n\,{}_2F_1\left(1,\frac{1+m+n}{2n};\frac{1+m+3n}{2n};c^2x^{2n}\right)\right)}{(1+m)(1+m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] (x*(d*x)^m*((1 + m + n)*(a + b*ArcTanh[c*x^n]) - b*c*n*x^n*Hypergeometric2F
1[1, (1 + m + n)/(2*n), (1 + m + 3*n)/(2*n), c^2*x^(2*n)]))/((1 + m)*(1 + m
 + n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\,\mathrm{arctanh}\left(c\,x^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^n)),x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*(d^m*n*integrate(x^m/(c*(m + 1)*x^n + m + 1), x) + d^m*n*integrate(x^m/
(c*(m + 1)*x^n - m - 1), x) + (d^m*x*x^m*log(c*x^n + 1) - d^m*x*x^m*log(-c*
x^n + 1))/(m + 1))*b + (d*x)^(m + 1)*a/(d*(m + 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n)),x, algorithm="fricas")
[Out] integral((d*x)^m*b*arctanh(c*x^n) + (d*x)^m*a, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{atanh}(cx^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**n)),x)
[Out] Integral((d*x)**m*(a + b*atanh(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n)),x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^n) + a)*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\left(a + b\,\mathrm{atanh}(c\,x^n)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c*x^n)),x)
[Out] int((d*x)^m*(a + b*atanh(c*x^n)), x)
```

3.242 $\int \frac{(dx)^m}{a+b\tanh^{-1}(cx^n)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(dx)^m}{a+b\tanh^{-1}(cx^n)},x\right)$$

[Out] Unintegrable((d*x)^m/(a+b*arctanh(c*x^n)),x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(dx)^m}{a+b\tanh^{-1}(cx^n)}\,dx$$

Verification is not applicable to the result.

[In] Int[(d*x)^m/(a + b*ArcTanh[c*x^n]),x]

[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x^n]), x]

Rubi steps

$$\int \frac{(dx)^m}{a+b\tanh^{-1}(cx^n)}\,dx = \int \frac{(dx)^m}{a+b\tanh^{-1}(cx^n)}\,dx$$

Mathematica [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\tanh^{-1}(cx^n)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^n]),x]

[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^n]), x]

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\,\mathrm{arctanh}\,(c\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x^n)),x)
[Out] int((d*x)^m/(a+b*arctanh(c*x^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arctanh(c*x^n) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arctanh(c*x^n) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b\operatorname{atanh}(cx^n)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**n)),x)
[Out] Integral((d*x)**m/(a + b*atanh(c*x**n)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arctanh(c*x^n) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a + b\operatorname{atanh}\left(c\,x^n\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c*x^n)),x)
[Out] int((d*x)^m/(a + b*atanh(c*x^n)), x)
```

3.243 $\int \frac{(dx)^m}{\left(a+b\tanh^{-1}(cx^n)\right)^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctanh(c*x^n))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x^n])^2,x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x^n])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}\,dx$$

Mathematica [A]
time = 1.24, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^n])^2,x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^n])^2, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\,\mathrm{arctanh}\left(c\,x^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x^n))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c*x^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 2*(c^2*d^m*x*e^(m*log(x) + 2*n*log(x)) - d^m*x*x^m)/(b^2*c*n*x^n*log(c*x^n
+ 1) - b^2*c*n*x^n*log(-c*x^n + 1) + 2*a*b*c*n*x^n) + integrate(-2*(c^2*d^m
*(m + n + 1)*e^(m*log(x) + 2*n*log(x)) - d^m*(m - n + 1)*x^m)/(b^2*c*n*x^n*
log(c*x^n + 1) - b^2*c*n*x^n*log(-c*x^n + 1) + 2*a*b*c*n*x^n), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**n))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x^n) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{\left(a+b\,\mathrm{atanh}\left(c\,x^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c*x^n))^2,x)
```

```
[Out] int((d*x)^m/(a + b*atanh(c*x^n))^2, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
       return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
       if expn.parent() is SR:
          return expn.operator() is None
       if expn.parent() in (ZZ, QQ, AA, QQbar):
          return expn in expn.parent() # Should always return True
       if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
          return expn in expn.parent().base_ring() or expn in expn.parent().gens()

       return False

    except AttributeError as error:
       print("Exception,AttributeError in  is_atom")
       print ("cought exception" , type(error).__name__ )
       return False


def expnType(expn):

    if debug:
       print (">>>>>Enter expnType, expn=", expn)
       print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
       return 1
    elif type(expn)==list:   #isinstance(expn,list):
       return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
       if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
          return 1
       else:
          return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
       if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
          return expnType(expn.operands()[0])    #expnType(expn.args[0])
       elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
          if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(4,m1)    #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(5,m1)    #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(6,m1)    #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```